Nonequilibrium dynamics of the chiral phase transition - a MC approach to discrete particle-field interactions

Wesp et al, arXi,v:1411.7979

by **C. Greiner**
in collaboration with A. Meistrenko, C. Wesp, and H. van Hees

Keystone, WWND 2015

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

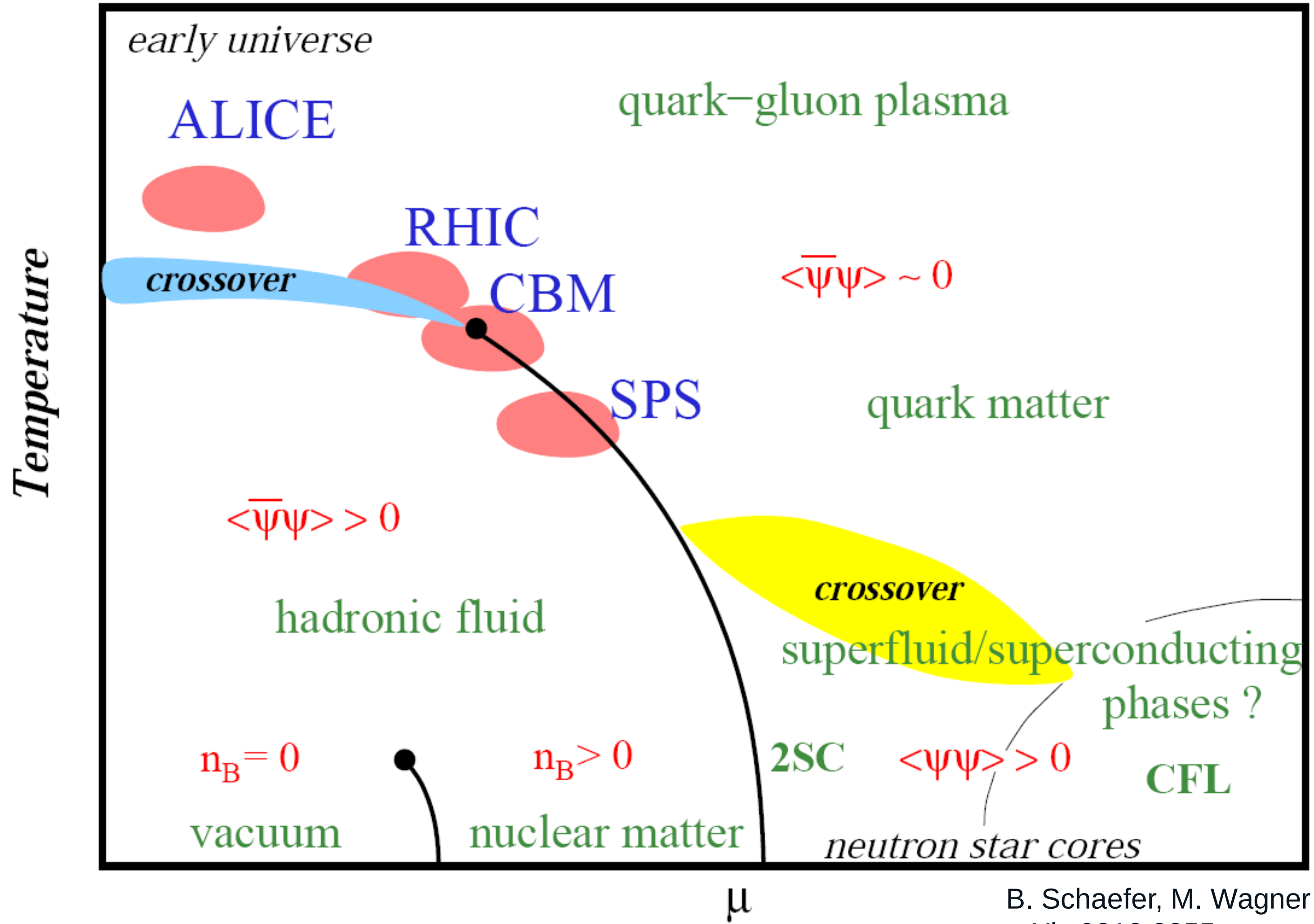
Deutsche Forschungsgemeinschaft DFG

H-QM Helmholtz Research School Quark Matter Studies

GEFÖRDERT VOM Bundesministerium für Bildung und Forschung

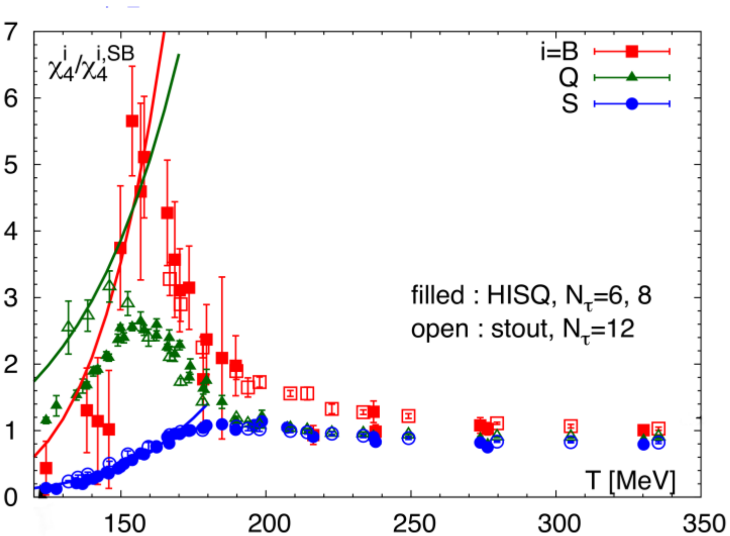
HIC for FAIR Helmholtz International Center

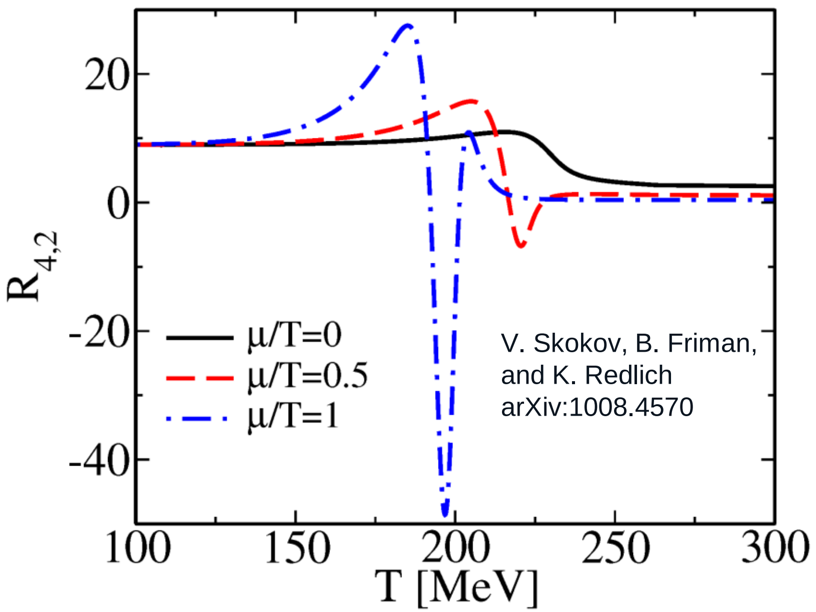
Motivation



B. Schaefer, M. Wagner
arXiv:0812.2855

Motivation





$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n}\left(p/T^4\right)}{\partial\left(\mu_B/T\right)^l \partial\left(\mu_s/T\right)^m \partial\left(\mu_Q/T\right)^n}$$

$$R_{4,2} = \frac{\chi_4^B}{\chi_2^B} = \frac{\langle\delta N^4\rangle}{\langle\delta N^2\rangle} - 3\langle\delta N^2\rangle$$

Model: Lagrangian

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ - g\left(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5\right)\right]\psi - \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U\left(\sigma,\vec{\pi}\right)$$

Yukawa coupling

O(N) theory / Φ^4 coupling

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - f_\pi m_\pi^2\sigma + U_0$$

parameters:

λ	$= 20$	self coupling
g	$= 3...6$	quark-sigma coupling
ν^2	$= f_\pi^2 - m_\pi^2/\lambda$	field shift term
f_π	$= 93\,MeV$	pion decay constant
m_π	$= 138\,MeV$	pion mass
U_0	$= m_\pi^4/(4\lambda) - f_\pi^2 m_\pi^2$	ground state

Mean-field + Vlasov + Interaction

$3D + 1$ grid for the σ-field (similar for $\vec{\pi}$):

$$\partial_\mu \partial^\mu \sigma + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2\right) \sigma - f_\pi m_\pi^2 + g \boxed{\langle \bar{\psi} \psi \rangle} = I \left(\sigma \leftrightarrow \psi \psi\right)$$

Quarks as test particles:

$$\left[\partial_t + \frac{p}{E_\psi \left(t, \vec{x}, \vec{p}\right)} \cdot \nabla_{\vec{x}} - \boxed{\nabla_{\vec{x}} E_\psi \left(t, \vec{x}, \vec{p}\right)} \nabla_{\vec{p}}\right] f_\psi \left(t, \vec{x}, \vec{p}\right) = C \left(\psi \psi \rightarrow \psi \psi \,, \sigma \leftrightarrow \psi \psi\right)$$

$$\boxed{\langle \bar{\psi} \psi \rangle \left(t, \vec{x}\right) = g d_\psi \sigma \left(t, \vec{x}\right) \int \frac{d^3 p}{\left(2\pi\right)^3} \frac{f_\psi \left(t, \vec{x}, \vec{p}\right) + f_{\bar{\psi}} \left(t, \vec{x}, \vec{p}\right)}{\sqrt{\vec{p}^2 + M_\psi^2 \left(t, \vec{x}, \vec{p}\right)}}}$$

