

Quasi-periodic oscillations of perturbed tori

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Abstract

- Axisymmetric hydrodynamical simulations of perturbed tori orbiting a black hole were performed.
- Equilibrium tori with a constant distribution of angular momentum in a pseudo-Newtonian potential (Kluźniak-Lee) were constructed.
- Epicyclic motion were triggered by adding sub-sonic velocity fields; radial, vertical and diagonal to the tori.
- L_2 norm of density ($\|\rho\|_2$) was measured as the perturbed tori evolved in time.
- Power spectrum of $\|\rho\|_2$ manifested eigenfrequencies of tori modes.

Modes

| Modes | Frequencies (Ω_K) | |
|---------------------------------|----------------------------|-----------------|
| | $r_c = 5.2 r_g$ | $r_c = 7.2 r_g$ |
| Radial (R)/harmonic | 0.65/1.3 | 0.76/1.52 |
| Vertical (V)/harmonic (V_h) | 1.0/2.0 | 1.0/2.0 |
| Breathing (B) | 1.65 | 1.69 |
| Plus (+) | 0.98 | 1.18 |
| x | 1.18 | 1.26 |

Table 1: Modes and frequencies. From left to right the columns are, modes with labels in parenthesis and theoretical values of frequencies (Ω_K) at r_c , where $r_g = 2GM/c^2$.

Models

| Model | Resolution | r/r_g | z/r_g | r_{in}/r_g | r_c/r_g | r_{out}/r_g | r_t/r_g | \mathcal{M} | r_t/r_c | Perturbation |
|-------|------------------|-------------|--------------|--------------|-----------|---------------|-----------|---------------|-----------|----------------------------|
| T1a | 296×228 | [4.8 - 5.6] | [-0.4 - 0.4] | 5.02 | 5.2 | 5.39 | 0.18 | 0.3 | 0.03 | vertical, radial, diagonal |
| T1b | 512×496 | [4.8 - 5.6] | [-0.4 - 0.4] | 5.02 | 5.2 | 5.39 | 0.18 | 0.3 | 0.03 | vertical, diagonal |
| T2a | 296×228 | [6.8 - 7.6] | [-0.4 - 0.4] | 7.02 | 7.2 | 7.39 | 0.18 | 0.3 | 0.02 | vertical, radial, diagonal |
| T2b | 512×496 | [6.8 - 7.6] | [-0.4 - 0.4] | 7.02 | 7.2 | 7.39 | 0.18 | 0.3 | 0.02 | vertical, diagonal |
| T3 | 512×512 | [4.5 - 6.0] | [-0.5 - 0.5] | 4.84 | 5.2 | 5.62 | 0.36 | 0.3 | 0.07 | vertical |

Table 2: The Parameter space of the simulation. From left to right the columns are, model name, resolution, range of the radial (r) and vertical (z) domain, inner radius (r_{in}), center of the torus (r_c), outer radius (r_{out}), cross section of torus r_t , magnitude of velocity perturbation (in Mach number \mathcal{M} at r_c) and ratio of the cross section of the torus to the distance from black hole (r_t/r_c). The type of perturbation is also listed. Models T1a-T2b are *thinner* tori and T3 *thicker* torus. For models T1a-T3, $r_g = 2GM/c^2$.

Simulations

Numerical code PLUTO was used to perform the simulations. Frequencies of various modes obtained from the simulations are accurate to the order of 0.03.

