A scaling relation between pA and AA collisions

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GB, D.Teaney, arXiv:1312.6770, Phys. Rev. C. **90**, 014905

Motivation

- Recent measurements of the two particle correlation function at the LHC and RHIC revealed a striking similarity between high multiplicity proton-nucleus (pA) and nucleus-nucleus (AA) collisions
- Same physics?? Collective flow in pA ?? Hydro in pA?? [Bozek et.al., Shuryak et. al., Kozlov et. al. , . . .]
- There are also some quantitative differences in the measurements

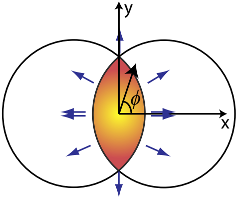
Idea: Come up with a framework that accounts for the similarities and the differences.

$\Rightarrow$ "Conformal dynamics"

Collective flow in nucleus-nucleus (AA) collisions

Flow in AA

- Key measurement: transverse momentum anisotropy



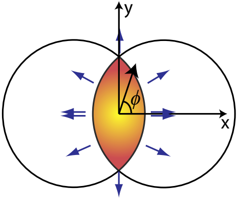
$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

- Interpretation:
  - system behaves as a fluid with low viscosity
  - different pressure gradients in x and y $\Rightarrow$ anisotropy in p_T
  - average eccentricity $\epsilon_2 \equiv \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle} \Rightarrow v_2$

    (linear response : "$v_2 = k\,\epsilon_2$")

Flow in AA

- Key measurement: transverse momentum anisotropy



$$\frac{dN}{d^2p_T} = \frac{dN}{p_T dp_T} \sum_{n=1}^{\infty} (1 + 2v_n \cos(n\phi))$$

- Interpretation:
  - system behaves as a fluid with low viscosity
  - different pressure gradients in x and $y \Rightarrow$ anisotropy in p_T
  - average eccentricity $\epsilon_2 \equiv \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle} \Rightarrow v_2$
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Flow in AA

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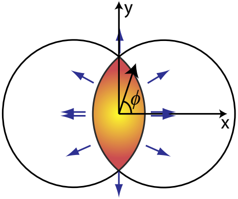

$$\frac{dN}{d^2p_T} = \frac{dN}{p_T dp_T}\left(1 + 2v_2\cos(2\phi) + 2v_3\cos(3\phi) + \dots\right)$$

- Interpretation:
  - system behaves as a fluid with low viscosity
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    (linear response : "$v_2 = k\,\epsilon_2$")

Flow in AA

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos(2\phi - 2\Psi_2) + 2v_3 \cos(3\phi - 3\Psi_3) + \dots$$

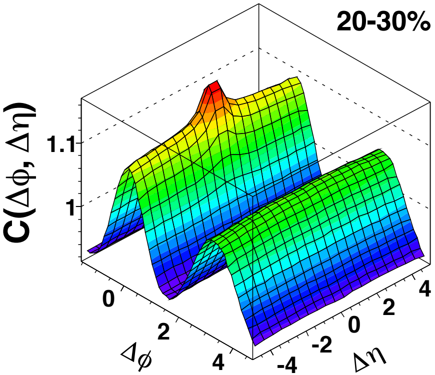
- The actual measurement: two particle correlation fnc.

$$\begin{aligned} C(\Delta\phi) &\propto \left\langle \frac{dN}{d\phi} \frac{dN}{d(\phi + \Delta\phi)} \right\rangle_{\Psi_2, \Psi_3, \dots} \\ &\propto 1 + 2\langle v_2^2 \rangle \cos(2\Delta\phi) + 2\langle v_3^2 \rangle \cos(3\Delta\phi) + \dots \end{aligned}$$

notation: $v_2\{2\} \equiv \sqrt{\langle v_2^2 \rangle}, \quad v_3\{2\} \equiv \sqrt{\langle v_3^2 \rangle}, \quad \dots$

Flow in AA

A typical measurement:

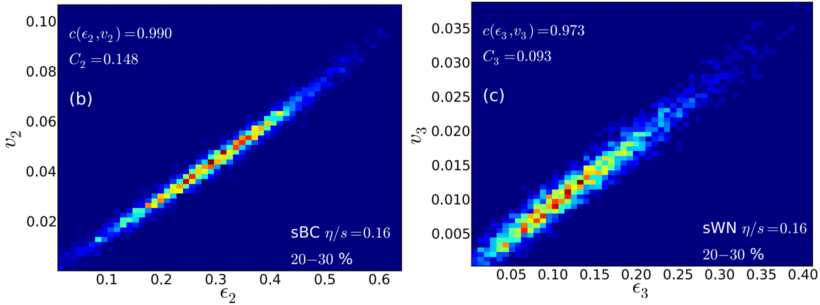


[ATLAS, PRC 86 014907]

$\Rightarrow$ extract $\langle v_2^2 \rangle$, $\langle v_3^2 \rangle$ from a Fourier fit

Flow in nucleus-nucleus (AA) collisions

The triumph of linear response:



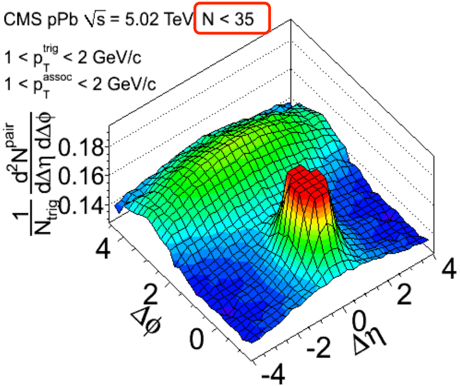
[Niemi et. al. PRC87 054901]

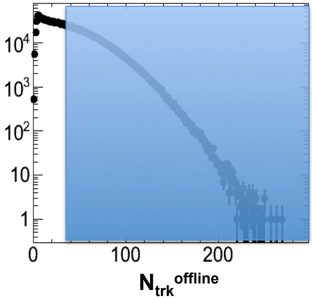
- To a good approximation:

$$v_2\{2\} = k_2\sqrt{\langle\epsilon_2^2\rangle}, \quad v_3\{2\} = k_3\sqrt{\langle\epsilon_3^2\rangle}$$

The recent proton-nucleus (pA) results

A typical event (low multiplicity)

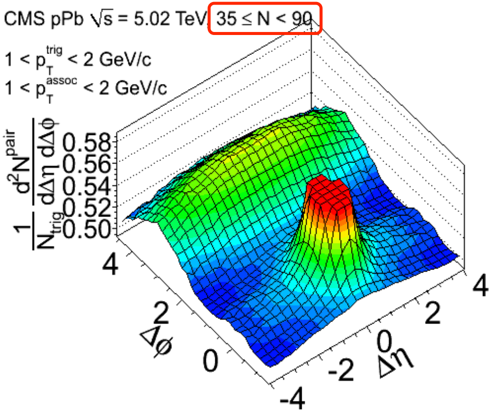


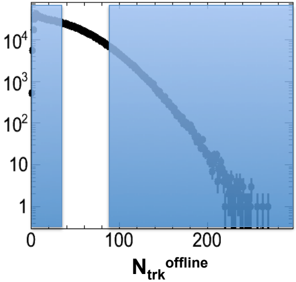


[data from CMS, slides from G. Roland, RBRC workshop Apr. 15-17, 2013, also PLB 724 213]

The recent pA results

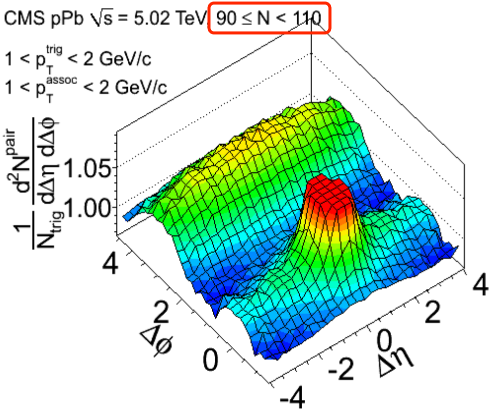
A typical event (higher multiplicity)

