Particle's Trajectory Around Static and Spherically Symmetric black hole in Massive Gravity Theory

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Abstract

We attempt to explain the trajectory of a particle around the massive object using an effective gravitational potential obtained from dRGT massive gravity theory. The dRGT massive gravity is the modification of the Einstein's general relativity (GR) by considering theory with a massive graviton, while in GR the graviton is massless. We start with finding the static and spherically symmetric black hole solutions of the modified Einstein equations in empty space. We found that at small scale, the solution recovers Einstein's gravity with small correction contributed from graviton mass. At large scale, the dominant contribution provides the accelerating expansion of the universe since the graviton mass serves as cosmological constant. The corrections in the solution may provide an opportunity to distinguish the particle motion between massive gravity and GR.

Keywords**:** massive gravity theory, static and spherically symmetric black hole

Introduction

The recent observations suggest that the universe is expanding with acceleration. However, the general relativity cannot describe this phenomenon. So, one modification to general relativity is to add mass to graviton resulting in massive gravity theory. The first attempt began in 1939, Fierz and Pauli introduced the mass term to the action of linear gravitational perturbation [1]. However, 1970 van Dam, Veltman, and Zakharov studied the Fierz-Pauli action by adding the symmetric source into the action [2-3]. They found the discontinuity when they limited graviton mass going to zero and the result is different from general relativity result called vDVZ discontinuity.

After that, Boulware and Deser found a ghost mode (called BD ghost) when they considered non-linear perturbations in Fierz-Pauli action [4]. At the same time, Veinstein [5] also found the action of non-linear perturbations should be considered. He proposed the mechanism to fix the vDVZ discontinuity. The massive gravity theory has been continuously developed in order to alleviate this problem in the theory

One of viable model of massive gravity theory without BD ghost is 4-dimensional dRGT massive gravity theory introduced by de Rham, Gabadadze, and Tolley in 2010 [6-8]. It was found that graviton mass in dRGT massive gravity played a role of cosmological constant driving the universe to expand with acceleration and it should explain the local gravity such as the solar system [9]. In this paper, the Mercury's trajectory will be considered as well.

Theory

We will find the static and spherically symmetric solutions of modified Einstein equations in empty space. The physical metric which can be written as

$$ds^2 = -n(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \qquad (1)$$

In this work, we will follow [10-12] by choosing the fiducial metric as

$$f_{\mu\nu} = diag(0,0,r^2,r^2 sin^2\theta) \qquad (2)$$

We can solve the solution of f(r) and the solution can be written as

$$f(r) = 1 - \frac{2M}{r} + \frac{\Lambda}{3}r^2 + \gamma r + \zeta \qquad (3)$$

where

$$\Lambda = 3{m_g}^2(1+\alpha+\beta) \qquad (4a)$$
$$\gamma = -c{m_g}^2(1+2\alpha+3\beta) \qquad (4b)$$
$$\zeta = c^2{m_g}^2(\alpha+3\beta) \qquad (4c)$$

M is a mass of massive source. This solution has represented the cosmological as well, namely Λ, in term of the graviton mass m_g other three parameters α,β,and c.

We will find the equations of motion by interpreting effective potential U_{eff}.

The first of all we will find total energy and angular momentum part by using killing vector

$$E = f(r)\frac{dt}{d\tau} \quad and \quad L = r^2\frac{d\phi}{d\tau} \tag{5}$$

Using line element for massive particle, $ds^2 = -d\tau^2$. We will get

$$\left(\frac{dr}{d\tau}\right)^2 + \frac{\Lambda}{3}r^2 + \gamma r + (\gamma L^2 - 2M)\frac{1}{r} + (1+\zeta)\frac{L^2}{r^2} - 2M\frac{L^2}{r^3} = E^2 - \frac{\Lambda}{3}L^2 - 1 \tag{6}$$

We define effective potential as

$$U_{eff} = \frac{\Lambda}{3}r^2 + \gamma r + (\gamma L^2 - 2M)\frac{1}{r} + (1+\zeta)\frac{L^2}{r^2} - 2M\frac{L^2}{r^3} \tag{7}$$

