Boltzmann dynamics, temperature dependence of energy loss and coalescence: Towards an understanding of the R_{AA} and v_2 for D-Mesons

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The 31[th] Winter Workshop on Nuclear Dynamics

Outline

- The puzzling Relation between R_{AA} and v_2 for Heavy Flavors
  - T-dependence of the interaction
  - Transport vs Fokker-Planck approach
  - Hadronization: Coalescence and Fragmentation
- $c\bar{c}$ angular correlation

Introduction Heavy Quarks

HQ are ideal probes to study the QGP

✓ $M_{HQ} >> \Lambda_{QDC}$ ($M_{charm} \cong 1.3$ GeV; $M_{bottom} \cong 4.2$ GeV)

✓ $M_{HQ} >> T$

Before the first experimental results at RHIC:

It was expected HQ not dragged by the expanding medium:

- *spectra close to the pp one-> large R_{AA}*

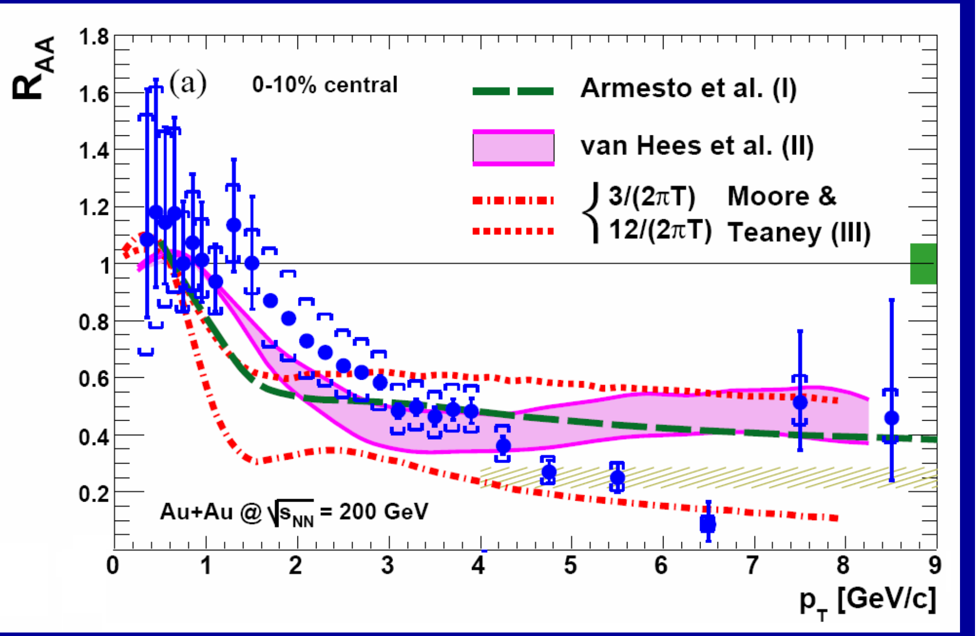
$$R_{AA}(p_T) = \frac{1}{N_{coll}} \frac{d^2N^{AA}/dp_T dy}{d^2N^{pp}/dp_T dy}$$

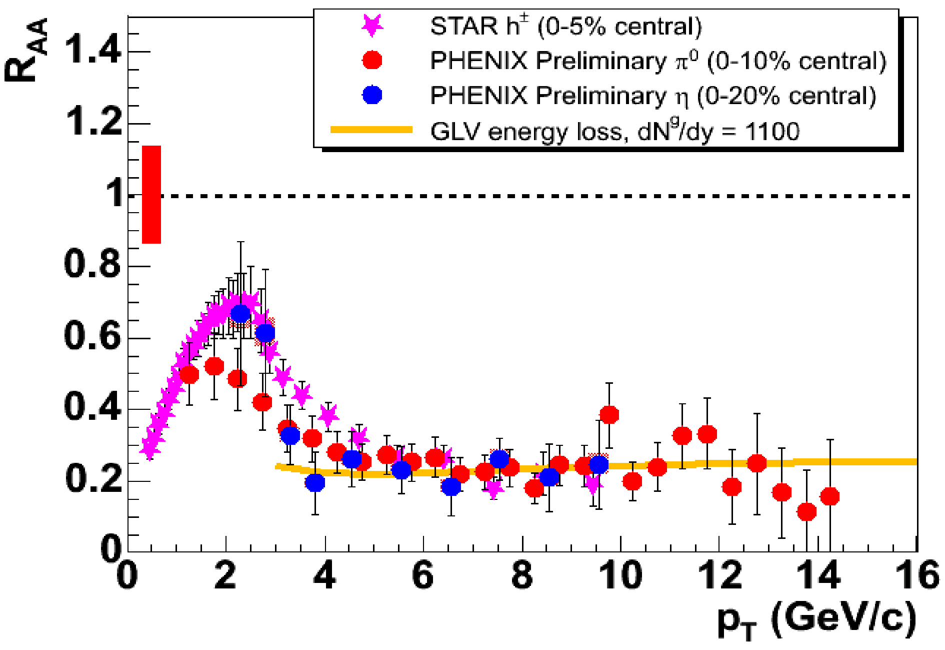
- *small elliptic flow v_2*

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

R_{AA} of Heavy Quarks

RHIC

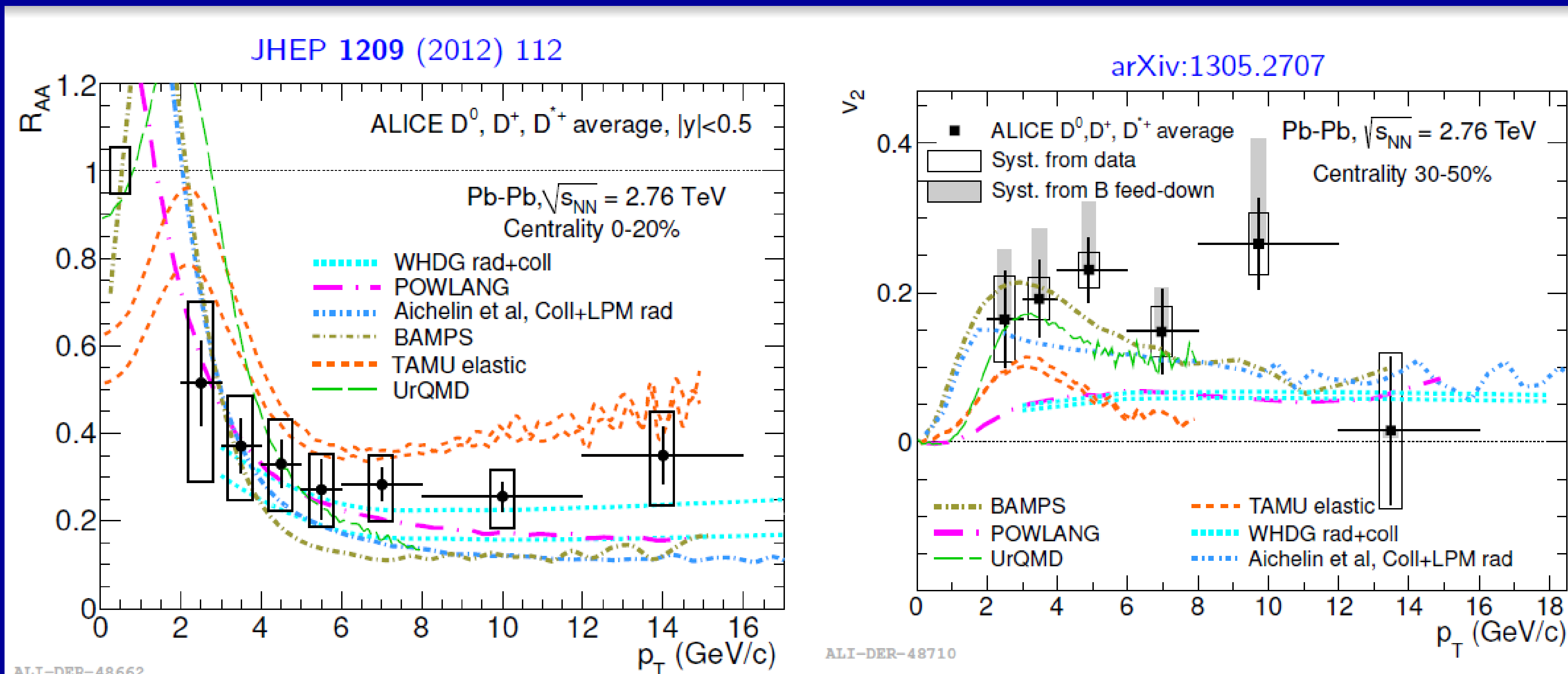




[PHENIX: *PRL98(2007)172301]*

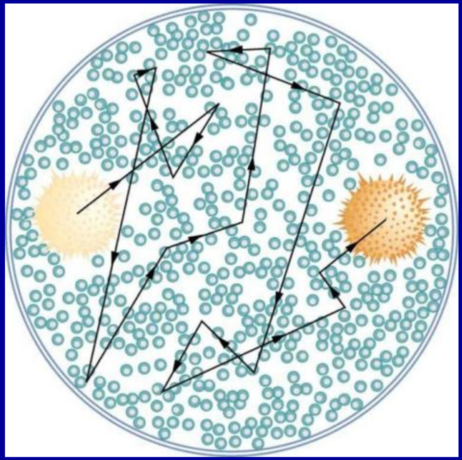
Small R_{AA} not so different from that of light flavor

R_{AA} and V_2



Again at LHC energy heavy flavor suppression is similar to light flavor: small R_{AA}, large v_2

Standard Description of HQ propagation in the QGP



Fokker-Plank approach

$$\frac{\partial f_{c,b}}{\partial t} = \gamma \frac{\partial (p f_{c,b})}{\partial p} + D \frac{\partial^2 f_{c,b}}{\partial p^2}$$

The interaction is encoded in the drag and diffusion coefficents

$$\gamma p = \int d^3k \left| M(k,p) \right|^2 p$$

$$D = \frac{1}{2} \int d^3k \left| M(k,p) \right|^2 p^2$$

Evaluated from scattering matrix $|M|^2$

drag evaluated from pqcd -> R_{AA} larger than exp. data
-> v_2 smaller than exp. data

Heavy Quark strongly dragged by interaction with light quarks, the real cross section is a K factor larger?

R_{AA} and v_2 correlation

The larger k the smaller the R_{AA} ,the larger the v_2

It is possible to reproduce R_{AA} multiplying the drag by a k-factor
It is not possible to reproduce both R_{AA} and v_2 with the same k-factor

Reproducing both is not only an issue of the strength of the interaction

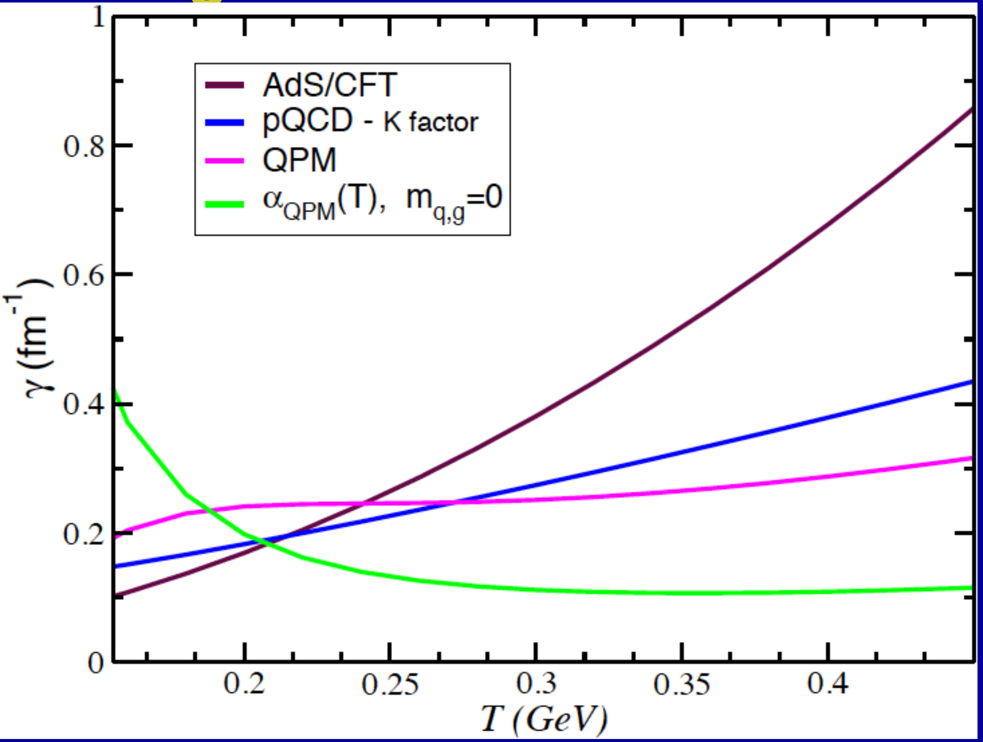
The temperature dependence of the interaction plays a rule

This is general, seen also for light quarks

[Scardina, Di Toro, Greco, PRC82(2010)]
[J.Liao and E. Shuryak PRL 102 (2009)]