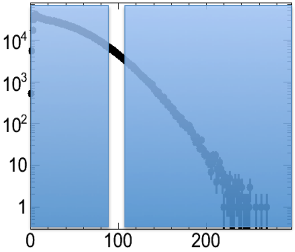




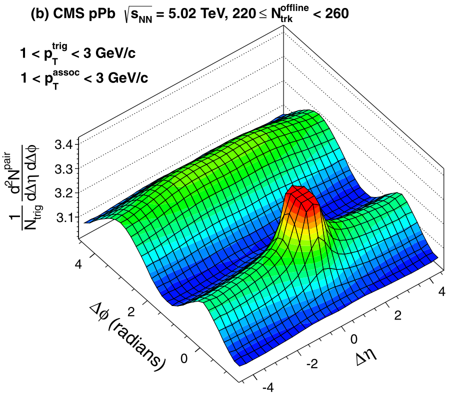
The recent pA results

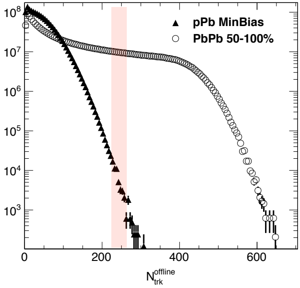
A somewhat rare event





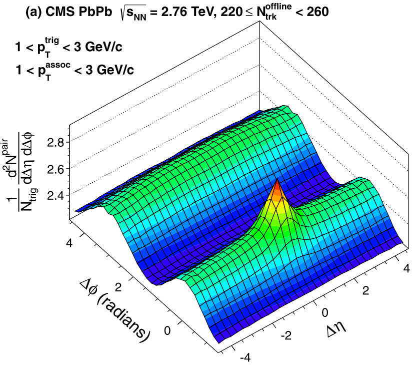
A very rare event (high multiplicity)



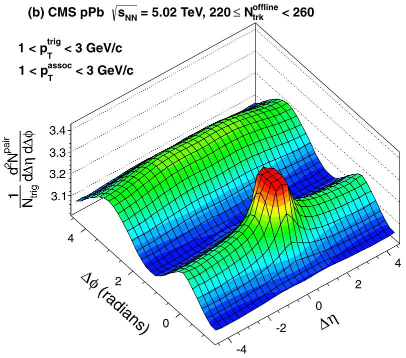


The recent pA results

Compare pA and AA at the same multiplicity



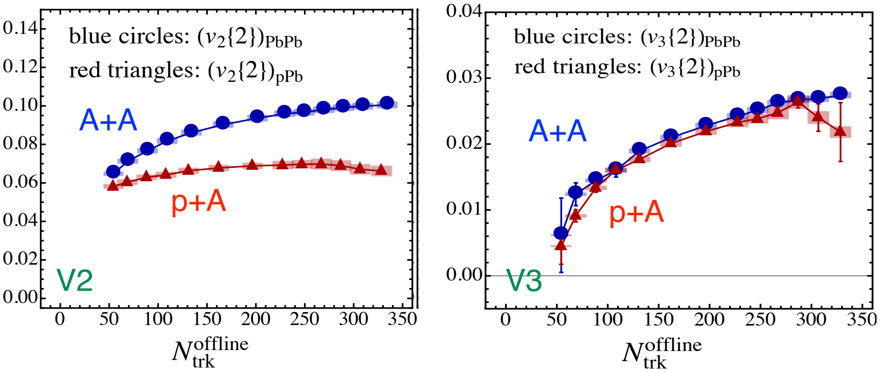
PbPb



pPb

The recent pA results

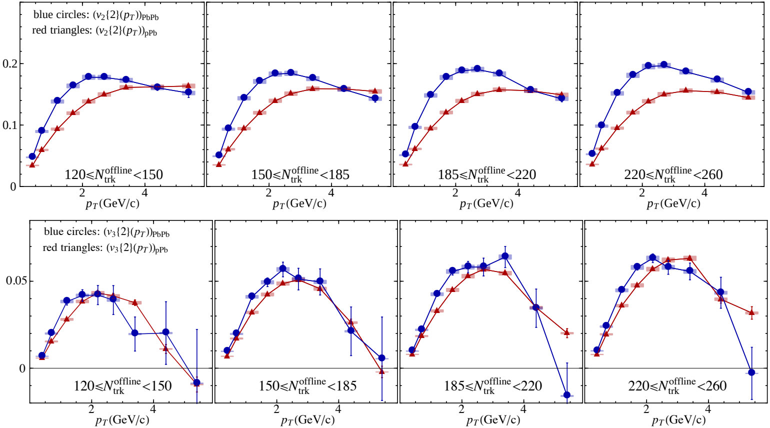
v_2 and v_3



$|\Delta\eta| > 2$, $0.3 < p_T < 3GeV$, PbPb: 2.76 TeV, pPb: 5.02 TeV

[CMS, PLB 724 213]

