

Fluctuations of conserved charges
and freeze-out conditions in heavy ion collisions

Claudia Ratti

University of Houston, Texas (USA)

S. Borsanyi, Z. Fodor, S. Katz, S. Krieg, C. R., K. Szabo, PRL 2014

Motivation

- ❖ Synergy between fundamental theory and experiment
- ❖ We can create the **deconfined phase of QCD** in the laboratory
- ❖ Lattice QCD simulations have reached unprecedented levels of accuracy
 - ➡ physical quark masses
 - ➡ several lattice spacings → continuum limit
- ❖ Can we learn something about hadronization from the synergy between **fundamental theory** and **experiment**?

The observables: fluctuations of conserved charges

- ❖ They can be calculated **on the lattice** as combinations of **quark number susceptibilities**
- ❖ They can be compared to experimental measurements (with some caveats)
- ❖ The chemical potentials are related:

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q;$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q;$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.$$

- ❖ susceptibilities are defined as follows:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p / T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

Relating lattice results to experimental measurement

❖ we can relate susceptibilities to moments of multiplicity distributions:

$$\text{mean : } M = \chi_1$$

$$\text{variance : } \sigma^2 = \chi_2$$

$$\text{skewness : } S = \chi_3 / \chi_2^{3/2}$$

$$\text{kurtosis : } \kappa = \chi_4 / \chi_2^2$$

$$S\sigma = \chi_3 / \chi_2$$

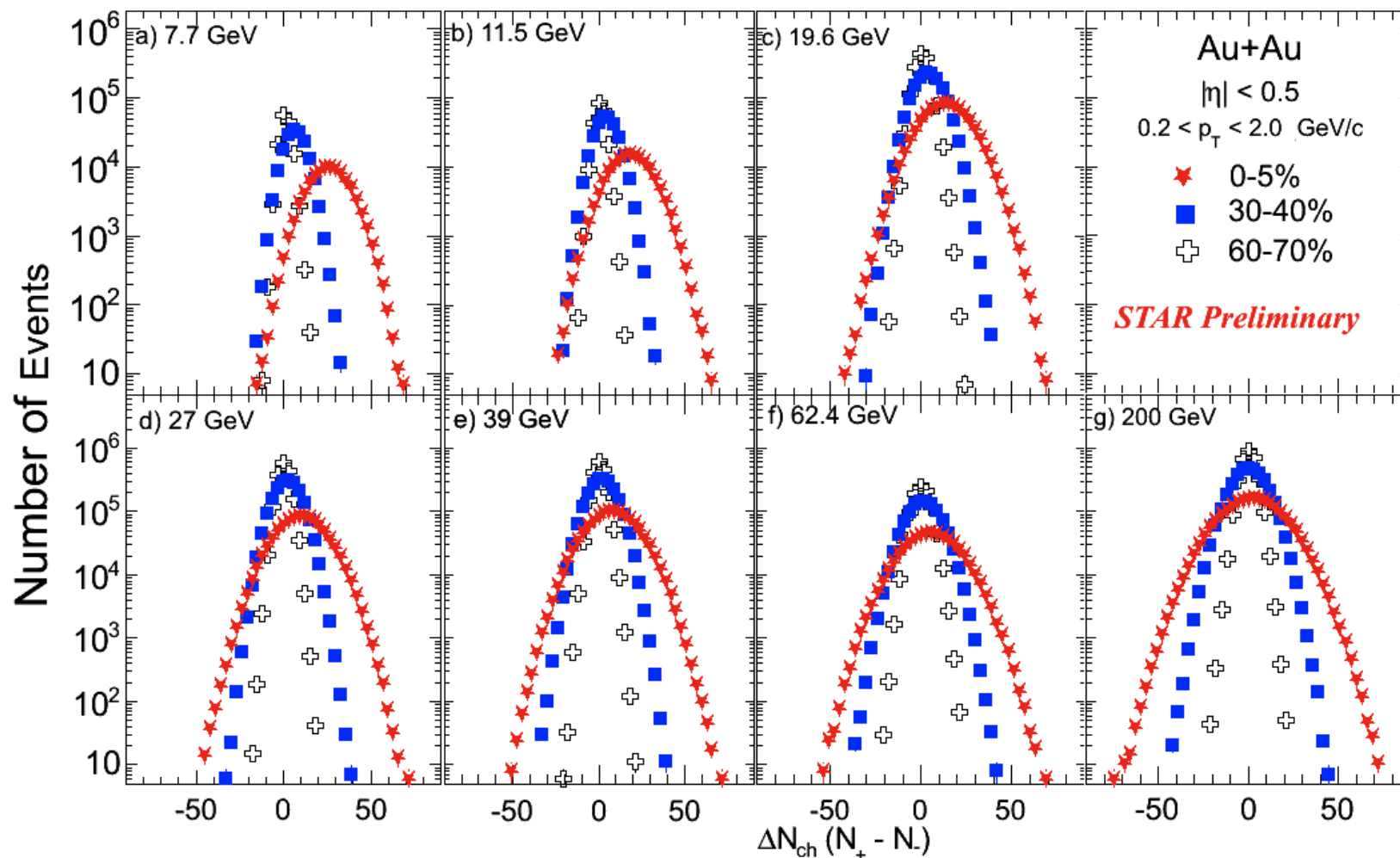
$$\kappa\sigma^2 = \chi_4 / \chi_2$$

$$M/\sigma^2 = \chi_1 / \chi_2$$

