Causality with Two Metrics

Ignacy Sawicki
Université de Genève

With Alexander Vikman

MARIE CURIE ACTIONS

Perturbations in Cosmology

$$\phi + \bar{\phi}(\tau) + \pi \qquad g_{\mu\nu} = \bar{g}_{\mu\nu}^{\mathrm{FRW}} + h_{\mu\nu}$$

$$S_{\phi 2} = \int \mathrm{d}\tau \mathrm{d}^3 x a^4 D\left(\dot{\pi}^2 - c_s^2 \partial_i \pi \partial^i \pi - m^2 \pi^2\right)$$

$$\mathcal{H} = D\left(\frac{\Pi^2}{D^2} + c_s^2 (\partial_i \pi)^2 + m^2 \pi^2\right)$$

$D < 0$: Ghosts

$c_s^2 < 0$: Gradient Instabilities

$m^2 < 0$: Tachyons

The Acoustic Metric $\mathcal{G}_{\mu\nu}$

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}(\phi, \partial_\mu\phi, \nabla_\mu\nabla_\nu\phi)$$

$$S_{\phi 2} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-\mathcal{G}}\left(\mathcal{G}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - m^2\pi^2\right)$$

$$\mathcal{G}_{\mu\nu}^{\mathrm{FRW}} \propto \begin{pmatrix} D & & & \\ & -Dc_s^2 & & \\ & & -Dc_s^2 & \\ & & & -Dc_s^2 \end{pmatrix}$$

$$\Box_{\mathcal{G}}\pi = m^2\pi$$

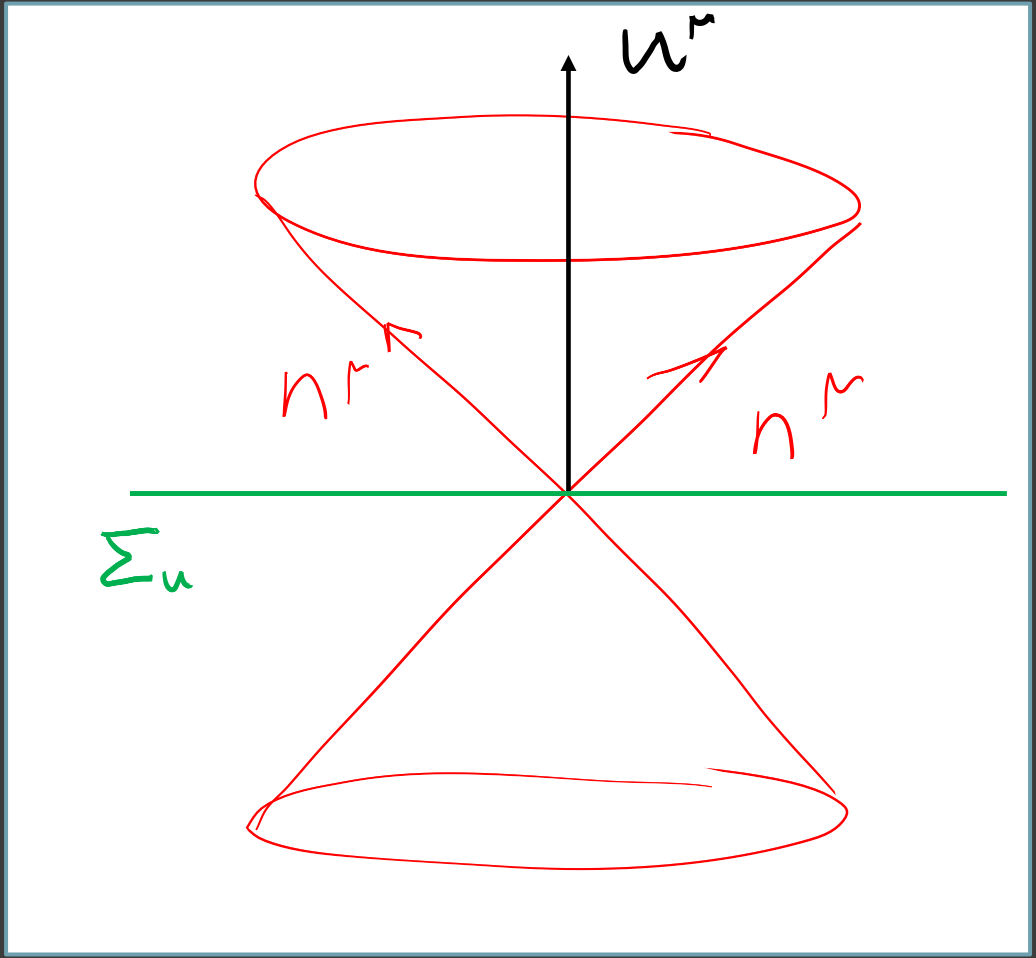
The Gist

- Each d.o.f. propagates in its own metric which can be different from $g_{\mu\nu}$
  - When is it consistent even to talk about common evolution?