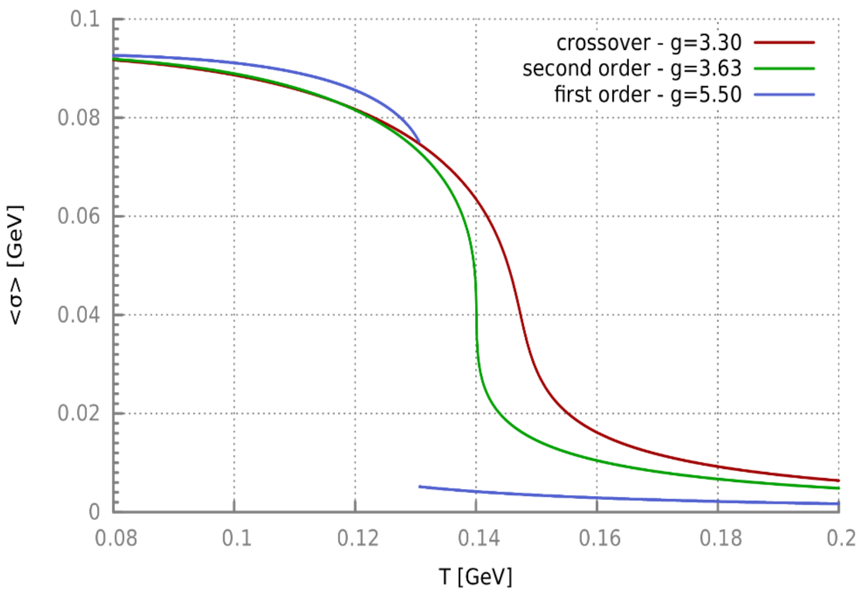
$$\boxed{M_\psi^2 \left(t, \vec{x}\right) = g^2 \left[\sigma^2 \left(t, \vec{x}\right) + \vec{\pi}^2 \left(t, \vec{x}\right)\right]}$$

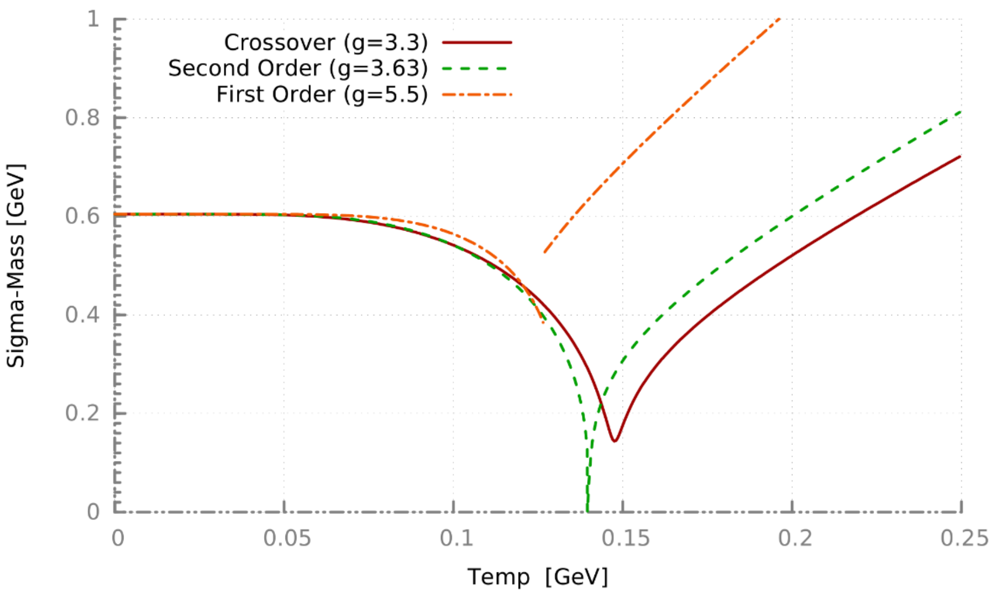
stochastic model for interaction rates between field and particles:

$$P \left(\Delta E \,, \Delta \vec{p} \,, \psi \psi \leftrightarrow \sigma\right)$$

Equilibrium Phase Diagram

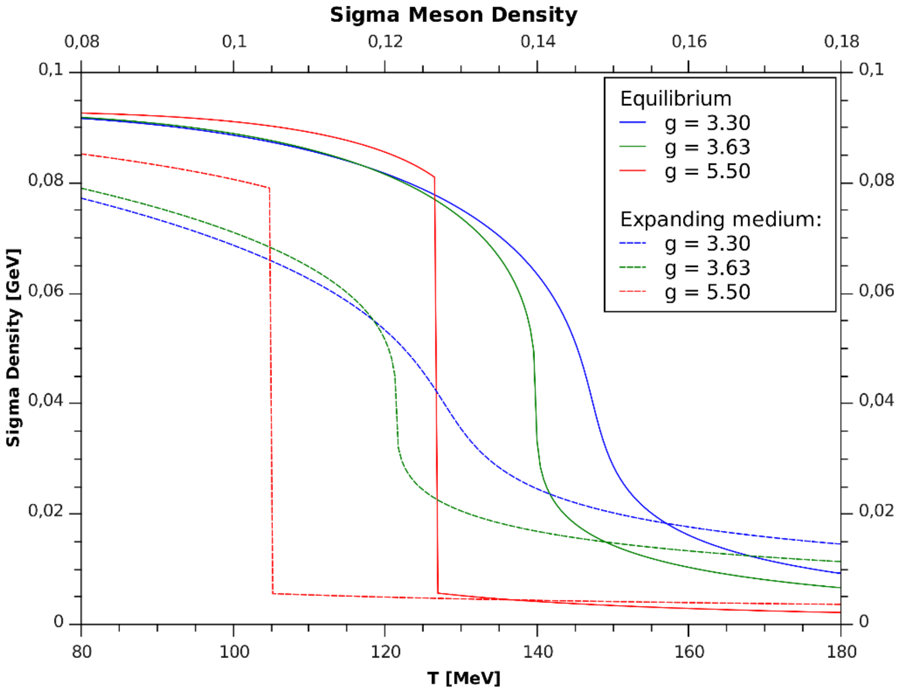
- Type of phase transition dependent on coupling strength





Problems: Non-Equilibrium Dynamics

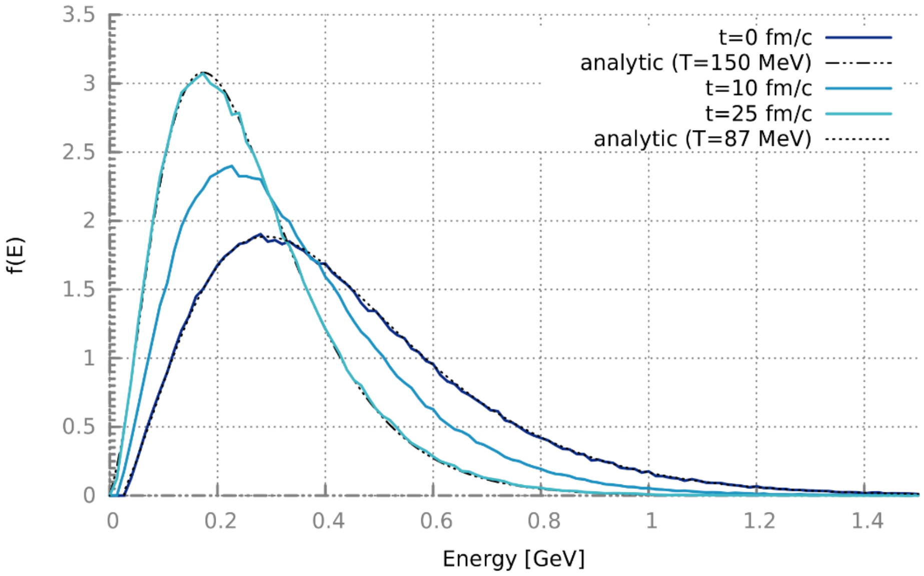
→ Without particle production,
the dynamics and the temperature of the phase transition is changed

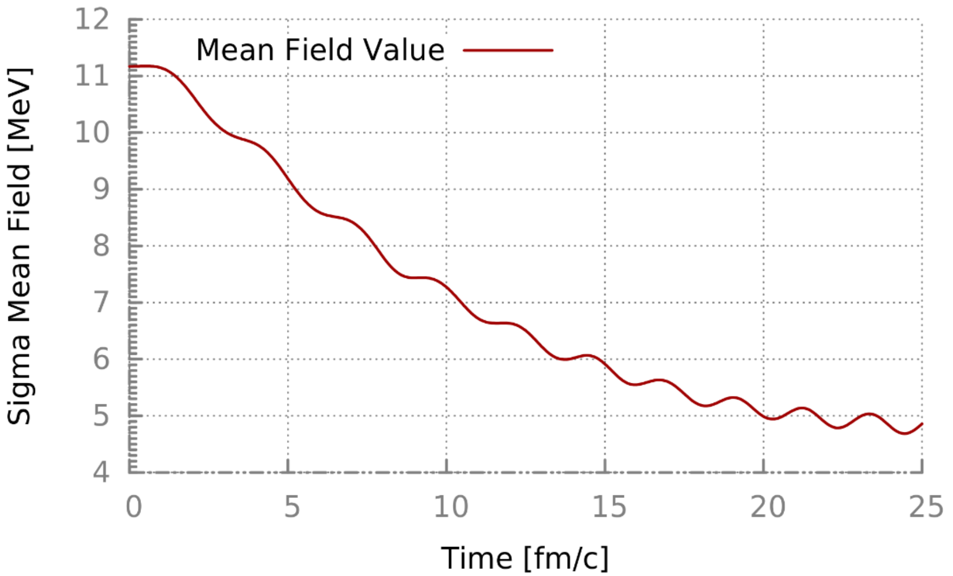


Phase transition for a slowly expanding box without particle production in comparison to full equilibrium

