N=2 Seven-Dimensional Gauged Supergravity from Eleven Dimensions

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Motivations and Backgrounds

- The AdS/CFT correspondence: recent interest in six-dimensional $N = (1,0)$ superconformal field theory (SCFT)
- Gauged supergravity:
  - $N = 2$ seven-dimensional gauged supergravity provides a gravity dual of $N = (1,0)$ SCFTs.
  - Pure $N = 2$ gauged supergravity cannot accommodate all of the possible $N = (1,0)$ SCFTs.
  - Matter-coupled gauged supergravity without topological mass term does not admit AdS_7 critical points.
- The gauged supergravity of interest here is $N = 2$ $SO(4)$ gauged supergravity in seven dimensions with topological mass term for the three-form field.

Eleven-dimensional supergravity

- Field content: (e_M^A, Ψ_M, A_{MNP}), $M, N = 0, \dots, 10$
- Bosonic action:

$$S = \frac{1}{2\kappa^2}\int d^{11}x \left[R * \mathbb{I} - *F \wedge F - \frac{\sqrt{2}}{3} F \wedge F \wedge A\right] \qquad (1)$$

with $F = dA$.

- Supersymmetry transformation to leading order in fermions:

$$\delta e_M^A = \frac{1}{2}\bar{\epsilon}\Gamma^A \Psi_M \qquad (2)$$

$$\delta A_{MNP} = -\frac{3\sqrt{2}}{4}\bar{\epsilon}\Gamma_{[MN}\Psi_{P]} \qquad (3)$$

$$\delta \Psi_M = D_M \epsilon + \frac{\sqrt{2}}{288}\left(\Gamma_M{}^{NPQR} - 8\delta_M^N \Gamma^{PQR}\right) F_{NPQR}\epsilon \qquad (4)$$

where $D_M\epsilon = \partial_\mu \epsilon + \frac{1}{4}\omega_M^{AB}\Gamma_{AB}\epsilon$.

- Field content: supergravity multiplet $(e^m_\mu, \psi^A_\mu, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma)$
  vector multiplet $(A_\mu, \lambda^A, \phi^i)$
- Gauged Lagrangian:

$$\begin{aligned}\mathcal{L} \;=\;& \frac{1}{2}R * \mathbb{I} - \frac{1}{2}e^\sigma a_{IJ} * F^I_{(2)} \wedge F^J_{(2)} - \frac{1}{2} * H_{(4)} \wedge H_{(4)} \\ & -\frac{5}{8} * d\sigma \wedge d\sigma - \frac{1}{2} * P^{ir} \wedge P_{ir} + \frac{1}{\sqrt{2}} H_{(4)} \wedge \omega_{(3)} \\ & -4hH_{(4)} \wedge C_{(3)} - V * \mathbb{I} \end{aligned} \qquad (5)$$

  where the scalar potential is given by

$$V = \frac{1}{4}e^{-\sigma}\left(C^{ir}C_{ir} - \frac{1}{9}C^2\right) + 16h^2 e^{4\sigma} - \frac{4\sqrt{2}}{3} h e^{\frac{3\sigma}{2}} C\,. \qquad (6)$$

- The potential admits a supersymmetric AdS_7 vacuum.

- Fermionic supersymmetry transformations:

$$\begin{aligned}
\delta\psi_\mu &= 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\epsilon - \frac{1}{240\sqrt{2}}e^{-\sigma}H_{\rho\sigma\lambda\tau}\times \\
&\quad \left(\gamma_\mu\gamma^{\rho\sigma\lambda\tau} + 5\gamma^{\rho\sigma\lambda\tau}\gamma_\mu\right)\epsilon \\
&\quad -\frac{i}{20}e^{\frac{\sigma}{2}}F^i_{\rho\sigma}\sigma^i\left(3\gamma_\mu\gamma^{\rho\sigma} - 5\gamma^{\rho\sigma}\gamma_\mu\right)\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon, \qquad (7)\\
\delta\chi &= -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon - \frac{i}{10}e^{\frac{\sigma}{2}}F^i_{\mu\nu}\sigma^i\gamma^{\mu\nu}\epsilon - \frac{1}{60\sqrt{2}}e^{-\sigma}H_{\mu\nu\rho\sigma}\gamma^{\mu\nu\rho\sigma}\epsilon \\
&\quad +\frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\epsilon - \frac{16}{5}e^{2\sigma}h\epsilon, \qquad (8)\\
\delta\lambda^r &= -i\gamma^\mu P^{ir}_\mu\sigma^i\epsilon - \frac{1}{2}e^{\frac{\sigma}{2}}F^r_{\mu\nu}\gamma^{\mu\nu}\epsilon - \frac{i}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\sigma^i\epsilon \qquad (9)
\end{aligned}$$