- Eigenvalue structure of the acoustic metric tells you about
  - Ghosts and instabilities
  - Causal evolution and Cauchy problem
  - And it’s all coordinate/observer invariant

- Clustering DE/MG typically has metrics which are not homogeneous/isotropic around non-linear sources
  - e.g Vainstein

Lightcones: characteristic surfaces for Maxwell



- Geometrical optics

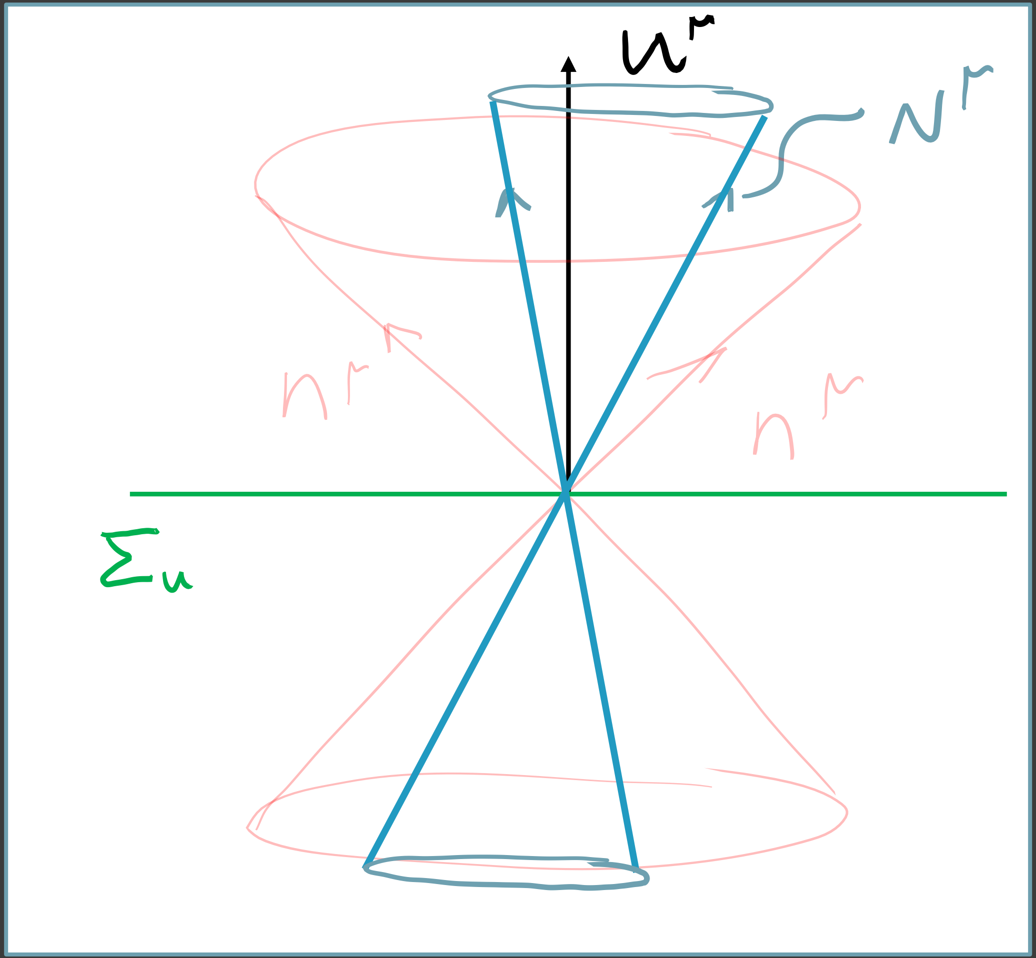
$$A_\mu = \bar{A}_\mu(x) e^{iS(x)}$$

$$p_\mu = -\partial_\mu S \qquad n^\mu = g^{\mu\nu} p_\nu$$

- Maxwell: $\nabla_\mu F^{\mu\nu} = 0$

$$g^{\mu\nu} \partial_\mu S \partial_\nu S = 0$$

Sound Cones: characteristic surfaces for scalars



- Eikonal ansatz

$$\pi = \mathcal{A}(x) e^{iS(x)/\epsilon}$$

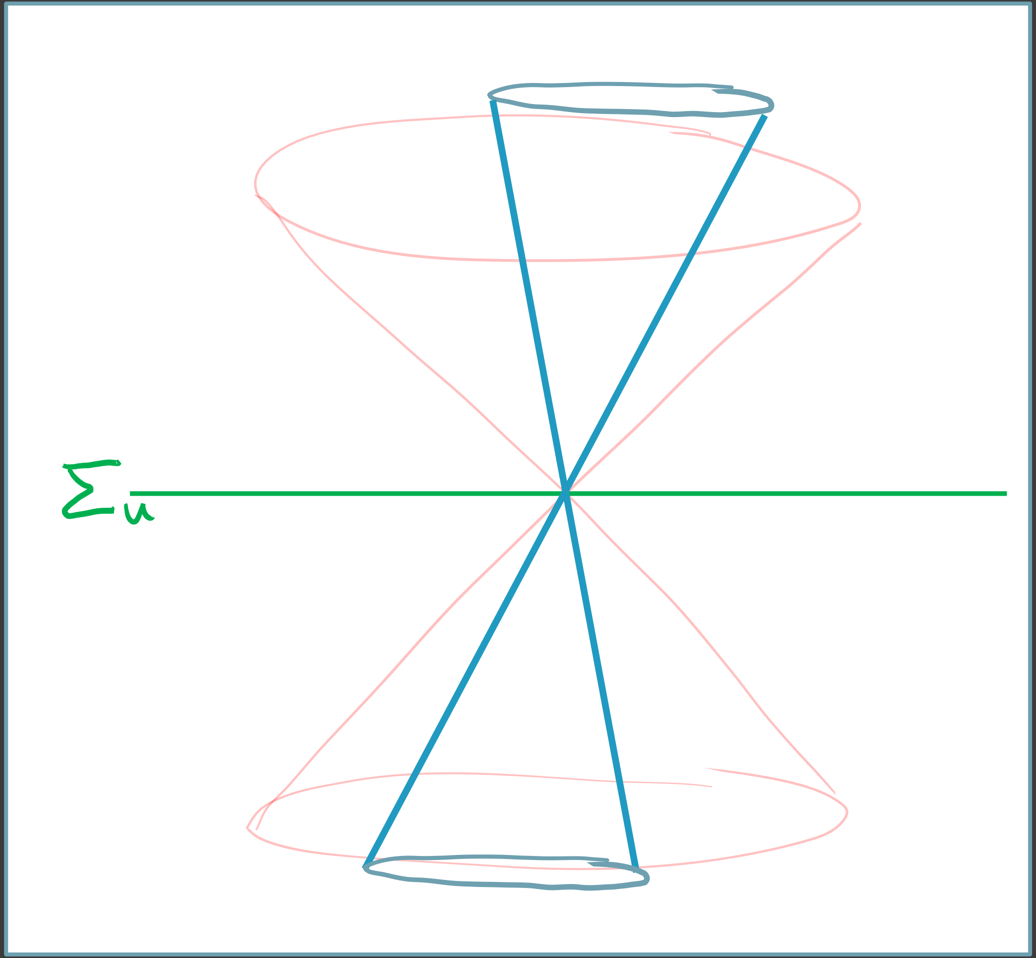
$$P_\mu \equiv -\partial_\mu S \qquad N^\mu \equiv \mathcal{G}^{\mu\nu} P_\nu$$