Transverse momentum dependence of v_2 and v_3



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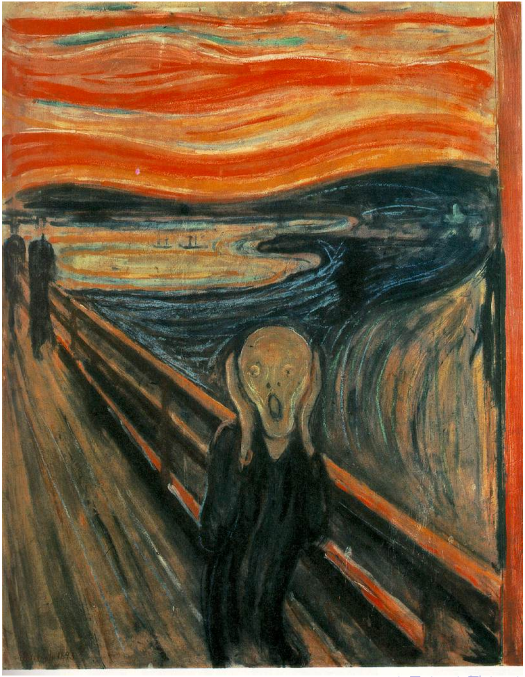


“Conformal dynamics” (as an elliptical cow approximation)

- Initial state: N_{clust} independently distributed clusters such that the multiplicity $N \propto N_{clust}$
- “Conformal dynamics”: The density of clusters sets a momentum scale: only scale other than the system size L

$$\tau_R \sim l_{mfp} \sim \frac{1}{T_i}$$

$\Rightarrow$ Universal Knudsen numbers at fixed multiplicity

$$\frac{l_{mfp}}{L} \propto \frac{1}{T_i L} = f\left(\frac{dN}{dy}\right)$$

$\Rightarrow$ The pA system is smaller but hotter

- Flow emerges as a collective response to the geometry:

$$v_{2,3} = \underbrace{k_{2,3}(l_{mfp}/L)}_{\text{response coefficient}} \times \underbrace{\epsilon_{2,3}}_{\text{geometry}}$$

(e.g. saturation inspired model: $N_{clust} = \pi Q_s^2 L^2 \Rightarrow \frac{l_{mfp}}{L} \propto \frac{1}{Q_s L} \propto \frac{1}{\sqrt{dN/dy}}$)

Linear response + conformal dynamics:

$$v_2 = k_2(dN/dy)\epsilon_2 \qquad v_3 = k_3(dN/dy)\epsilon_3$$

How different are the geometries?

Independent cluster model [Bhalero, Ollitrault]

- Distribution of clusters:

$$n(\boldsymbol{x}) = \bar{n}(\boldsymbol{x}) + \delta n(\boldsymbol{x}) \quad , \quad \langle \delta n(\boldsymbol{x}) \delta n(\boldsymbol{y}) \rangle = \bar{n}(\boldsymbol{x}) \delta^{(2)}(\boldsymbol{x} - \boldsymbol{y})$$

- Flow is sourced both by
  - average geometry $\bar{n}(\boldsymbol{x})$
  - fluctuations $\delta n(\boldsymbol{x})$

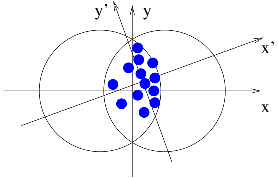


figure: Bhalerao, Ollitrault, nucl-th/0607009

- $\epsilon_2 \to v_2$: driven by average geometry and fluctuations (AA)
  fluctuations (pA)
- $\epsilon_3 \to v_3$: driven by fluctuations (AA and pA)

Eccentricity and elliptic flow

- Linear response: $v_2 = k_2 \sqrt{\langle \epsilon_2^2 \rangle}$
- Conformal scaling: $k_{2,pA} = k_{2,AA} \equiv k_2(dN/dy)$
- Eccentricity in non-central AA

$$(\epsilon_2\{2\})^2_{AA} = \epsilon_s^2 + \langle \delta\epsilon_2^2 \rangle$$

- Eccentricity in pA:

$$\begin{aligned} (\epsilon_2\{2\})^2_{pA} &= \langle \delta\epsilon_2^2 \rangle \\ \langle \delta\epsilon_2^2 \rangle &= \frac{\langle r^4 \rangle}{N_{\text{clust}} \langle r^2 \rangle^2} \end{aligned}$$

