Relaxing the limits on inflationary magnetogenesis

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Magnetic fields are everywhere

From the Earth all the way out to remote protogalaxies and recently in the intergalactic space (?)

Origin as yet unknown

Late-time or early time (?)

Many scenarios, but the issue is still open
Facts & Open Questions

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Early-time Magnetogenesis

Galactic dynamo requirements

Minimum coherence length

\((\lambda_B)_0 \sim 10 \text{ Kpc}\)

Strength-range

\(10^{-22} \text{ G} \lesssim B_0 \lesssim 10^{-12} \text{ G}\)

Post-inflationary \(B\)-Fields

Problem

Too small correlation lengths:

\((\lambda_B)_0 \ll 10 \text{ Kpc}\)

Solution

“Inverse cascade” (?)

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Typically too weak:

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Outside classical EM (?)
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# Inflationary Magnetogenesis

## Advantage

Very large correlation lengths: \((\lambda_B)_0 \gg 10 \text{ Kpc}\)

## Disadvantage

Extremely weak today: \(B_0 \lesssim 10^{-53} \text{ G}\)

### Reason

Adiabatic magnetic decay

\[ B \propto a^{-2} \]

Throughout the lifetime of the universe

On all scales
Inflationary Magnetogenesis

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## Inflationary Magnetic Fields

### On spatially flat FRW backgrounds

\[
\dot{B}_a + 5 H \dot{B}_a + 3(1 - w) H^2 B_a - D^2 B_a = J_a ,
\]

to linear order. Setting \( B_a = a^2 B_a , \)

\[
B''_a - a^2 D^2 B_a = a^2 J_a .
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### During inflation

The universe is a very poor conductor (i.e. \( J_a = 0 \)). Therefore.

\[
B'''_a - a^2 D^2 B_a = 0 .
\]

Then, at horizon crossing,

\[
B(k) = a^2 B(k) = C_1 \cos(k \eta) + C_2 \sin(k \eta) .
\]

### Well outside the horizon

When \( \lambda_B \gg \lambda_H \Leftrightarrow k \eta \ll 1 , \)

\[
a^2 B(k) = C_1 + C_2 k \eta , \quad \text{with} \quad a = a(\eta) .
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Large-scale, Post-inflationary Magnetic Evolution (I)

On subhorizon scales
- Electric currents are formed
- The currents freeze the $B$-fields into the matter
- The magnetic flux remains conserved
- The $B$-fields decay adiabatically

On superhorizon scales
- There are no electric currents
- The magnetic fields are causally disconnected
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Large-scale, Post-inflationary Magnetic Evolution (II)

Evolution during reheating & dust \((a \propto \eta^2)\)

\[
B = - (3B_* + \eta_* B'_*) \left( \frac{a_*}{a} \right)^2 + (4B_* + \eta_* B'_*) \left( \frac{a_*}{a} \right)^{3/2}.
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Evolution during radiation \((a \propto \eta)\)

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Superadiabatic amplification when \(\lambda_B \gg \lambda_H\)

As long as \(4B_* + \eta_* B'_* \neq 0\) and \(2B_* + \eta_* B'_* \neq 0\),

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B \propto a^{-3/2} \quad \text{and} \quad B \propto a^{-1}.
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Strong inflationary amplification \((B \propto a^{-1/2})\)

\[B_0 \sim 10^{-2} \text{ G } \quad \text{(for } M \sim 10^{17} \text{ GeV } \& \text{ } T_{RH} \sim 10^{10} \text{ GeV)}\]

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Reverse engineering the galactic-dynamo constraints

Today the dynamo requires

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Residual magnetic field

$$(\lambda_B)_0 \sim 10 \text{ Kpc} \implies T_{HC} \sim 10^{-6} \text{ GeV}$$

$$B_0 \approx 10^{-33} \left(\frac{M}{10^{17}}\right)^{2/3} \left(\frac{T_{RH}}{10^{10}}\right)^{1/3} \text{ G}.$$ 

Subsequent amplification

Spherically symmetric galactic collapse $\implies B_0 \sim 10^{-29} \text{ G}$

Anisotropic galactic collapse $\implies B_0 \sim 10^{-27} \text{ G}$

Additional amplification (?)

When $w = 1$ (stiff matter) $\implies B = \text{constant}$

Stiff-matter era (from, say, $T_{RH} \sim 10^{10} \text{ GeV}$ to $T \sim 10^{4} \text{ GeV}$)

$B_0 \sim 10^{-21} \text{ G}$
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Summary

By appealing to causality

- The large-scale magnetic decay may slow down
- Strong inflationary amplification may not be necessary
- Conventional magnetogenesis might still work
Thanks

and

Merry Christmas