Mie Scattering by Small Spherical Particles

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Outline

1. Motivation

1. Theories

2. Results and discussions

3. Future work

Motivation

Figure 1. Color of the sky.

Motivation

Figure 2. Gold nanoparticles of difference sizes, and hence difference colors.

Y. Richad. Optocapacitance Shines new light on the brain. 2015;. [cited 2015 May 10]. Available from: http://blogs.scientificamerican.com/

Motivation

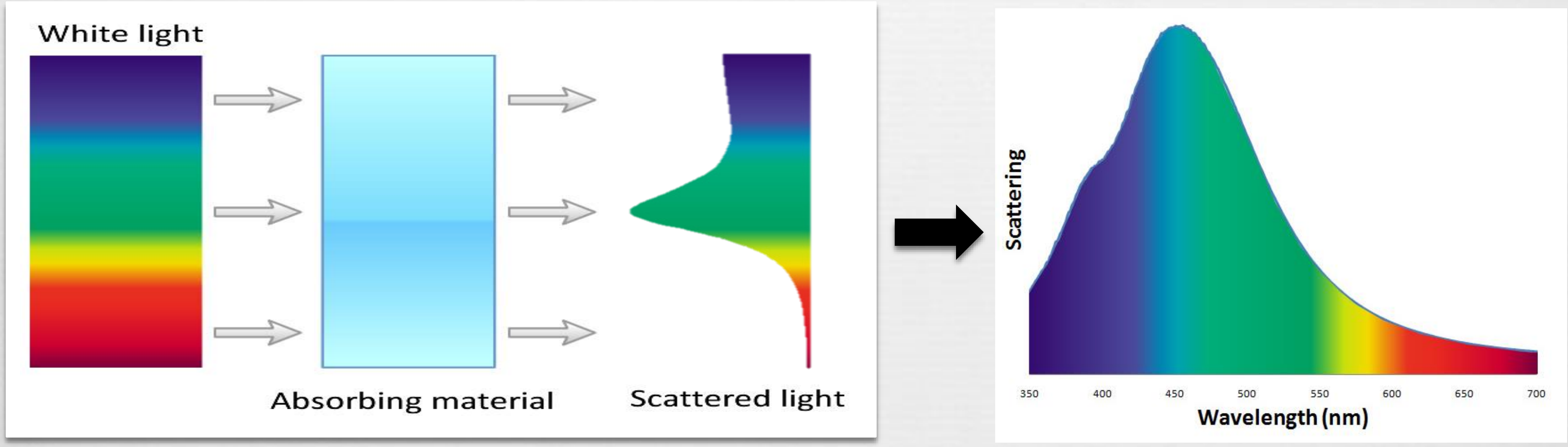


Figure 3. Spectrum of white light interacting with large silver particles.