T- dependence of the Drag Coefficient

Drag Coefficient



pQCD *(Combridge cross-section)*

$$a_{pQCD} = \frac{4p}{11 ln\left(2pTL^{-1}\right)} \quad , \quad m_D^2 = 4pa_{pQCD}(T)\,T$$

AdS/CFT

$$g_{AdS/CFT} = k\frac{T^2}{M}$$

[Akamatsu-Hatsuda-Hirnao, PRC79 (09) 054907]
[S. K. Das PRC89 (2014) 054912]

$a_{QPM}(T)$, $m_{q,g}=0$

we mean simply the coupling of the QPM, but with a bulk of massless q and g

Quasi-Particle-Model *(fit to lQCD e,P)*

$$g^2_{QP}(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

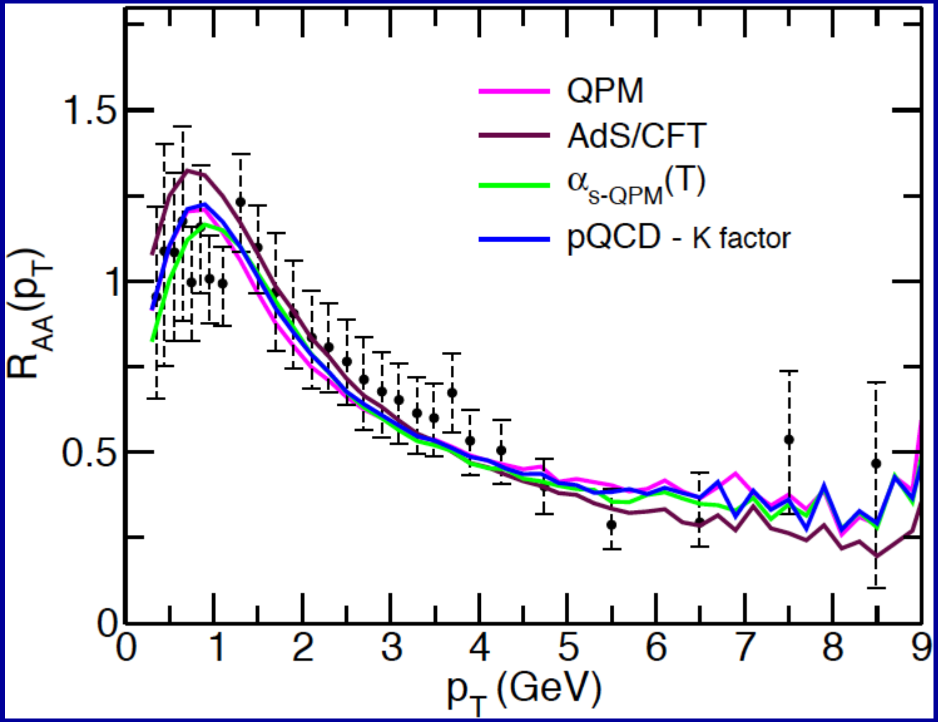
$$m_g^2 = \frac{1}{6}\left(N_c + \frac{1}{2}N_f\right)g^2T^2 \qquad m_q^2 = \frac{N_c^2 - 1}{8N_c}g^2T^2($$

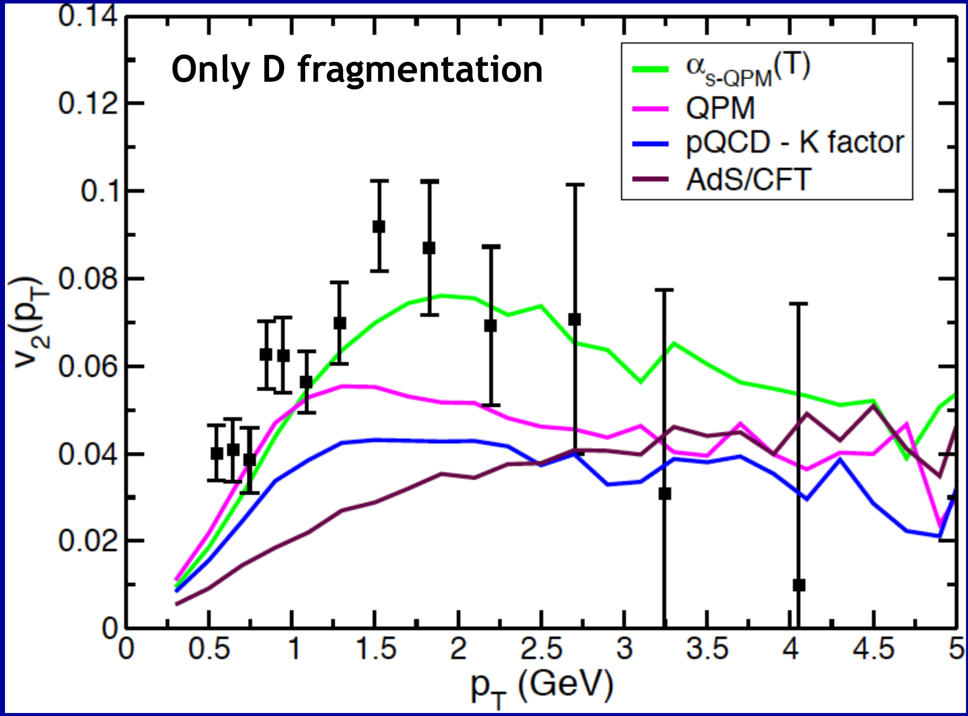
[S.Plumari et al PRD 84 094004 (2011)]

Impact of T-dependence of the Drag

Au+Au@200AGeV, b=8 fm

Interaction rescaled to have very similar R_{AA} for all the cases

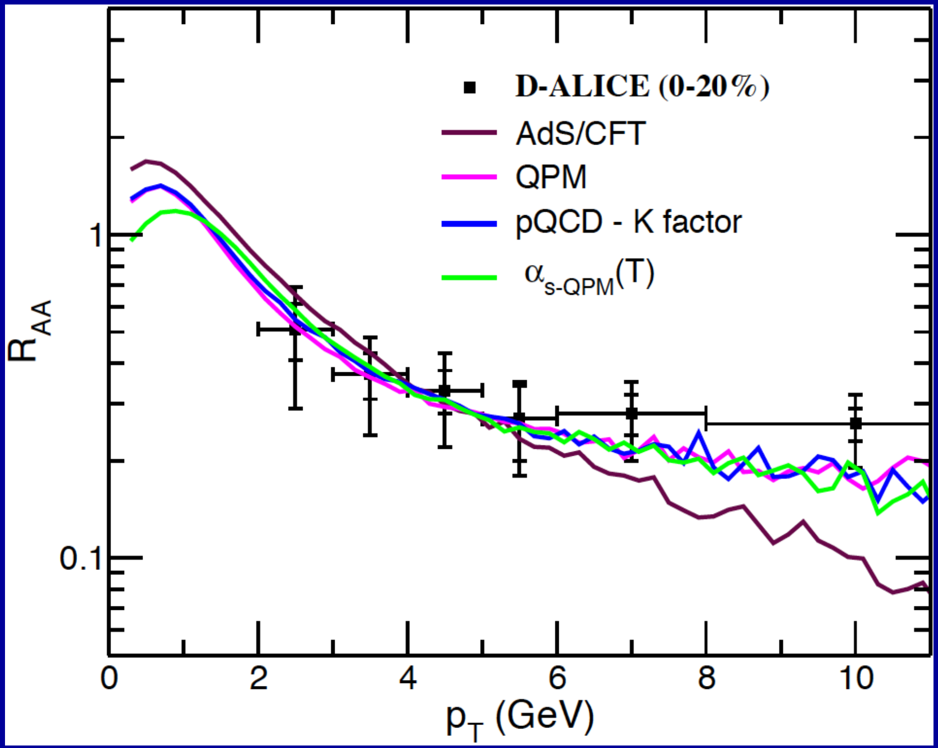


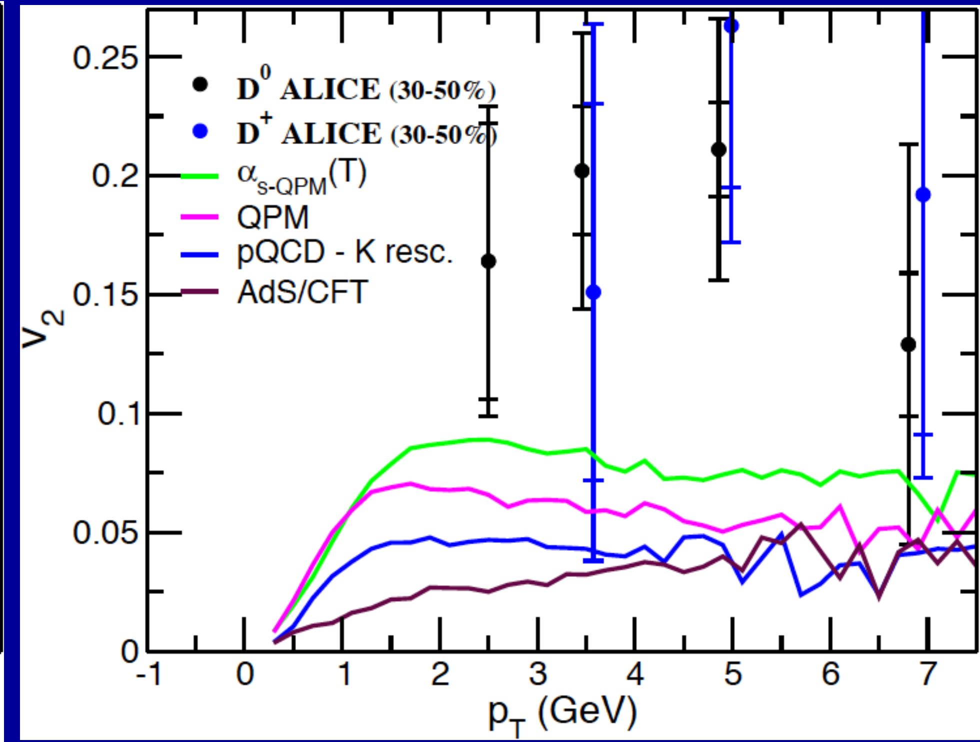


- $R_{AA}(p_T)$ well reproduced whatever is the T-dependence
- At fixed $R_{AA}(p_T)$ -> $v_2(p_T)$ quite larger if T -> T_c

Impact of T-dependence of the Drag

LHC - Pb+Pb@2.76ATeV

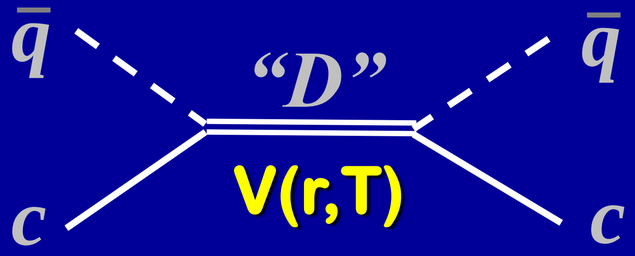




- Similar trends as for RHIC case

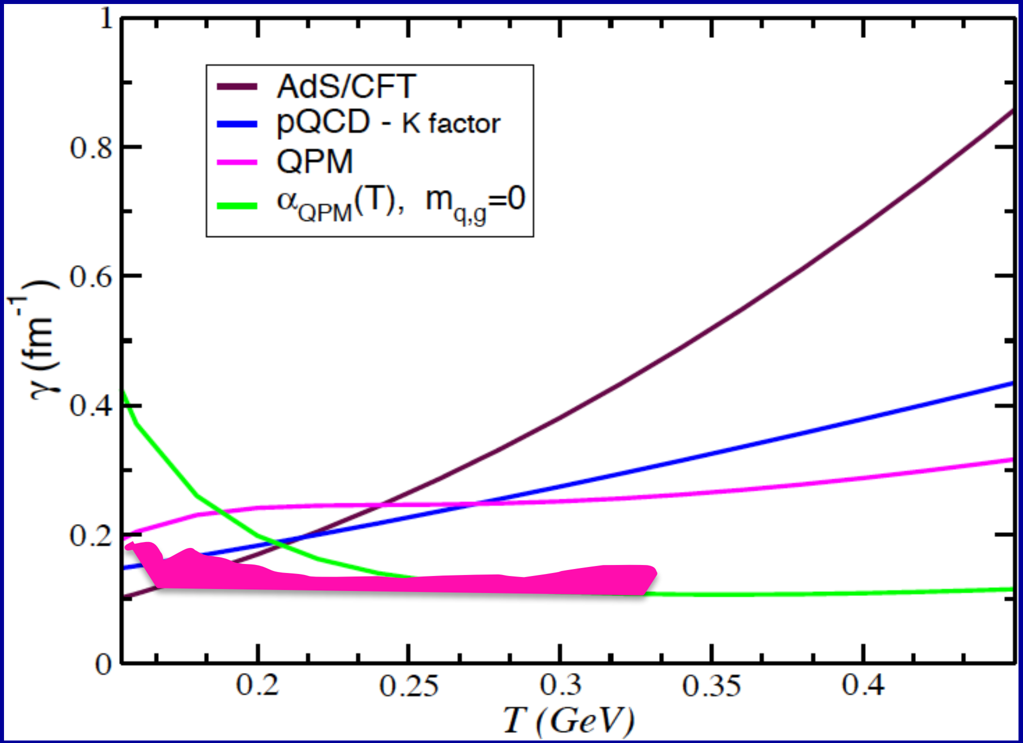
T-matrix approach: scattering under V(r,T)

Hadronic bound states can survive at temperature larger than T_c



The interaction potential V(r,T) can be evaluated employing T-matrix scattering theory

Resonant Scattering



[T-matrix,PRL 100 (2008) V. Greco et al.]