| Model | Trend of perturbation | Frequencies (Ω_K) | Modes |
|-------|-----------------------|--|---|
| T1a | radial | 0.65/1.3, 1.01, 1.68 | R/harmonic, +, B |
| | vertical | 1.67, 1.99 | B, V_h |
| | diagonal | 0.65/1.3, 1.0, 1.68, 1.99 | R/harmonic, +, B, V_h |
| T1b | vertical | 0.65, 1.01, 1.66, 1.99 | R, +, B, V_h |
| | diagonal | 0.65/1.29, 1.0, 1.68, 1.99 | R/harmonic, +, B, V_h |
| | vertical | 1.71, 1.99 | B, V_h |
| T2a | radial | 0.77/1.53, 1.15, 1.71 | R/harmonic, +, B |
| | vertical | 1.71, 1.99 | B, V_h |
| | diagonal | 0.77/1.52, 1.16, 1.73, 1.99 | R/harmonic, +, B, V_h |
| T2b | vertical | 0.76, 1.18, 1.71/3.37, 1.99/4.03 | R, +, B/harmonic, V_h /harmonic |
| | diagonal | 0.76/1.52, 1.16, 1.72, 2.0 | R/harmonic, +, B, V_h |
| T3 | vertical | 0.64, 1.02, 1.2/2.36, 1.66/3.33, 1.95/3.89 | R, +, x/harmonic, B/harmonic, V_h /harmonic |

Table 3: Frequencies and corresponding modes of oscillating tori. From left to right: model name, type of perturbation, frequencies (Ω_K) at r_c obtained from simulations and modes corresponding to solid lines in PSDs, respectively.

References

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Thinner Tori

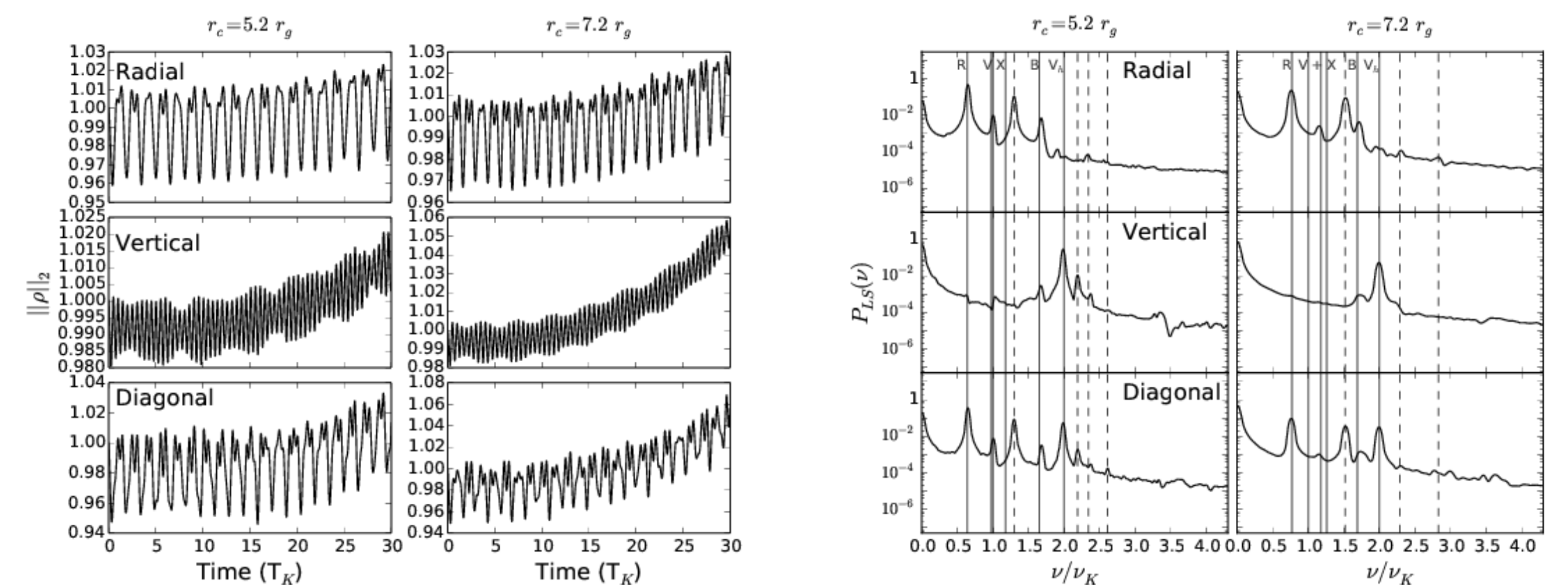


Fig. 1: Left: $\|\rho\|_2$ as a function of Keplerian time (T_K at r_c) for models T1a and T2a. Right: PSDs of $\|\rho\|_2$. Solid lines correspond to the theoretical values of fundamental eigenfrequencies and dashed lines to additional frequencies in the simulations.