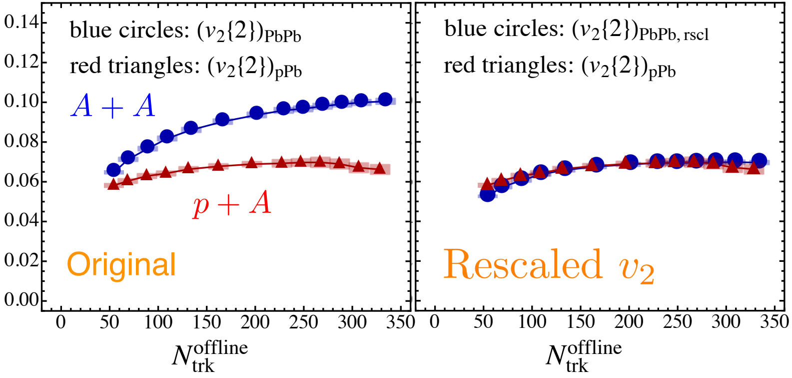
Eccentricity and elliptic flow

- In order to compare the elliptic flow in pPb and PbPb justly one should "remove" the overall geometry from AA and isolate the fluctuation driven part:

$$\Rightarrow (v_2\{2\})_{\mathrm{PbPb,rscl}} \equiv \sqrt{\frac{\langle \delta\epsilon_2^2 \rangle}{(\epsilon_2\{2\})^2_{PbPb}}} (v_2\{2\})_{\mathrm{PbPb}}$$

- Conformal dynamics suggest that $(v_2\{2\})_{\mathrm{PbPb,rscl}} = (v_2\{2\})_{\mathrm{pPb}}$ at the same multiplicity

Eccentricity and elliptic flow



- The scaling factor $\sqrt{\frac{\langle \delta\epsilon_2^2 \rangle}{(\epsilon_2\{2\})^2_{PbPb}}}$ is a *nontrivial* function of multiplicity and is calculated by Glauber model (not a fit!).
- No fine tuning!

Eccentricity and elliptic flow

- Don't know the cluster distribution for pA.
  Does it matter??

Eccentricity and elliptic flow

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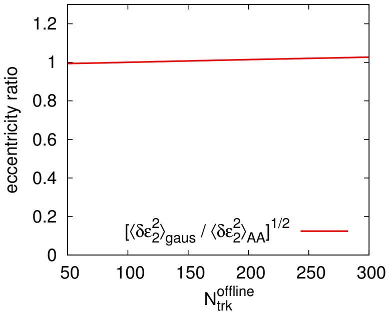
Eccentricity and elliptic flow

- Don't know the cluster distribution for pA.
  Does it matter?? NO!
- Two very different distributions: $\sqrt{\frac{\langle \delta\epsilon_2^2 \rangle_{\text{hard-sphere}}}{\langle \delta\epsilon_2^2 \rangle_{\text{Gaussian}}}} \approx 0.85$
- Gaussian seems plausible. Compare with nuclear geometry

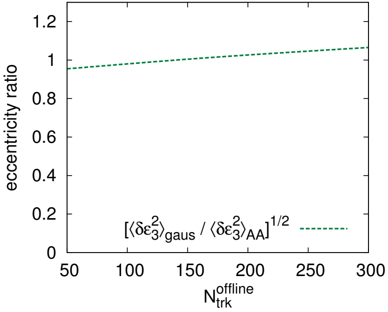


Triangular flow

- Linear response: $v_3 = k_3\sqrt{\langle\epsilon_3^2\rangle}$
- Conformal scaling: $(v_3\{2\})_{pA} = k_3\sqrt{\langle\delta\epsilon_3^2\rangle_{pA}} \approx (v_3\{2\})_{AA} = k_3\sqrt{\langle\delta\epsilon_3^2\rangle_{AA}}$

$$\langle\delta\epsilon_3^2\rangle = \frac{\langle r^6\rangle}{N_{\rm clust}\langle r^2\rangle^3}$$

- Compare $\langle\delta\epsilon_3^2\rangle_{pA}$ with that of nuclear geometry



Triangular flow

Expect v_3s to be the same.