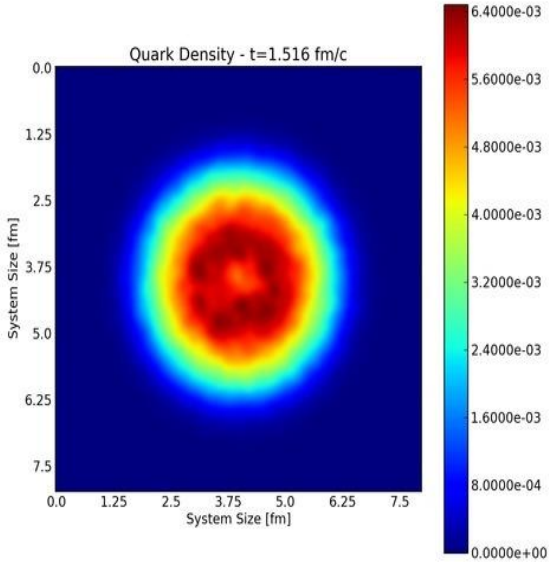
… Problems

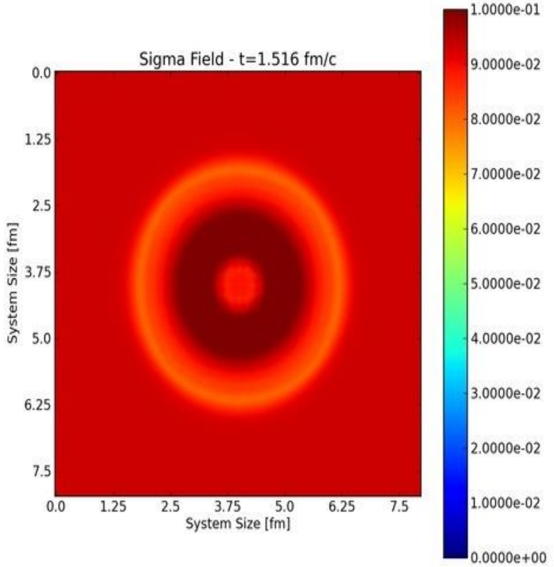
- Cooling the quarks in a thermal box does not lead to a phase transition
- Chemical processes and damping are missing!

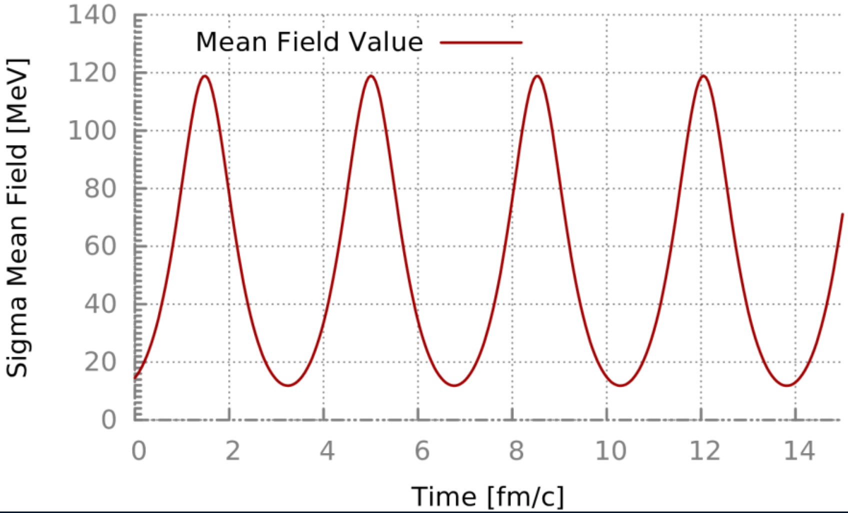




…Problems: Non-Equilibrium Oscillations



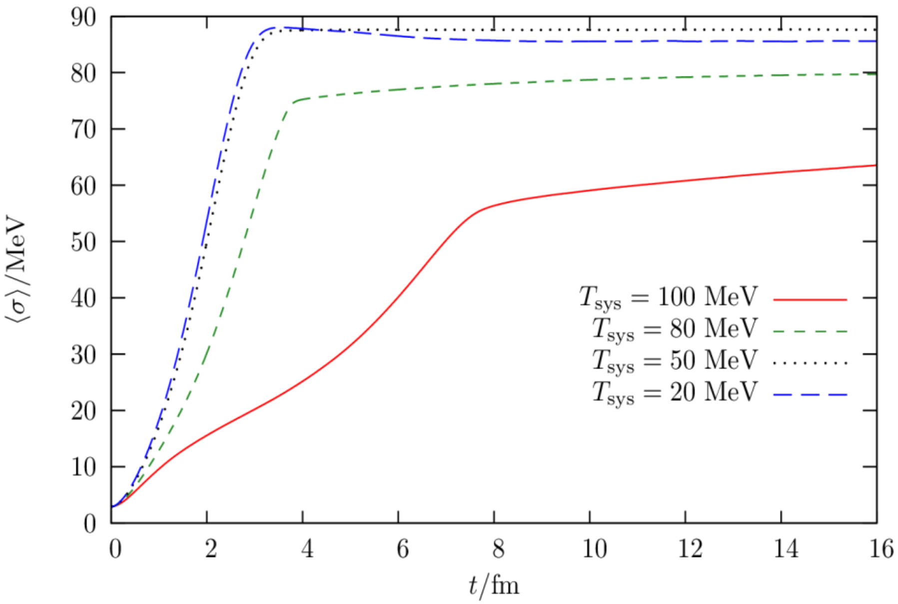




→ Rapid expansion of matter lead to fast oscillations in undamped fields

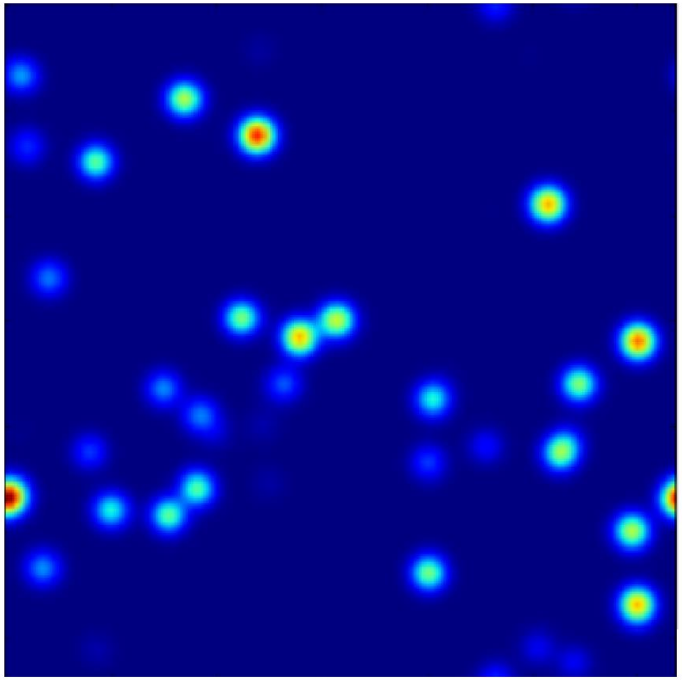
→ Global oscillations of the mean-field do not equilibrate

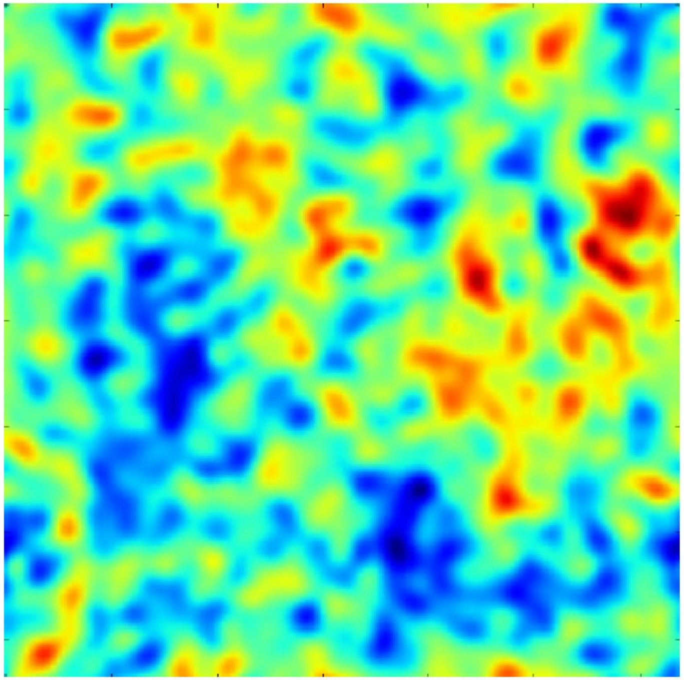
Approach to Field-Equilibration



Chiral fields are coupled to hydro-bath via Langevin equation
(M. Nahrgang, Phys.Lett. B711 (2012) 109-116)

3D Field Dynamics





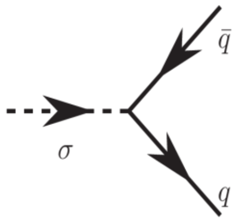
dynamic $\bar{\psi}\psi \leftrightarrow \sigma$ interaction

Noise is generated by particle-interactions, not by a noise-term!

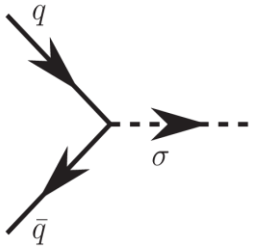
Implementation of Particle Production

- For thermal behavior, particle production and annihilation has to be implemented

- Mean-Field Damping:



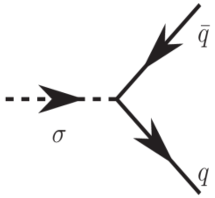
- Field Excitation:



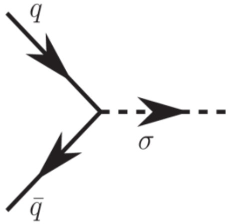
- Implemented with detailed balance
- Energy and momentum conservation at all times
- Interactions are discrete in time!