$N = 2\ SO(4)$ 7D gauged supergravity from 11 dimensions

- Reduction ansatz:

$$\begin{aligned} d\hat{s}_{11}^2 &= \Delta^{\frac{1}{3}} ds_7^2 + \frac{2}{g^2}\Delta^{-\frac{2}{3}} X^3 \left[X \cos^2\xi + X^{-4}\sin^2\xi \tilde{T}^{-1}_{\alpha\beta}\mu^\alpha\mu^\beta \right] d\xi^2 \\ &\quad -\frac{1}{g^2}\Delta^{-\frac{2}{3}} X^{-1}\tilde{T}^{-1}_{\alpha\beta}\sin\xi\mu^\alpha d\xi D\mu^\beta \\ &\quad +\frac{1}{2g^2}\Delta^{-\frac{2}{3}} X^{-1}\tilde{T}^{-1}_{\alpha\beta}\cos^2\xi D\mu^\alpha D\mu^\beta, \qquad (10) \\ \hat{F}_{(4)} &= F_{(4)}\sin\xi + \frac{1}{g} X^4 \cos\xi * F_{(4)} \wedge d\xi \\ &\quad + \frac{1}{g^3}\Delta^{-2} U \cos^3\xi d\xi \wedge \epsilon_{(3)} \end{aligned}$$

$F_{(4)}$ ansatz continue

$$
\begin{aligned}
&+\frac{1}{3!g^3}\epsilon_{\alpha\beta\gamma\delta}\Delta^{-2}X^{-3}\sin\xi\cos^4\xi\mu^\kappa\left[5\tilde{T}^{\alpha\kappa}X^{-1}dX+D\tilde{T}^{\alpha\kappa}\right]\\
&\wedge D\mu^\beta\wedge D\mu^\gamma\wedge D\mu^\delta\\
&+\frac{1}{2g^3}\epsilon_{\alpha\beta\gamma\delta}\Delta^{-2}\cos^3\xi\mu^\kappa\mu^\lambda\left[\cos^2\xi X^2\tilde{T}^{\alpha\kappa}D\tilde{T}^{\beta\lambda}-\sin^2\xi X^{-3}\delta^{\beta\lambda}D\tilde{T}^{\alpha\kappa}\right.\\
&\left.-5\sin^2\xi\tilde{T}^{\alpha\kappa}X^{-4}\delta^{\beta\lambda}dX\right]\wedge D\mu^\gamma\wedge D\mu^\delta\wedge d\xi+\frac{1}{2g^2}\cos\xi\epsilon_{\alpha\beta\gamma\delta}\times\\
&\left[\frac{1}{2}\cos\xi\sin\xi X^{-4}D\mu^\gamma-\left(X^{-4}\sin^2\xi\mu^\gamma+X^2\cos^2\xi\tilde{T}^{\gamma\kappa}\mu^\kappa\right)d\xi\right]\\
&\wedge F^{\alpha\beta}_{(2)}\wedge D\mu^\delta
\end{aligned}
\tag{11}
$$

7D $SO(4)$ gauged sueprgravity

- Lagrangian:

$$\begin{aligned}\mathcal{L}_7 &= R*\mathbb{I} - \frac{1}{4}X^{-2}\tilde{T}^{-1}_{\alpha\gamma}\tilde{T}^{-1}_{\beta\delta} * F^{\alpha\beta}_{(2)} \wedge F^{\gamma\delta}_{(2)} - \frac{1}{2}X^4 * F_{(4)} \wedge F_{(4)} \\ &\quad -\frac{1}{4}\tilde{T}^{-1}_{\alpha\beta} * D\tilde{T}_{\beta\gamma} \wedge \tilde{T}^{-1}_{\gamma\delta} D\tilde{T}_{\delta\alpha} + \frac{1}{8}\epsilon_{\alpha\beta\gamma\delta} C_{(3)} \wedge F^{\alpha\beta}_{(2)} \wedge F^{\gamma\delta}_{(2)} \\ &\quad -5X^{-2} * dX \wedge dX - \frac{1}{2} g F_{(4)} \wedge C_{(3)} - V * \mathbb{I} \end{aligned} \tag{12}$$

with the scalar potential

$$V = \frac{1}{2}g^2\left[X^{-8} - 2X^{-3}\tilde{T}_{\alpha\alpha} + 2X^2\left(\tilde{T}_{\alpha\beta}\tilde{T}_{\alpha\beta} - \frac{1}{2}\tilde{T}^2_{\alpha\alpha}\right)\right]. \tag{13}$$