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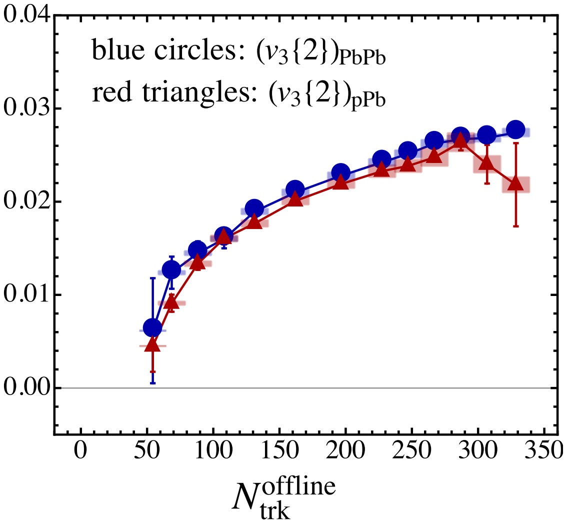


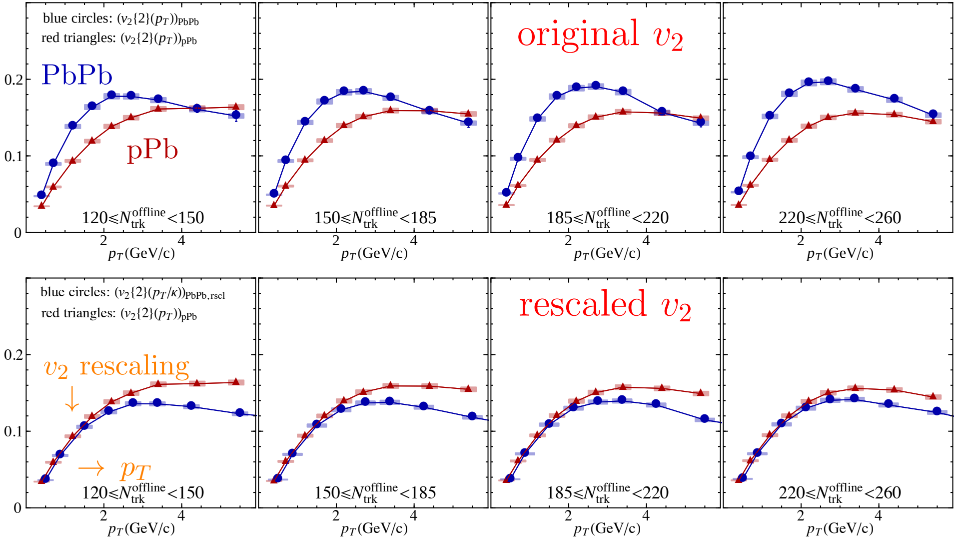
Transverse momentum dependence of the flow

- Scaling argument (dictated by "conformal dynamics"):

$$v_2(p_T) = \underbrace{\xi_2}_{\text{response coef.}} \times \underbrace{\epsilon_2}_{\text{geometry}} \times \underbrace{f_2\left(\frac{p_T}{\langle p_T\rangle}\right)}_{\text{universal function at fixed dN/dy}}$$