Dynamical Interactions between Particles and Fields

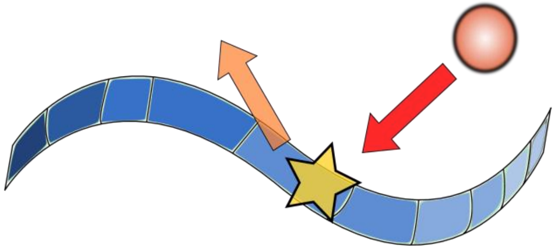
- Particle Annihilation and Creation is a discrete process
- Field damping / dissipation by pair production



- Field excitation by pair annihilation



- Energy and momentum conservation at all times



$$P\left(\Delta E, \Delta \boldsymbol{P}, \Delta t\right)$$

Challenge: Discrete Interactions between Particles and Fields

- Energy and momentum of a field:

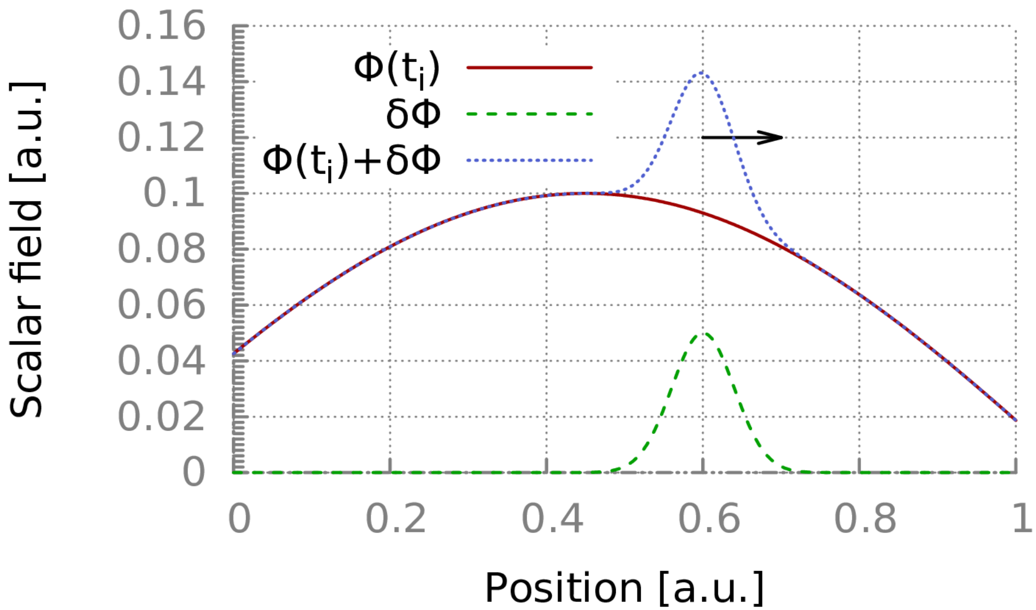
$$E = \int_V \mathrm{d}^3x \, \epsilon(\boldsymbol{x})$$
$$= \int_V \mathrm{d}^3x \, \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\left(\vec{\nabla}\phi\right)^2 + U(\phi)\right],$$
$$\boldsymbol{P} = \int_V \mathrm{d}^3x \, \boldsymbol{\Pi}(\boldsymbol{x}) = \int_V \mathrm{d}^3x \, \dot{\phi}\vec{\nabla}\phi,$$

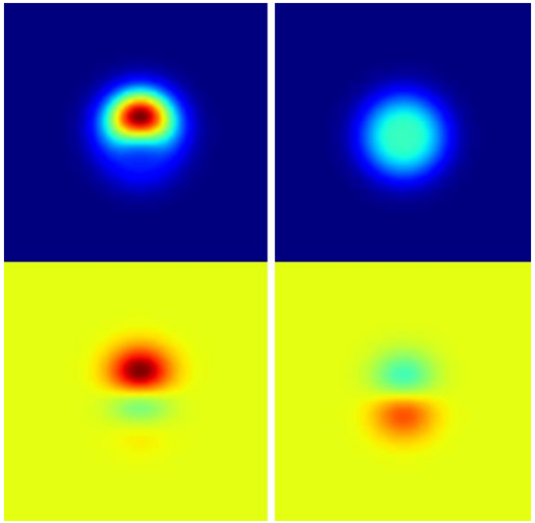
- Interaction by energy momentum exchange

$$\Delta E(t_k) = E\left[\phi(\boldsymbol{x}, t_k) + \delta\phi(\boldsymbol{x}, t_k)\right] - E\left[\phi(\boldsymbol{x}, t_k)\right],$$
$$\Delta \boldsymbol{P}(t_k) = \boldsymbol{P}\left[\phi(\boldsymbol{x}, t_k) + \delta\phi(\boldsymbol{x}, t_k)\right] - \boldsymbol{P}\left[\phi(\boldsymbol{x}, t_k)\right].$$