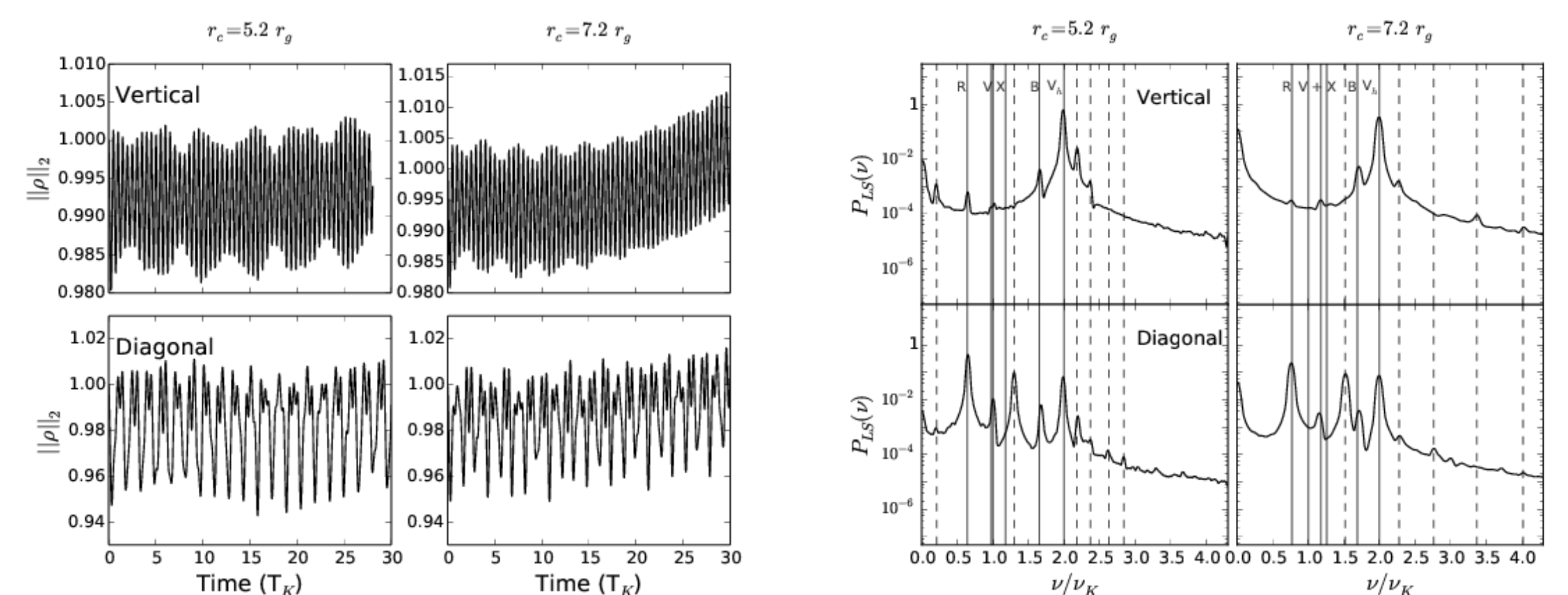


Fig. 2: Same as Fig. 1 for models T1b and T2b, respectively.