Then we rewrite the effective potential to consider easily

$$\begin{aligned} V_{eff} = \frac{r_v U_{eff}}{r_s} &= (1+\alpha+\beta)\left(\frac{r}{r_v}\right)^2 \\ &- c(1+2\alpha+3\beta)\left(\frac{r}{r_v}\right) \\ &- \left[1 - \frac{(1+2\alpha+3\beta)}{c}\frac{L^2\varepsilon}{r_s}\right]\left(\frac{r_v}{r}\right) \\ &+ [1+\varepsilon(\alpha+3\beta)]\frac{L^2 r_v}{r_s r^2} - \frac{L^2 r_v}{r^3} \end{aligned} \tag{8}$$

Where

$$r_s = 2M,\ \ r_v = \left(\frac{r_s}{{m_g}^2}\right)^{1/3},\ \ \varepsilon = \frac{c^2 r_s}{{r_v}^3} = c^2{m_g}^2$$

Results and Discussion

We will consider some cases in each limit of parameters with $r \ll r_v \ and\ \varepsilon \ll 1$

Case 1: L is very small, we get

$$V_{eff} = -\frac{r_v}{r} \tag{9}$$

If a particle is near the source, the particle will move to the source.

Case 2: $L\ is\ very\ large\ and\ L^2\varepsilon \ll 1$, we get

$$V_{eff} = -\left(\frac{r_v}{r}\right) + \frac{L^2 r_v}{r_s r^2} - \frac{L^2 r_v}{r^3} \tag{10}$$

This case is normally in GR. There are two solutions,

$$r = \frac{L^2}{r_s} \pm \frac{L}{r_s}\sqrt{L^2 - 3{r_s}^2} \tag{11}$$

We have plot effective potential and radius by using Mercury's information. Graph shows both stable and unstable orbit

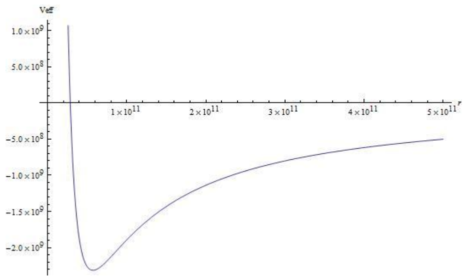

Figure 1. Graph an effective potential versus radius shows stable orbit of Mercury. The radius is about $10^{10}\ m$

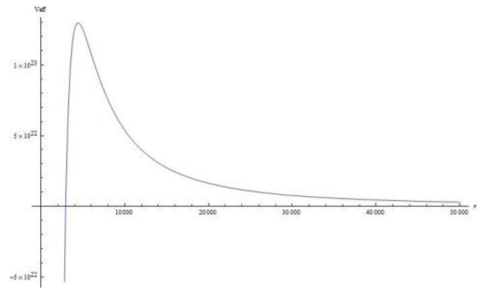

Figure 2. Graph an effective potential versus radius shows unstable orbit of Mercury. The radius is about $10^4\ m$

Case 3: $L\ is\ very\ large\ and\ L^2\varepsilon \sim 1$, we get

$$V_{eff} = -\left[1 - \frac{(1+2\alpha+3\beta)}{c}\frac{L^2\varepsilon}{r_s}\right]\left(\frac{r_v}{r}\right) + \frac{L^2 r_v}{r_s r^2} - \frac{L^2 r_v}{r^3} \tag{12}$$

Two solutions are

$$r = \frac{L^2}{r_s}\left[\frac{1 \pm \sqrt{1 - 3\frac{{r_s}^2}{L^2}\left(1 - (1+2\alpha+3\beta)\frac{\varepsilon L^2}{c r_s}\right)}}{1 - (1+2\alpha+3\beta)\frac{\varepsilon L^2}{c r_s}}\right] \tag{13}$$

We found that the solutions not only depend on angular momentum and r_s but also depend on the parameters $\alpha,\ \beta, and\ c$ which are in the correct term. However, this case can be also reduced to GR case when m_g goes to zero. And we also found stable and unstable orbit as in GR case.

Conclusions

We found the solution of static and spherically symmetric of the modified Einstein equations in empty space and found that the effective potential which is the same as one in GR plus some correction terms from massive gravity. Finally, this model shows both stable and unstable orbit of mercury as same as in GR prediction

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