Refinements in the Jungle Universes

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Abstract

How can effective barotropic matter emerge from the interaction of cosmological fluids in an isotropic and homogeneous cosmological model ?

The dynamics of homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker universes is a natural special case of generalized Lotka-Volterra systems where each of the universes fluid components can be seen as a competitive species in a predator-prey model. (Jungle universe : [7])

In addition to numerical simulations illustrating this behaviour among the barotropic fluids filling the universe, we analytically pinpoint that effective time-dependent barotropic indices can arise from a physical coupling between those fluids whose dynamics could then look like that of another type of cosmic fluid, such as a cosmological constant.

Since the nature of dark energy is still unknown, this dynamical approach could help understanding some of the properties of dark matter and dark energy at large cosmological scales.

Introduction

Einstein's general relativity for gravitation has led him to study the dynamics of the universe. His cosmology describes an isotropic, homogeneous and static universe, while the current ΛCDM model includes a possible accelerated expansion supported by the observation of distant supernovae [1, 2] and the cosmic microwave background [3]. This late-time cosmic acceleration could be explained by dark energy whose nature still remains undetermined.

Several possible explanations have been proposed to account for this acceleration, from modifications of gravity - with $f(R)$ gravity, scalar-tensor theories, braneworlds - to new cosmological fluids such as generalised Chaplygin gas, scalar field with various couplings with matter, or more naturally non-gravitational couplings between the constituents of the universe [4]. But the form of these time-dependent [5] and often linear [6] interactions generally lacks physical justifications, see [8, 9] for exact solutions with interacting fluids and physical discussion on energy exchanges.

After a reformulation of the standard cosmological model in terms of the density proportion of the constituents filling the universe, we study a natural quadratic coupling between cosmological fluids suggested by the intrinsic Lotka-Volterra structure of the fields equations. This coupling leads to an effective dynamical behaviour [7]. We eventually propose the basis for a new interpretation of the acceleration of the universe.

1 The standard universe is a generalised predator-prey system

The dynamics of homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker universes is a special case of generalised Lotka-Volterra system where the competitive species are the barotropic cosmological fluids filling the universe, as it will be underlined in this section.

The field equations of the standard cosmological model can be written as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\sum_{i=1}^{n-1}\left(\rho_i + \frac{3p_i}{c^2}\right) + \frac{c^2}{3}\Lambda \tag{1}$$

$$H^2 \;=\; \frac{8\pi G_N}{3}\sum_{i=1}^{n-1}\rho_i + \frac{c^2}{3}\,\Lambda - \frac{c^2}{a^2}k. \tag{2}$$

Let us consider barotropic fluids with equation of state

$$\text{for } i = 1,\ldots,n+1, \qquad p_i = \omega_i \rho_i\, c^2$$

where each fluid $i = 1,\ldots,n$ can be baryonic matter, radiation, dark matter, dark energy, etc. In the previous equations, $i = n$ would be associated to the cosmological constant Λ with $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G_N}$ and $\omega_\Lambda = -1$, while $i = n+1$ could be associated to curvature introducing $\rho_k = -\frac{3c^2 k}{8\pi G_N}a^{-2}$ and $\omega_k = -\frac{1}{3}$.

A change of variable for the time parameter, $\lambda = \ln(a)$, and a reformulation in terms of relative abundances Ω_i, instead of densities ρ_i, lead to the following dynamics for each independent cosmological fluid governed by a continuity equation of type $\dot\rho_i = -3H(\rho_i + \frac{p_i}{c^2})$:

$$\text{for } i = 1,\ldots,n+1, \quad \frac{\mathrm{d}\Omega_i}{\mathrm{d}\lambda} = \Omega_i\left[-(1+3\,\omega_i) + \sum_{j=1}^{n}(1+3\,\omega_j)\,\Omega_j\right]$$