The resonant scattering tends to compensate the decrease by the density scatters because takes into account that V(r,T) becomes stronger close to Tc

↓

T-matrix approach produces a quite large v_2 because of the T-dependence of interaction

Transport theory

$$p^{\mu}\partial_{\mu} f(x,p) + M(X)\partial_{\mu} M(X)\partial_{p}^{\mu} f(X,p) = C_{22}$$

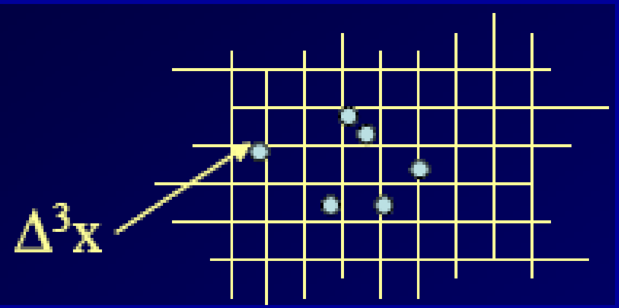
Free-streaming — Mean Field — Collisions

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla_r f + \vec{F}\cdot\nabla_p f = C_{22}$$

Classic Boltzmann equation

Describes the evolution of the one body distribution function f(x,p)

It is valid to study the evolution of both bulk and Heavy quarks



To solve numerically the B-E we divide the space into a 3-D lattice and we use the standard test particle method to sample f(x,p)

Transport theory

✓ Collision integral (stochastic algorithm)

$$C_{22} = \int d^3k \left[\omega(p+k,k) f_{HQ}(p+k) - \omega(p,k) f_{HQ}(p) \right]$$

$$\omega(p,k) = g \int \frac{d^3q}{(2\pi)^3} f'(q) v_{rel} \sigma_{p,q \to p-k,q+k}$$

ω(p,k) is the transition rate for collisions of HQ with heath bath changing the HQ momentum from p to p-k

✓ Collision integral (stochastic algorithm)

$$P_{22} = \frac{\Delta N_{coll}}{\Delta N_{HQ} \Delta N_g} = v_{rel} \sigma_{p,q \to p-k,q+k} \frac{\Delta t}{\Delta^3 x}$$

Fokker Planck equation

HQ interactions are conveniently encoded in transport coefficients that are related to elastic scattering matrix elements on light partons.

The Fokker Planck eq can be derived from the B-E

B-E

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}\right)f\left(x,p,t\right)=C_{22} \quad \Rightarrow \quad \frac{\partial}{\partial t}f\left(p,t\right)=C_{22}$$

$$C_{22}=\int d^3k\left[\omega(p+k,k)f(p+k)-\omega(p,k)f(p)\right]$$

If |k|<< |P|

$$\omega(p+k,k)f(p+k)\approx\omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(\omega f)$$

Fokker Planck equation

$$\omega(p+k,k)f(p+k) \approx \omega(p,k)f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

$$C_{22} \cong \int d^3k \left[k_i \frac{\partial}{\partial p_i}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} \right] \omega(p,k) f(p)$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \frac{\partial}{\partial \mathbf{p}_i}\left[\mathbf{A}_i(\mathbf{p})\mathbf{f} + \frac{\partial}{\partial \mathbf{p}_j}\left[\mathbf{B}_{ij}(\mathbf{p})\mathbf{f}\right]\right]$$

where we have defined the kernels

$$\mathbf{A}_i = \int \mathbf{d}^3\mathbf{k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k}_i \quad \rightarrow \textbf{Drag Coefficient}$$

$$\mathbf{B}_{ij} = \int \mathbf{d}^3\mathbf{k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k}_i\mathbf{k}_j \quad \rightarrow \textbf{Diffusion Coefficient}$$

Where B_{ij} can be divided in a longitudinal and in a transverse component B_0 , B_1

[B. Svetitsky PRD 37(1987)2484]

Langevin Equation

The Fokker-Planck equation is equivalent to an ordinary stochastic differential equation →

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k$$

Γ is the deterministic friction (drag) force

C_{ij} is a stochastic force in terms of independent Gaussian-normal distributed random variable $\rho=(\rho_x,\rho_y,\rho_z)$

$$P(\rho) = \left(\frac{1}{2\pi}\right)^3 exp(-\frac{\rho^2}{2})$$

ρ obey the relations:

$$<\rho_i(t)>=0 \qquad <\rho_i(t)\rho_k(t')>=\delta(t-t')\delta_{jk}$$

the covariance matrix and Γ are related to the diffusion matrix and to the drag coefficient by

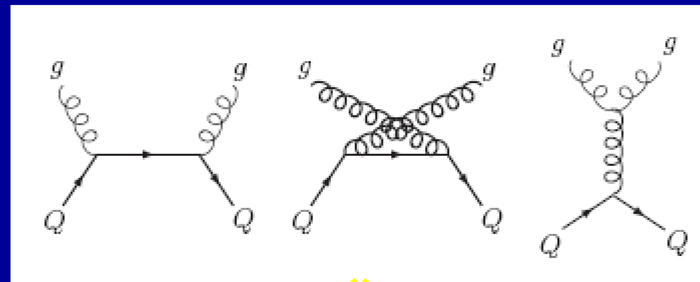
$$C_{jk} = \sqrt{2B_0} P_{jk}^{\perp} + \sqrt{2B_1} P_{jk}^{\parallel}$$

$$A_i = p_j \Gamma - \xi C_{lk} \frac{\partial C_{ij}}{\partial p_l}$$

Evaluation of Drag and diffussion

Common approach between LV and BM

Langevin approach



Boltzmann approach

For Collision Process the A_i and B_{ij} can be calculated as following :

M -> A_i, B_{ij}

M -> σ

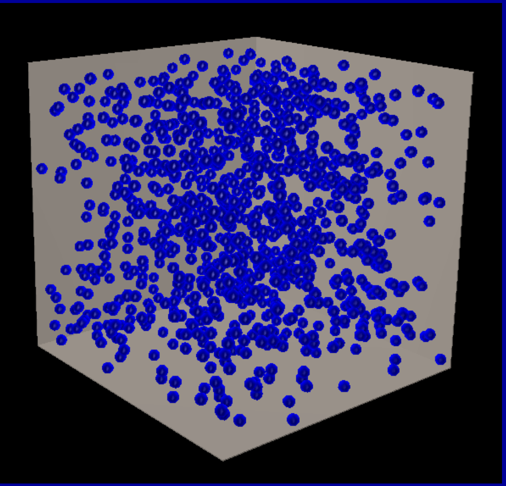
$$A_i = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_c}\sum|M|^2$$

$$(2\pi)^4\delta^4(p+q-p'-q')f(q)[(p-p')_i] = \langle\langle(p-p')_i\rangle\rangle$$

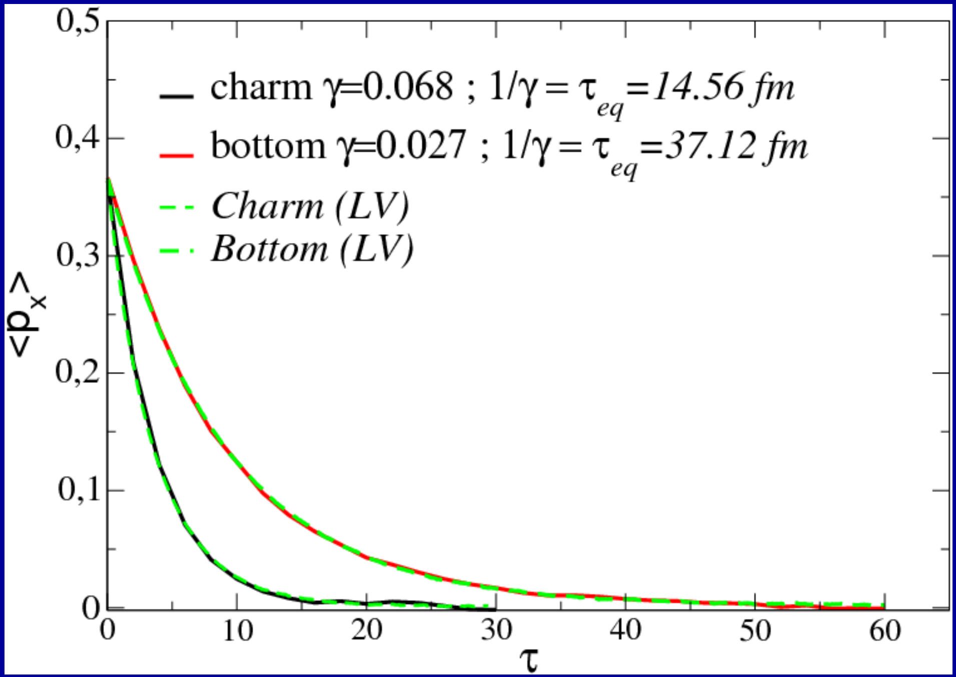
$$\sigma_{gc\to gc} = \frac{1}{16\pi\left(s-M_c^2\right)^2}\int_{-(s-M^2)^2/s}^{0} dt\sum|M|^2$$

$$B_{ij} = \frac{1}{2}\langle\langle(p-p')_i(p'-p)_j\rangle\rangle$$

Mean momentum evolution in a static medium



We consider as initial distribution in p-space a δ(p-1.1GeV) for both C and B with p_x=(1/3)p



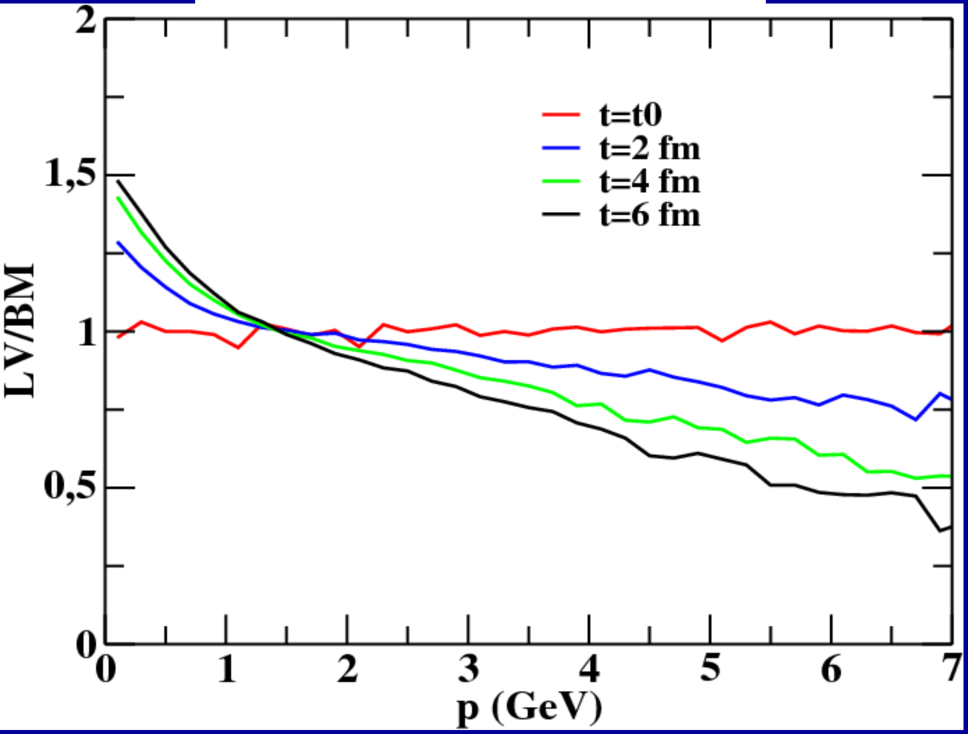
Each component of average momentuma evolves according to $<p_i>=p^0_i\exp(-\gamma t)$ where 1/ γ is the relaxation time to equilibrium (τ)