$$S\sigma^3 / M = \chi_3 / \chi_1$$

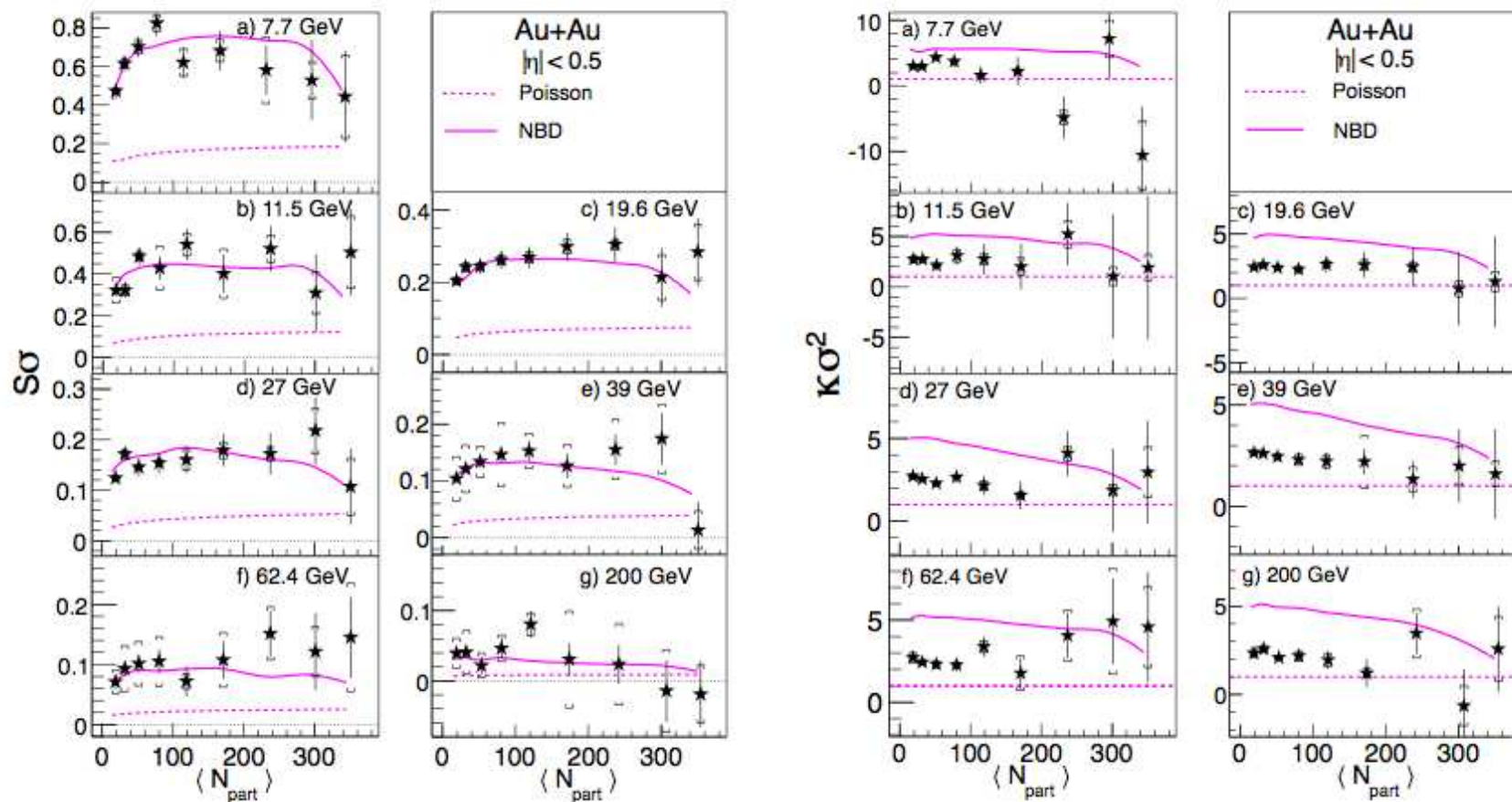
F. Karsch (2012)

Experimental measurement I



Star Collaboration: [arXiv 1212.3892](https://arxiv.org/abs/1212.3892)

Experimental measurement II



Star Collaboration: PRL 2014

Caveats

- ❖ Effects due to volume variation because of finite centrality bin width [V. Skokov, B. Friman, K. Redlich, PRC \(2013\)](#)
- ❖ Finite reconstruction efficiency
- ❖ Spallation protons
- ❖ Canonical vs Grand Canonical ensemble
- ❖ Proton multiplicity distributions vs baryon number fluctuations
- ❖ Final-state interactions in the hadronic phase [J. Steinheimer *et al.*, PRL \(2013\)](#)

Caveats

- ❖ Effects due to volume variation because of finite centrality bin width [V. Skokov, B. Friman, K. Redlich, PRC \(2013\)](#)
 - ➡ Experimentally corrected by centrality-bin-width correction method
- ❖ Finite reconstruction efficiency
 - ➡ Experimentally corrected based on binomial distribution [A. Bzdak, V. Koch, PRC \(2012\)](#)
- ❖ Spallation protons
 - ➡ Experimentally removed with proper cuts in p_T
- ❖ Canonical vs Grand Canonical ensemble
 - ➡ Experimental cuts in the kinematics and acceptance [V. Koch, S. Jeon, PRL \(2000\)](#)
- ❖ Proton multiplicity distributions vs baryon number fluctuations
 - ➡ Numerically very similar once protons are properly treated [M. Asakawa and M. Kitazawa, PRC \(2012\)](#), [M. Nahrgang *et al.*, 1402.1238](#)
- ❖ Final-state interactions in the hadronic phase [J. Steinheimer *et al.*, PRL \(2013\)](#)
 - ➡ Consistency between different charges = fundamental test

Relations between chemical potentials

❖ μ_B , μ_S and μ_Q are NOT independent:

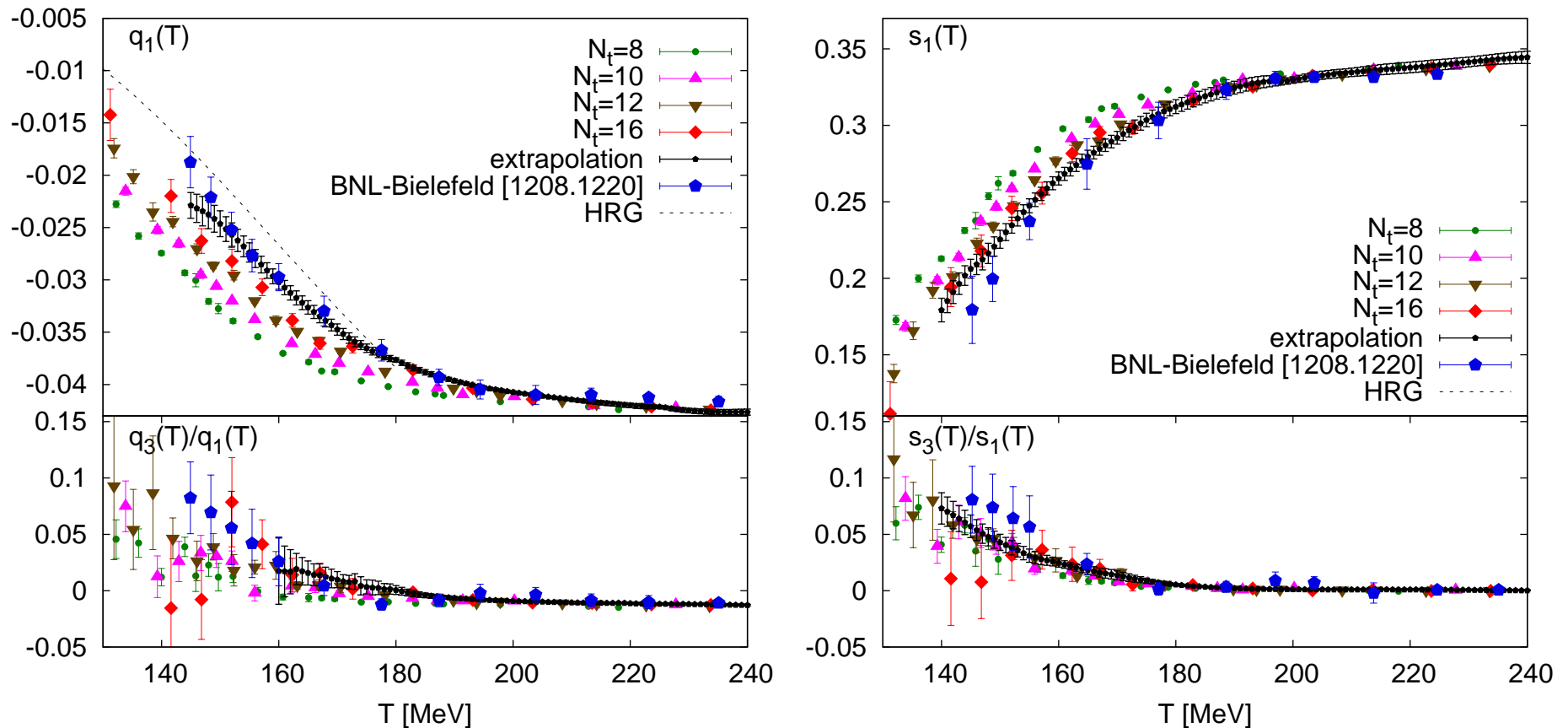
$$\langle n_S \rangle = 0 \quad \langle n_Q \rangle = \frac{Z}{A} \langle n_B \rangle \quad \Rightarrow \quad \frac{Z}{A} = 0.4$$

❖ By expanding n_B , n_S and n_Q up to μ_B^3 we get:

$$\mu_Q(T, \mu_B) = q_1(T)\mu_B + q_3(T)\mu_B^3 + \dots$$

$$\mu_S(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + \dots$$

Taylor coefficients: results



WB Collaboration: PRL (2013)

- ❖ μ_Q turns out to be very small
- ❖ Agreement between WB and BNL-Bielefeld collaborations

Thermometer and Baryometer

❖ R_{31}^B : thermometer

$$R_{31}^B(T, \mu_B) = \frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)} = \frac{\chi_4^B(T, 0) + \chi_{31}^{BQ}(T, 0)q_1(T) + \chi_{31}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)} + \mathcal{O}(\mu_B^2)$$

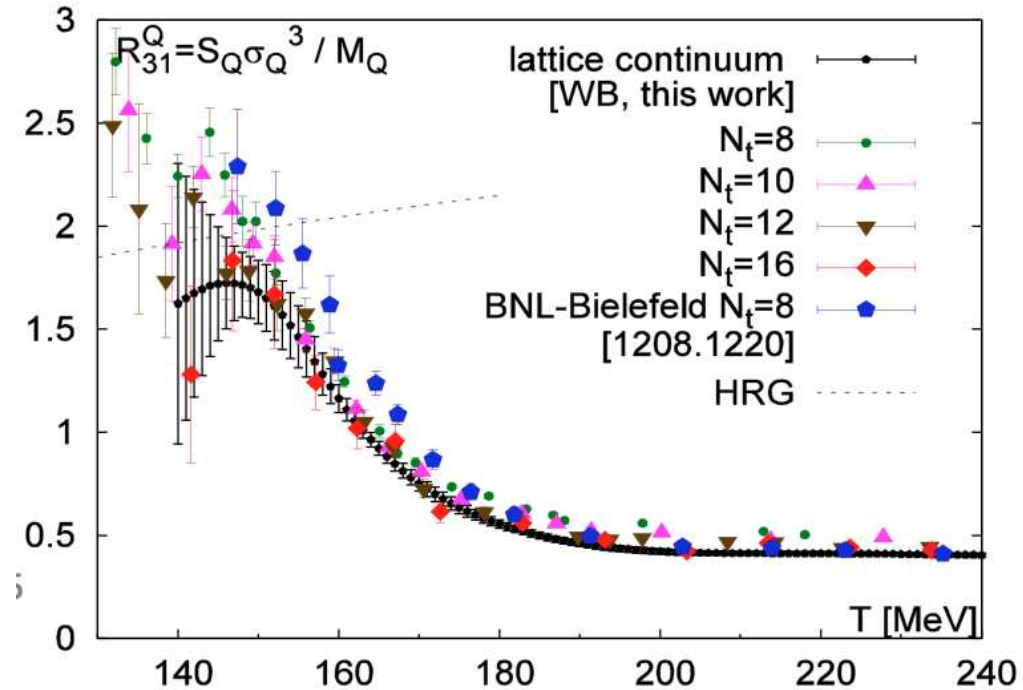
❖ Expand numerator and denominator around $\mu_B = 0$: ratio is independent of μ_B