Mapping of fields and supersymmetric vacuum

- The reduced theory is exactly the same as the 7D gauged supergravity provided that the following identifications are made

$$H_{(4)} \to \frac{F_{(4)}}{\sqrt{2}}, \qquad C_{(3)} \to \frac{C_{(3)}}{\sqrt{2}},$$
$$F^I = \frac{1}{4}\Gamma^I_{\alpha\beta}F^{\alpha\beta}_{(2)} \qquad \text{or} \qquad F^{\alpha\beta}_{(2)} = -\frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\Gamma^I_{\gamma\delta}F^I,$$
$$X = e^{-\frac{\sigma}{2}}\,. \tag{14}$$

- The theory admits a supersymmetric AdS_7 critical point with 16 supercharges: $\sigma = 0$, $T_{\alpha\beta} = \delta_{\alpha\beta}$, $L_{AdS_7} = \frac{1}{4h}$. The 11D geometry is $AdS_7 \times S^4$.
- This critical point is dual to $N = (1, 0)$ SCFTs in six dimensions.

- Unbroken $SO(3)_{\text{diag}} \subset SO(3) \times SO(3)$: $T_{\alpha\beta} = \text{diag}(\delta_{ab}e^{-\phi}, e^{3\phi})$
  The 7D metric: $ds_7^2 = e^{2A(r)}dx_{1,5}^2 + dr^2$
- BPS equations:

$$\phi' = -4e^{-\frac{\sigma}{2}-3\phi}\left(e^{4\phi}-1\right)h, \tag{15}$$

$$\sigma' = \frac{8}{5}e^{-\frac{\sigma}{2}-3\phi}\left(1+3e^{4\phi}-4e^{\frac{5}{2}\sigma+3\phi}\right)h, \tag{16}$$

$$A' = \frac{4}{5}e^{-\frac{\sigma}{2}-3\phi}\left(1+3e^{4\phi}+e^{\frac{5}{2}\sigma+3\phi}\right) \tag{17}$$

- Solution: introducing a new coordinate $\tilde{r}$ via $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$,

$$16h\tilde{r} = \ln\left[\frac{1+e^{\phi}}{1-e^{\phi}}\right] - 2\tan^{-1}\phi + C_1, \tag{18}$$

$$\sigma = \frac{2}{5}\left[\phi - \ln\left[1+12C_2-12C_2e^{4\phi}\right]\right], \tag{19}$$

$$A = \frac{1}{4}\left[\phi - 2\ln(1-e^{4\phi})\right] - \frac{1}{8}\sigma\,. \tag{20}$$

- UV geometry:

$$ds_7^2 = e^{\frac{2r}{L_{AdS_7}}} dx_{1,5}^2 + dr^2 \qquad (21)$$

which is dual to the $N = (1,0)$ SCFT.

- IR geometry:

$$C_2 = 0 \quad : \qquad ds_7^2 = (4hr - C)^2 dx_{1,5}^2 + dr^2, \qquad (22)$$

$$C_2 \neq 0 \quad : \qquad ds_7^2 = (4hr - C)^{\frac{3}{4}} dx_{1,5}^2 + dr^2 \qquad (23)$$

where C is a constant related to C_1.

- Both of them are dual to the non-conformal $N = (1,0)$ gauge theory in six dimensions.

Outlook

- A more general $SO(4) \sim SU(2) \times SU(2)$ gauged supergravity contains two independent gauge couplings.
- The gauged supergravity obtained from a truncation of the $N = 4$ gauged supergravity only accounts for the special case of equal $SU(2)$ gauge couplings.
- Look for new (possible) AdS_7 geometries from string/M-theory.
- Find supergravity descriptions of recently found AdS_7 critical points from massive type IIA theory in ten dimensions.
- Find possible gravity duals of many $N = (1,0)$ SCFT found recently from F-theory.

Thank You For Your Attention.