D. Nathalie. Color engineering. 2015;. [cited 2015 May 10]. Available from: http://nanocomposix.com/

Mie Scattering

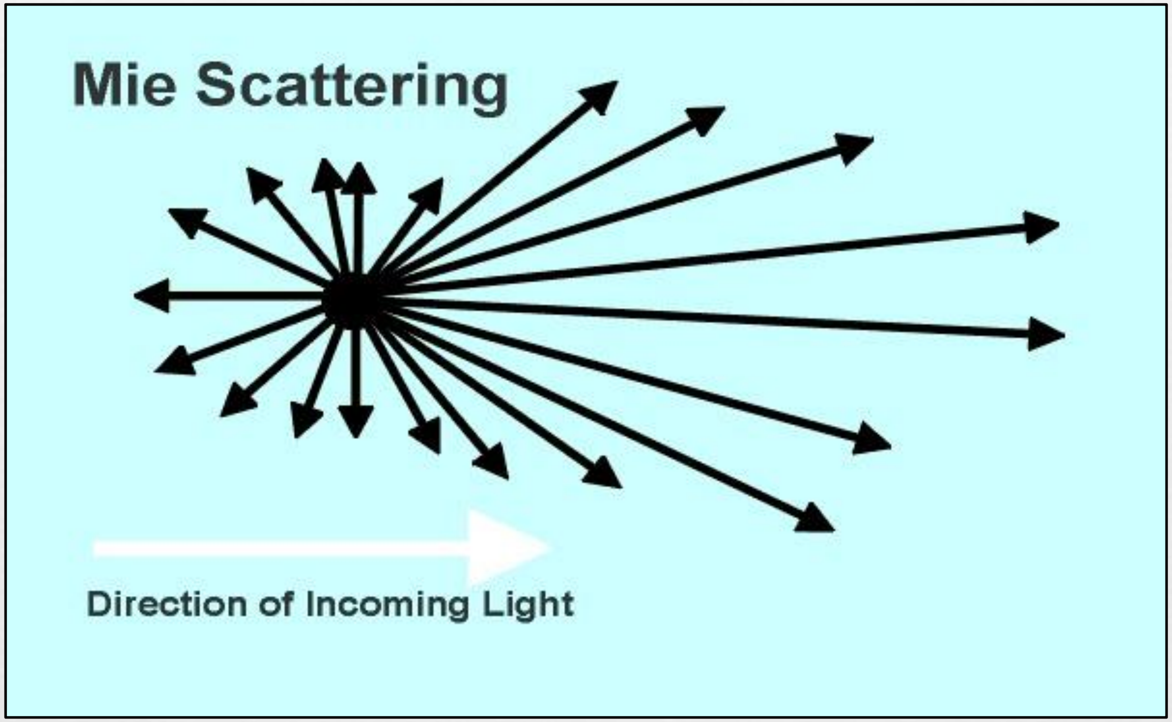


Figure 4. Pattern of Mie scattering

C. H. Keith. How many color has the sky?. 2008;. [cited 2015 May 10]. Available from: http://www.islandnet.com/

Mie scattering

Maxwell's equations for an electromagnetic wave propagating.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{E} = 0$$

ϵ_0 and μ_0 are the dielectric permittivity and magnetic permeability of the free space respectively

Mie scattering

In Mie theory, the light scattered from the spherical particles can be solved using the scalar Helmholtz equation.

$$\nabla^2\Psi + k^2 m^2 \Psi = 0 \longrightarrow 1$$

whose solution is given by

$$r\Psi(r,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{n=0}^{l}\Big[c_n\psi_n(kmr) + d_n\chi_n(kmr)\Big]\Big[P_n^l(cos\theta)\Big]\Big[a_l sin(l\phi) + b_l cos(l\phi)\Big]$$

Mie scattering

Using the solution of the Helmholtz equation, one can calculate the efficiency cross section for extinction Q_{ext} and scattering Q_{sca} of the spherical particle.

$$Q_{ext} = \frac{2}{q^2} \sum_{n=1}^{\infty} (2n+1) Re(a_n + b_n) \longrightarrow 2$$

$$Q_{sca} = \frac{2}{q^2} \sum_{n=1}^{\infty} (2n+1)(\mid a_n \mid^2 + \mid b_n \mid^2) \longrightarrow 3$$

Near the dipole scattering resonance, only the term n=1 needs to be considered.

V. I. Krasovskii, and V. A. Karavanskii, "Surface Plasmon Resonance of Metal Nanoparticles for Interface Characterization", ISSSN **17**, 2008.

Mie scattering

The scattering amplitudes a_n and b_n are defined as follows,

$$a_n = \frac{\psi_n'(kma)\psi_n(ka) - m\psi_n(kma)\psi_n'(ka)}{\psi_n'(kma)\xi_n(ka) - m\psi_n(kma)\xi_n'(ka)}$$

$$b_n = \frac{m\psi_n'(kma)\psi_n(ka) - \psi_n(kma)\psi_n'(ka)}{m\psi_n'(kma)\xi_n(ka) - \psi_n(kma)\xi_n'(ka)}$$

The refractive index m is linked to the dielectric function $\epsilon(\omega)$ via a simple relation, which is

$$\boxed{m = \sqrt{\epsilon(\omega)}} \longrightarrow 4$$

Dielectric function

In the Lorentz model, the dielectric function of non-conducting materials can be expressed as

$$\epsilon(\omega) = 1 - \frac{{\omega_p}^2}{\omega^2 + i\gamma\omega}$$

→ 5

The damping constant of electron oscillation

$$\gamma = \frac{V_f}{l_e} + \frac{2V_f}{a}$$

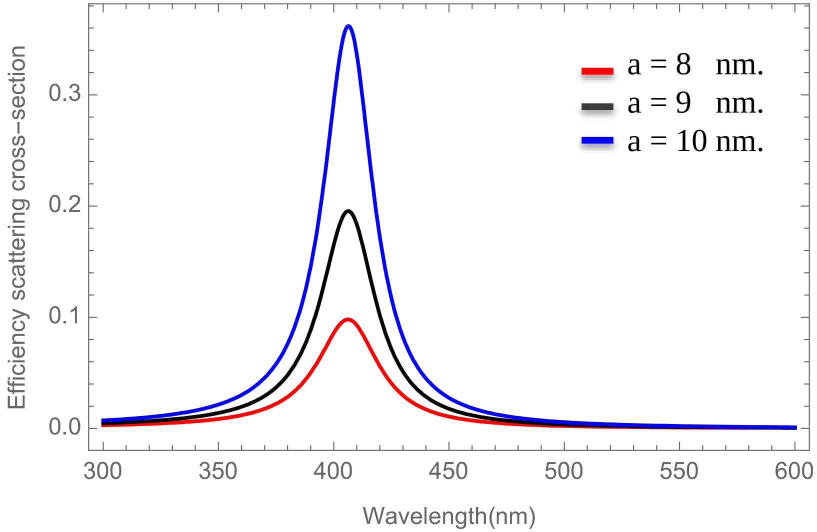
→ 6

ε = dielectric function
ω_p = plasma frequency
ω = angular frequency
γ = damping constant
V_f = Fermi velocity
l_e = mean free path of bulk metal

