Acoustically generated gravitational waves at a first order phase transition

PRL 112, 041301 (2014) [arXiv:1304.2433],
arXiv:1504.03291 + work in progress

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Texas 2015, Geneva, 16 December 2015

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Motivation and context

- GWs are a unique and promising test of high energy physics (advanced LIGO and VIRGO; eLISA scheduled for 2034)
- First order PTs involve bubbles nucleating and growing: bubble collisions produce gravitational waves
- Standard Model EW PT is a crossover, but first order common in extensions (singlet, 2HDM, ...) 
  Andersen, Laine et al., Kozaczuk et al., Kamada and Yamada, Carena et al., Bödeker et al., Damgaard et al.
- First order PT around the EW scale could give right conditions for baryogenesis (but would then not give a good signal for GWs)
- What physics can we extract from the GW power spectrum at EW scales?
Envelop approximation

Kosowsky, Turner and Watkins; Kamionkowski, Kamionkowsky and Turner

- Thin-walled bubbles, no fluid
- Bubbles expand with velocity $v_w$
- Stress-energy tensor $\propto R^3$ on wall
- Overlapping bubbles $\rightarrow$ GWs
- Keep track of solid angle
- Collided portions of bubbles source gravitational waves
- Resulting power spectrum is simple
  - One scale ($R_*$)
  - Two power laws ($k^3, k^{-1}$)
  - Amplitude
    $\Rightarrow$ 4 numbers define spectral form
The envelope approximation makes predictions

Espinosa, Konstandin, No and Servant; Huber and Konstandin

4-5 numbers parametrise the transition:

- $\alpha$, vacuum energy fraction
- $v_w$, bubble wall speed
- $\kappa$, conversion efficiency to fluid KE
- Transition rate:
  - $H_*$, Hubble rate at transition
  - $\beta$, bubble nucleation rate

Energy in GWs ($\Omega_{GW} = \rho_{GW}/\rho_{Tot}$):

$$\Omega_{GW}^{envelope} \approx \frac{0.11 v_w^3}{0.42 + v_w^2} \left( \frac{H_*}{\beta} \right)^2 \frac{\kappa^2 \alpha^2}{(\alpha + 1)^2}$$
Envelope approximation power laws do not depend on nucleation

Work in progress!

- Re-implemented the method of Huber and Konstandin
- Bubbles nucleated at the same time have same power laws as bubbles nucleated ‘properly’
- Can re-weight from equal time nucleation case to unequal time
Our approach: field+fluid system

- Scalar $\phi$ + ideal fluid $u^\mu$ (treated using standard SR hydro Wilson and Matthews)
- Split stress-energy tensor $T^{\mu\nu}$ into field and fluid bits
  
  Ignatius, Kajantie, Kurki-Suonio and Laine

  \[
  \partial_\mu T^{\mu\nu} = \partial_\mu (T^{\mu\nu}_{\text{field}} + T^{\mu\nu}_{\text{fluid}}) = 0
  \]

- Parameter $\eta$ sets the scale of friction due to plasma

  \[
  \partial_\mu T^{\mu\nu}_{\text{field}} = \eta u^\mu \partial_\mu \phi \partial^\nu \phi \quad \partial_\mu T^{\mu\nu}_{\text{fluid}} = -\eta u^\mu \partial_\mu \phi \partial^\nu \phi
  \]

- Effective potential $V(\phi, T)$ can be kept simple

  \[
  V(\phi, T) = \frac{1}{2} \gamma (T^2 - T_0^2) \phi^2 - \frac{1}{3} AT \phi^3 + \frac{1}{4} \lambda \phi^4
  \]

  - $\gamma, T_0, A, \lambda$ chosen to match scenario of interest

- Equations of motion (+ continuity equation)

  \[
  \partial_\mu \partial^\mu \phi + \frac{\partial V(\phi, T)}{\partial \phi} = -\eta u^\mu \partial_\mu \phi
  \]

  \[
  \partial_\mu \left\{ [\epsilon + p] u^\mu u^\nu - g^{\mu\nu} [p - V(\phi, T)] \right\} = \left( \eta u^\mu \partial_\mu \phi + \frac{\partial V(\phi, T)}{\partial \phi} \right) \partial^\nu \phi
  \]
Here, $\eta = 0.2$ (deflagration)
Metric perturbations evolve as

\[ \ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT} \]

equivalently Garcia-Bellido and Figueroa; Easther, Giblin and Lim

\[ \ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi \tau_{ij} \]

and project \( h_{ij}(k) = \Lambda_{ij,lm}(k) u_{ij}(k) \) later

Consider only terms at leading order in the perturbation \( h_{ij} \)

\[ \tau^f_{ij} = W^2 (\epsilon + p) V_i V_j \quad \tau^\phi_{ij} = \partial_i \phi \partial_j \phi \]

Power \( \rho_{GW} = T_{00}^{grav} \) per logarithmic interval,

\[ \frac{d\rho_{GW}}{d \ln k} = \frac{1}{32\pi GV} \frac{k^3}{(2\pi)^3} \int d\Omega \Lambda_{ij,lm}(k) \dot{u}_{ij}(t, k) \dot{u}^*_{lm}(t, k) \]
Simulations at $1024^3$, deflagration, fluid kinetic energy density, $\sim 250$ bubbles
How the sources behave over time