which reads as a predator-prey system, with a community matrix $\mathbf{A}$ and a capacity vector $\mathbf{r}$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{bmatrix}\Omega_1\\ \vdots\\ \Omega_n\\ \Omega_k\end{bmatrix} = \mathrm{diag}\begin{bmatrix}\Omega_1\\ \vdots\\ \Omega_n\\ \Omega_k\end{bmatrix}\left(\underbrace{\begin{bmatrix}1+3\,\omega_1 & \ldots & 1+3\,\omega_n & 0\\ \vdots & \ddots & \vdots & \vdots\\ 1+3\,\omega_1 & \ldots & 1+3\,\omega_n & 0\\ 1+3\,\omega_1 & \ldots & 1+3\,\omega_n & 0\end{bmatrix}}_{\mathbf{A}\text{ matrix}}\begin{bmatrix}\Omega_1\\ \vdots\\ \Omega_n\\ \Omega_k\end{bmatrix} + \underbrace{\begin{bmatrix}-(1+3\,\omega_1)\\ \vdots\\ -(1+3\,\omega_n)\\ -(1+3\,\omega_k)\end{bmatrix}}_{\mathbf{r}\text{ vector}}\right)$$

in $\mathbb{R}^{n+1}$. The search for equilibria in a Lotka-Volterra system governing the evolution of $n+1$ independent fluids, with a $(n+1)^{\text{th}}$ fluid made of curvature with index k, consists in solving $\dot\Omega = 0 = \mathrm{diag}(\Omega)\,(\mathbf{A}\,\Omega + \mathbf{r})$, see e.g. [10] chap. 4. If all fluids interact with each other only gravitationally, then the system $(\mathbf{A}\,\Omega + \mathbf{r}) = 0$ has no solution, since the capacity vector does not lie in the image of the community matrix. The flat Minkowski spacetime is the solution of $\mathrm{diag}(\Omega) = 0$, while all other equilibria correspond to universes containing a single fluid j, i.e. when $\Omega_j \neq 0$ but $\Omega_{i\neq j} = 0$, then $\mathrm{diag}(\Omega)$ is a zero divisor matrix.

The usual asymptotic states of FLRW universes, such as Einstein-de Sitter ($\Omega_m \neq 0$), de Sitter ($\Omega_\Lambda \neq 0$) or Milne ($\Omega_k \neq 0$) universes, thus appear as particular equilibria between cosmic species, where generally one species dominates the others, see Fig. 1. These equilibria correspond to Ω vectors with only one non-zero value which equals 1 because of (2). Consequently, in the absence of non-gravitational interactions, they must lie on the axes of (Ω_i, Ω_j)-planes for all $i, j = 0,\ldots,n+1$.

2 Jungle Universes : cooperation and competition among cosmic fluids

A universe without any interaction between cosmological fluids apart from a gravitational one seems a little awkward, whereas a natural coupling between cosmic fluids leads to a much richer dynamics.

Various types of coupling, most of the time linear in the densities [6], have been proposed by several authors, see e.g. [5], but very few with non-linear interactions have been studied [11, 12, 13]. Except for scalar fields, the non-linearity is introduced in the equations of state [14] but rarely as an interaction between fluids [15, 16]. A natural non-linear coupling preserving the intrinsic Lotka-Volterra structure previously mentioned has been studied in [7]. This "jungle coupling", defined by $Q(\epsilon_{ij}) = \epsilon_{ij}\,\Omega_i\,\Omega_j$, naturally vanishes in the absence of one of the interacting fluids, and the interaction rate increases with the coupled densities. Interactions which are proportionnal to the product of these densities also provide one of the best observational fits for holographic dark energy models coupled