Discrete Interaction: local field excitations

Interactions lead to discrete field excitations



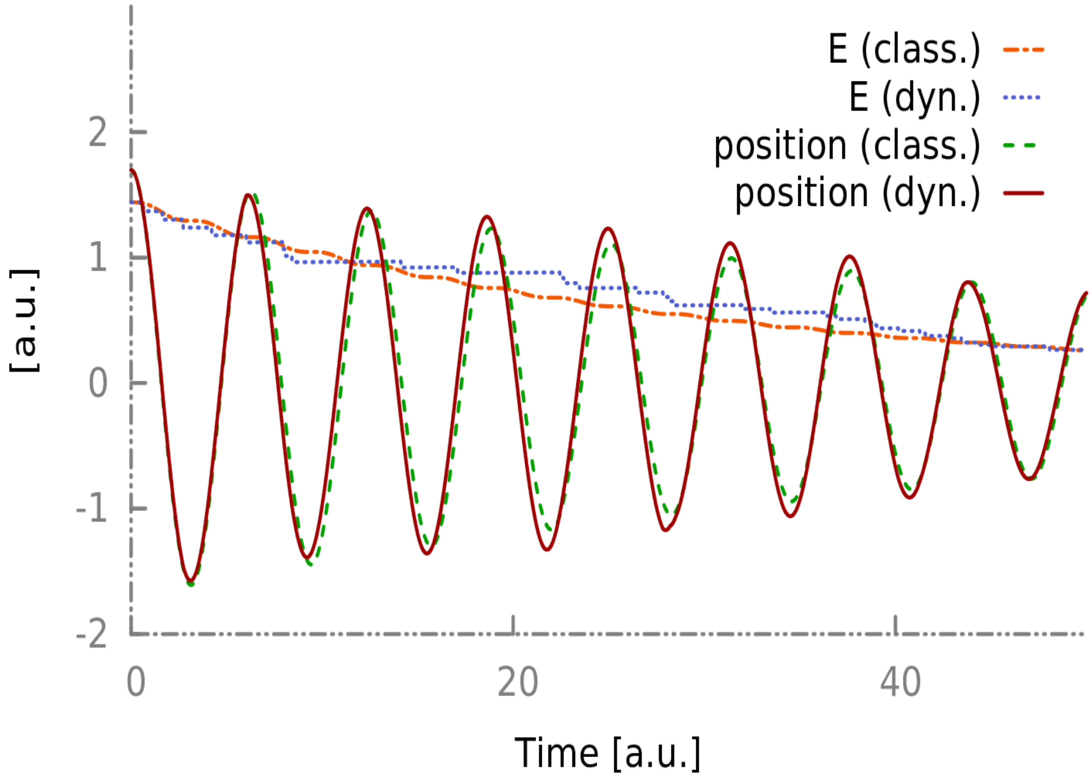


Interactions are given by probability distribution

$$P\left(\Delta E, \Delta \boldsymbol{P}, \Delta t\right)$$

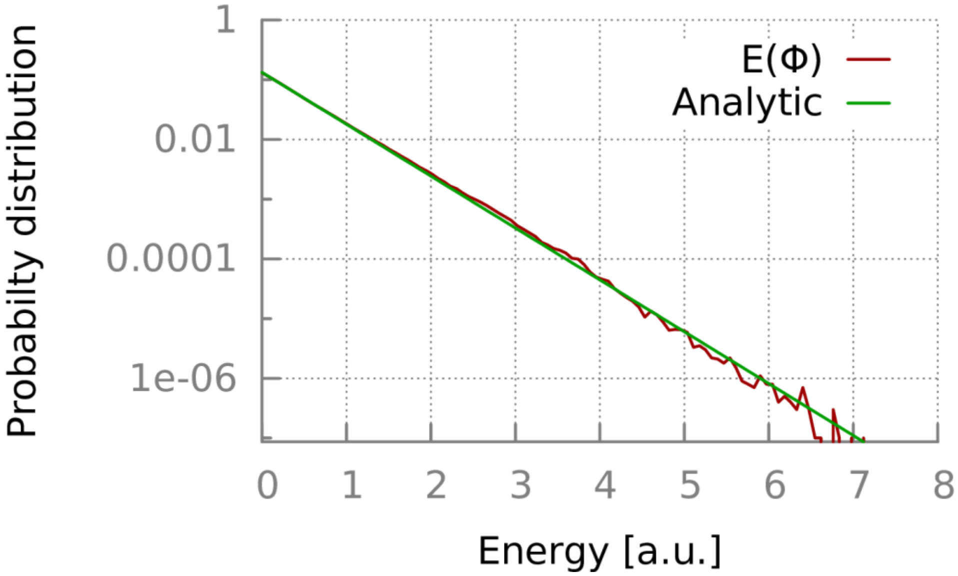
Scalar Damped Harmonic Oscillator

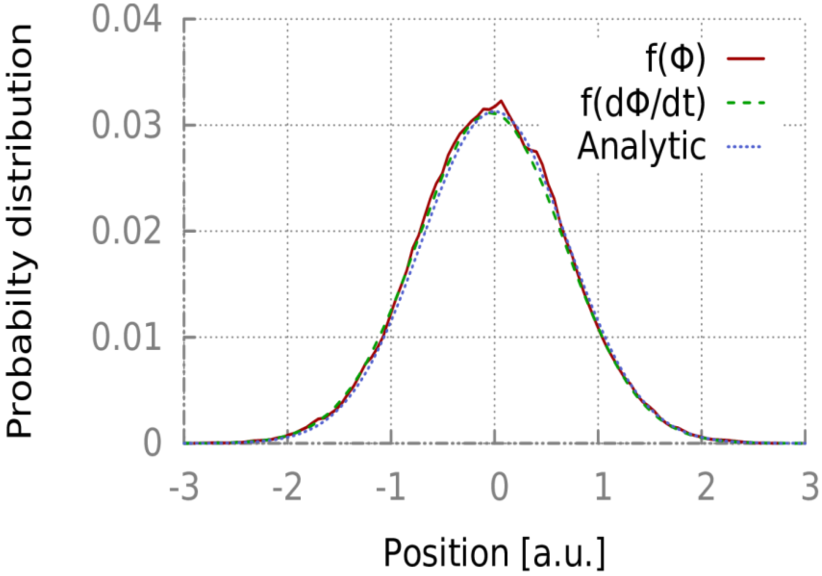
$$P\left(\Delta E, \Delta t\right) = \delta(\Delta E - \overline{\Delta E})\left(\gamma \cdot \Delta t \frac{E(t)}{\overline{\Delta E}}\right) + \delta(\Delta E)\left(1 - \gamma \cdot \Delta t \frac{E(t)}{\overline{\Delta E}}\right)$$



Scalar Langevin Equation with Heatbath

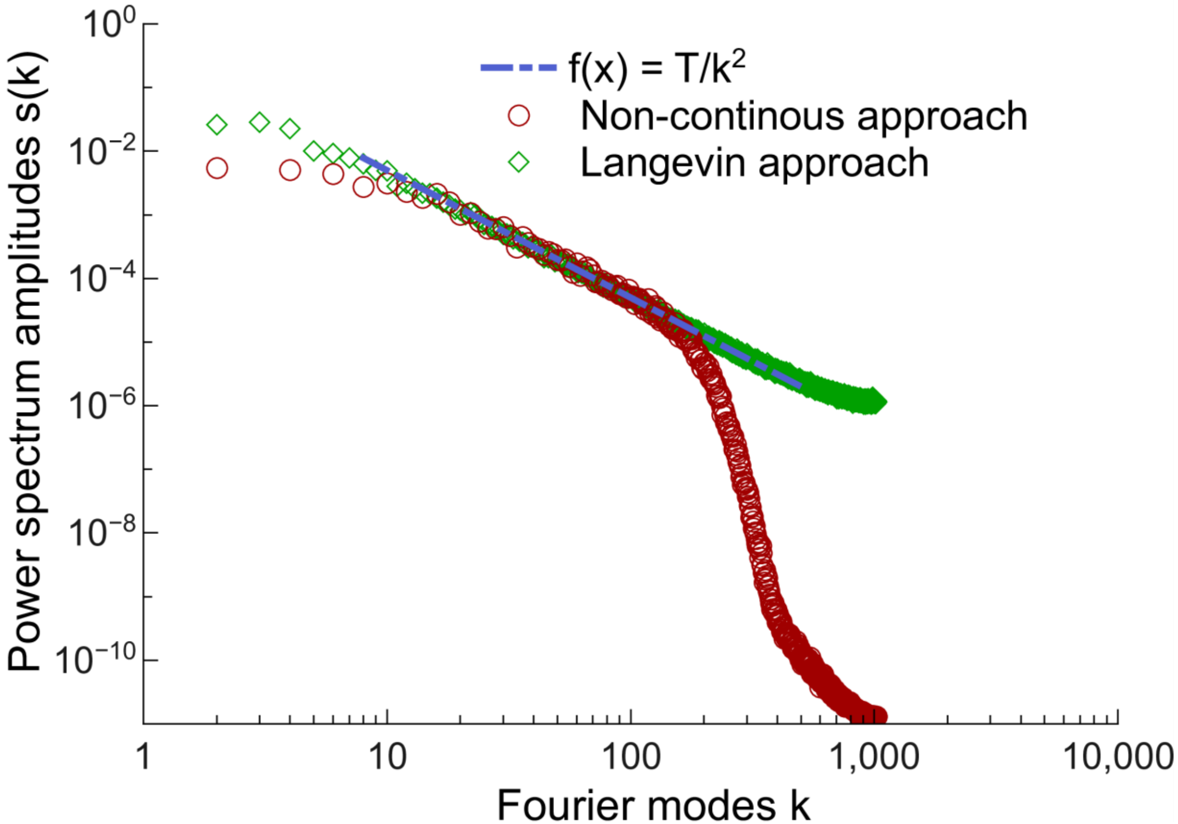
$$P(x, \Delta E, \Delta t) = \delta\left(\Delta E - \dot{\phi}(x,t) \cdot \Delta t \cdot \kappa \cdot \xi(x,t)\right)$$
$$+\delta(\Delta E - \overline{\Delta E})\left(\frac{\gamma \cdot \Delta t}{\overline{\Delta E}} E(x,t)\right) + \Pr_0 \ \delta(\Delta E)$$





1D Wave Equation a la Langevin

$$P(x, \Delta E, \Delta t) = \delta\left(\Delta E - \dot{\phi}(x,t) \cdot \Delta t \cdot \kappa \cdot \xi(x,t)\right)$$
$$+\delta(\Delta E - \overline{\Delta E}) \left(\frac{\gamma \cdot \Delta t}{\overline{\Delta E}} E(x,t)\right) + \underset{0}{\mathrm{Pr}}\ \delta(\Delta E)$$



Linear Sigma Model with quark interactions

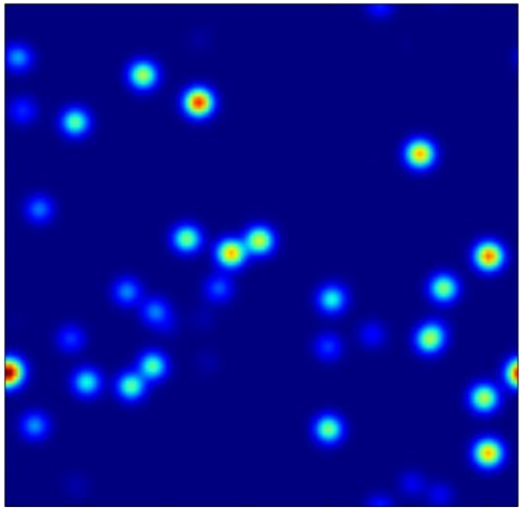
$$P\left(\Delta E, \Delta \boldsymbol{P}, \Delta t\right) =$$

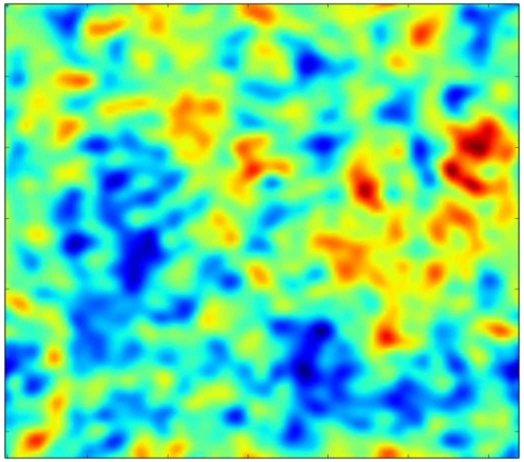
$$\sum_{i,j}^{N_{\text{cell}}} \delta\left(\Delta E - \sqrt{s}\right) \delta\left(\Delta \boldsymbol{P} - \left(\boldsymbol{p}_i + \boldsymbol{p}_j\right)\right) \frac{\hat{\sigma}_{\bar{q}q \to \sigma} v_{\text{rel}}(s) \Delta t}{\Delta V N_{\text{test}}}$$

$$+ \delta\left(\Delta E - E_\sigma\right) \delta\left(\Delta \boldsymbol{P} - \boldsymbol{P}_\sigma\right) \frac{\Gamma_\sigma(m_\sigma)\ n_\sigma(\phi(\boldsymbol{x}), t) \Delta t}{\Delta V}$$

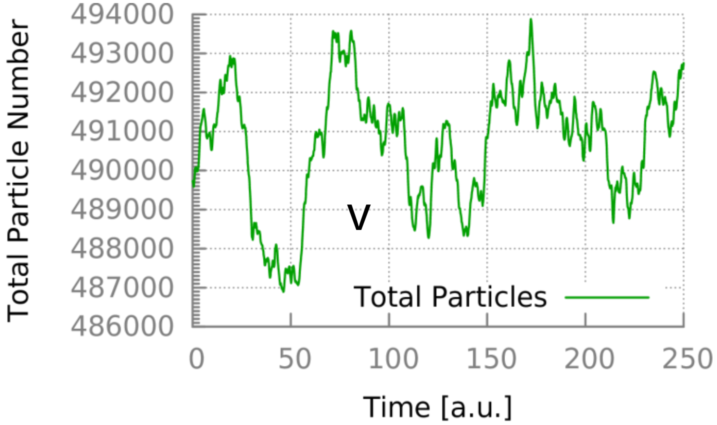
$$+ \Pr_0 \delta\left(\Delta E\right) \delta\left(\Delta \boldsymbol{P}\right)$$

- Interaction are modeled with microscopic cross-sections
- Dynamical creation of fluctuations
- Fields show thermal distributions
- Thermalization and Chemicalization between fields and particles