- EoM : $\mathcal{G}^{\mu\nu} \nabla_\nu \nabla_\mu \pi + \cdots = 0$

$$\mathcal{G}^{\mu\nu} P_\mu P_\nu = 0$$
$$\mathcal{G}^{-1}_{\mu\nu} N^\mu N^\nu = 0$$

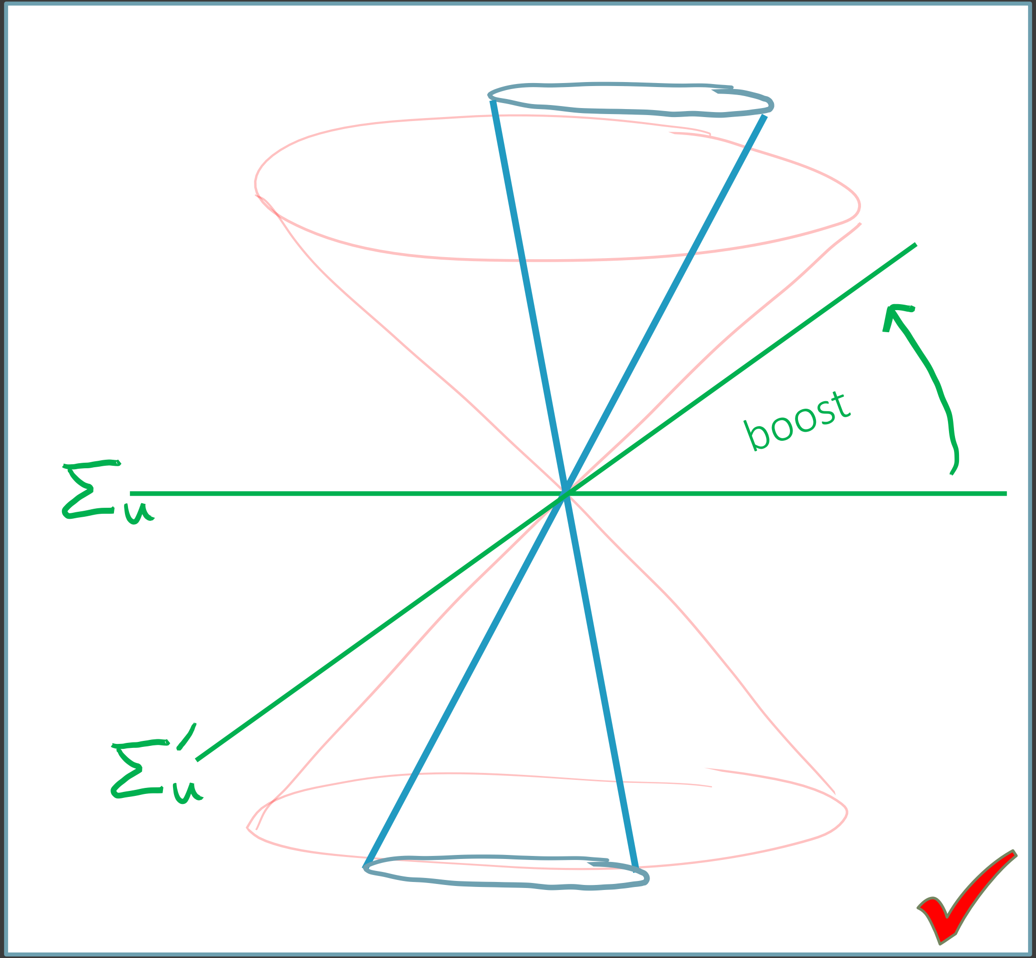