❖ R_{12}^B : baryometer

$$R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \frac{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

❖ Expand numerator and denominator around $\mu_B = 0$: ratio is proportional to μ_B

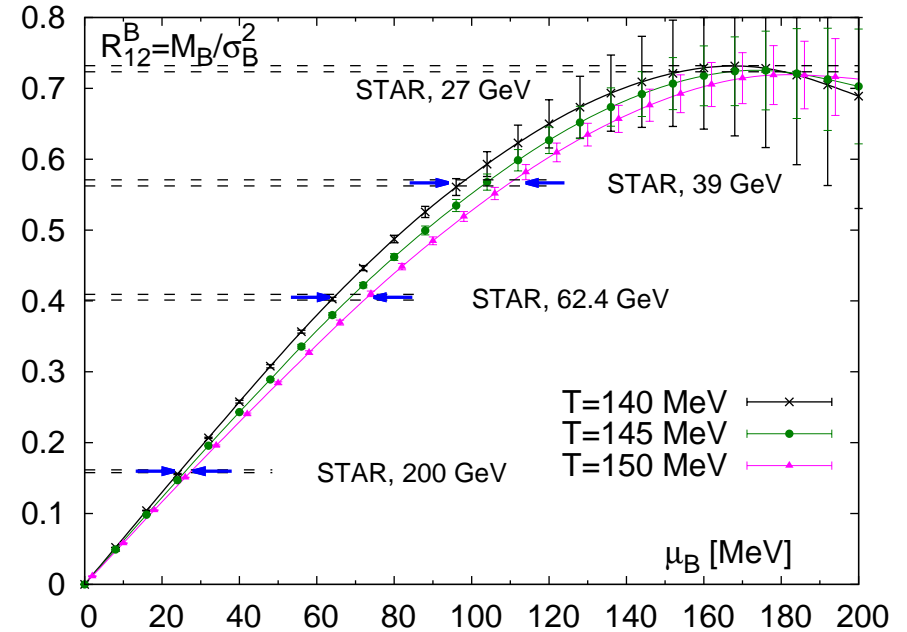
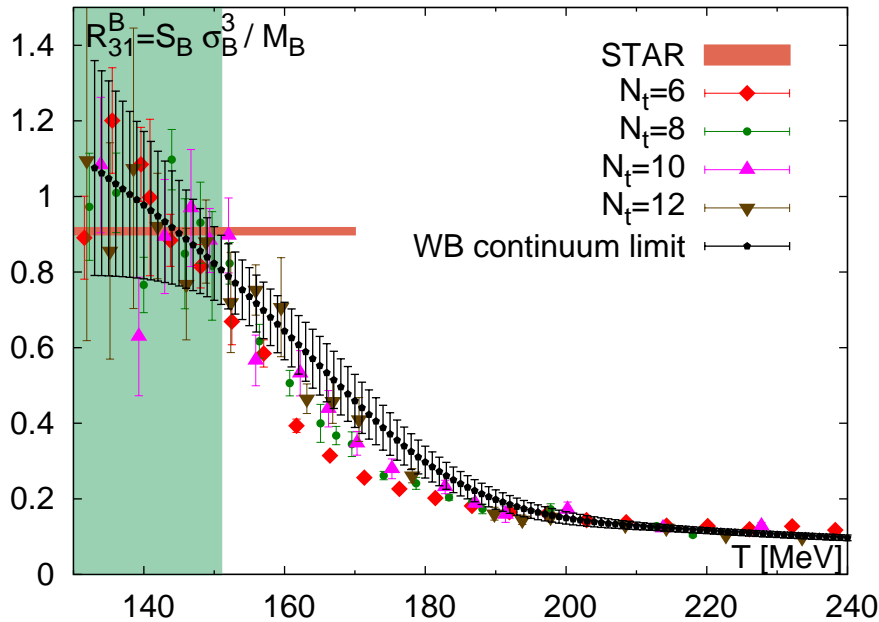
Thermometer from electric charge



WB Collaboration: PRL (2013)

- ❖ Would be **ideal observable**: less ambiguous than baryon number vs proton
- ❖ Present uncertainty in the experimental data does not allow to extract T_f