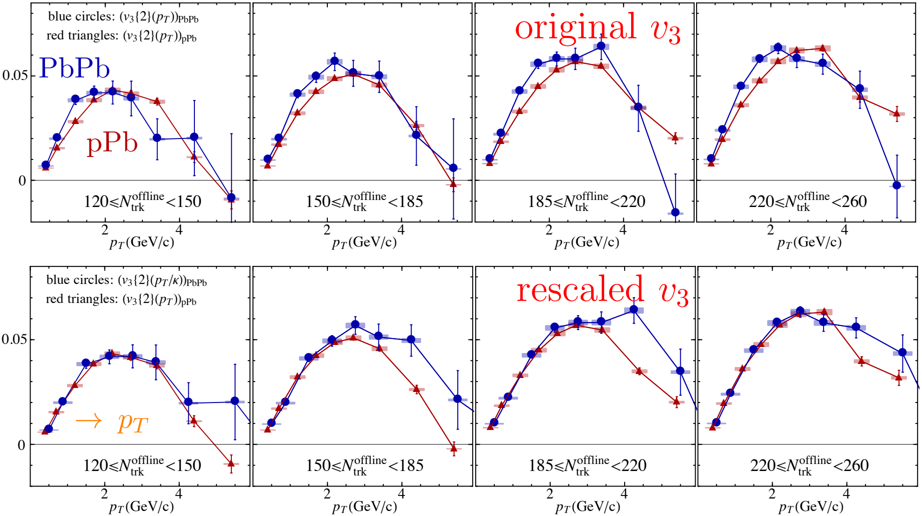
- Input: $\frac{\langle p_T\rangle_{\mathrm{pPb}}}{\langle p_T\rangle_{\mathrm{PbPb}}} \simeq 1.25$ (ALICE, arXiv:1307.1094)
- Expect:
  - $\frac{L_{PbPb}}{L_{pPb}} = \frac{T_{i\,pPb}}{T_{i\,PbPb}} \simeq 1.25$ (pA is smaller and hotter)
  - $\left[v_2\{2\}(p_T)\right]_{pPb} = \left[v_2\{2\}\left(\frac{p_T}{\kappa}\right)\right]_{PbPb,\mathrm{rscl}}$

Scaling of $v_2(p_T)$



Notice the slopes at small p_T!

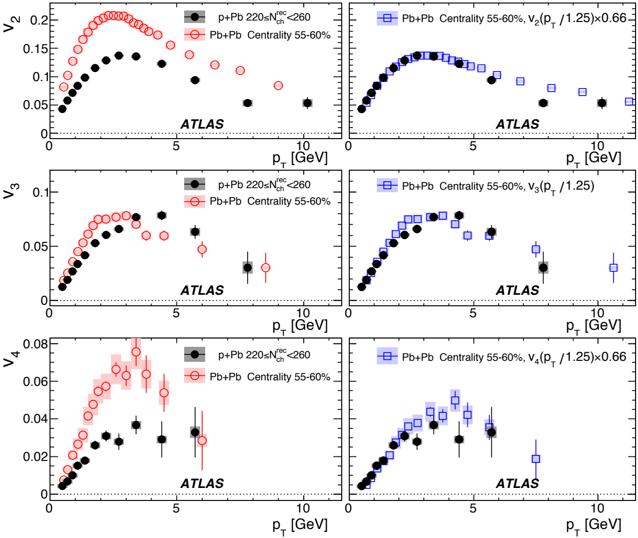
Scaling of $v_3(p_T)$



Notice the slopes at small p_T!

• ATLAS recently adopted and extended our analysis

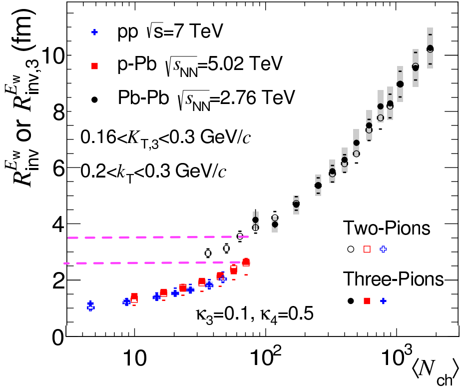
recoil subtraction ⇒ remarkable agreement even at larger p_T!



[ATLAS, PRC 90, 044906]

HBT radius

- The recent ALICE measurement reveals that $\frac{R_{PbPb}}{R_{pPb}} \simeq 1.4$ at the highest multiplicity measured (ALICE, arXiv:1404.1194)

