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DE LA RECHERCHE À L'INDUSTRIE

Neutrinos beyond the linear regime: a new theoretical approach

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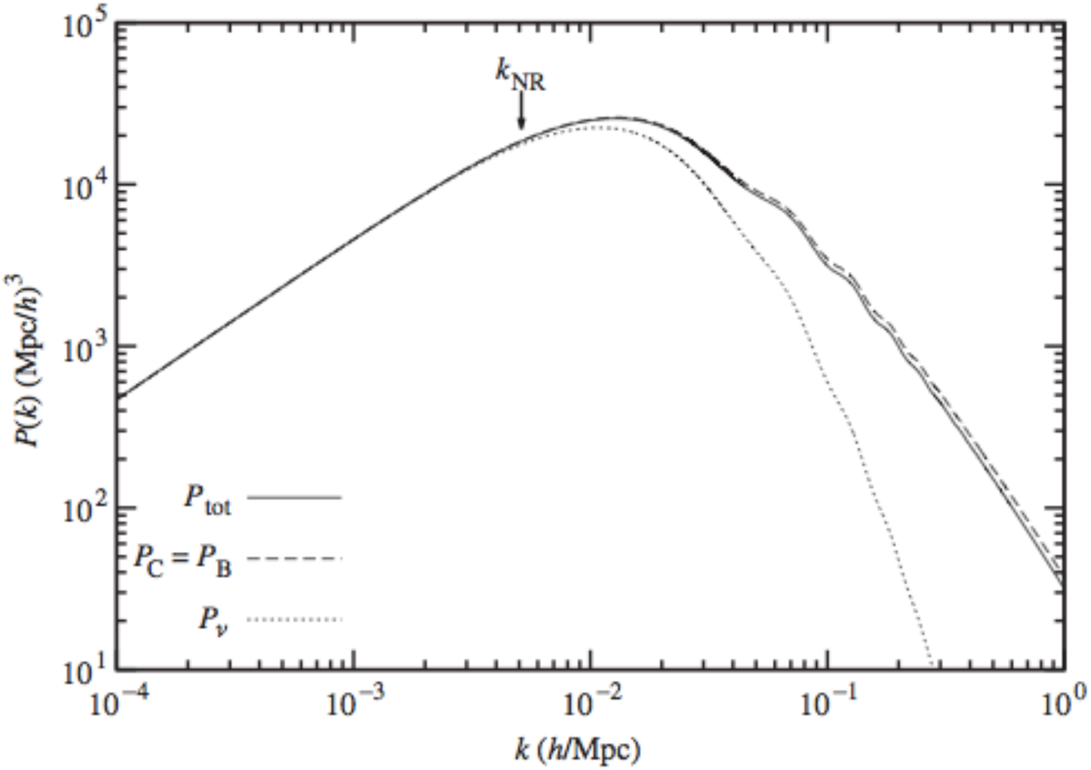
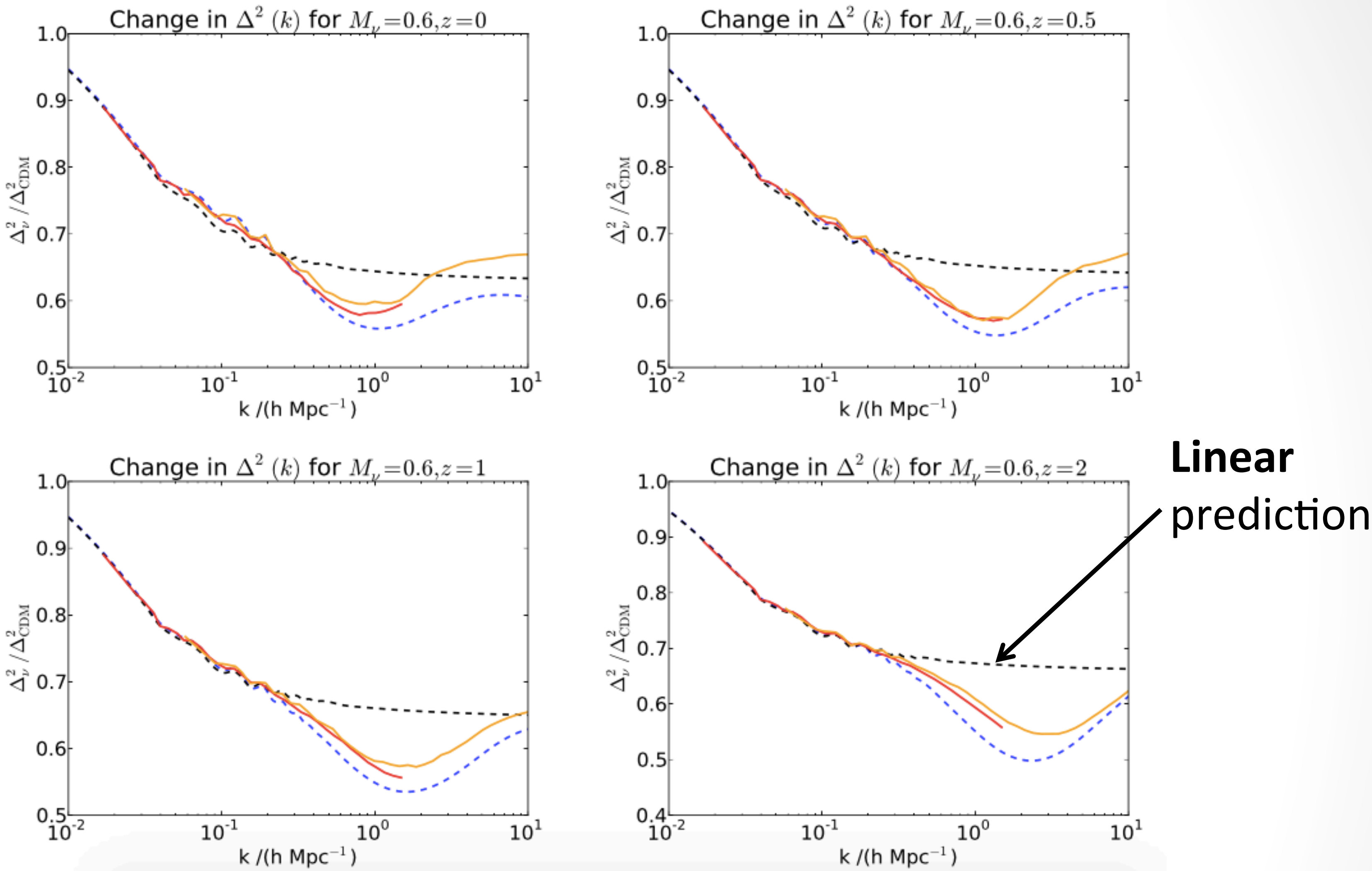