Extracting freeze-out parameters from baryon number

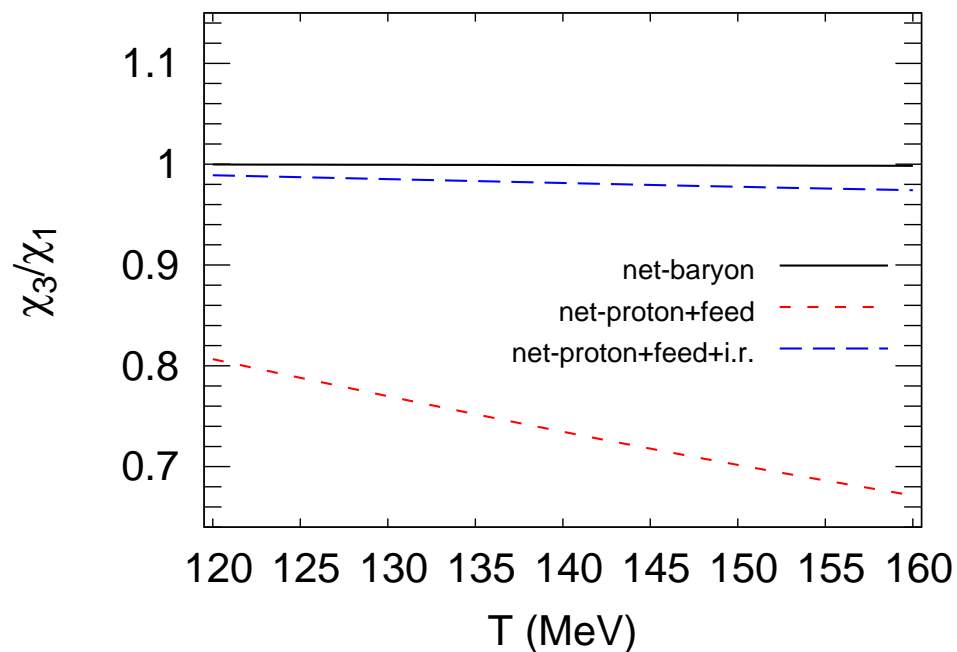


WB Collaboration: PRL (2014); STAR data from 1309.5681

Upper limit: $T_f \leq 151 \pm 4$ MeV

\sqrt{s} [GeV]	μ_B^f [MeV]
200	25.8 ± 2.7
62.4	69.7 ± 6.4
39	105 ± 11
27	-

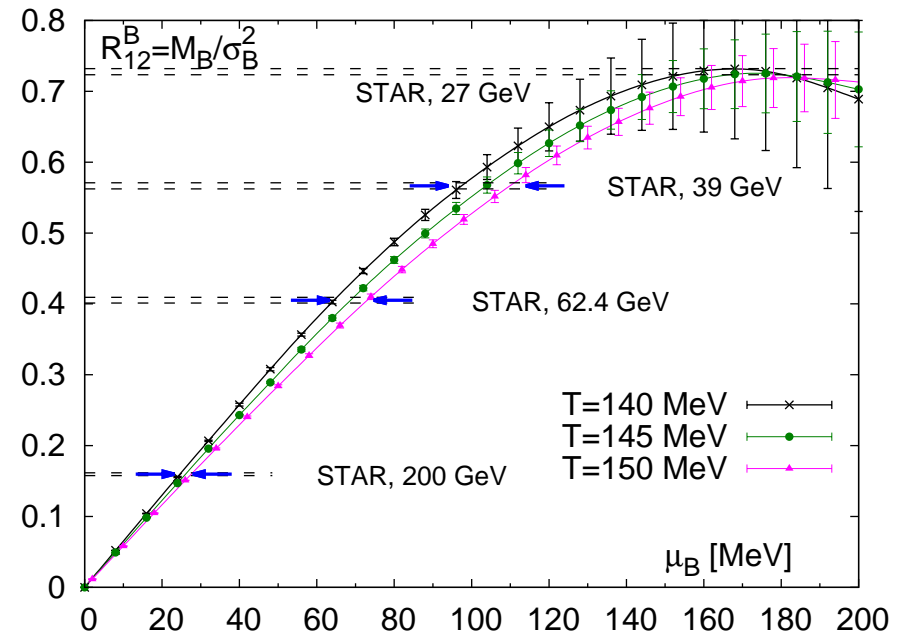
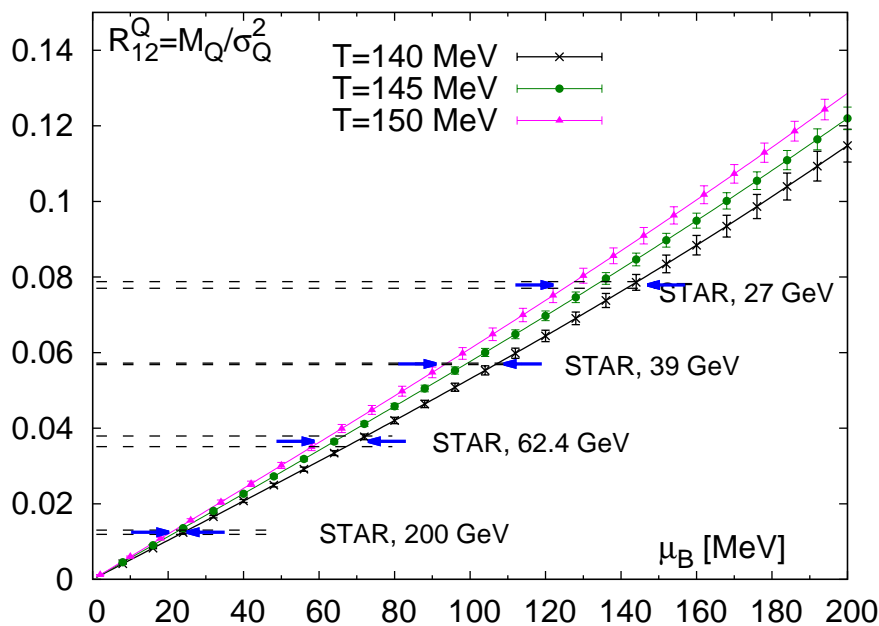
Baryon number vs protons.



M. Nahrgang, M. Bluhm, P. Alba, R. Bellwied, C. R., arXiv:1402.1238

- ❖ Study in the HRG model
- ❖ Take into account feed-down from resonance decay
- ❖ Take into account resonance regeneration \rightarrow Isospin randomization

Extracting freeze-out μ_B from electric charge

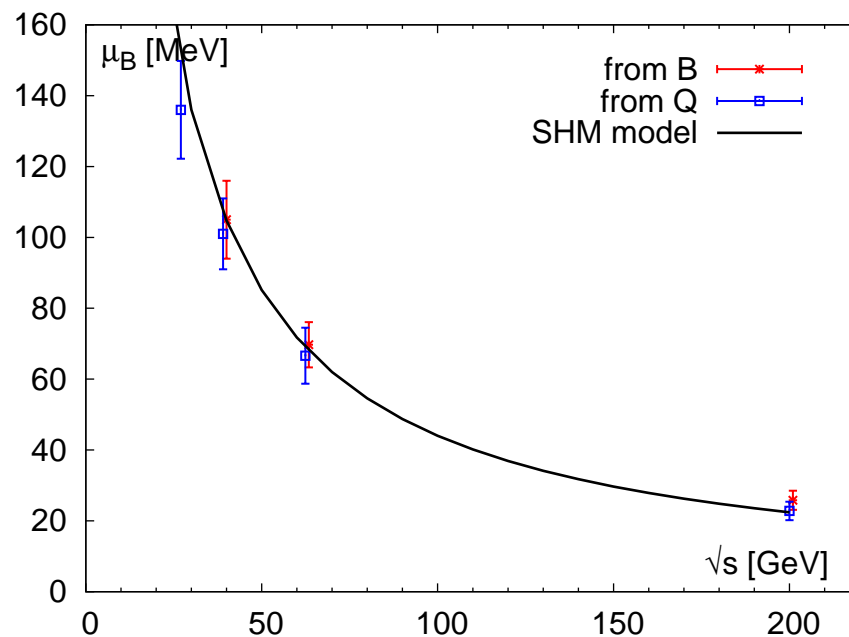


WB Collaboration: PRL (2014); STAR data from 1309.5681 and 1402.1558

- ❖ It is of fundamental importance to test the **consistency** between the freeze-out parameters obtained with **different conserved charges**
- ❖ This consistency check validates the method and shows equilibration of the medium

Consistency is found!