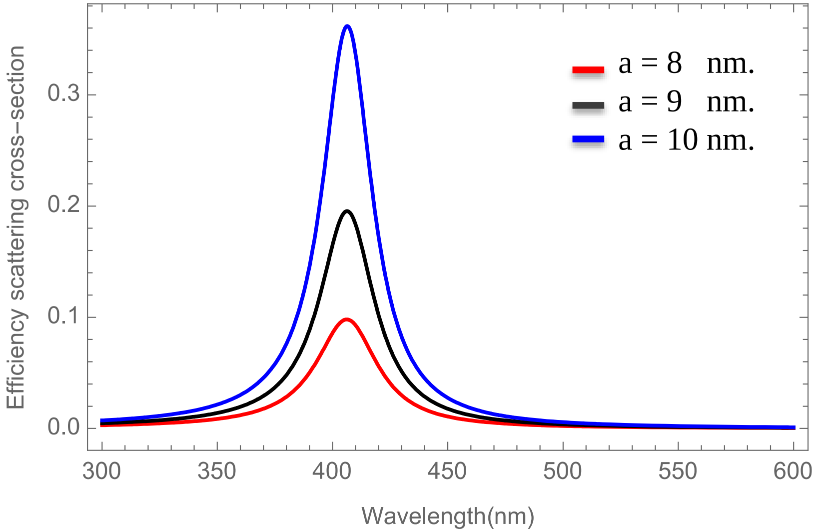
F. Xaiofeng, Z. Weitao,and J. S. David, "Light scattering and surface plasmons on small spherical particles", Science & Application **3**, 2014

Results and Discussion

Mie scattering

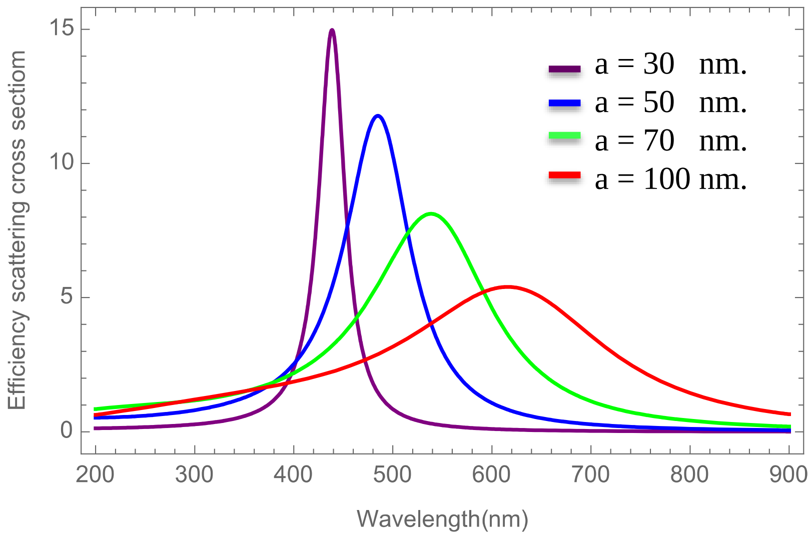


Rayleigh scattering

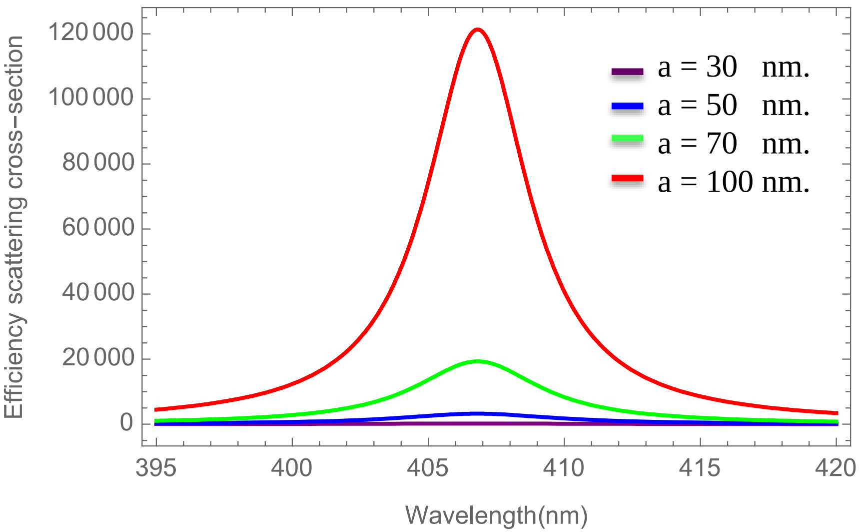


Results and Discussion

Mie scattering



Rayleigh scattering



Future Work

I. Compare this theoretical study with the experimental result and possibly this will lead to a new way to obtain the size of nano-particles

I. Change some parameters, e.g. the type of medium or type of nano-particles to study how the spectrum changes

THANK YOU

FOR

YOUR ATTENTION