Figure 6.4 Matter power spectrum at redshift zero for a ΛCDM model with three degenerate massive neutrino species ($m_\nu = 0.3$ eV), compared to the individual power spectrum of CDM, baryon and neutrino density perturbations. In this model k_{nr} is equal to $5.1 \times 10^{-3} h$/Mpc (see Eq. (5.94)). For wavenumbers $k > k_{nr}$, neutrino perturbations remain smaller than CDM and baryon perturbations, because of their low growth rate after the nonrelativistic transition.

Source: *Neutrino Cosmology* (book)
(J.Lesgourgues, G. Mangano, G. Miele and S. Pastor, 2013)



Numerical simulations showing the effect of massive neutrinos on the nonlinear matter power spectrum (m = 0.6 eV)

Authors: S. Bird *et al.* (arXiv: 1109.4416)

a) The standard description of the nonlinear growth of structure

Review on the subject → arXiv: 1311.2724 (F.Bernardeau)

- In cold fluids, **the velocity dispersion is negligible**.

→ No curl mode can be generated in the velocity field u_i .

→ The velocity field is entirely characterized by its **divergence:**

$$\theta(x^i,t) = \frac{1}{aH}\frac{\partial u_i(x^i,t)}{\partial x^i}.$$

→ In reciprocal space, the system can be rewritten **compactly** with the help of the variable

$$\Psi_a(\mathbf{k},\eta) \equiv \left(\delta(\mathbf{k},\eta), -\theta(\mathbf{k},\eta)\right)^{\mathrm{T}}.$$

- The resulting equation is:

$$\boxed{a\frac{\partial\Psi_a}{\partial a}(\mathbf{k},\eta) + \Omega_a{}^b(\eta)\,\Psi_b(\mathbf{k},\eta) = \gamma_a{}^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1,\eta)\Psi_c(\mathbf{k}_2,\eta)}.$$

b) The standard description of massive neutrinos

All details can be found e.g. in *Neutrino Cosmology*
(J.Lesgourgues, G. Mangano, G. Miele and S. Pastor, 2013).

- Neutrinos are described via their **phase-space** distribution function.

- The quantities useful for cosmology are obtained by computing the moments of the distribution function:

$$T_{\mu\nu}(\eta, x^i) = \int \mathrm{d}^3 p_i \, (-g)^{-1/2} \frac{p_\mu p_\nu}{p^0} \, f(\eta, x^i, p_i).$$

- The distribution function satisfies the **Vlasov equation**,

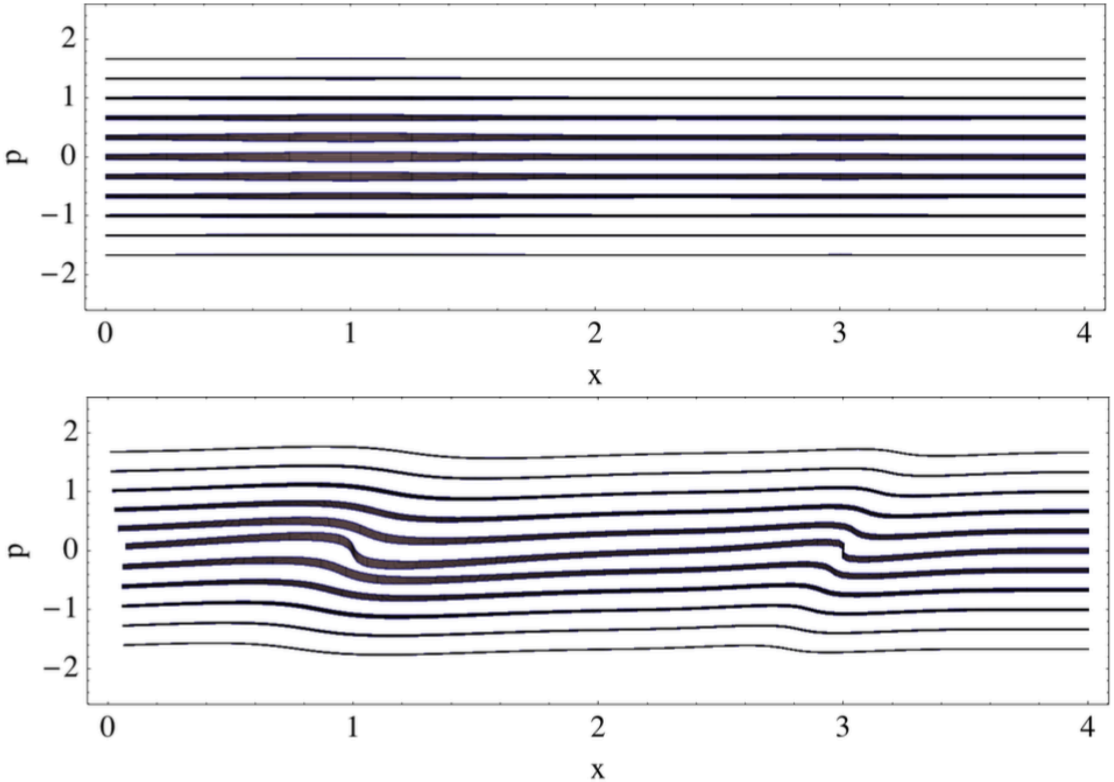
$$\frac{\mathrm{d}f(x, q, \eta)}{\mathrm{d}\eta} = 0.$$

- In **linear** perturbation theory, $f = f_0(1 + \Psi)$.