Superluminality and Causality

Subluminal



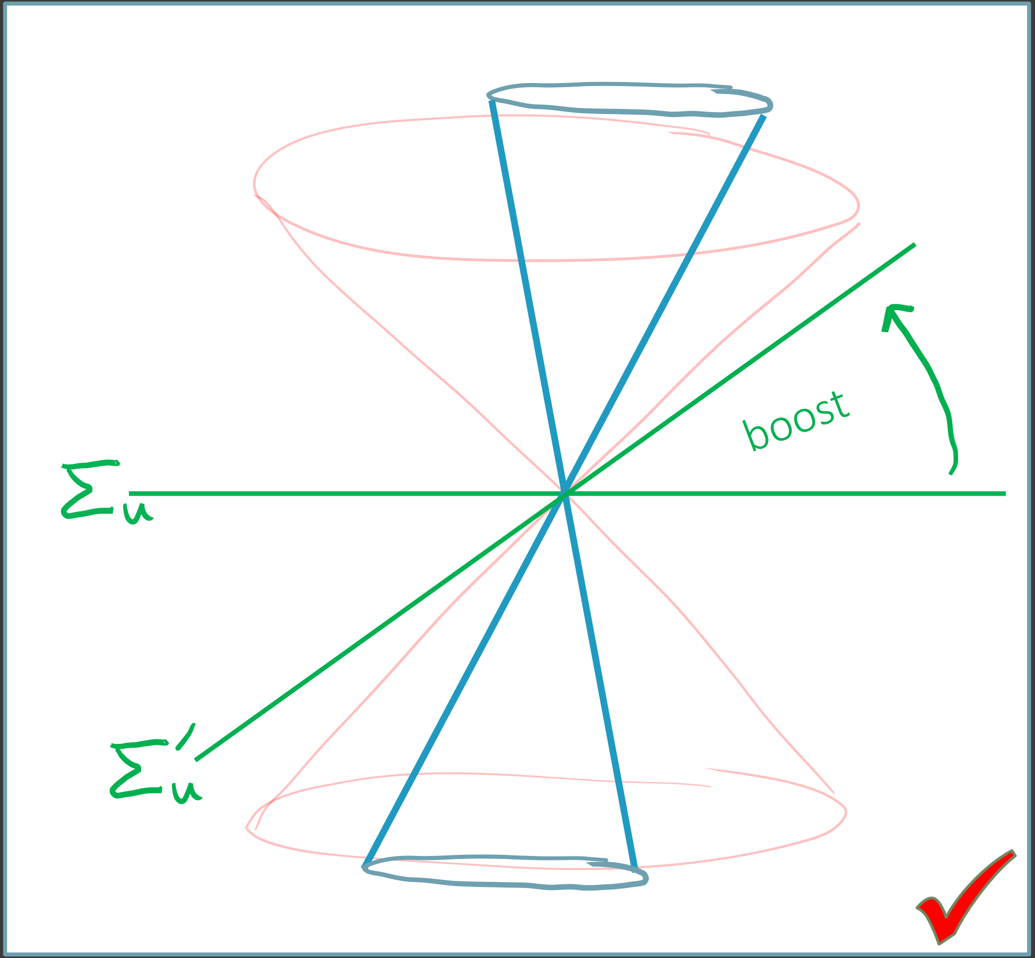
Superluminality and Causality

Subluminal

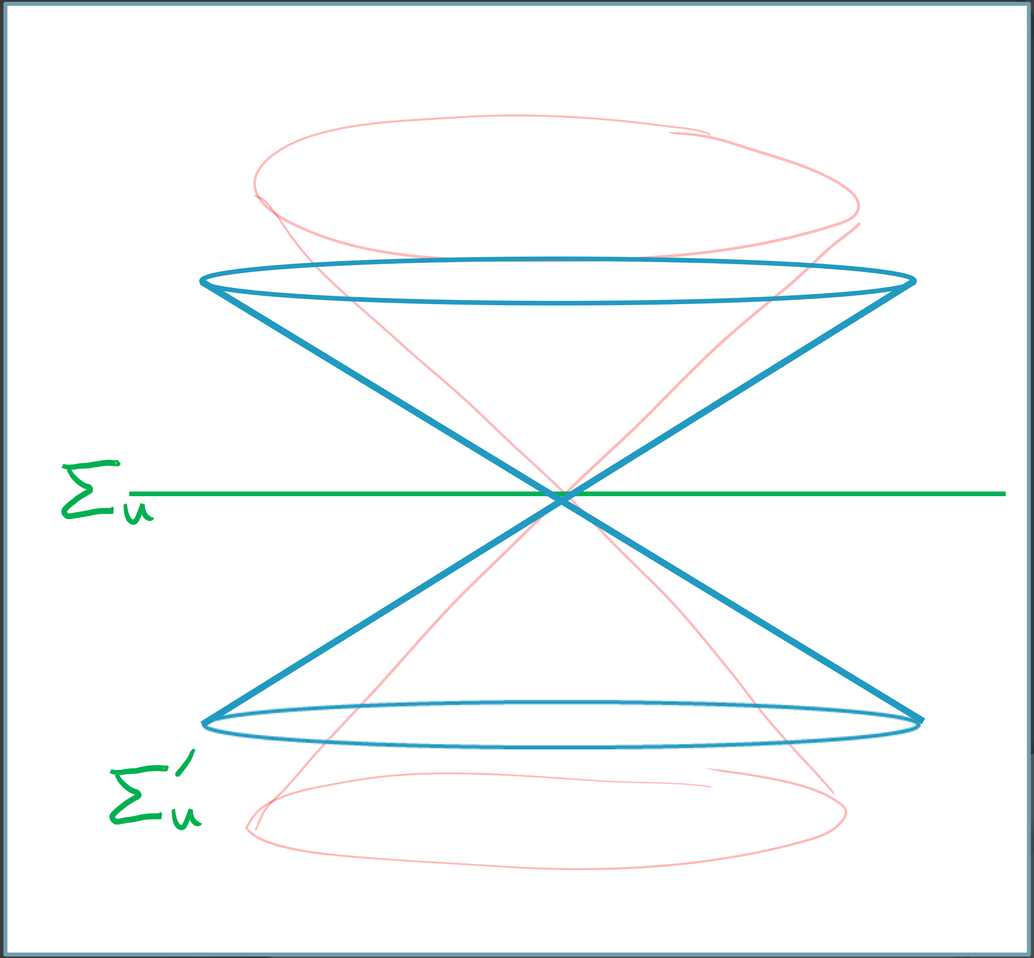


Superluminality and Causality