Thicker Torus

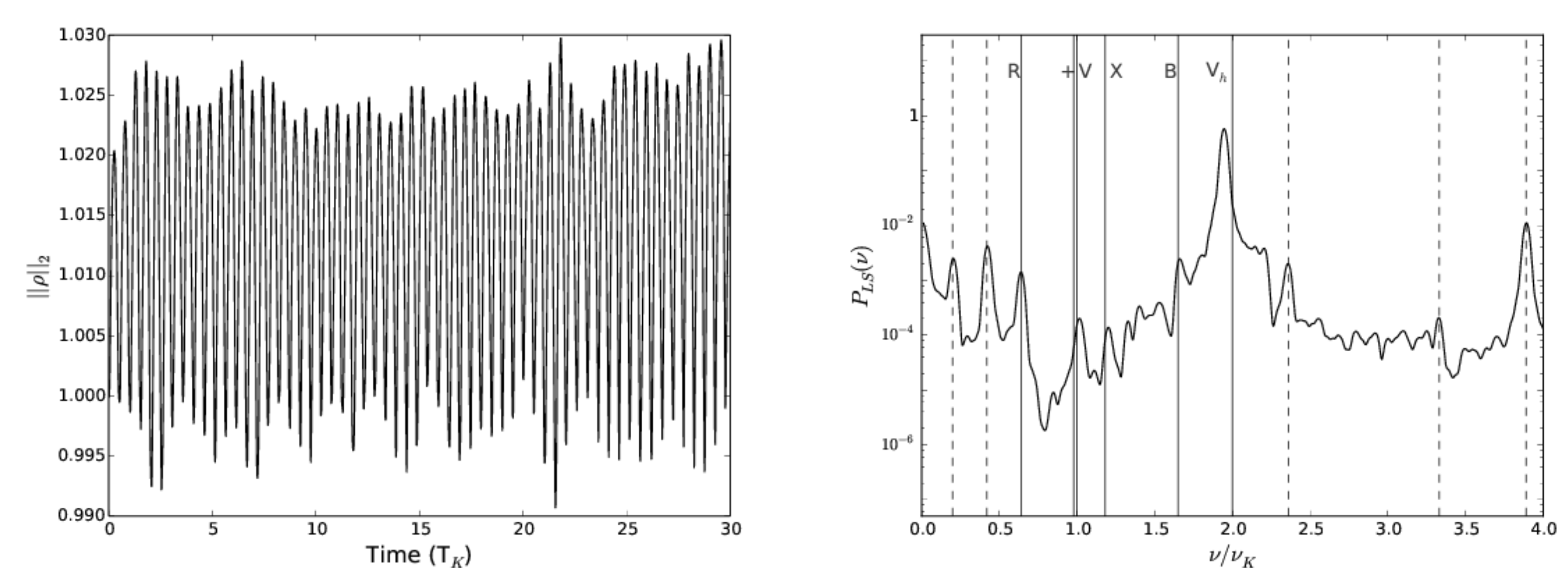


Fig. 3: Same as Fig. 1 for model T3. $r_c = 5.2 r_g$

Conclusions

- ✓ The thinner tori (T1a-T2b) manifests radial mode (R)/harmonic, plus-mode (+), breathing mode (B)/harmonic and vertical mode (V_h)/harmonic. The plus-mode and radial epicyclic mode appear in an approximate 3:2 ratio. Vertical epicyclic motion was not excited by radial perturbation.
- ✓ Vertical oscillation of thicker torus (T3) exhibits radial mode (R), plus-mode (+), breathing mode (B)/harmonic, x-mode/harmonic and vertical mode (V_h)/harmonic.
- ✓ Presence of radial mode and plus-mode in vertical oscillations of thinner tori (T1b & T2b) and thicker torus (T3) are numerical in origin and not a result of coupling between radial and vertical epicyclic modes, since simulations of unperturbed tori (T1b, T2b & T3) shows the presence of radial mode (R), plus-mode (+) and breathing mode (B).
- ✓ Results from our simulations are relevant in the context of high-frequency quasi-periodic oscillations (HF QPOs) observed in stellar-mass black hole binaries.