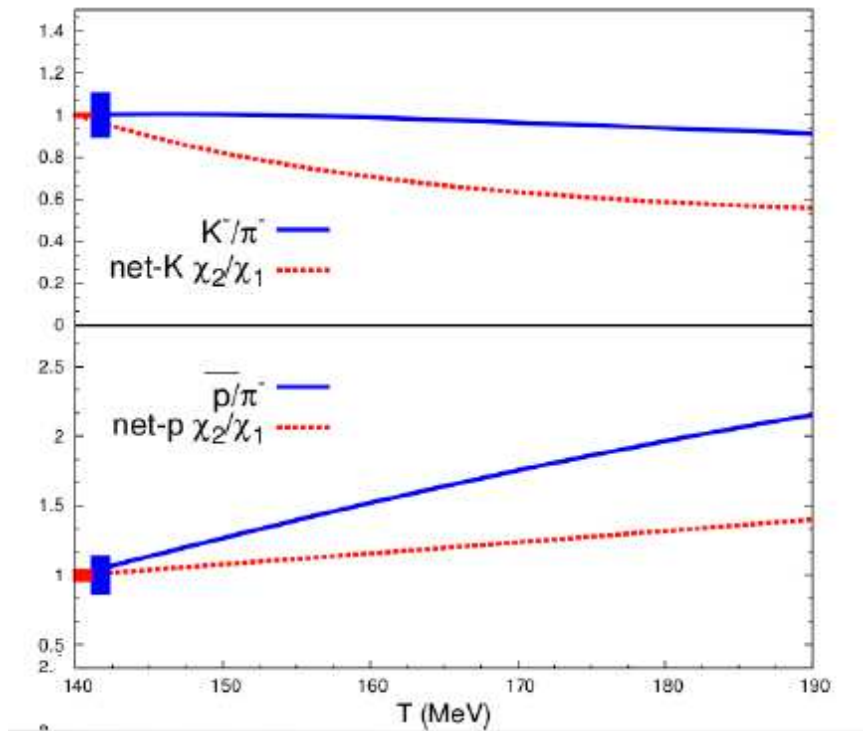
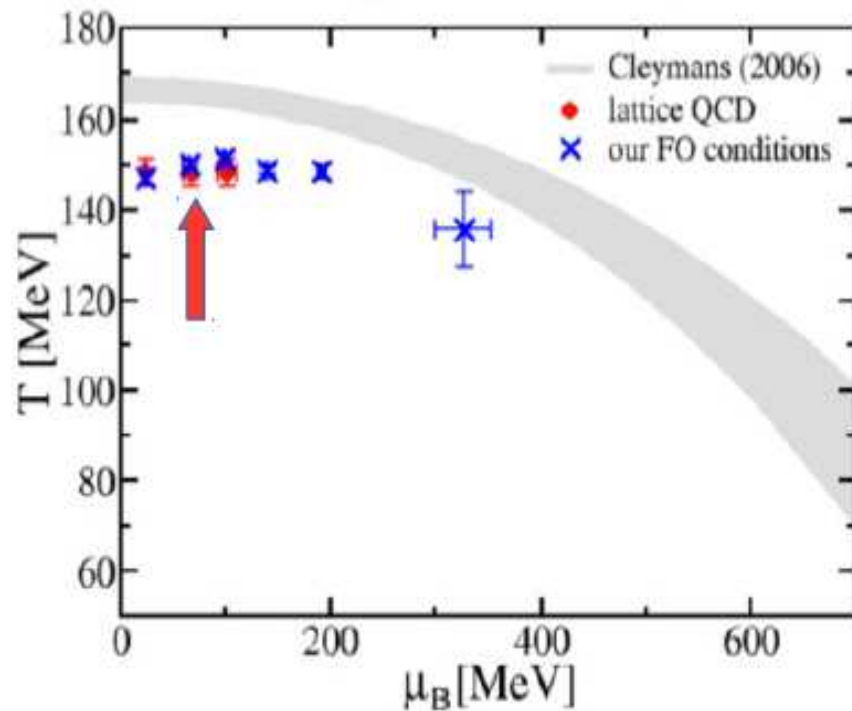
$\sqrt{s} [GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8 ± 2.7	22.8 ± 2.6
62.4	69.7 ± 6.4	66.6 ± 7.9
39	105 ± 11	101 ± 10
27	-	136 ± 13.8



Lattice: WB Collaboration: PRL (2014); SHM: Andronic *et al.*, NPA (2006)

HRG model analysis

- ❖ Experimental cuts in acceptance and momentum
- ❖ Resonance decay and regeneration

