Bulk viscosity-driven suppression of shear viscosity effects on the flow harmonics at RHIC

arXiv:1411.2574

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Outline

1. Heavy-Ion Collisions
2. Effects of Viscosity
3. Viscous Rel. Hydro Event by Event
4. Results
5. Conclusions
Heavy ion collisions are modeled through

- **Initial Condition**: Pre-equilibrium state using gluon saturation models/Glauber-like models
- **Viscous hydrodynamical evolution**/Lattice Equation of State
- **Hadronization mechanism**: Cooper Frye including viscous corrections
- **Hadronic afterburner**
Event-by-event NeXus initial conditions and 3+1 ideal relativistic hydrodynamics fit the flow harmonics well.

Shear Viscosity in Heavy-Ion Collisions

- Resistance against the deformation of a fluid
  \[ \Pi_{\text{Navier–Stokes}}^{\mu\nu} \sim \eta \partial\langle \mu u^\nu \rangle \]

- PHSD (PRC87(2013)064903)
- AdS/CFT -KSS limit (Kovtun,Son,Stairnets PRL94(2005)111601)
- UrQMD (Demir, Bass PRL(2009)102)
- semi-QGP- \( \kappa = 32 \) (Hidaka,Pisarski PRD81(2010)076002)
- Also, Csernai,Kapusta,Mclerran PRL 97, 152303 (2006) (not shown)

- Dyson-Schwinger Yang-Mills (arXiv:1411.7986)
Bulk Viscosity in Heavy-Ion Collisions

- Resistance against the radial expansion or compression of a fluid $\Pi_{\text{Navier-Stokes}} \sim -\zeta(\partial_\mu u^\mu)$
- Evolution with a non-zero $\zeta/s$ slows down the expansion of the fluid.
- Previous assumption: $\zeta/s$ is negligible in hydrodynamics studies of heavy-ion collisions
Bulk Viscosity in Heavy-Ion Collisions

- Resistance against the radial expansion or compression of a fluid $\Pi_{\text{Navier-Stokes}} \sim -\zeta(\partial_\mu u^\mu)$
- Peak at $T_c$?

Peak also seen in:
JNH, PRL 103 (2009) 172302,
Kharzeev JHEP 0809 (2008) 093

- HRG+HS (Kadam and Mishra arXiv:1408.6329)
- PHSD (PRC 87, 064903 (2013))
- non-conformal holographic model (Finazzo, Rougemont, Noronha - to appear shortly)
- 14 mom. (Denicol et al, PRC90(2014)024912)
Second-order Transport Coefficients

Equations of Motion - 2nd order
Denicol et al, PRD85(2012)114047

\[ \dot{\Pi} + \frac{\Pi}{\tau_{\Pi}} = -\frac{\zeta}{s} \theta + -\delta_{\Pi \Pi} \Pi \theta + \lambda_{\Pi \pi} \tau_{\mu \nu} \sigma_{\mu \nu} \]
\[ + \phi_1 \Pi^2 + \phi_3 \pi_{\mu \nu} \pi_{\mu \nu} \]
\[ (1) \]

\[ \dot{\pi}^{\langle \mu \nu \rangle} + \frac{\pi_{\mu \nu}}{\tau_{\pi}} = \frac{2\eta}{s} \sigma_{\mu \nu} - \frac{4}{3} \pi_{\mu \nu} \theta \]
\[ + 2\pi^{\langle \mu \nu \rangle \alpha} + \phi_7 \tau^{\langle \mu \pi \alpha \rangle \nu} + \lambda_{\pi \pi} \Pi \sigma_{\mu \nu} - \tau_{\pi \pi} \tau^{\langle \mu \sigma \nu \rangle \alpha} \]
\[ + \phi_6 \Pi^{\pi^{\mu \nu}} \]
\[ (2) \]

v-USPhydro - in black
MUSIC non-zero terms - red
MUSIC zero terms - gray
Shear+Bulk Direct Coupling Terms

$$\phi_6 \Pi \pi^{\mu\nu}$$
in $\pi^{\mu\nu}$ evolution

$$\lambda_{\pi \Pi} \Pi \sigma_{\mu\nu}$$
in $\pi^{\mu\nu}$ evolution

$$\lambda_{\Pi \pi} \pi^{\mu\nu} \sigma_{\mu\nu}$$
in $\Pi$ evolution

Finazzo, Rougemont, Noronha to appear shortly

Denicol et al, PRC90(2014)024912
Molnar et al, PRD89(2014)074010

Denicol et al, PRC90(2014)024912
Molnar et al, PRD89(2014)074010
Relaxation Times