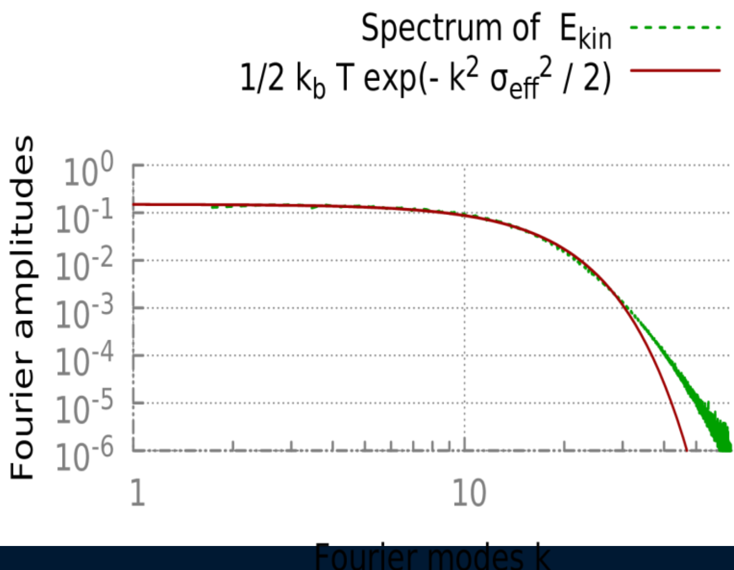


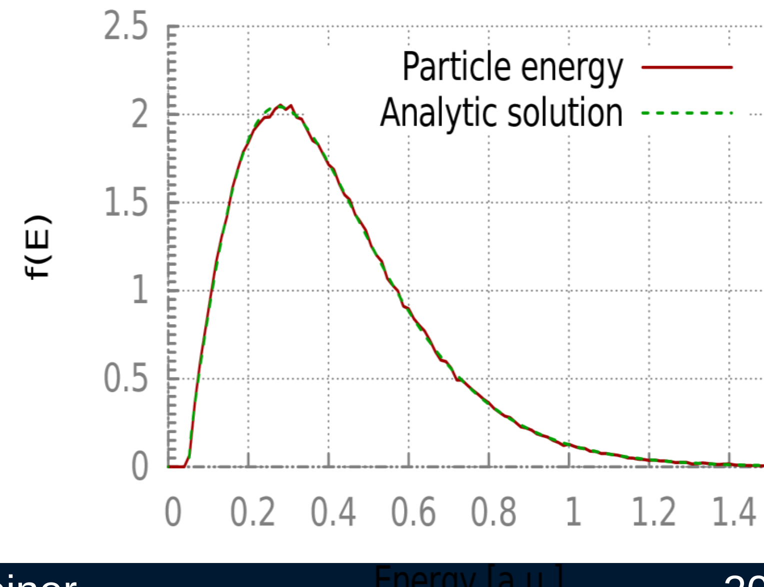


Linear Sigma Model with Quarks



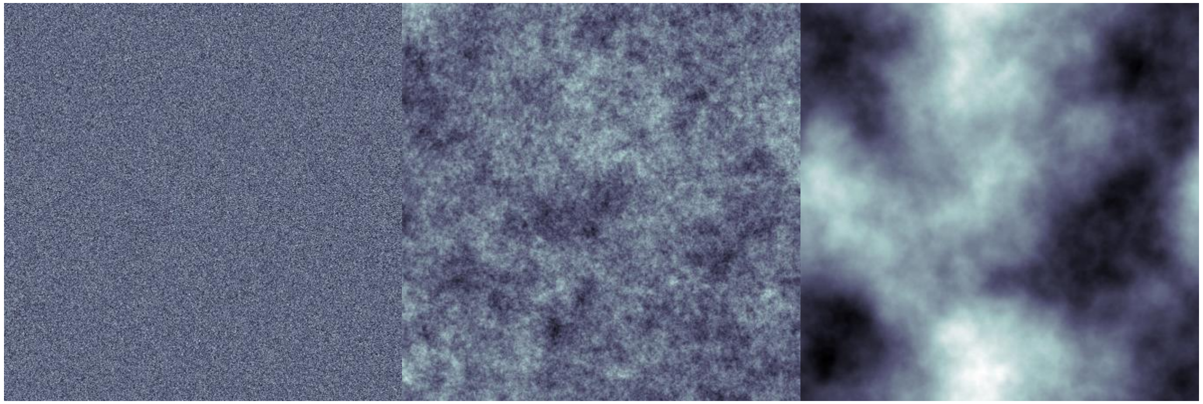
$$P\left(\Delta E, \Delta \boldsymbol{P}, \Delta t\right) =$$
$$\sum_{i,j}^{N_{\text{cell}}} \delta\left(\Delta E - \sqrt{s}\right) \delta\left(\Delta \boldsymbol{P} - (\boldsymbol{p}_i + \boldsymbol{p}_j)\right) \frac{\hat{\sigma}_{\bar{q}q\to\sigma} v_{\text{rel}}(s) \Delta t}{\Delta V N_{\text{test}}}$$
$$+ \delta\left(\Delta E - E_\sigma\right) \delta\left(\Delta \boldsymbol{P} - \boldsymbol{P}_\sigma\right) \frac{\Gamma_\sigma(m_\sigma)\; n_\sigma(\phi(\boldsymbol{x}), t) \Delta t}{\Delta V}$$
$$+ \Pr_0 \delta\left(\Delta E\right) \delta\left(\Delta \boldsymbol{P}\right)$$



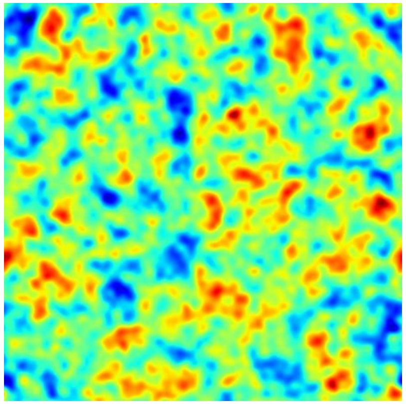


White / Colored Noise

$$S\left(\mathbf{k}\right) = \left|\mathcal{F}\left[\phi(\mathbf{x})\right](\mathbf{k})\right|^2 = \left|\int_{-\infty}^{\infty} \phi(\mathbf{k}) \cdot e^{-2\pi i \mathbf{x}\cdot\mathbf{k}}\ \mathrm{d}^3\mathbf{x}\right|^2$$



$\alpha = 0$ $\alpha = 1$ $\alpha = 2$



$$\frac{1}{2}\mathcal{F}\left|\dot{\phi}^2(\mathbf{x})\right|(\mathbf{k}) = \frac{k_B T}{2}\exp\left(-\frac{\mathbf{k}^2\sigma^2}{2}\right)$$

$$S(\mathbf{k}) \approx S_0 \cdot |\mathbf{k}|^{-\alpha}$$

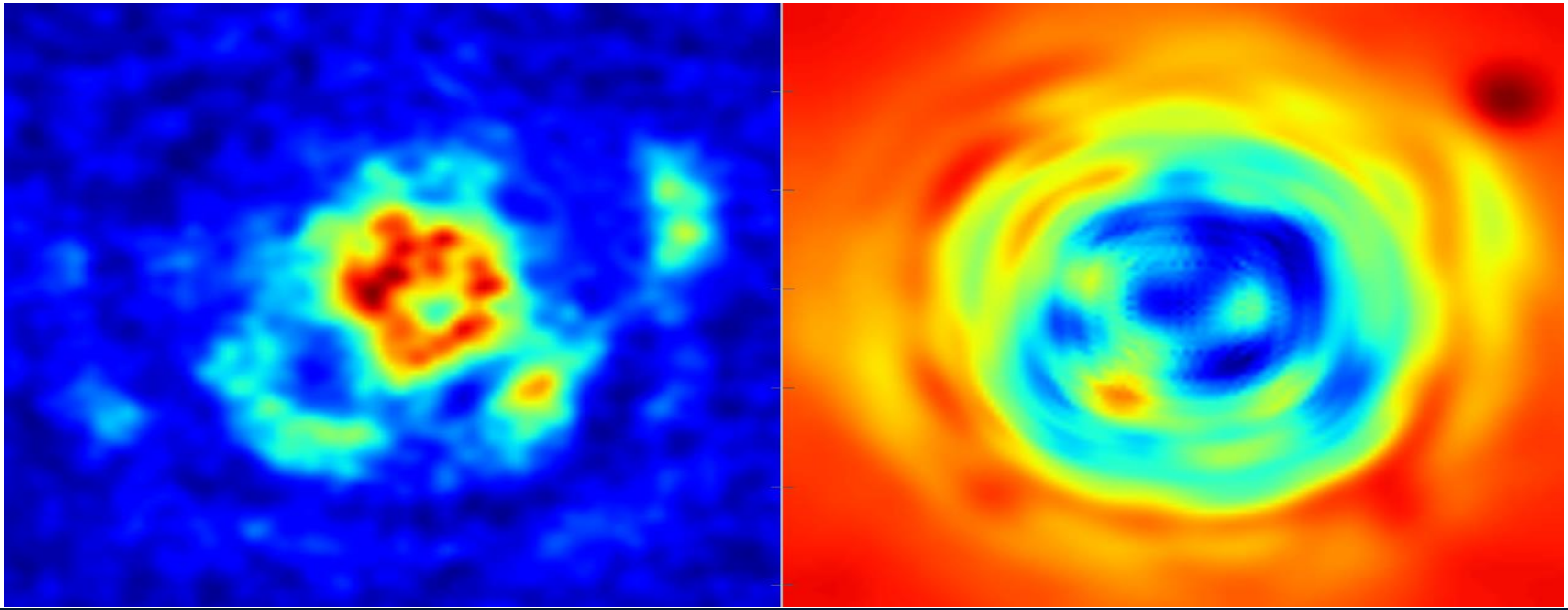
Outlook: Exploding Matter Droplets

$$\mathrm{Pr}\left(\bar{q}q \to \sigma\right) = \hat{\sigma}_{\bar{q}q\to\sigma}\; v_{\mathrm{rel}} \frac{\Delta t}{\Delta V N_{\mathrm{test}}}$$

$$\sigma_{\bar{q}q\to\sigma}(s) = \frac{\bar{\sigma}\;\Gamma^2}{\left(\sqrt{s}-m_\sigma\right)^2 + \left(\frac{1}{2}\Gamma\right)^2}$$

$$m_q = g^2\left(\sigma^2 + \pi^2\right)$$

$$\Gamma_\sigma = \frac{g^2}{8\pi m_\sigma}\sqrt{1-\frac{4m_q^2}{m_\sigma^2}}$$



Summary and outlook

- linear sigma model with Yukawa coupling to quarks
- 3D+1 non-equilibrium transport model:
  1. mean field + quarks with stochastic interaction
  2. mean field + mesons + quarks from 2PI

Outlook:

- dynamical simulation with collision terms
  - inelastic processes important
- study of net-quark number fluctuations

Thank you for your attention!