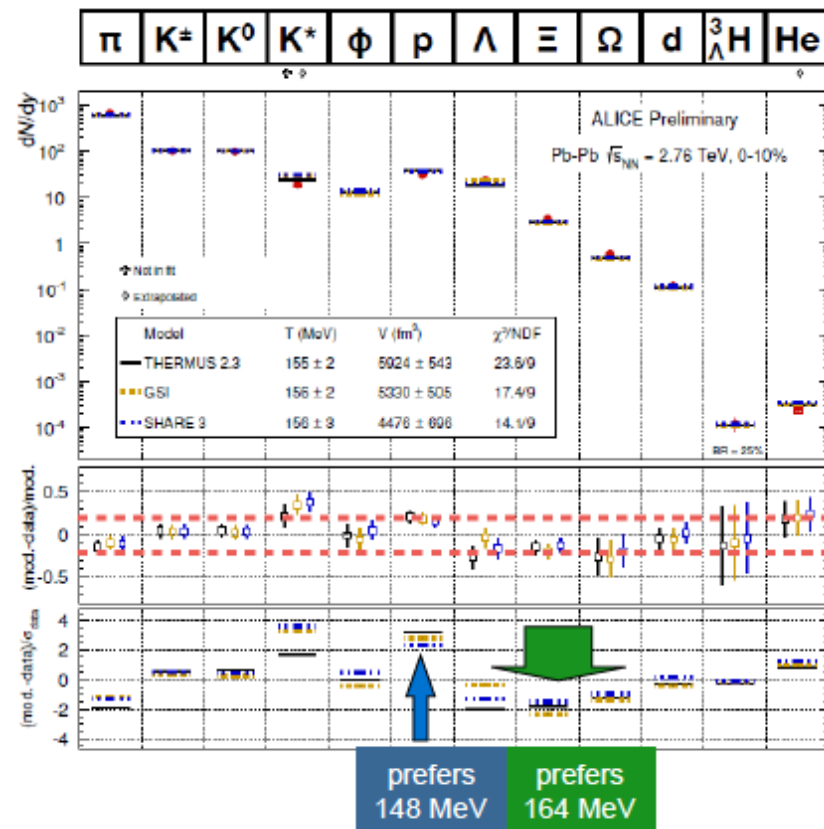
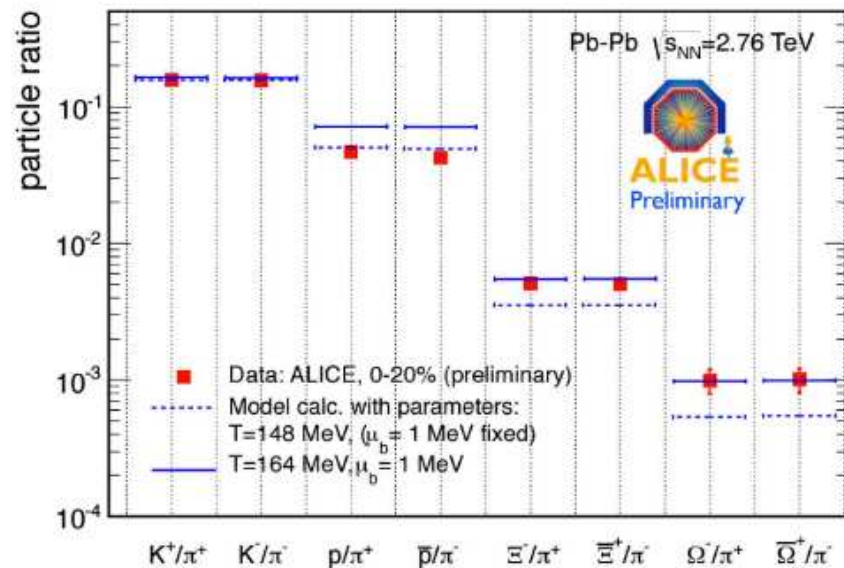


P. Alba *et al.*, PLB 2014

P. Alba *et al.*, in preparation

Freeze-out temperature from yields

- ❖ Fit to yields of identified particles: Statistical Hadronization Model (SHM)
- ❖ Model-dependent. Parameters: freeze-out **temperature** and **chemical potential**



R. Preghenella for ALICE, SQM 2012

M. Floris, QM 2014.

Fluctuations from yields

■ Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3} (\langle p \rangle + \langle N \rangle + \langle \Lambda + \Sigma_0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + \langle \Xi^- \rangle + \langle \Xi^0 \rangle + \langle \Omega^- \rangle + \overline{par})$$

■ Net strangeness

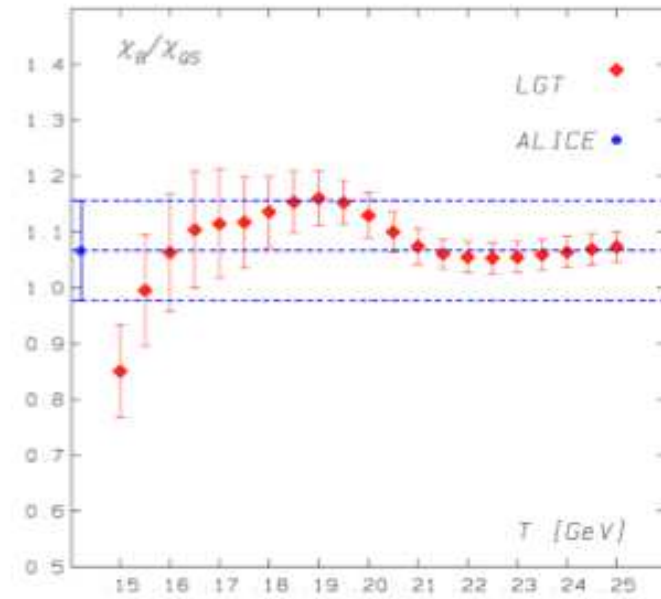
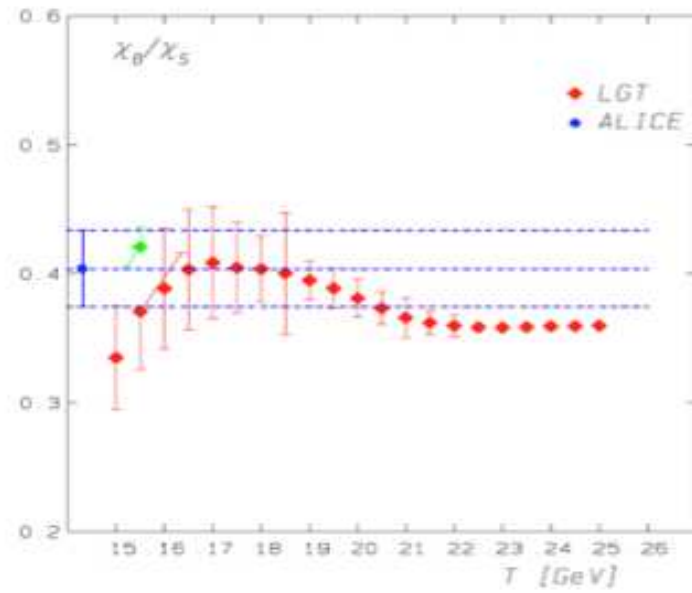
$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3} (\langle K^+ \rangle + \langle K_S^0 \rangle + \langle \Lambda + \Sigma_0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + 4\langle \Xi^- \rangle + 4\langle \Xi^0 \rangle + 9\langle \Omega^- \rangle + \overline{par} \\ - (\Gamma_{\varphi \rightarrow K^+} + \Gamma_{\varphi \rightarrow K^-} + \Gamma_{\varphi \rightarrow K_S^0} + \Gamma_{\varphi \rightarrow K_L^0}) \langle \varphi \rangle)$$

■ Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3} (\langle K^+ \rangle + 2\langle \Xi^- \rangle + 3\langle \Omega^- \rangle + \overline{par} \\ - (\Gamma_{\varphi \rightarrow K^+} + \Gamma_{\varphi \rightarrow K^-}) \langle \varphi \rangle - (\Gamma_{K_0^* \rightarrow K^+} + \Gamma_{K_0^* \rightarrow K^-}) \langle K_0^* \rangle)$$

K. Redlich *et al.*, 2014

Fluctuations from yields



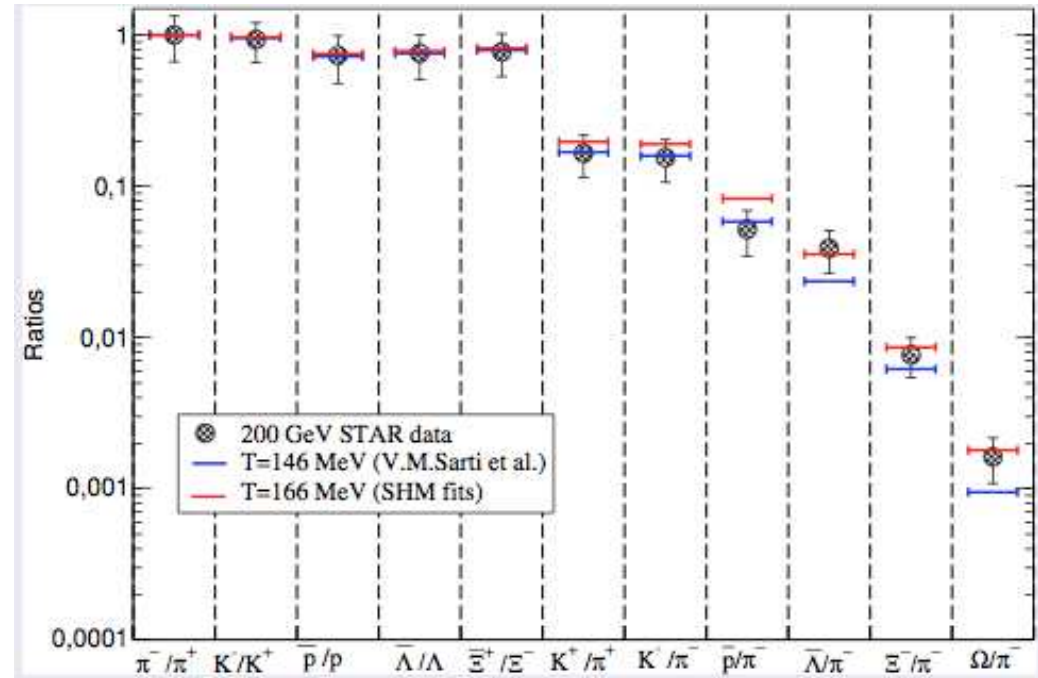
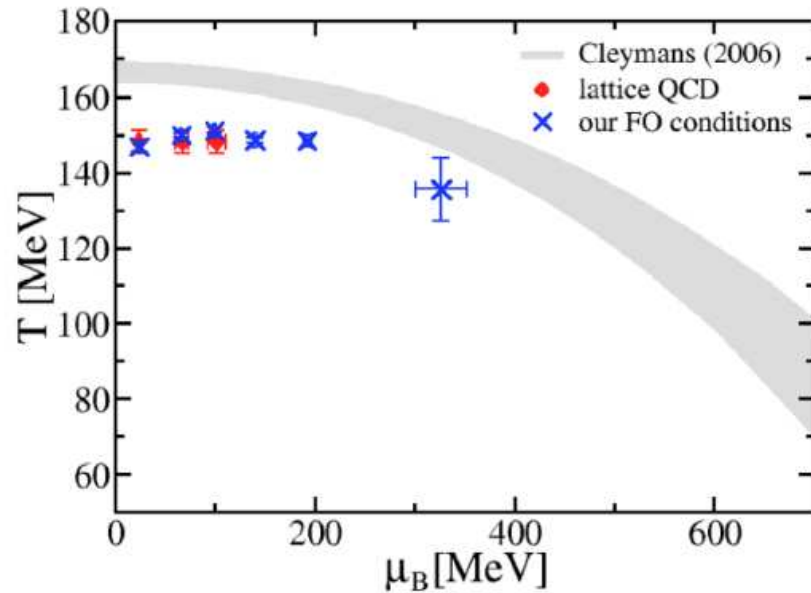
K. Redlich *et al.*, 2014

Conclusions

- ❖ It is possible to extract freeze-out parameters from first principles
- ❖ Higher order fluctuations of baryon number:
 - ⇒ $R_{31}^B(T, \mu_B) = \frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)}$: Thermometer
 - ⇒ $R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)}$: Baryometer
- ❖ Higher order fluctuations of electric charge:
 - ⇒ independent measurement
 - ⇒ $R_{12}^Q(T, \mu_B) = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)}$: Baryometer
- ❖ The freeze-out parameter sets obtained from B and Q are consistent with each other
- ❖ Looking forward to strangeness fluctuation data!

HRG model and particle ratios

- Freeze-out conditions in agreement with HRG model analysis



P. Alba *et al.*, arXiv:1403.4903.

- Yields of strange particles would need a higher T_{ch}