Subluminal



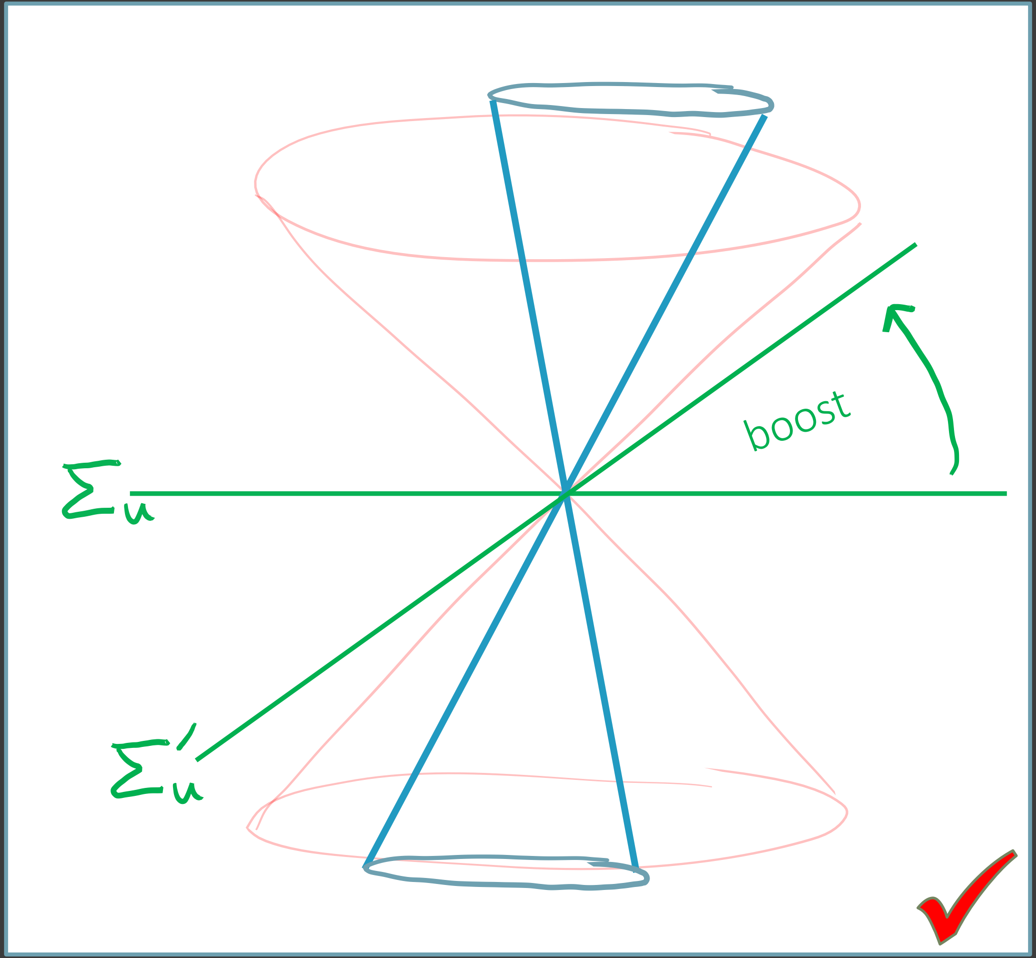
Superluminal



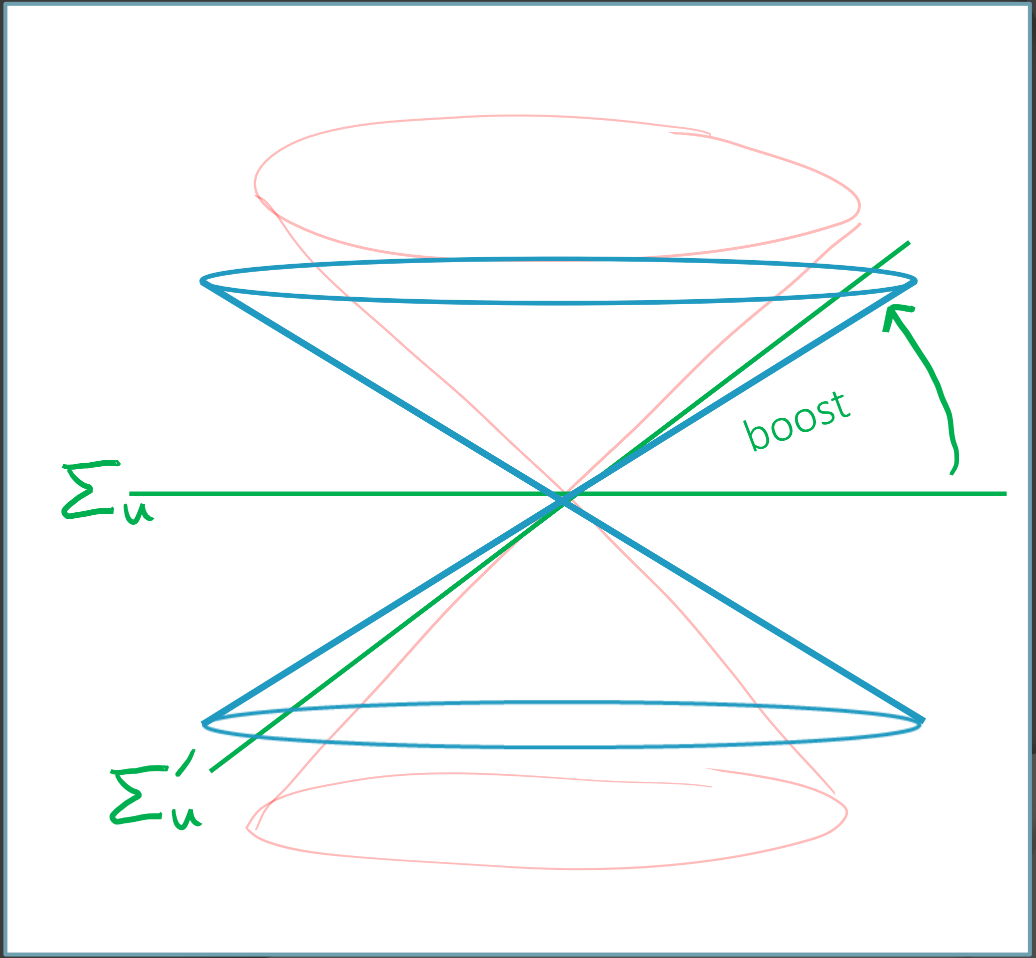
Superluminality and Causality

Babichev, Mukhanov and Vikman (2006)