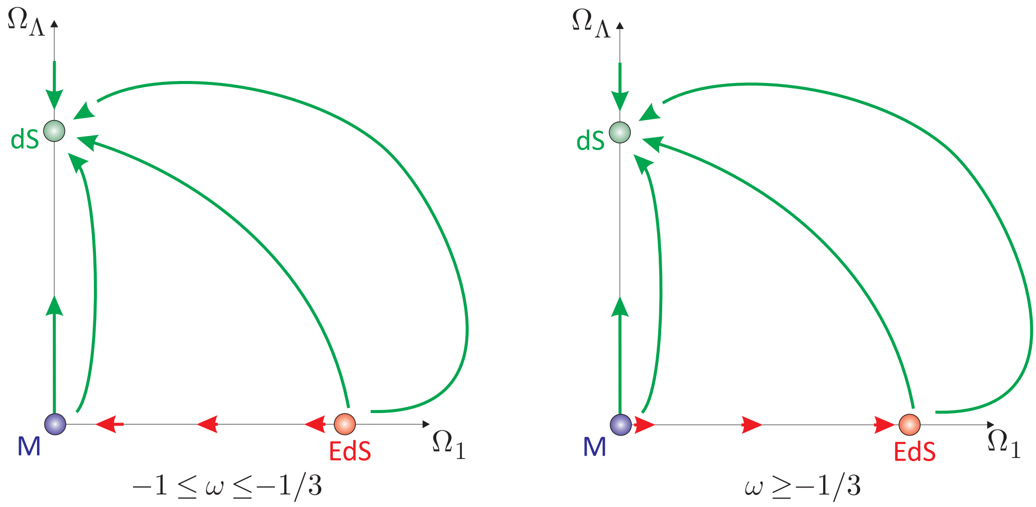

Figure 1: Dynamics in the $(\Omega_1, \Omega_2) = (\Omega_1, \Omega_\Lambda)$-plane, considering a species 1, a cosmological constant and curvature. Time increases when following the arrows and we define $\omega = \omega_1$ (e.g. $\omega = \omega_m = 0$ for baryonic matter in the ΛCDM model). Common universes such as Einstein-de Sitter (EdS), de Sitter (dS) or Milne (M) universes are equilibria whose stability depends on the nature of fluid 1, i.e. on the value of ω.

to dark matter [13] and are the bests, with respect to linear couplings, to alleviate the coincidence problem with common time-dependent equations of state [17]. The quadratic coupling term $Q(\epsilon_{ij})$ added to and substracted from the conservation equations of the interacting fluids i and j

$$\frac{\mathrm{d}\Omega_{i/j}}{\mathrm{d}\lambda} = \Omega_{i/j}\left[-(1+3\,\omega_{i/j}) + \sum_{l=1}^{n}(1+3\,\omega_l)\,\Omega_l\right] \;+/-\; Q(\epsilon_{ij})$$

partially breaks the degeneracy of the community matrix $\mathbf{A}$. Equilibria may be located outside the axes of (Ω_i, Ω_j)-planes. The associated dynamics can contain limit cycles (Fig. 2), universes with several fluids in equilibrium, and even chaos which has been conjectured for more than four fluids in interaction. The dynamical behaviours in this so-called Jungle Universe could answer some questions such as the coincidence problem.

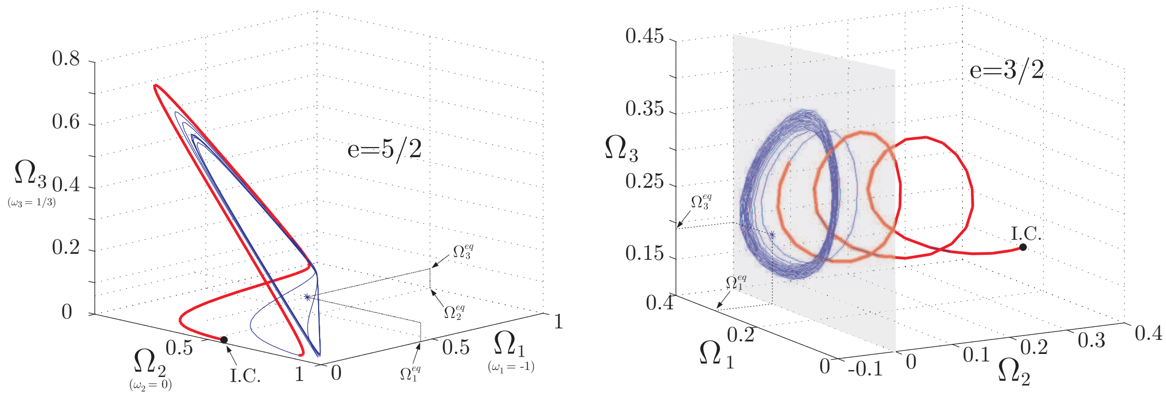

Figure 2: Evolution of the three coupled relative abundances, in the 3D phase space, with coupling constants : $\epsilon_{12} = \epsilon_{13} = \epsilon_{23} =$ e. The beginning of the orbits are marked by a bold red line. Initial conditions are indicated by I.C. and a black dot. Relevant equilibria are indicated by stars.