$\tau_b/\tau_c=2.55\cong m_b/m_c$

For a very inclusive quantity BM and LV give same result

Boltzmann vs Langevin (Charm)

$$\frac{dN}{d^3p}^{Langevin} \bigg/ \frac{dN}{d^3p}^{Boltzmann}$$



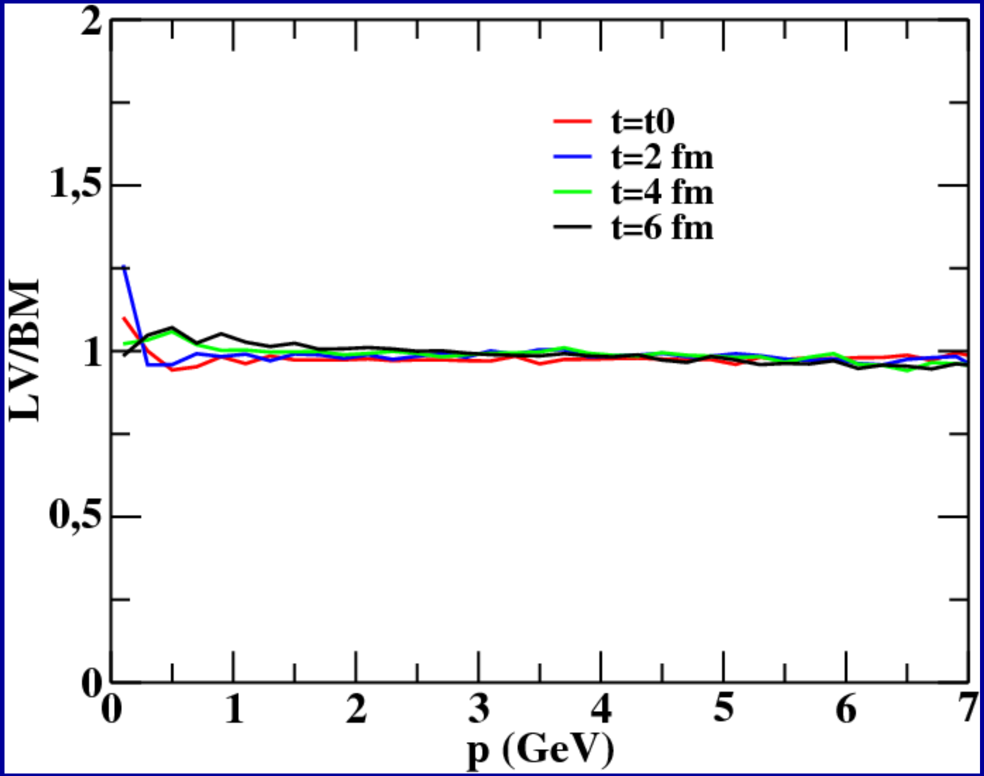
We have plotted the results as a ratio between LV and BM at different time to quantify how much the ratio differs from 1

[S. K. Das , F. Scardina, V. Greco PRC90 044901 (2014)]

Is the charm really Heavy and its scattering soft ?

We studied the effect of the mass and of the momentum transferred on the approximation involved in the F-P

Boltzmann vs Langevin (Bottom)



In bottom case Langevin approximation gives results similar to Boltzmann

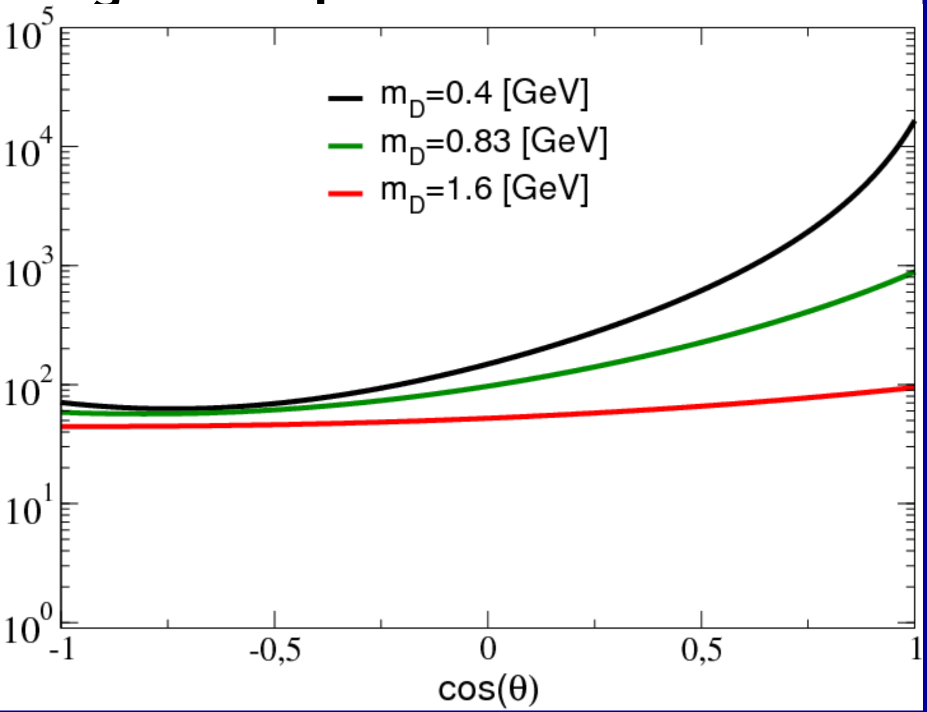
The Larger M the Better Langevin approximation works

[S. K. Das , F. Scardina, V. Greco PRC90 044901 (2014)]

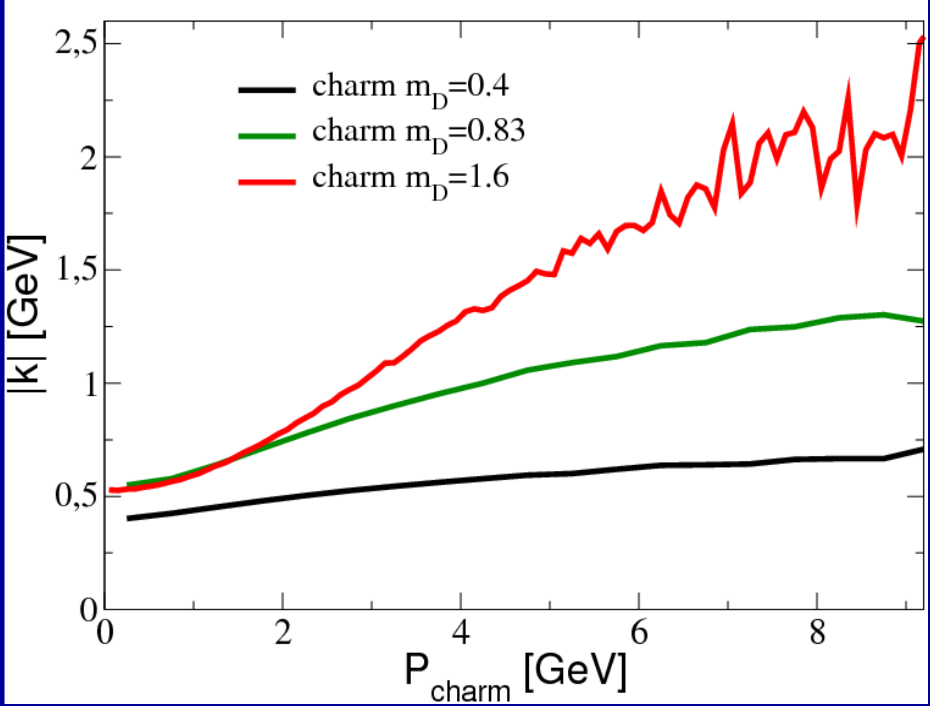
Boltzmann vs Langevin (Charm)

- simulating different average momentum transfer

Angular dependence of σ

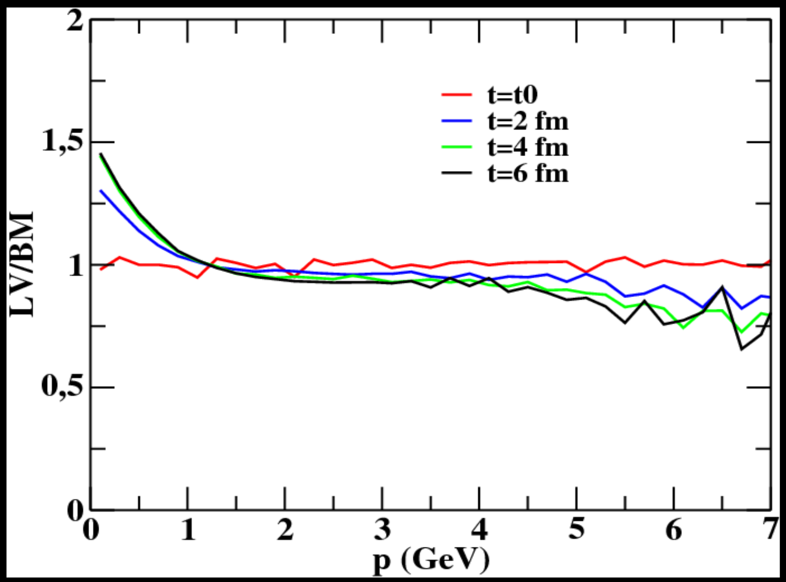


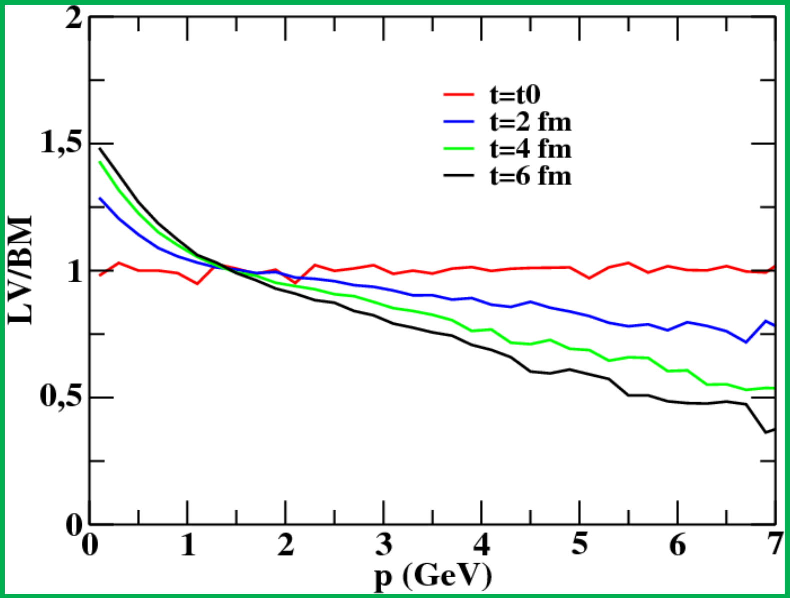
Mometum transfer vs P

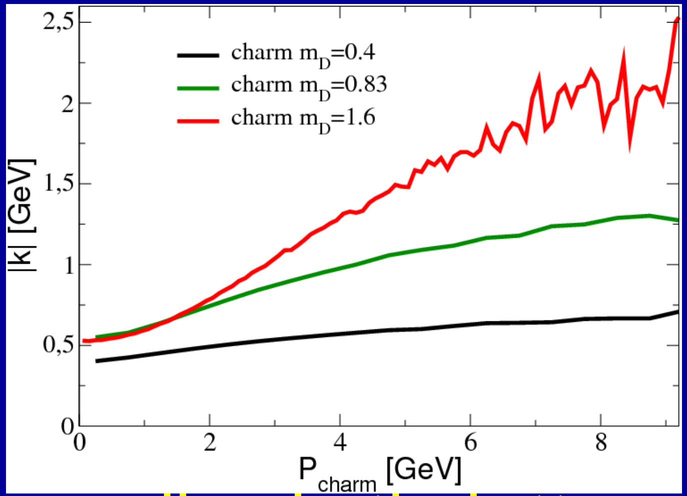


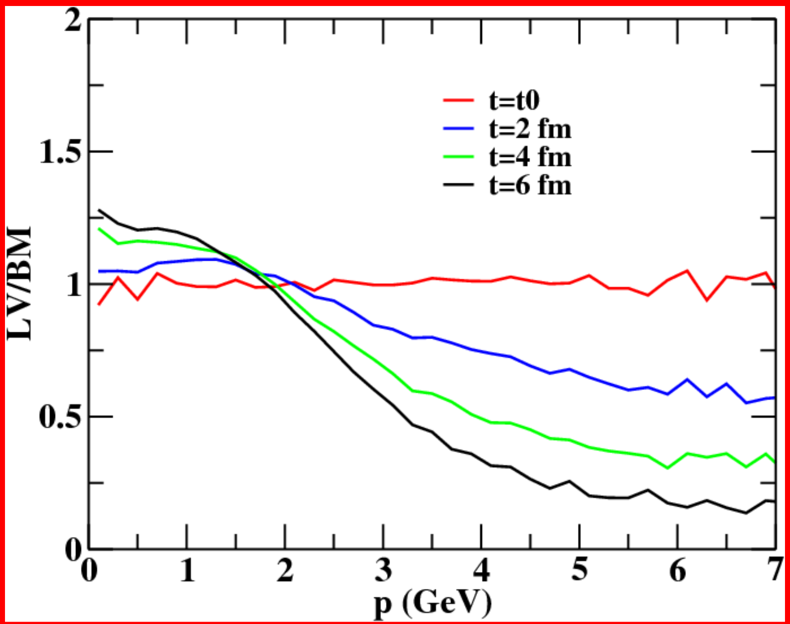
Decreasing m_D makes the σ more anisotropic ➡ Smaller average momentum transfer

Boltzmann vs Langevin (Charm)