- Explicit calculations are done with the help of **Boltzmann codes** (integration of the Boltzmann hierarchy).

c) Proposition of a new approach

NEUTRINOS AS A COLLECTION OF FLOWS



initial momentum (label of the flow)

- Total distribution function: $f^{\mathrm{tot}}(\eta, \mathbf{x}, \mathbf{p}) = \sum_{\vec{\tau}} f_{\vec{\tau}}(\eta, \mathbf{x}, \mathbf{p}).$
- One density field per flow: $n_c(\eta, \mathbf{x}; \vec{\tau}) = \int \mathrm{d}^3 p_i \; f_{\vec{\tau}}(\eta, x^i, p_i).$

momentum field of the flow

- In each flow, $f_{\vec{\tau}}(\eta, x^i, p_i) = n_c(\eta, \mathbf{x}; \vec{\tau}) \delta_{\mathrm{D}}(p_i - P_i(\eta, \mathbf{x}; \vec{\tau})).$

- In each flow, $P_i(\eta, \mathbf{x}; \vec{\tau}) = \dfrac{\int \mathrm{d}^3 p_i \ f_{\vec{\tau}}(\eta, x^i, p_i) p_i}{\int \mathrm{d}^3 p_i \ f_{\vec{\tau}}(\eta, x^i, p_i)}$.

arbitrary function

- More generally, $\mathcal{F}\left[P_i(\eta, \mathbf{x})\right] n_c(\eta, \mathbf{x}) = \int \mathrm{d}^3 p_i \ f(\eta, x^i, p_i) \ \mathcal{F}\left[p_i\right]$.

- The physical quantities of interest can be expressed in terms of our fields:

$$\underbrace{\int \mathrm{d}^3 p_i \, f^{\mathrm{tot}}(\eta, x^i, p_i) \ \mathcal{F}(p_i)}_{\text{Boltzmann approach}} = \underbrace{\int \mathrm{d}^3 \tau_i \, n_c(\eta, \mathbf{x}; \tau_i) \ \mathcal{F}(P_i(\eta, \mathbf{x}; \tau_i))}_{\text{our approach}}.$$

- **In each flow**, the equation of motion of the density field is

$$\boxed{\frac{\partial}{\partial \eta} n_c + \frac{\partial}{\partial x^i}\left(\frac{P^i}{P^0} n_c\right) = 0},$$

where $P^i = g^{ij} P_j$ and P^0 is defined so that $P^\mu P_\mu = -m^2$.

- **In each flow,** $T^{\mu\nu} = -P^\mu J^\nu$.