Further dynamical properties are explained in [7]. In the last section, we will employ the time evolution of the dynamical behaviour of interacting cosmological fluids to look for the possibility of an effective dark energy.

3 Camouflage in the jungle : could dark energy emerge from the jungle coupling ?

The observed accelerated expansion of the universe suggests the existence of a cosmological constant or at least dark energy of unknown nature. Could a coupling among cosmic fluids lead to a similar behaviour instead ?

Using the formalism of Jungle Universes, we suggest hereafter that the acceleration of the expansion could result from a special interaction between fluids of known properties. The interaction term in the continuity equation of a fluid i reads

$$\dot{\rho}_i = -3H(\rho_i + \frac{p_i}{c^2}) + \sum_{j=1}^{n} \epsilon_{ij} H\, \Omega_j\, \rho_i.$$

It actually modifies its equation of state which then describes a barotropic fluid with an effective time-dependent barotropic index

$$\omega_i^{\text{eff}} = \omega_i - \sum_{j=1}^{n} \frac{\epsilon_{ij}}{3}\, \Omega_j.$$

Consequently a dominant negative term can appear in the second member of (1), which is equivalent to an acceleration $\ddot{a} > 0$.

As an illustration, consider three dark matter fluids with the first two and last two interacting with each other. Indexing them from 1 to 3, with $\epsilon_{12} = -2$ and $\epsilon_{23} = -3$, the mutual interactions make the third fluid asymptotically dominate the others in terms of density with an effective barotropic index close to -1, see Fig. 3. Thus fluid 3 dynamically behaves nearly as a cosmological constant. In the same way, the first fluid looks like a perfect gas while the second one still behaves as a dark matter.

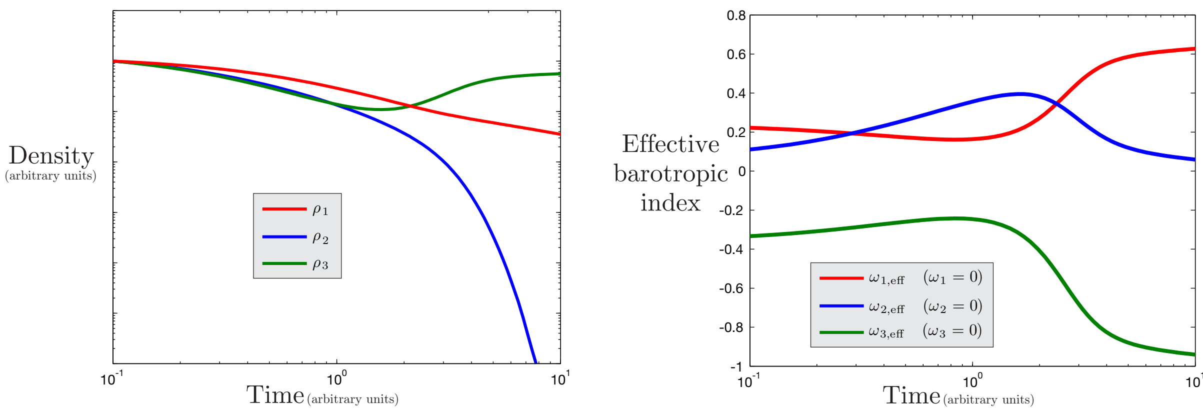

Figure 3: Interaction between three dark matter fluids : evolution of their density and effective barotropic index. Coupling constants : $\epsilon_{12} = -2$ and $\epsilon_{23} = -3$.

Couplings between barotropic cosmological fluids change their observed behaviour and can influence the global dynamics of the universe. The acceleration of the universe could then be explained in a natural way without introducing unknown types of energy.

Conclusion

The natural generalised Lotka-Volterra structure of the evolution of the universe constituent densities has enabled us to describe the isotropic and homogenous universe as a generalised predator-prey system. The effective barotropic indices that emerge from the natural quadratic jungle coupling between cosmological fluids induce various dynamics of the evolution of the scale factor and fluid densities, from limit cycles to possible chaos. We have made a simulation of an effective cosmological constant from a coupling between three dark matter fluids. Similar interactions could explain the observed acceleration of the universe. Comparisons with cosmological data are therefore planned to be processed.

Keywords. Cosmology, dynamical systems, Jungle Universe, cosmological coupling, interaction, dark energy

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