- \( \overline{U}_f \) is the rms fluid velocity; \( \overline{U}_\phi \) the analogous field quantity

- Constructed from \( \tau_{fi}^f \) and \( \tau_{ii}^\phi \), they indicate how strong each source is

\[
(\bar{\epsilon} + \bar{p})\overline{U}_f^2 = \frac{1}{V} \int d^3x W^2(\epsilon + p) \quad (\bar{\epsilon} + \bar{p})\overline{U}_\phi^2 = \frac{1}{V} \int d^3x (\partial_i \phi)^2
\]

\( t = (T^{-1}_c) \)

\( N_b = 988 \)

\( N_b = 37 \)
So does the envelope approximation really work?

- Compare field+fluid simulation with envelope approximation
- Nucleate 125 bubbles in same locations

Power laws for fluid source totally different
- Field source OK (overestimated), but will be subdominant anyway
Acoustic waves source linear growth of gravitational waves

- Sourced by $T_{ij}^f$ only ($T_{ij}^\phi$ source is small constant shift)

- Source generically scales as $\rho_{GW} \propto t \left[ G \xi_f \left( \bar{\epsilon} + \bar{p} \right)^2 U_f^4 \right]$
• Weak transition: \( \alpha_{TN} = 0.01, \, v_w = 0.44 \)
• Power law behaviour above peak (approximately \( k^{-1} \) here)
• “Ringing” due to simultaneous bubble nucleation, not physically important
• Power is in the longitudinal modes – acoustic waves, not turbulence
• If we know \( \frac{dV^2}{d \ln k} \), can work out \( \frac{\dot{\rho}_{GW}}{d \ln k} \)
GW power spectra and power laws

- Sourced by $T_{ij}^f$ only

- Approximate $k^{-3}$ power spectrum
- Finite size of box means that we choose not to probe behaviour below peak $k$
Fluid characteristic length scale is imprinted in GW power spectrum

Define the fluid integral scale

\[
\xi_f = \frac{1}{\langle V^2 \rangle} \int \frac{d^3 k}{(2\pi)^3} |k|^{-1} P_V(k)
\]

and the analogous quantity \(\xi_{GW}\) for the gravitational wave power spectrum.

This length scale is what sets the peak of the fluid power spectrum.
Latest results: $4200^3$ using PRACE access

Some results from our latest runs ($4200^3$ lattice) – also work in progress!

- Curves every $1000/T_c$
- Here $v_w = 0.68$ (detonation)
- Friction parameter now dimensionless, damping term $\eta \rightarrow \eta \phi^2 / T$
Summary and outlook

- New source of GWs: sound waves from colliding bubble droplets
- Rate of GW energy production is generically $\rho_{GW} \propto t [G \xi_f (\bar{\epsilon} + \bar{p})^2 U_f^4]$
- Large enhancement over envelope approximation at EW scale → good news for models that do not produce strongly first-order PTs
- Power laws different from envelope approximation
- Functional form of power spectrum still a broken power law
- Currently trying to understand power laws with larger simulations – 18M CPU hours awarded by PRACE
- Building a science case for eLISA – Caprini et al.
Most realtime lattice simulations in the early universe have a single [nontrivial] length scale

Here, many length scales important

Simulations in arXiv:1504.03291 are with $2400^3$ lattice, $\delta x = 2/T_c$

$\rightarrow$ approx 200k CPU hours each ($\sim$ 3M total)
• Most power is in the longitudinal modes – acoustic waves, not turbulence
• System is quite linear. Reynolds number is $\sim 100$ due to discretisation.
GW power spectra – field and fluid sources

- By late times, fluid source dominates at all length scales
- $500/T_c$, $1000/T_c$, $1500/T_c$ (‘before’, ‘during’, ‘after’ collision)
- Fluid source shown by dashed lines, total power solid lines
Lifetime of sound waves and increase in GW power

- Does the acoustic source matter?
- Sound is damped by (bulk and) shear viscosity  
  \[ \left( \frac{4}{3} \eta_s + \zeta \right) \nabla^2 V_i^i + \ldots \Rightarrow \tau_{\eta}(R) \sim \frac{R^2 \epsilon}{\eta_s} \]
  
  \[ \frac{1}{H_*} \frac{\eta_s}{\epsilon} \sim 10^{-11} \frac{v_w}{H_*} \left( \frac{T_c}{100 \text{ GeV}} \right) \]

  the Hubble damping is faster than shear viscosity damping.

- Does the acoustic source enhance GWs?
  - Yes, we have
    \[ \Omega_{GW} \approx \left( \frac{\kappa \alpha}{\alpha + 1} \right)^2 (H_* \tau_{H_*})(H_* \xi_f) \Rightarrow \frac{\Omega_{GW}}{\Omega_{GW}^{\text{envelope}}} \gtrsim 60 \frac{\beta}{H_*}. \]