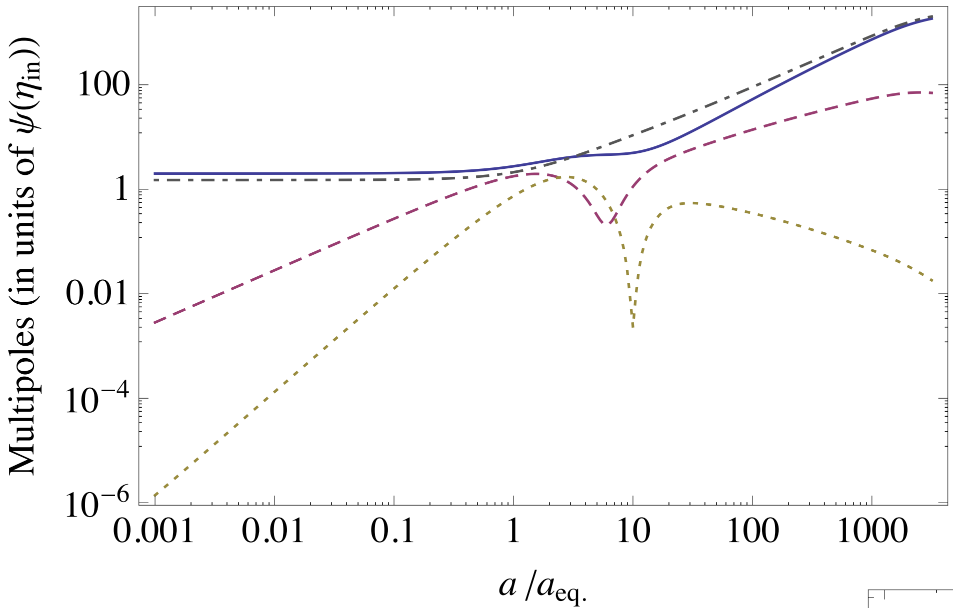
energy-momentum tensor

particle four-current

Combined conservation laws impose

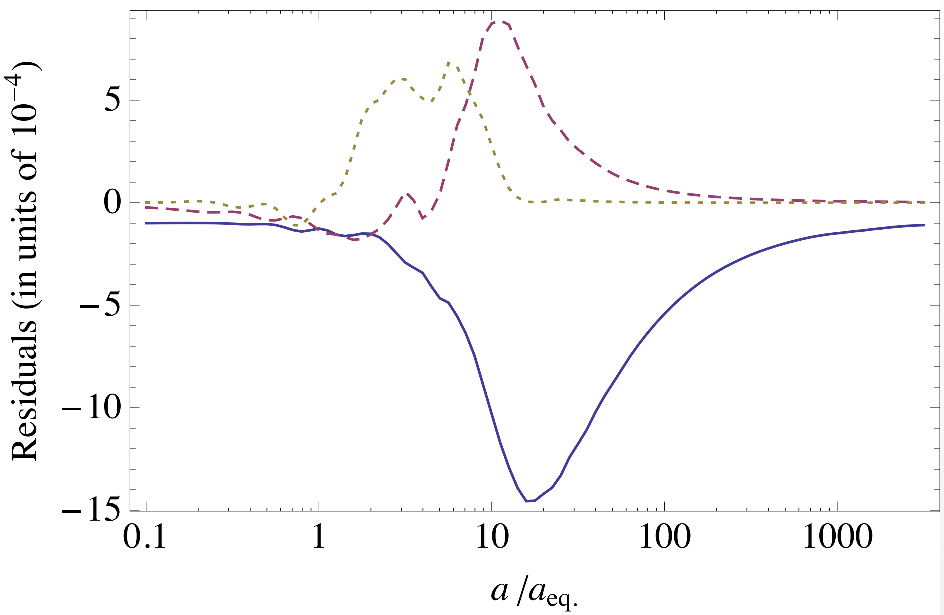
$$\boxed{P^\nu \partial_\nu P_i = \frac{1}{2} P^\sigma P^\nu \partial_i g_{\sigma\nu}}.$$

$(l_{\max} = 6, N_\mu = 12, N_q = N_\tau = 40, k = k_{eq} = 0.01h/\mathrm{Mpc}, m = 0.3\ \mathrm{eV})$.



Multipoles computed using **our approach**.
Solid line: energy density contrast.
Dashed line: velocity divergence.
Dotted line: shear stress

Relative differences (in units of 10^{-4}) in comparison with the Boltzmann approach



More details can be found in arXiv: 1311.5487 (H. Dupuy and F. Bernardeau).

USEFUL PROPERTIES ON SUBHORIZON SCALES

- In a perturbed Friedmann-Lemaître metric, the equations read:

$$\frac{\partial}{\partial \eta} n_c + \frac{\partial}{\partial x^i}\left(\frac{P^i}{P^0} n_c\right) = 0,$$

$$\frac{\partial P_i}{\partial \eta} + \frac{P^j}{P^0}\frac{\partial P_i}{\partial x^j} = a^2(\eta)\left[-P^0 \partial_i A + P^j \partial_i B_j + \frac{1}{2}\frac{P^j P^k}{P^0}\partial_i h_{jk}\right].$$

- In the **subhorizon** limit, they become:

$$\mathcal{D}_\eta n_c + \partial_i (V_i n_c) = 0,$$

initial momentum of the flow

$$\mathcal{D}_\eta P_i + V_j \partial_j P_i = \tau_0 \partial_i A + \tau_j \partial_i B_j - \frac{1}{2}\frac{\tau_j \tau_k}{\tau_0}\partial_i h_{jk},$$

with $\tau_0 = -\sqrt{m^2 a^2 + \tau_i^2}, \quad \mathcal{D}_\eta = \frac{\partial}{\partial \eta} - \frac{\tau_i}{\tau_0}\frac{\partial}{\partial x^i}$

and $V_i = -\frac{P_i - \tau_i}{\tau_0} + \frac{\tau_i}{\tau_0}\frac{\tau_j (P_j - \tau_j)}{(\tau_0)^2}.$ peculiar velocity

GENERALIZATION 1: NO CURL MODES IN THE MOMENTUM FIELD

- On **subhorizon scales** the curl field, defined as

$$\Omega_i = \epsilon_{ijk}\partial_k P_j ,$$

obeys the equation $\mathcal{D}_\eta \Omega_k + V_i \partial_i \Omega_k + \partial_i V_i \Omega_k - \partial_i V_k \Omega_i = 0.$

➡ The curl field is only sourced by itself.