Subluminal

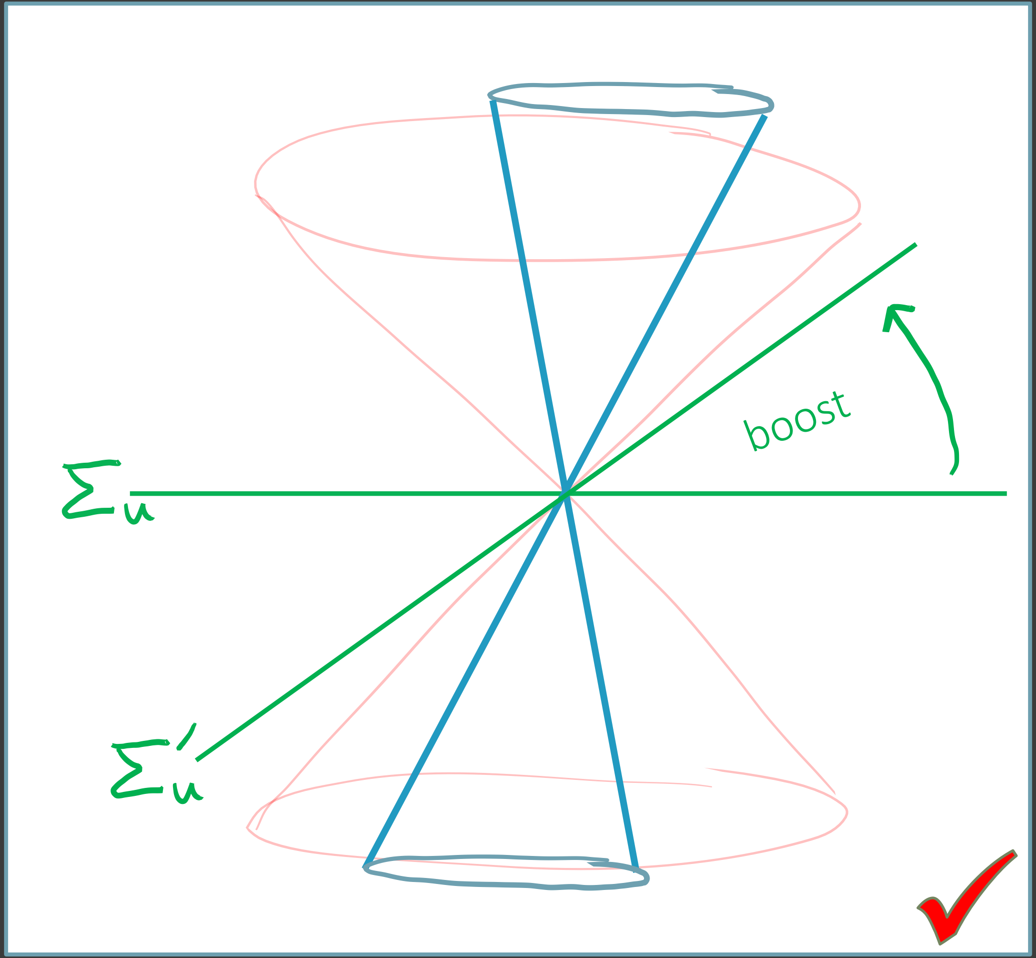


Superluminal

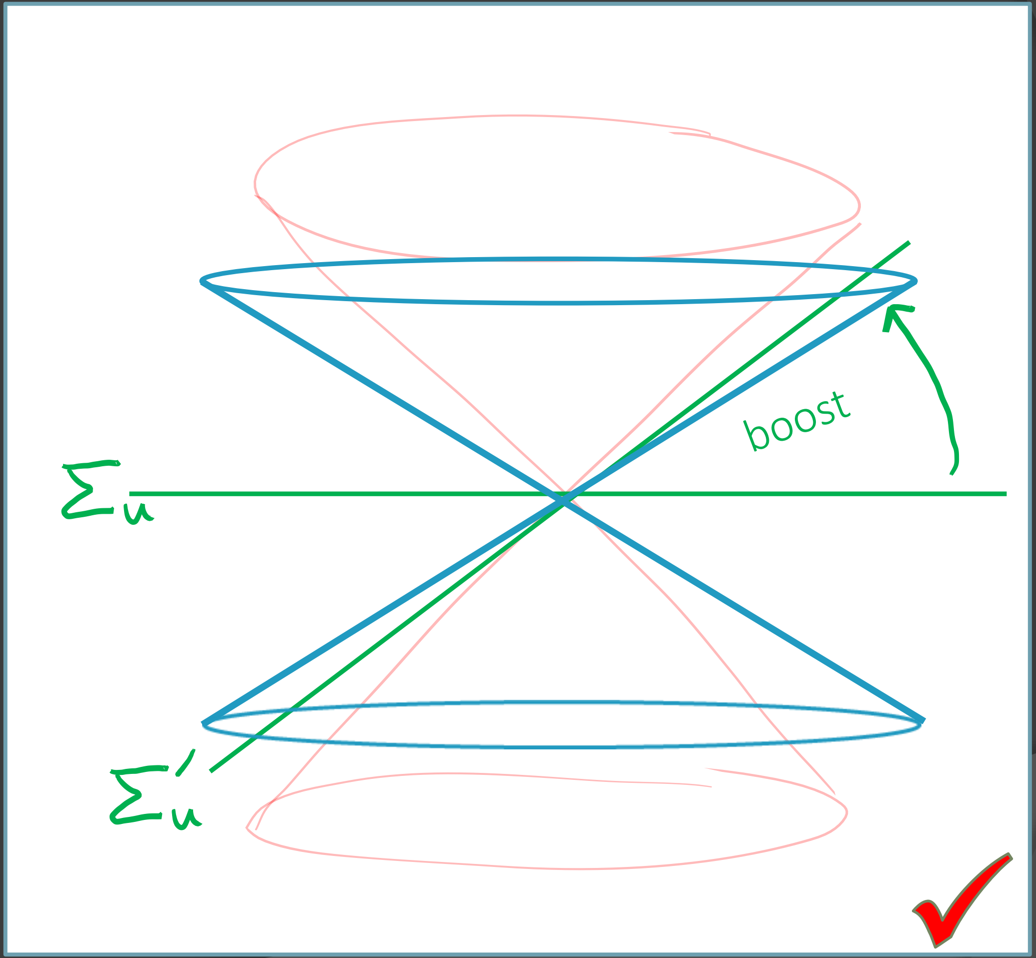


Superluminality and Causality

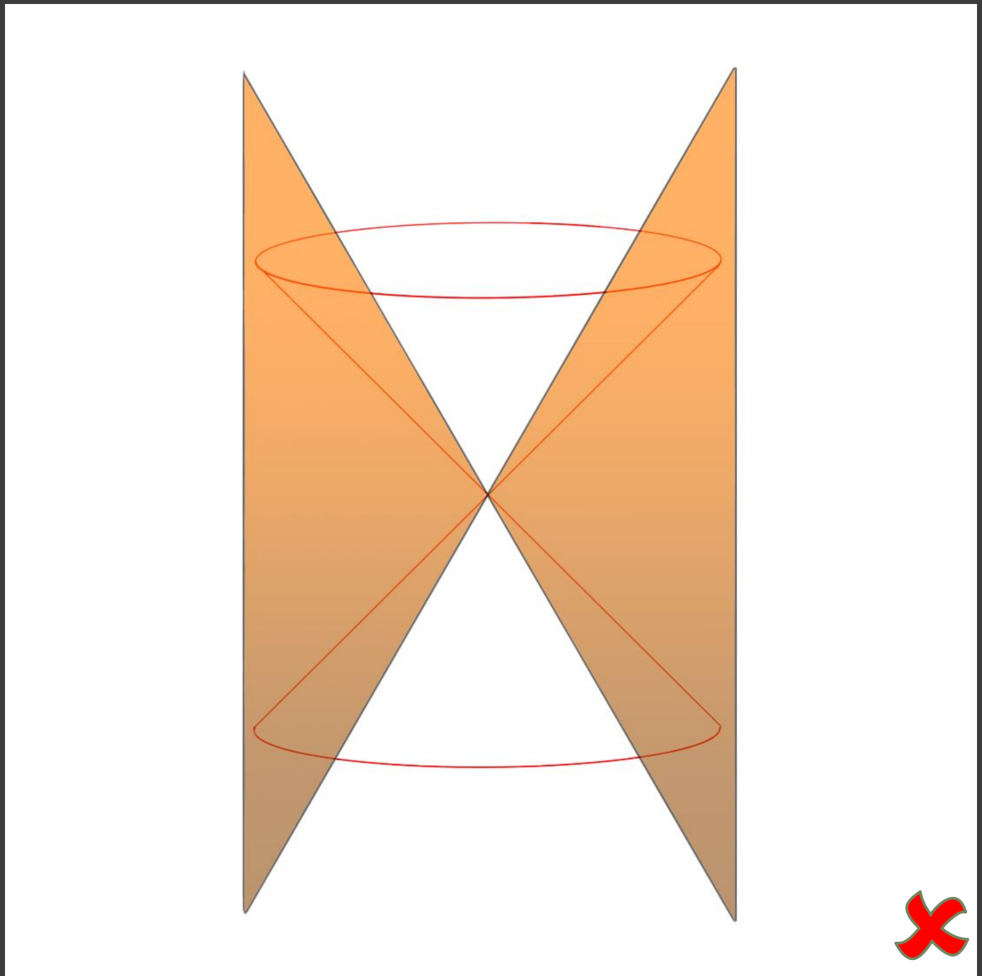
Subluminal

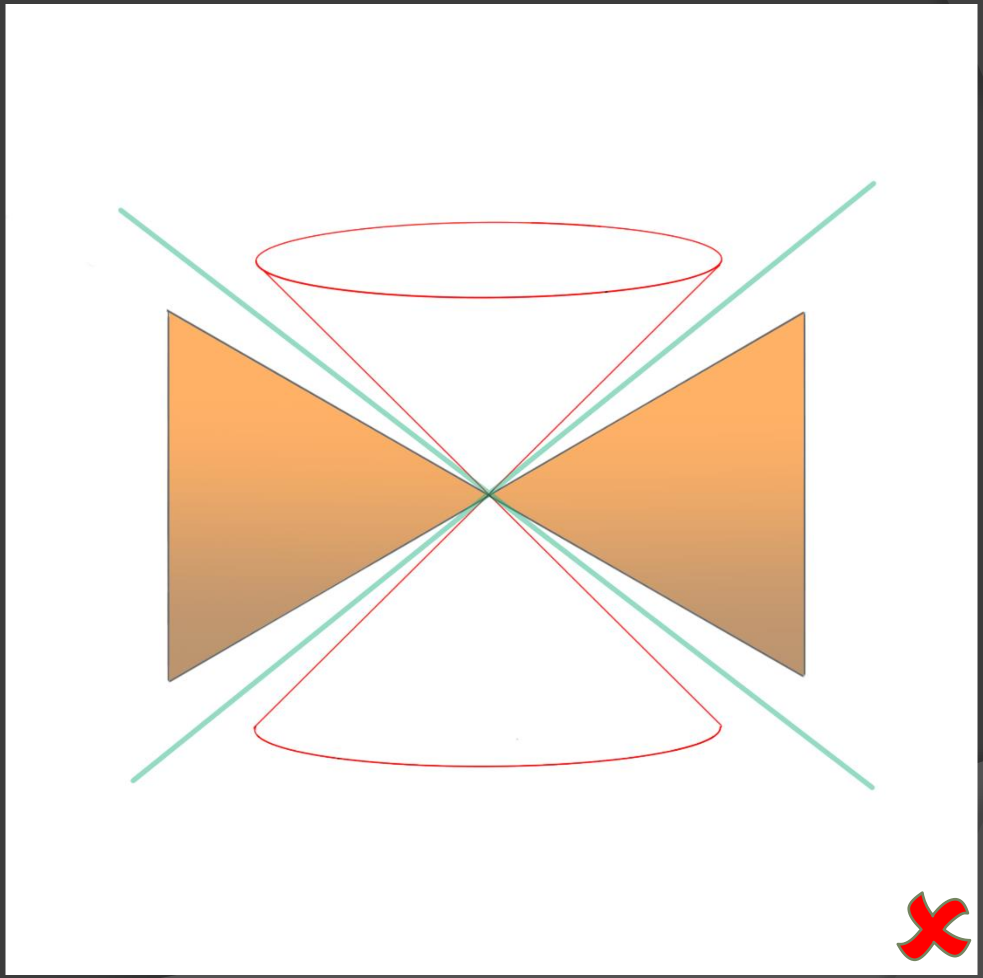


Superluminal



Inescapable Violations of Causality





Cone Existence + Causality ⇒ Stability

$$\mathcal{G}_{\mu\nu} p^\mu p^\nu = 0$$

Diagonalise*

$$\mathcal{G}_{IJ} p^I p^J = \lambda_0 (p^0)^2 + \sum_1^3 \lambda_i (p^i)^2$$

Cone exists

No $\lambda_I = 0$

Exactly *one* $\lambda_I > 0$

Common TL directions