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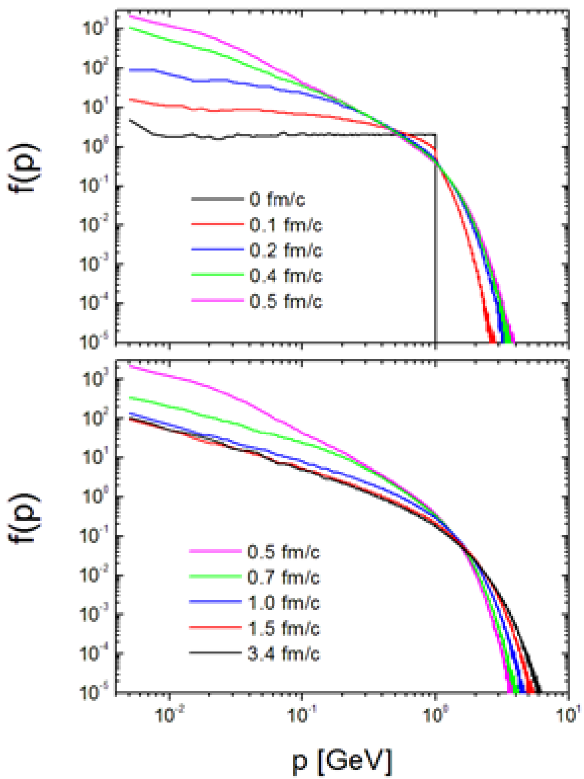
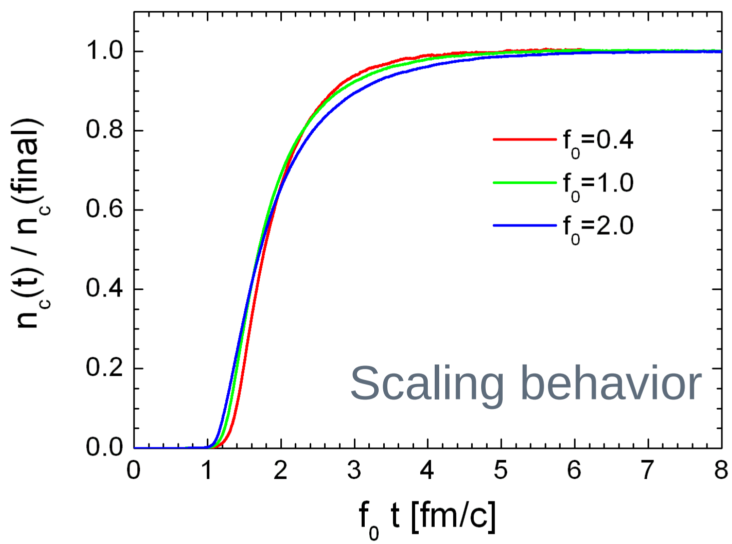
Gluon thermalization with Bose-Einstein condensation

Z. Xu et al, arXiv:1410.5616

From $f_{init}(\vec{p}) = f_0\theta(Q_s - p)$ to $f_{eq}(\vec{p}) = \frac{1}{e^{p/T}-1} + (2\pi)^3 n_c \delta^{(3)}(\vec{p})$



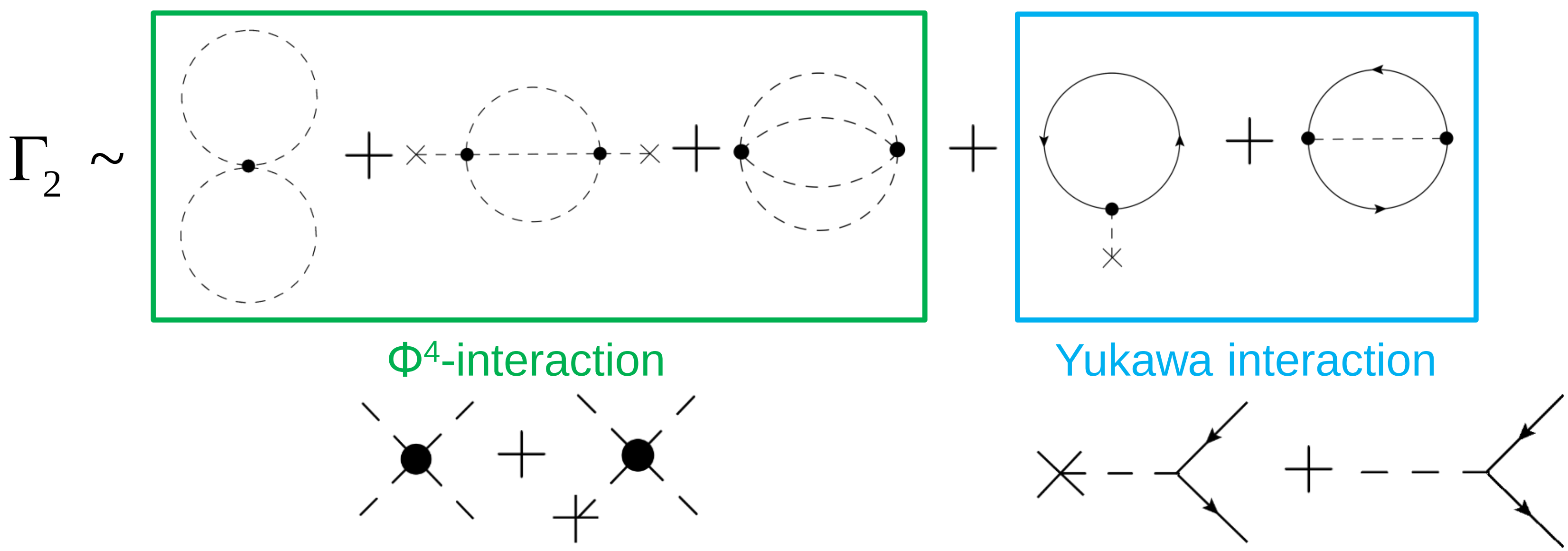


New (early) **phase**:
Emergence of
Bose-Einstein Condensation

2PI effective action

$$\Gamma[\phi,\psi,\bar{\psi},G,D] = S_{cl}[\phi] + \frac{i}{2}\cdot Tr\log G^{-1} + \frac{i}{2}\cdot TrG_0^{-1}G$$
$$- i\cdot Tr\log D^{-1} - i\cdot TrD_0^{-1}D + \Gamma_2[\phi,\psi,\bar{\psi},G,D]$$

relevant 2PI-diagrams (3 loops):

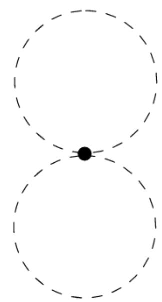


2PI effective action – effective mass

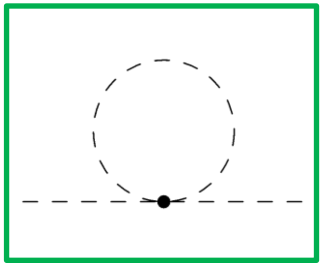
local part of the self-energy from Hartree approximation:

$$iG^{-1}_{\phi\phi} = iG^{-1}_{0,\phi\phi} - i\Pi_{\phi\phi}\,, \qquad \Pi_{\phi\phi} = 2i\frac{\delta\Gamma_2}{\delta G}$$

$$\Gamma \sim \int_{\mathcal{C}} d^4x\, G_{\phi_i\phi_i} G_{\phi_j\phi_j} \sim$$



$\longrightarrow$



effective mass from the gap equation of the propagator: $iG^{-1}_{\phi\phi} = k^2 - M^2_\phi$

$$M^2_\sigma(x) = \lambda\left(3\sigma^2 + 3\pi^2 - \nu^2\right) + \boxed{3\lambda G_{\sigma\sigma} + 3\lambda G_{\pi\pi}}$$
$$M^2_\pi(x) = \lambda\left(\sigma^2 + 5\pi^2 - \nu^2\right) + \boxed{\lambda G_{\sigma\sigma} + 5\lambda G_{\pi\pi}}$$

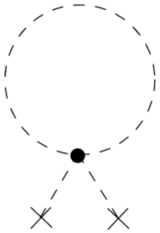
Mean-field EoM

$3D + 1$ simulation:

$$\partial_\mu \partial^\mu \sigma + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + 3G_{\sigma\sigma} + 3G_{\pi\pi} \right) \sigma - f_\pi m_\pi^2 + g \left\langle \bar{\psi} \psi \right\rangle = 0$$