➡ For adiabatic initial conditions, the **comoving momentum field can be written as a gradient**.

➡ As the velocity field of cold dark matter, it is **entirely characterized by its divergence**.

GENERALIZATION 2: COMPACT FORM OF THE EQUATIONS

- By analogy with cold dark matter, we introduce

$$\theta_{\tau_i}(\eta, x^i) = -\frac{P_{i,i}(\eta, x^i; \tau_i)}{ma\mathcal{H}}, \quad \delta_{\tau_i}(\eta, x^i) = \frac{n_c(\eta, x^i; \tau_i)}{n_c^{(0)}(\tau_i)} - 1.$$

- In reciprocal space, it gives

$$\left(a\partial_a - \mathrm{i}\frac{\mu k \tau}{\mathcal{H}\tau_0}\right)\delta_{\vec{\tau}}(\mathbf{k}) - \frac{ma}{\tau_0}\left(1 - \frac{\mu^2\tau^2}{\tau_0^2}\right)\theta_{\vec{\tau}}(\mathbf{k}) = \frac{ma}{\tau_0}\int \mathrm{d}^3\mathbf{k}_1 \mathrm{d}^3\mathbf{k}_2 \alpha_R(\mathbf{k}_1, \mathbf{k}_2; \vec{\tau})\delta_{\vec{\tau}}(\mathbf{k}_1)\theta_{\vec{\tau}}(\mathbf{k}_2),$$

$$\left(1 + a\frac{\partial_a \mathcal{H}}{\mathcal{H}} + a\partial_a - \mathrm{i}\frac{\mu k \tau}{\mathcal{H}\tau_0}\right)\theta_{\vec{\tau}}(\mathbf{k}) + \frac{k^2}{ma\mathcal{H}^2}\mathcal{S}_{\vec{\tau}}(\mathbf{k}) = \frac{ma}{\tau_0}\int \mathrm{d}^3\mathbf{k}_1 \mathrm{d}^3\mathbf{k}_2 \beta_R(\mathbf{k}_1, \mathbf{k}_2; \vec{\tau})\theta_{\vec{\tau}}(\mathbf{k}_1)\theta_{\vec{\tau}}(\mathbf{k}_2).$$

GENERALIZATION 2: COMPACT FORM OF THE EQUATIONS

- Considering *N* flows, it is useful to introduce the 2*N*-uplet

$$\Psi_a(\mathbf{k}) = (\delta_{\tau_1}(\mathbf{k}), \theta_{\tau_1}(\mathbf{k}), \ldots, \delta_{\tau_n}(\mathbf{k}), \theta_{\tau_n}(\mathbf{k}))^T.$$

- The resulting equations is

$$\boxed{a\frac{\partial \Psi_a}{\partial a}(\mathbf{k}, \eta) + \Omega_a{}^b(\mathbf{k}, \eta)\,\Psi_b(\mathbf{k}, \eta) = \gamma_a{}^{bc}(\mathbf{k}_1, \mathbf{k}_2, \eta)\Psi_b(\mathbf{k}_1, \eta)\Psi_c(\mathbf{k}_2, \eta).}$$

➡ The relativistic equation of motion is **formally the same as the equation describing cold dark matter**.

- This study is presented in arXiv: 1411.0428 (H. Dupuy and F. Bernardeau).

CONCLUSIONS AND PERSPECTIVES

- We proposed a new approach to study massive neutrinos beyond the linear regime.
- Principle of the method: describing neutrinos as a collection of flows.
- How could we make the explicit computation of the nonlinear matter power spectrum feasible?
- What are the scales at which both nonlinearities and relativistic effects are relevant?
- What is the number of flows necessary for the method to be satisfactory with a given precision?
- What is the most efficient way of discretizing momenta?