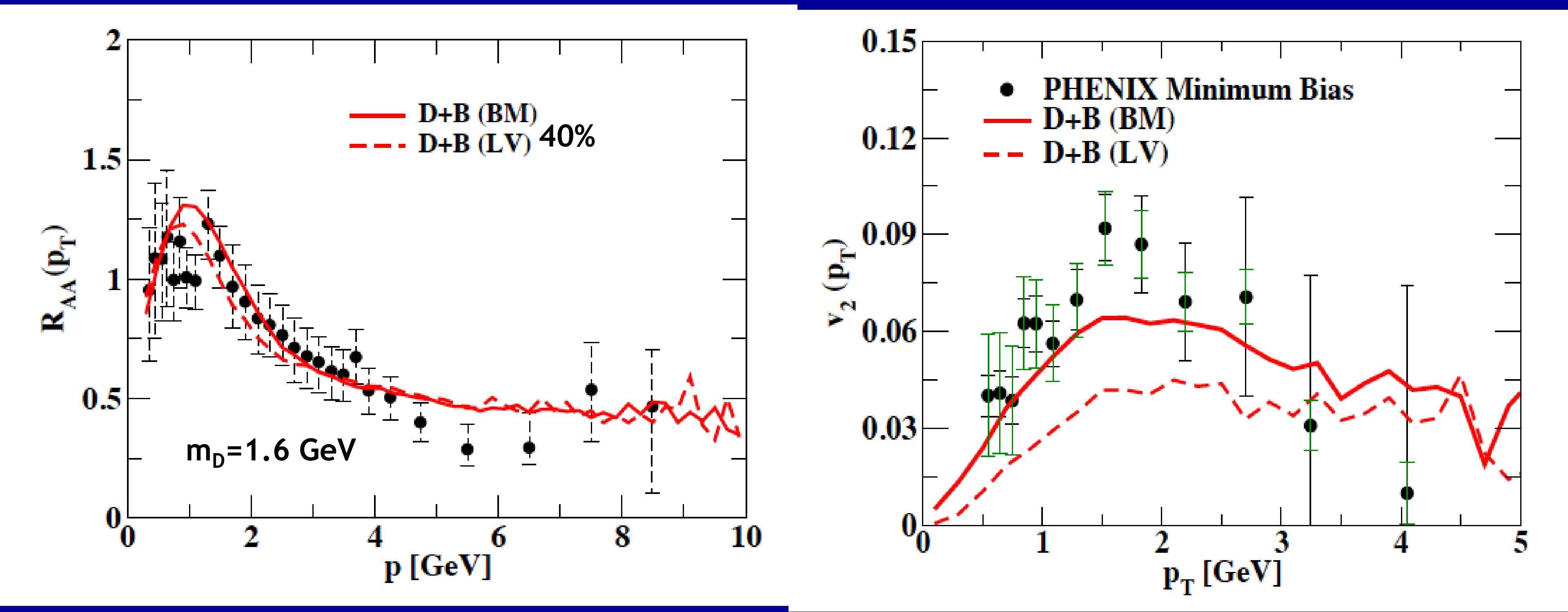




The smaller <k> the better Langevin approximation works

R_{AA} and v_2 Boltzmann vs Langevin

Au+Au@200AGeV, b=8 fm



- ✓ Fixed same $R_{AA}(p_T)$ [reduce γ by 40%] → $v_2(p_T)$ 35% higher (m_D=1.6 GeV)
  - dependence on the specific scattering matrix (isotropic case -> larger effect)

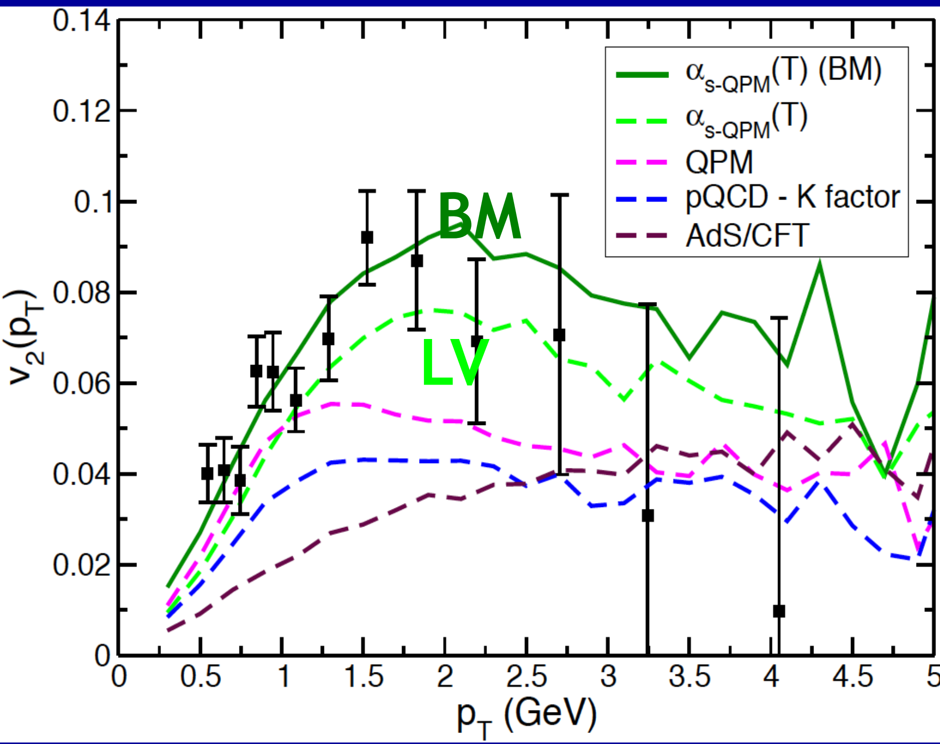
Hadronization by coalescence not included

[S. K. Das , F. Scardina, V. Greco PRC90 044901 (2014)]

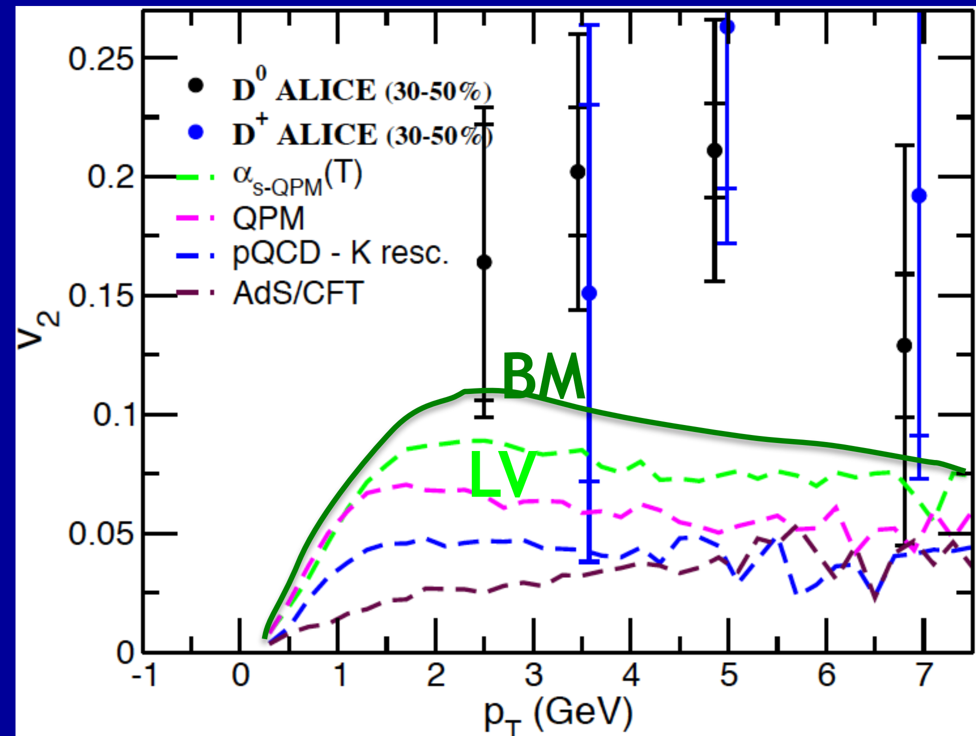
v_2 Boltzmann vs Langevin

Impact of the Boltzmann dynamics for $\alpha_{QPM}(T)$ case

Au+Au@200AGeV, b=8 fm

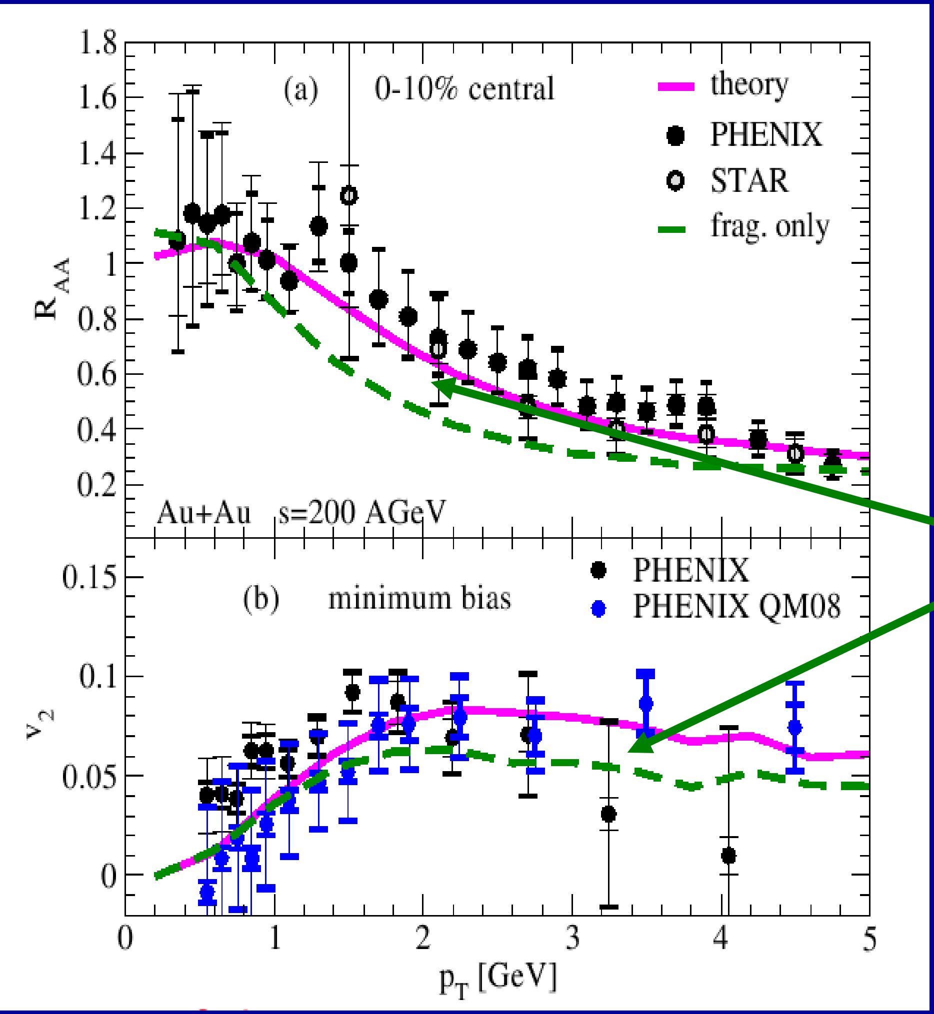


LHC - Pb+Pb@2.76ATeV



No coalescence included, only fragmentation

Impact of hadronization mechanism



Impact of hadronization

Coalescence increase

both R_{AA} and v_2

reverse the correlation

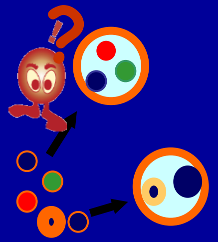
toward agreement with data

Hees-Mannarelli-Greco-Rapp, PRL100 (2008)