$\lambda_0 > 0$

$$c_s^2 = -\frac{\lambda_i}{\lambda_0}$$

$(+---)$

Example: $\mathcal{L} = K(X)$

$2X \equiv (\partial_\mu \phi)^2$

$$\mathcal{G}_{\mu\nu} = K_{,X} g_{\mu\nu} + K_{,XX} \partial_\mu \phi \partial_\nu \phi$$

- $\partial_\mu \phi$ timelike: type I

  $\lambda_0 = K_X + 2XK_{XX} \qquad \lambda_i = -K_X$

- $\partial_\mu \phi$ spacelike: type I

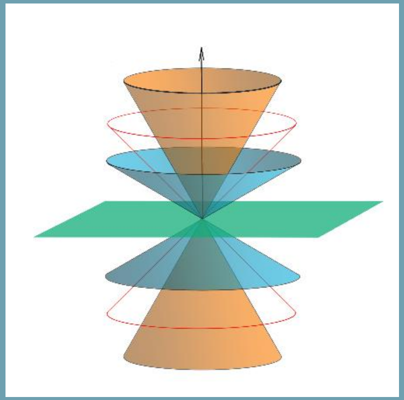
  $\lambda_0 = K_X \qquad \lambda_{2,3} = -K_X \qquad \lambda_1 = -(K_X + 2XK_{XX})$

- $\partial_\mu \phi$ null ($X = 0$): type III

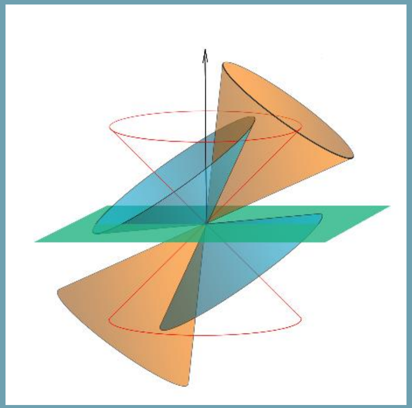
*Classification of Cone Geometries

Hall 1972

Type I: 4 real λ, no null



Type II: 2 λ complex, no null



$$|\mathcal{G}_{\mu\nu} - \lambda g_{\mu\nu}| = 0$$

Type III: 2 repeated λ
2 null e-vectors

Type IV: 3 repeated λ
3 null e-vectors

The Take Away

- Eigenvalues of acoustic metrics tell you everything about the free theory
  - Cauchy problem, stability
  - Non-linear/non-FRW backgrounds: (an)isotropisation?
- Causality constrains the relationship between the two metrics
  - Are there dynamical mechanisms which ensure it be preserved?