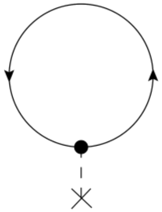
$$\partial_\mu \partial^\mu \vec{\pi} + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + G_{\sigma\sigma} + 5G_{\pi\pi} \right) \vec{\pi} + g \left\langle \bar{\psi} i \gamma_5 \vec{\tau} \psi \right\rangle = 0$$

$$G_{\phi\phi}\left(t, \vec{x}\right) = \frac{1}{2} \int \frac{d^3 p}{\left(2\pi\right)^2} \frac{1 + 2N_\phi\left(t, \vec{x}, \vec{p}\right)}{\sqrt{\vec{p}^2 + M_\phi^2\left(t, \vec{x}\right)}} \quad \text{with} \quad \phi \in \{\sigma, \pi\}$$



$$\left\langle \bar{\psi} \psi \right\rangle \left(t, \vec{x}\right) = g d_\psi \sigma\left(t, \vec{x}\right) \int \frac{d^3 p}{\left(2\pi\right)^3} \frac{f_\psi\left(t, \vec{x}, \vec{p}\right) + f_{\bar{\psi}}\left(t, \vec{x}, \vec{p}\right)}{\sqrt{\vec{p}^2 + M_\psi^2\left(t, \vec{x}, \vec{p}\right)}} ,$$

$$\left\langle \bar{\psi} i \gamma_5 \vec{\tau} \psi \right\rangle \left(t, \vec{x}\right) = g d_\psi \vec{\pi}\left(t, \vec{x}\right) \int \frac{d^3 p}{\left(2\pi\right)^3} \frac{f_\psi\left(t, \vec{x}, \vec{p}\right) + f_{\bar{\psi}}\left(t, \vec{x}, \vec{p}\right)}{\sqrt{\vec{p}^2 + M_\psi^2\left(t, \vec{x}, \vec{p}\right))}}$$

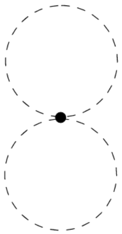


Vlasov-Boltzmann approach

self-consistent mass in the Vlasov equations of σ, $\vec{\pi}$ and ψ:

$$\left[\partial_t + \frac{p}{E_\phi\left(t,\vec{x},\vec{p}\right)} \cdot \nabla_{\vec{x}} - \nabla_{\vec{x}} E_\phi\left(t,\vec{x},\vec{p}\right) \nabla_{\vec{p}}\right] f_\phi\left(t,\vec{x},\vec{p}\right) = 0$$

$$M_\sigma^2\left(x\right) = \lambda\left(3\sigma^2 + 3\pi^2 - \nu^2\right) + 3\lambda G_{\sigma\sigma} + 3\lambda G_{\pi\pi}$$
$$M_\pi^2\left(x\right) = \lambda\left(\sigma^2 + 5\pi^2 - \nu^2\right) + \lambda G_{\sigma\sigma} + 5\lambda G_{\pi\pi}$$

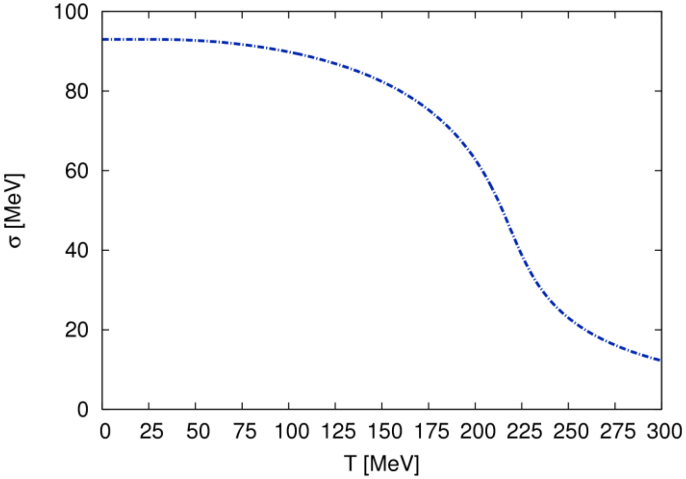


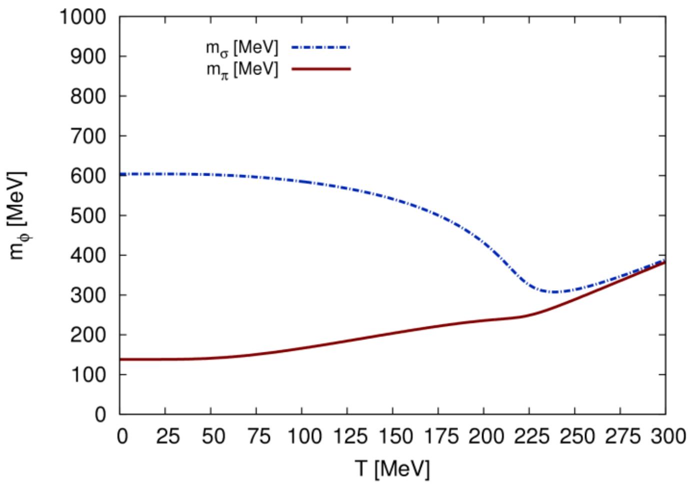
$$\left[\partial_t + \frac{p}{E_\psi\left(t,\vec{x},\vec{p}\right)} \cdot \nabla_{\vec{x}} - \nabla_{\vec{x}} E_\psi\left(t,\vec{x},\vec{p}\right) \nabla_{\vec{p}}\right] f_\psi\left(t,\vec{x},\vec{p}\right) = 0$$

$$M_\psi^2\left(t,\vec{x}\right) = g^2\left[\sigma^2\left(t,\vec{x}\right) + \vec{\pi}^2\left(t,\vec{x}\right)\right]$$

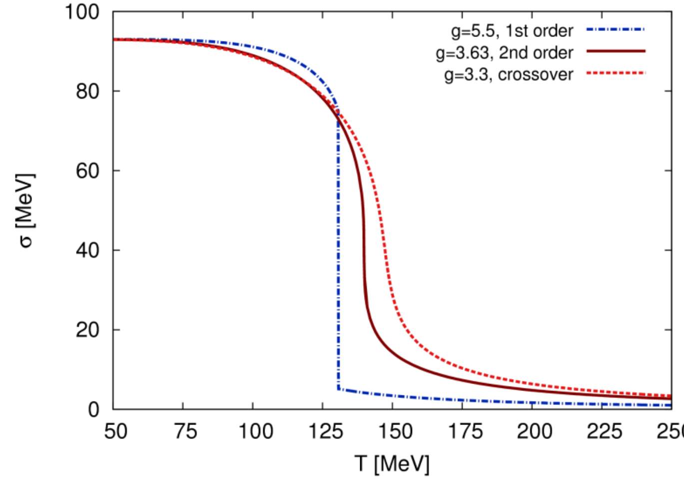
Equilibrium properties

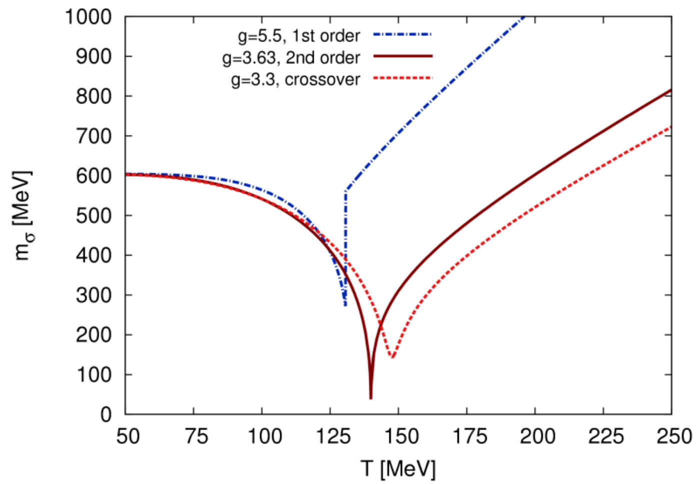
mean field + mesons





mean field + quarks





Dissipation kernel

Langevin equation (1D classical case):

$$m\ddot{x}\left(t\right) + 2\int_0^t dt\,\Gamma\left(t-t'\right)\dot{x}\left(t'\right) - F\left(x\right) = \xi\left(t\right)$$

Mean-field equation with a dissipation kernel:

$$\Gamma_2 \sim$$

$$\partial_\mu\partial^\mu\sigma + D\left(t,\vec{x}\right) + \lambda\left(\sigma^2 + \vec{\pi}^2 - \nu^2 + \frac{3}{2}G_{\sigma\sigma} + \frac{3}{2}G_{\pi\pi}\right)\sigma - f_\pi m_\pi^2 + g\left\langle\bar{\psi}\psi\right\rangle = 0$$

$$D\left(t,\vec{x}\right) \sim \int \frac{d^3k}{\left(2\pi\right)^3} e^{i\vec{k}\cdot\vec{x}} \int \frac{dk^0}{2\pi} \frac{\mathcal{M}\left(t,\vec{x},\vec{k}\right)}{E_k} \partial_t\sigma\left(t,\vec{k}\right)\pi\delta\left(E_k - k^0\right)$$

Vlasov-Boltzmann approach

Mean-field equation with a dissipation kernel:

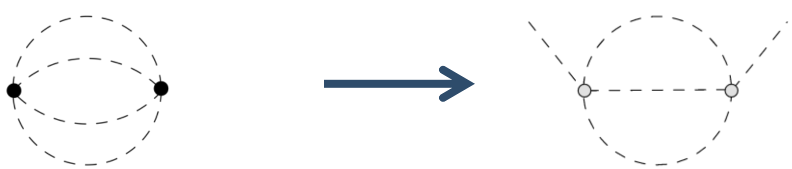
$\Gamma_2 \sim$

$$D\left(t,\vec{x}\right) \sim \int \frac{d^3k}{\left(2\pi\right)^3} e^{i\vec{k}\cdot\vec{x}} \int \frac{dk^0}{2\pi} \frac{\mathcal{M}\left(t,\vec{x},\vec{k}\right)}{E_k} \partial_t \sigma\left(t,\vec{k}\right) \pi\delta\left(E_k - k^0\right)$$

C. Greiner, B. Müller arXiv:hep-th/9605048

Vlasov-Boltzmann equation with $2 \to 2$ collision integral:

$\Gamma_2 \sim$



complexity of collision integrals: $\mathcal{O}\left(N^9\right) \quad \to \quad \mathcal{O}\left(N^6 - N^7\right)$ (with MC and symmetry)