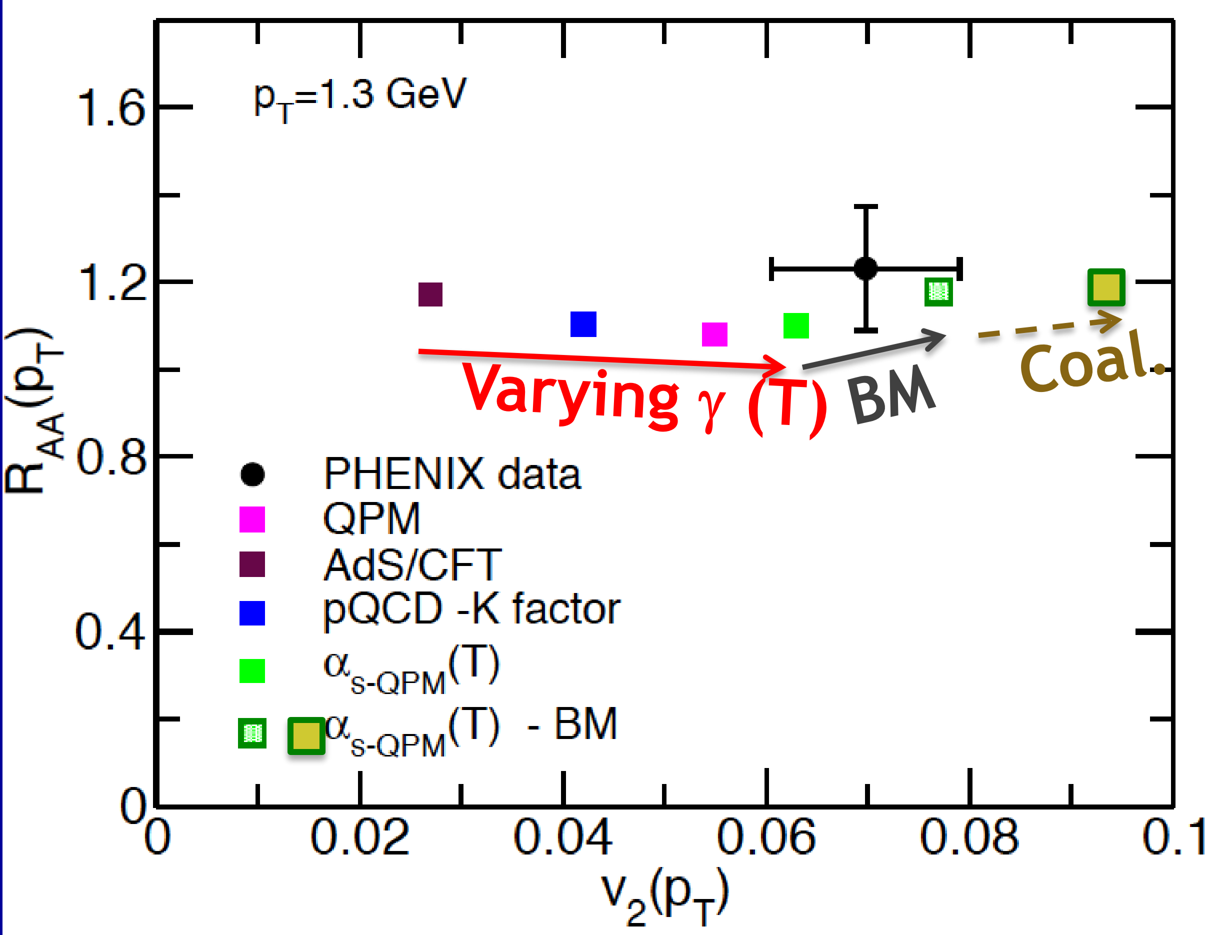
$$\frac{d^3N_{D,B}}{d^3P} = C_{D,B}\int_{\Sigma} f_{c,b} \otimes f_{\bar{q}} \otimes \Phi_M + \int_{\Sigma} f_{c,b} \otimes D_{c,b\to D,B}$$

f_q from π, K

Greco,Ko,Levai - PRL90



Summary on the build-up of v_2 at fixed R_{AA}

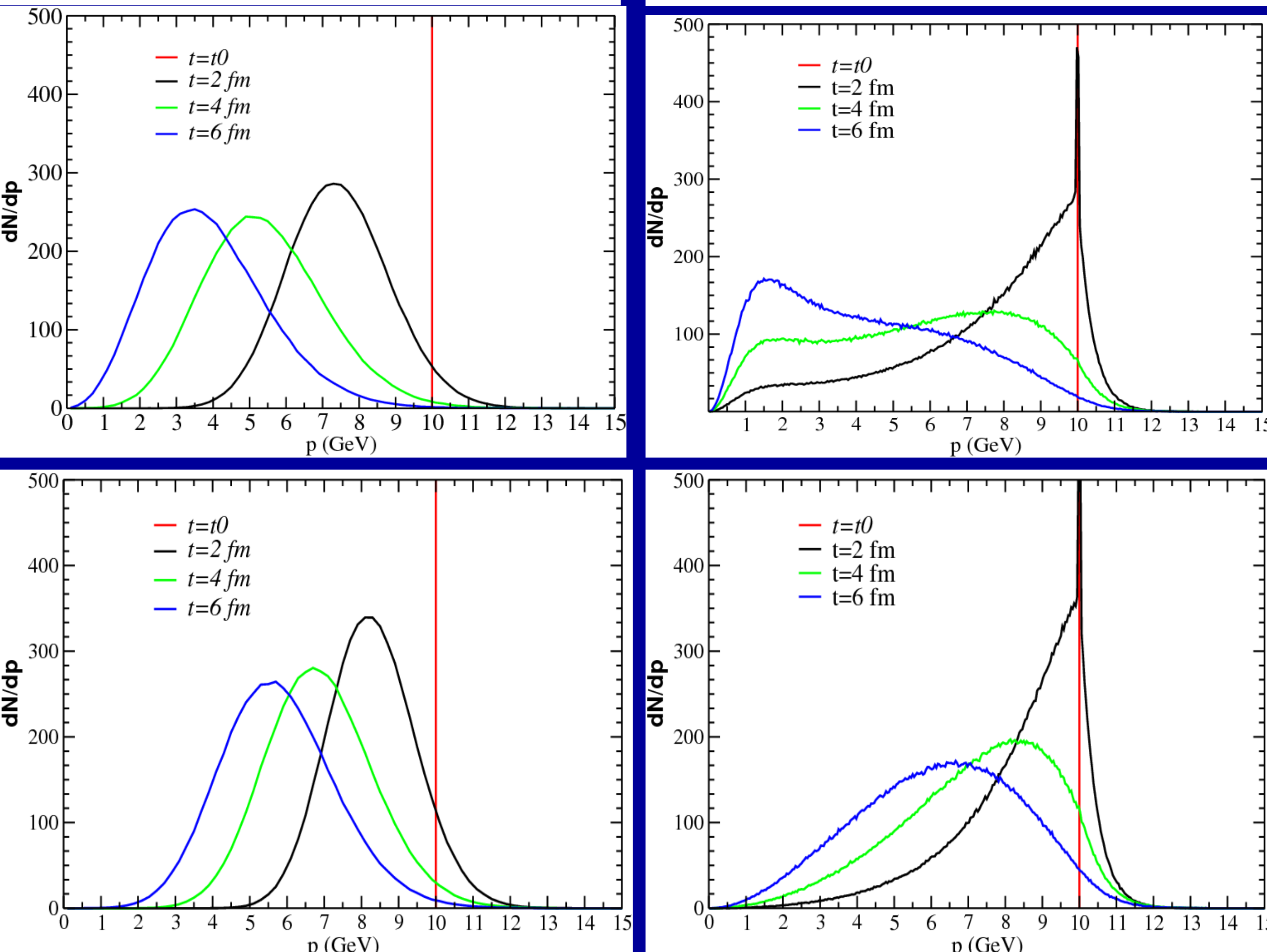


R_{AA} and V_2 are correlated but still one can have
R_{AA} about the same while V_2 can change up to a factor 3:
$\gamma(T)$ + Boltzmann dynamics+ hadronization

Energy loss of a single HQ

T=400 MeV

Langevin	Boltzmann



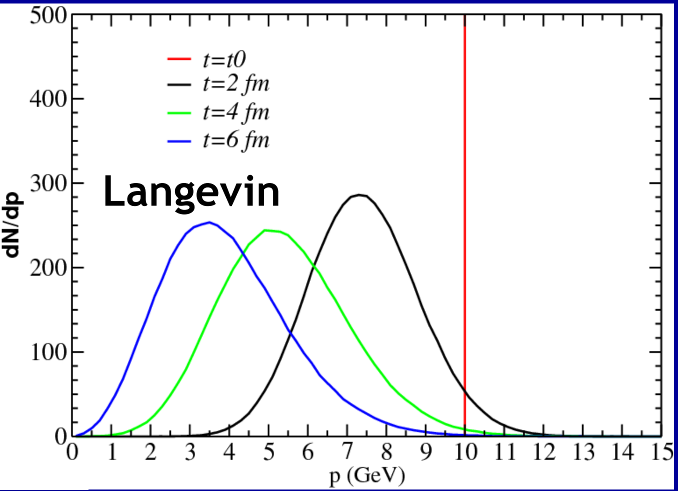
Charm $M_c/T \approx 3$

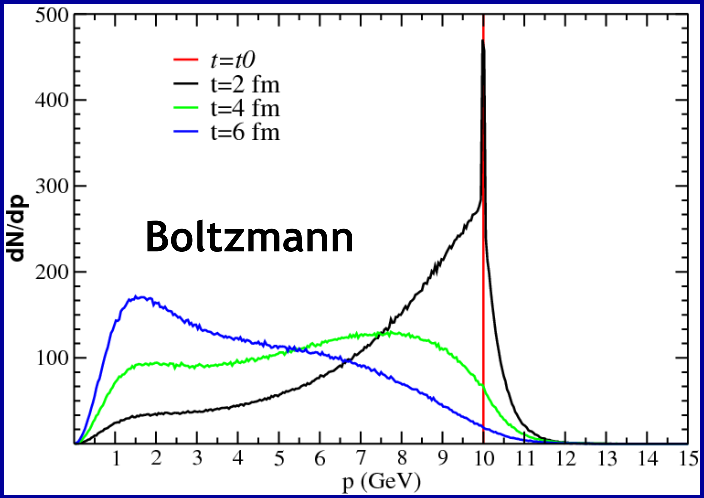
Bottom $M_b/T \approx 10$

T=400 MeV Mc/T≈3 Mb/T≈10 [F. ScardinaJ.Phys.Conf.Ser. 535 (2014) 012019]

Back to Back correlation

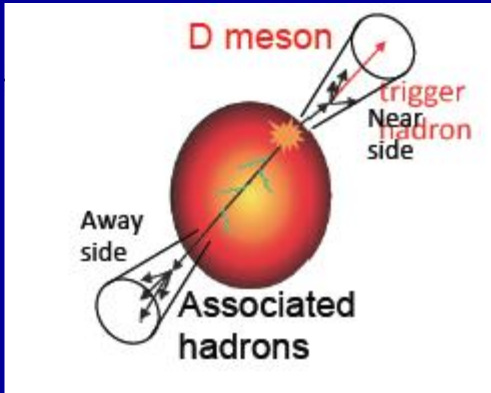
Back to back correlation observable could be sensitive to such a detail





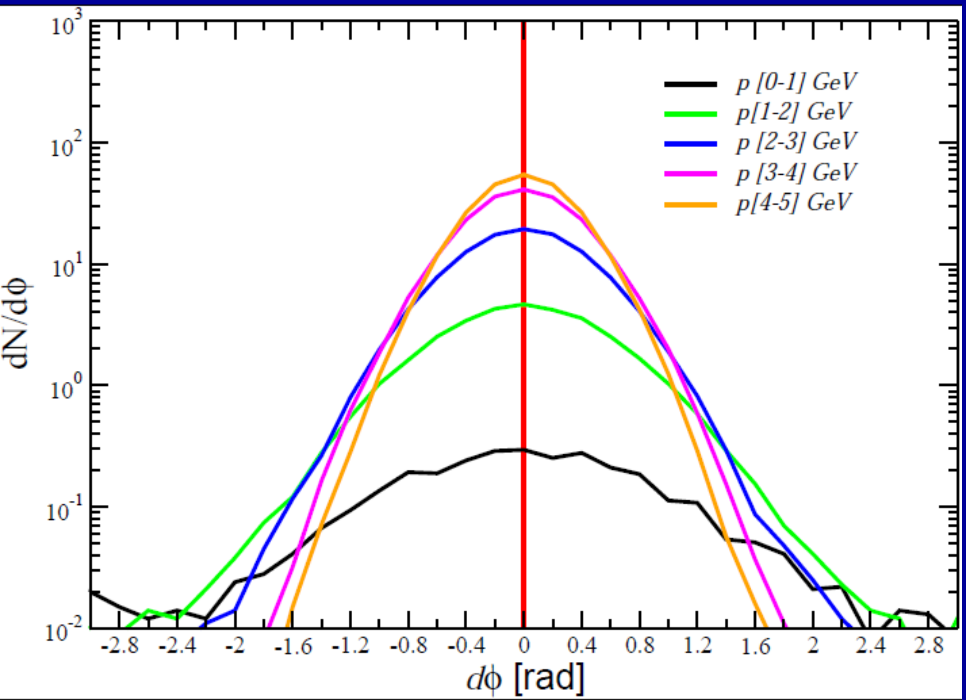
Initial δ(p=10) can be tought as a **Near side** charm with momentum equal 10

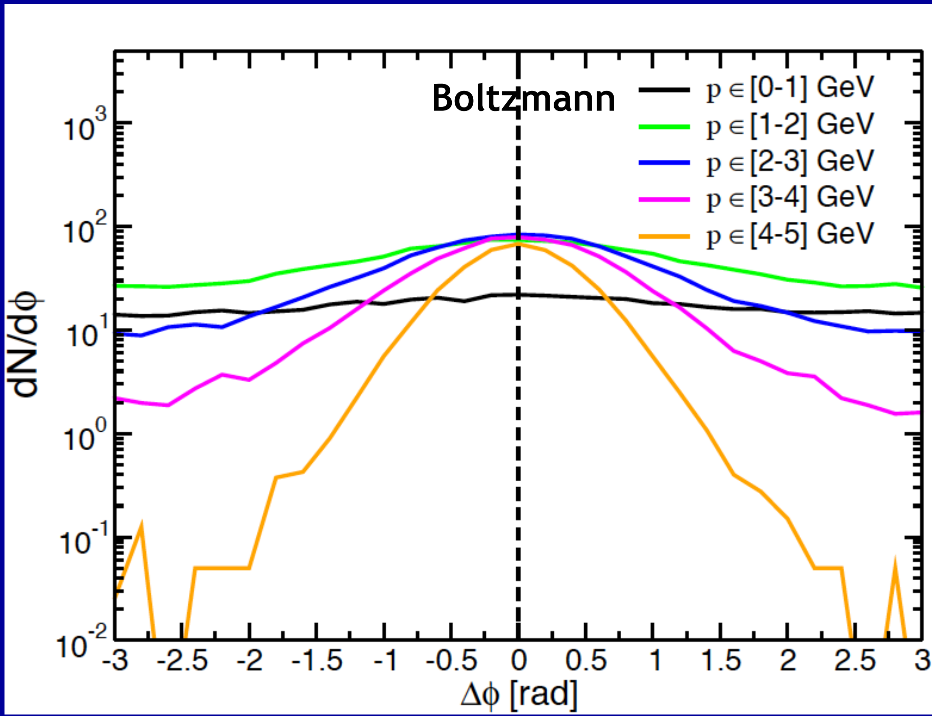
Final distribution can be tought as the momentum probabilty distribution to find an **Away side** charm