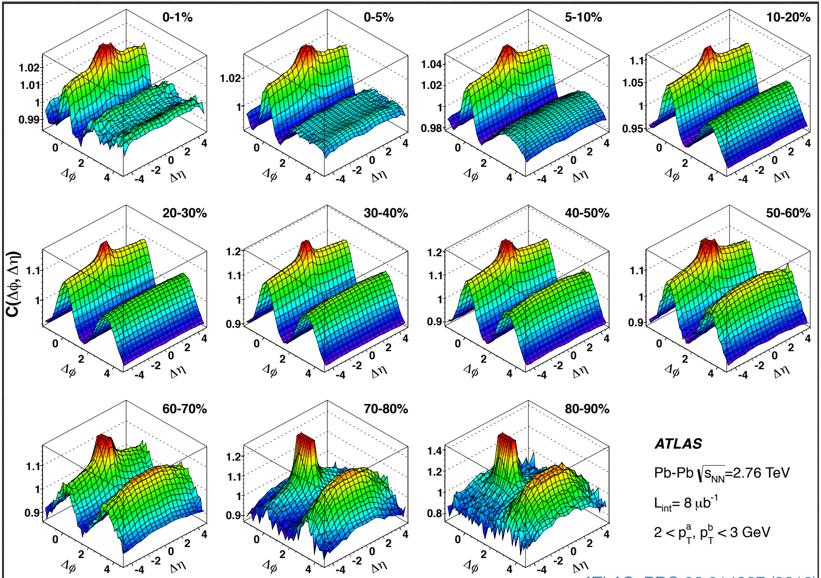


- Compare with the conformal scaling result $\frac{L_{AA}}{L_{pA}} \simeq 1.25$

Conclusions

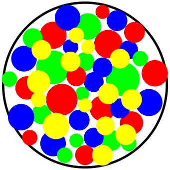
- The similarities *as well as* the differences between the high multiplicity pA and AA can be explained in a *quantitative* fashion by a simple conformal scaling framework.
- Universal Knudsen number (l_{mfp}/L) at fixed multiplicity (pA is smaller but hotter).
- No need to fine tune parameters.
- It seems phenomenologically reasonable to conclude that the flow in pA and AA stem from the same physics.
- Not necessarily hydrodynamics, viscous corrections can be large.

Flow in AA



ATLAS, PRC 86 014907 (2012)

- eg. saturation inspired model (early times):



[fig: L. McLerran]

  - Cluster density $\leftrightarrow$ saturation momentum: $Q_s^2 = \frac{N_{clust}}{\pi L^2}$
  - Mean free path, relaxation time (at early times): $\tau_R \sim \frac{l_{mfp}}{L} \propto \frac{1}{Q_s L} = \frac{1}{\sqrt{dN/dy}}$

- eg. Bjorken expansion (later times):
  - For flow a more relevant scale is $\tau \sim L$
  - Viscous corrections, etc: $\frac{l_{mfp}}{L} \propto \frac{1}{T(\tau)L} \propto \frac{1}{(dN/dy)^{1/3}}$
  - Consistent with more complicated hydro models

Jet energy loss heuristics (à la BDMPS)

- Different scales are involved:
  - Formation length, $l_{form} \propto \frac{\omega}{k_\perp^2}$
  - Mean free path, l_{mfp}
  - System size L
- Transverse momentum is accumulated by random walk, $\hat{q} \equiv d\langle k_\perp^2 \rangle / dt$
- Depending ω of radiated gluon, spectrum is different
  - $\omega \frac{\mathrm{d}N_g}{\mathrm{d}\omega\,\mathrm{d}z} \sim \frac{\alpha_\mathrm{s}}{\ell_\mathrm{mfp}}$ $\quad (\omega < \hat{q}\,\ell_\mathrm{mfp}^2)$ (Bethe-Heitler)
  - $\omega \frac{\mathrm{d}N_g}{\mathrm{d}\omega \mathrm{d}z} \sim \alpha_\mathrm{s}\sqrt{\frac{\hat{q}}{\omega}}$ $\quad (\hat{q}\,\ell_\mathrm{mfp}^2 < \omega < \hat{q}\,L^2)$ (LPM)
  - $\omega \frac{\mathrm{d}(\Delta N_g)}{\mathrm{d}\omega} \sim \alpha_\mathrm{s} \frac{(\hat{q}L^2)^2}{\omega^2}$ $\quad (\omega > \hat{q}\,L^2)$ ("deep LPM")

Jet energy loss heuristics for pA

- Depending on the energy, E, of the hard parton the total energy loss is:

$$\Delta E \sim \alpha_s \sqrt{E\hat{q}}\, L \quad (E < \hat{q}\, L^2) \quad , \quad \Delta E \sim \alpha_s\, \hat{q} L^2 \quad (E > \hat{q}\, L^2)$$

- Conformal scaling: $\hat{q}_{pA} = \kappa^3 \hat{q}_{AA}$
- Semi-qualitative predictions:
  - Larger transverse momentum broadening in pA
  - The transition from $\Delta E \propto L$ regime to L^2 regime requires a *larger* parton energy!