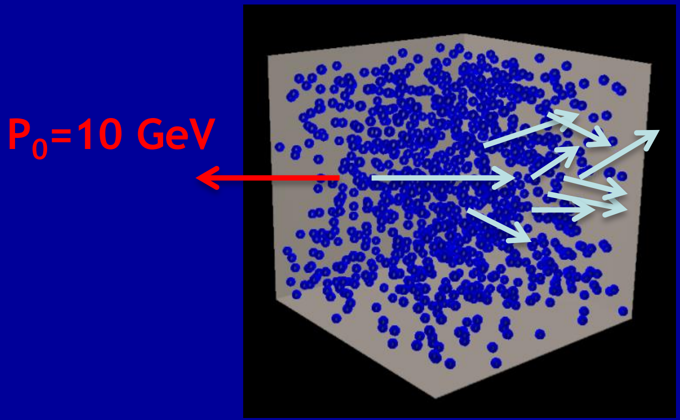


Boltzmann implies a larger momentum spread

Langevin vs Boltzmann angular correlation







The larger spread of momentum with the Boltzmann implicates a large spread in the angular distributions of the Away side charm

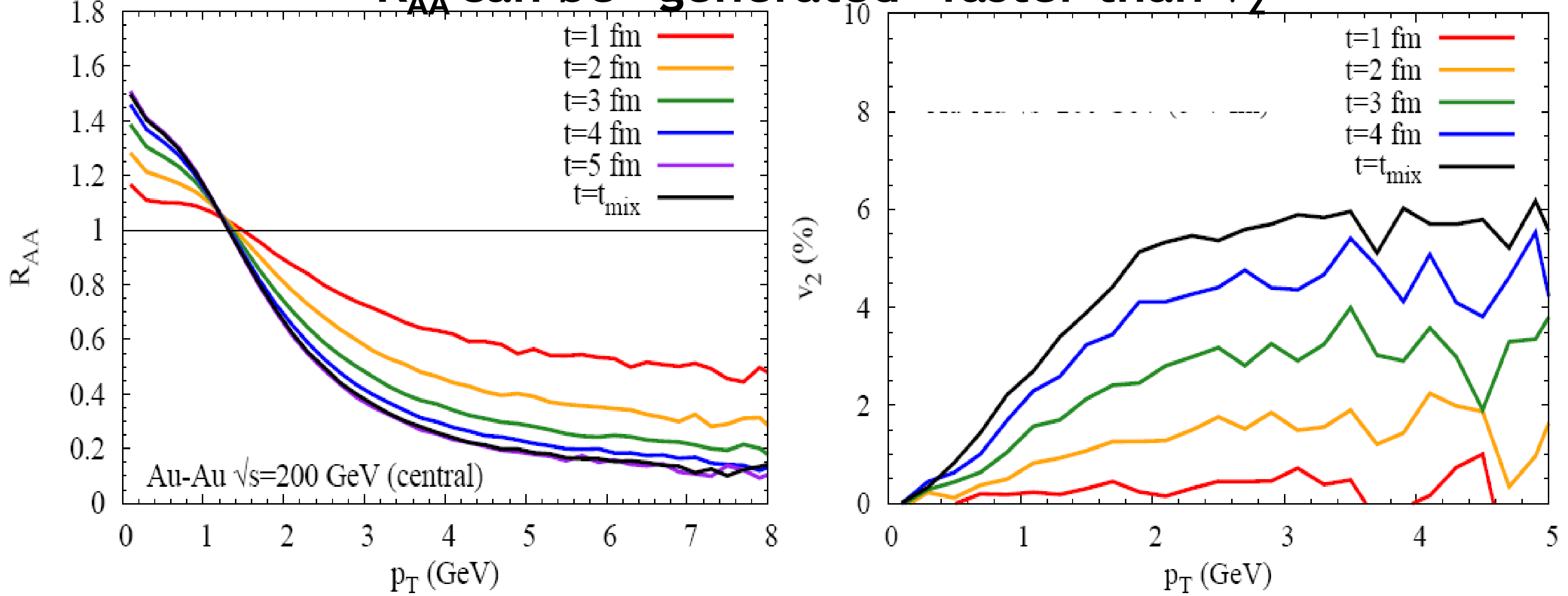
Striking difference also at $\Delta\phi$ =0:

- The evolution of the yield from 2 to 5 GeV is about 50 times different

Summary

- ✓ The exp. data for R_{AA} and v_2 seem to indicate an interaction about constant in T
- ✓ Boltzmann is more efficient in building up the v_2 related to HQ
- ✓ The more one looks at differential observables R_{AA}->V_2-> $dN_{cc}/d\Delta\phi$ the more the differences between the BM and F-P approach increases
- ✓ We can realize that charm in hot QGP is not that heavy and the motion not really Brownian
- ✓ Very similar dynamics for Bottom at least for R_{AA} and V_2

R_{AA} can be "generated" faster than v_2

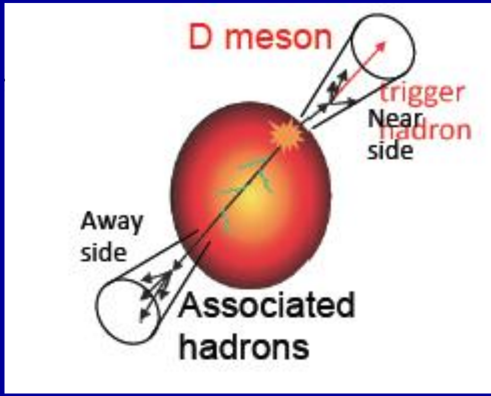


For heavy quark fragmentation, we are using Peterson fragmentation :

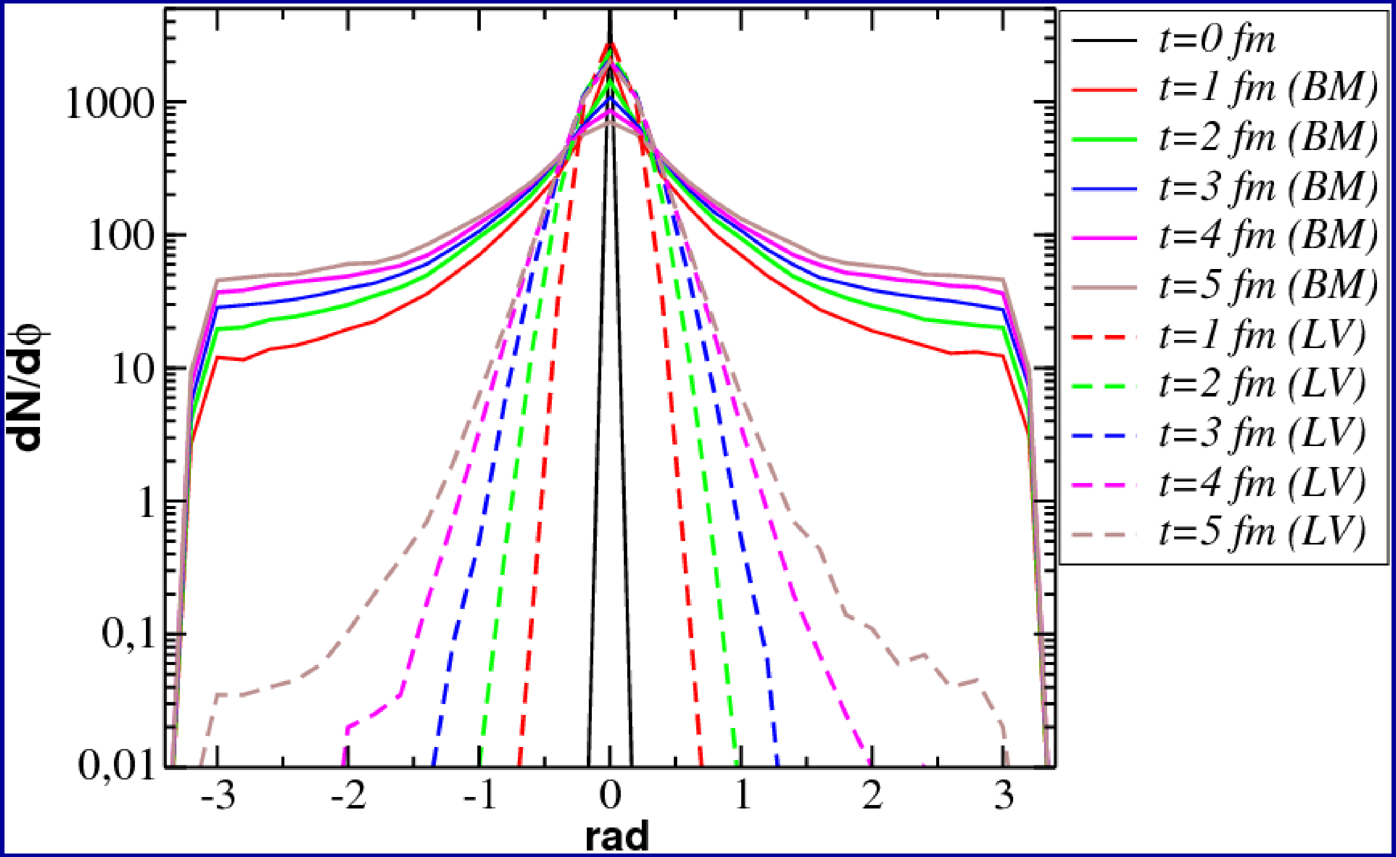
$$f(z) \propto \frac{1}{[z[1 - \frac{1}{z} - \frac{\epsilon_c}{1-z}]^2]} \qquad (6)$$

for charm quark $\epsilon_c = 0.04$. For bottom quark $\epsilon_c = 0.005$.

Back to Back correlation



The larger spread of momentum with the Boltzmann implicates a large spread in the angular distributions of the Away side charm



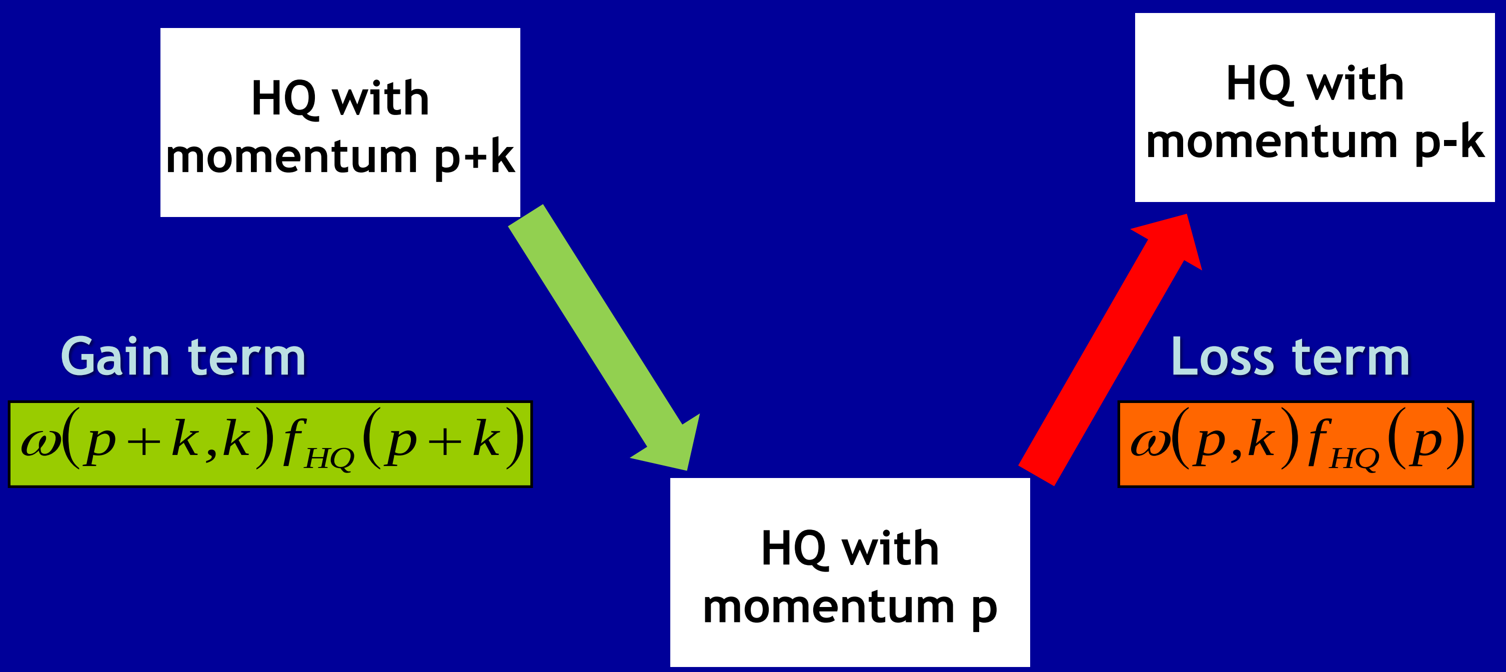
Transport theory

✓ **Collision integral**

$$C_{22} = \int d^3k \left[\omega(p+k,k) f_{HQ}(p+k) - \omega(p,k) f_{HQ}(p) \right]$$

$$\omega(p,k) = g \int \frac{d^3q}{(2\pi)^3} f'(q) v_{rel} \sigma_{p,q \to p-k,q+k}$$

$\omega(p,k)$ is the transition rate for collisions of HQ with heath bath changing the HQ momentum from p to p-k



Element of momentum space with momentum p

Transport theory

✓ Collision integral (stochastic algorithm)

Assuming two particle

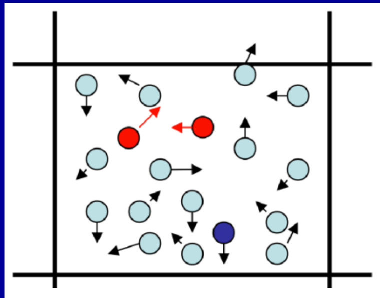
- In a volume Δ^3x in space
- momenta in the range $(P,P+\Delta^3P)$; $(q,q+\Delta^3q)$

$$\frac{(2\pi)^3\Delta N_{coll}}{\Delta t\Delta^3x\Delta^3p} = g\frac{\Delta^3q}{(2\pi^3)} f_{HQ}(P) f_g(q) v_{rel}\sigma_{p,q\rightarrow p-k,q+k}$$

collision rate per unit phase space for this pair

$$\frac{(2\pi)^3\Delta N_{coll}}{\Delta t\Delta^3x\Delta^3P} = g\frac{\Delta^3q}{(2\pi^3)}\frac{(2\pi)^3\Delta N_{HQ}}{\Delta^3x\Delta^3P}\frac{(2\pi)^3\Delta N_g}{\Delta^3x\Delta^3q} v_{rel}\sigma_{p,q\rightarrow p-k,q+k}$$

$$P_{22} = \frac{\Delta N_{coll}}{\Delta N_{HQ}\Delta N_g} = v_{rel}\sigma_{p,q\rightarrow p-k,q+k}\frac{\Delta t}{\Delta^3x}$$



Δ^3x

$\Delta t \rightarrow 0$

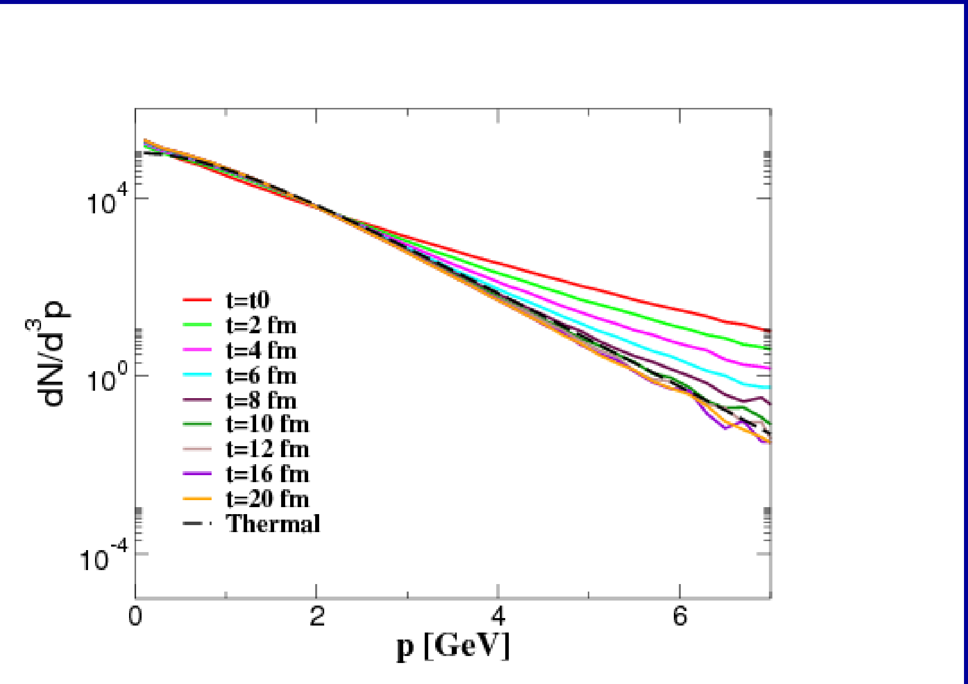
$\Delta^3x \rightarrow 0$

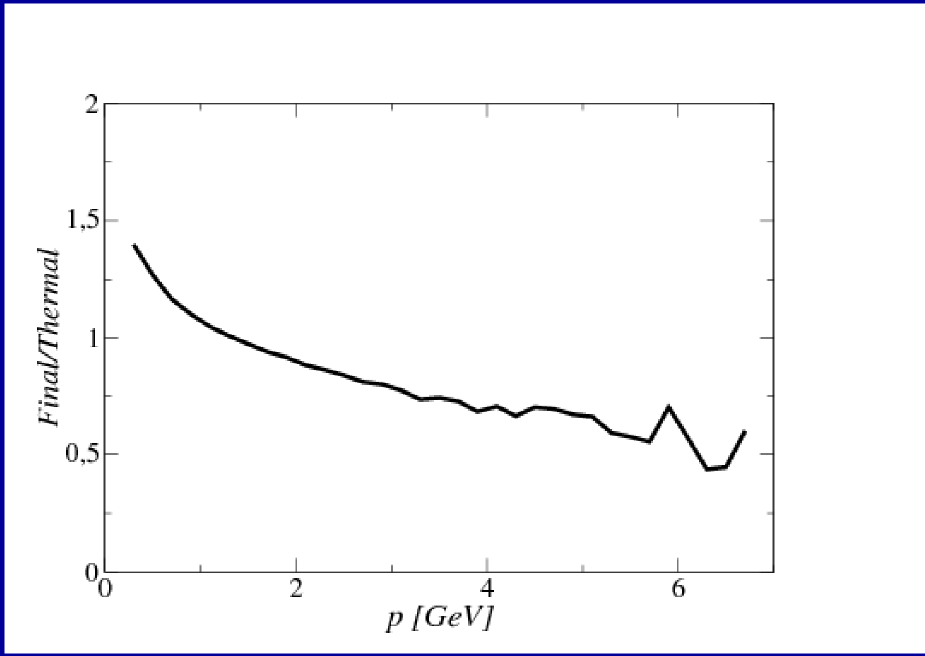
Exact solution

Charm propagation with the langevin eq

We solve Langevin Equation in a box in the identical environment of the B-E

Bulk composed only by gluon in Thermal equilibrium at T= 400 MeV.





The long-time solution of the Fokker Planck equation does not reproduces the equilibrium distribution (we are away from thermalization around 35-40 % at intermediate pt).

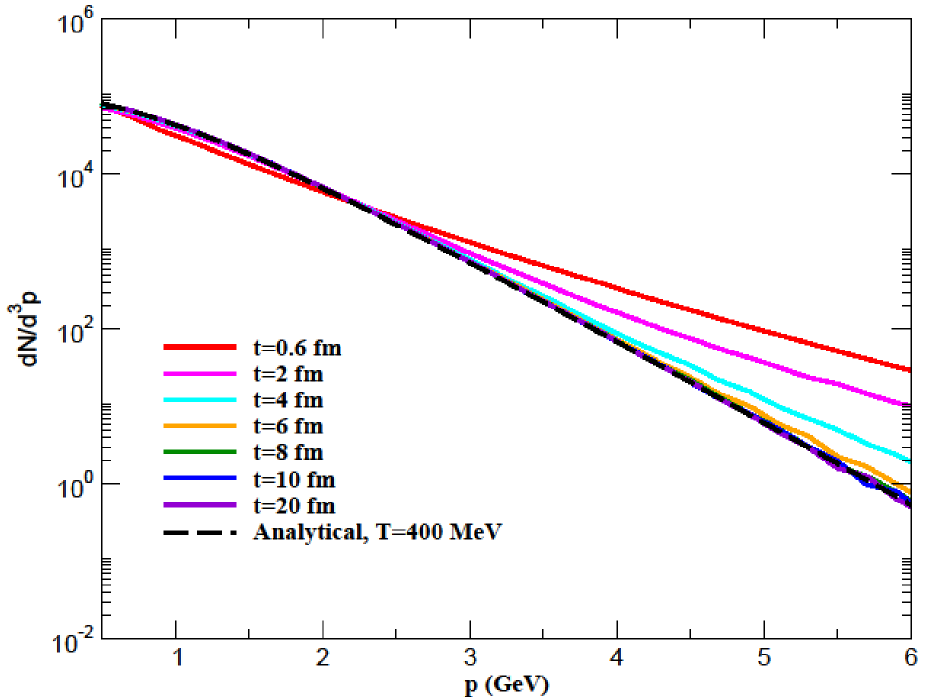
This is however a well-know issue related to the Fokker Planck

Charm propagation with the langevin eq

The long time solution is recovered relating the Drag and Diffusion coefficent by mean of the fluctaution dissipation relation $A(E)ET - D(E) + T(1-\xi)D'(E) = 0$

✓ Imposing the simple relativistic dissipation-fluctuation relations

D=Constant
A= D/ET from FDT

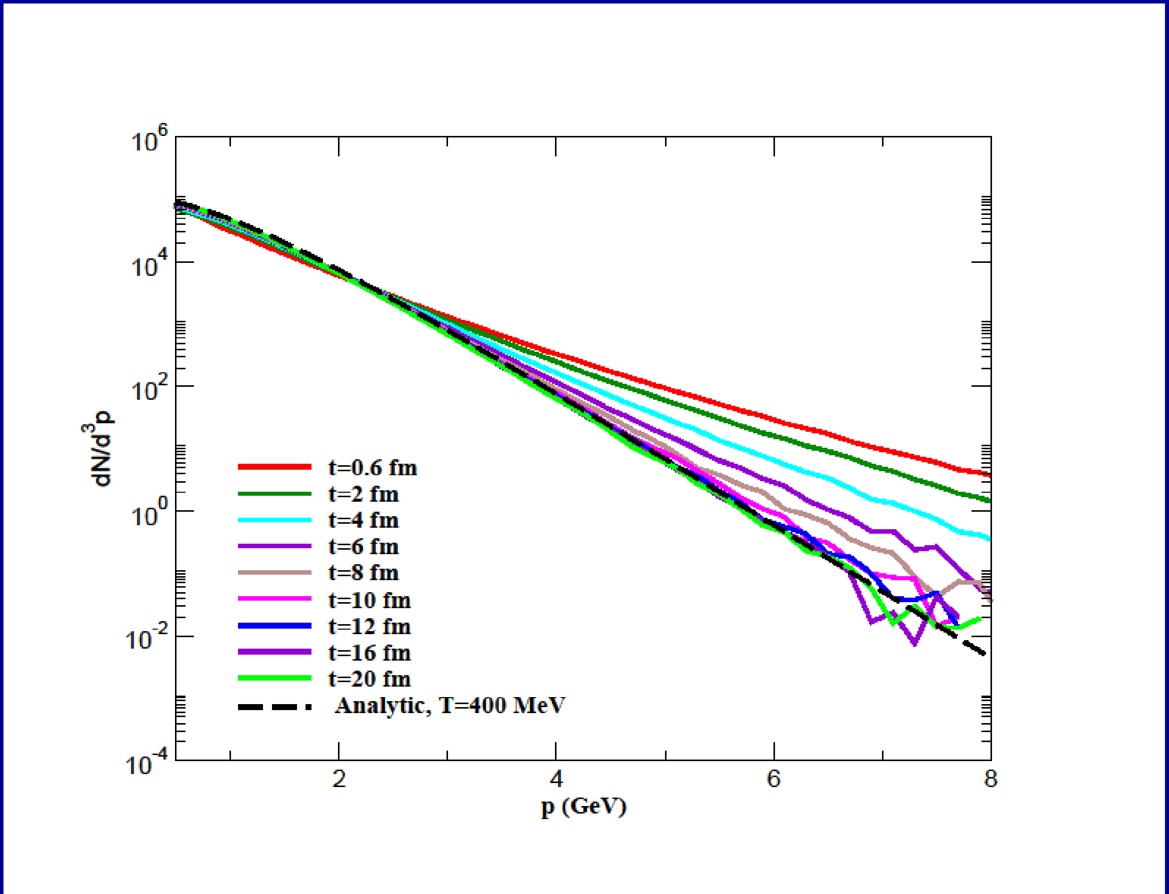


Charm propagation with the langevin eq

✓ **Imposing the full relativistic dissipation-fluctuation relations**

D(E)

$$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$



Boltzmann vs Langevin (Bottom)