- $\tau_\pi$ for HRG+HS+QGP
  PRL105(2010)162501

- $\tau_\Pi$ HRG+HS+QGP
  Huang et al,PRC83(2011)024906

- 14 moment

- nonconformal AdS
  (Finazzo, Rougemont, Noronha to appear shortly)
Given the Glauber Initial Condition $\tau = 1 \text{fm}$
Viscosity in Heavy-Ion Collisions

(b) ideal $\tau=6\text{fm}$

(c) bulk $\tau=6\text{fm}$

(e) shear+bulk $\tau=6\text{fm}$

(d) shear $\tau=6\text{fm}$
Motivation

Write a modular event-by-event 2+1 hydrodynamical code that runs ideal & viscous hydro with nonzero $\zeta/s$ and $\eta/s$

- Initial conditions easily implemented from other sources.
- Equations of motion solved using Smoothed Particle Hydrodynamics (SPH) - quick comp. time and avoids numerical viscosity/grid size issues.
- Coupled to UrQMD - results shown here without decays.

\[ T^{\mu\nu} = \varepsilon u^\nu u^\mu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \]
Description of Shear and Bulk Viscosity

$$\eta_s(T > T_{tr}) = -0.289 + 0.288 \left( \frac{T}{T_{tr}} \right) + 0.0818 \left( \frac{T}{T_{tr}} \right)^2$$

$$\eta_s(T < T_{tr}) = 0.681 - 0.0594 \left( \frac{T}{T_{tr}} \right) - 0.544 \left( \frac{T}{T_{tr}} \right)^2$$

JNH


$$\tau_\pi = 5 \eta / (\varepsilon + p)$$

PRL105, 162501 (2010)

$$\left( \frac{\zeta}{s} \right) = 0.5 \frac{\eta}{s} \left( \frac{1}{3} - c_s^2 \right), \quad \tau_\Pi = 9 \frac{\zeta}{\varepsilon - 3p}$$

BuchelPLB663(2008)286
Huang,Kodama,Koide,RischkePRC83(2011)024906
Effects of viscosity with hydrodynamics (hydro only)

Compare percentage change of mean and variance in the presence of shear+bulk vs. bulk only (or shear only)

Effects of shear on $\Pi$
- The mean has almost no variation
- Shear increases the variation in bulk (at late times)
- Variation decreases significantly at early times

Effects of bulk on $\pi^{00}$ and $\pi^{12}$
- Bulk suppresses the $\pi^{\mu\nu}$
- Largest effect at late times.
- Variation decreases across the board
Cooper-Frye Freeze-out

Overview

\[
\left( E p \frac{dN}{d^3p} \right)_i = g_i \int \Sigma \, d\Sigma \mu p^\mu f_i
\]

Particle distribution function:

\[
f_k^{(i)} = f_{0k}^{(i)} + \delta f_k^{(i)}
\]

\[
f_{0k}^{(i)} = \left( \exp\left[ E_i / T \right] + a_i \right)^{-1}
\]

Fermions: \( a_i = 1 \), Bosons: \( a_i = -1 \)
Boltzmann gas: \( a_i = 0 \)

Note that majority of viscous effects come from \( \delta f \).

Schenke, Jeon, Gale, PRC85(2012)024901
Truncating in momentum space up to the 2nd order and using the orthogonality relations from the basis:

\[
f^{(i)}_k = f^{(i)}_{0k} + \delta f^{(i)}_{k}^{Bulk} + \delta f^{(i)}_{k}^{Shear},
\]

\[
\delta f^{(i)}_{k}^{Shear} = \frac{f^{(i)}_{0k}}{2 (\varepsilon_i + P_i) T^2} \frac{\eta_i}{\eta} \pi^{\mu\nu} k_{i,\mu} k_{i,\nu},
\]

\[
\delta f^{(i)}_{k}^{Bulk} = f^{(i)}_{0k} \prod \left[ B^{(i)}_0 + D^{(i)}_0 u \cdot k_i + E^{(i)}_0 (u \cdot k_i)^2 \right]
\]

- \( E_{0,i}, D_{0,i}, B_{0,i} \): functions of mass \( m_i \) and \( T \)- determined through basis
Dependence on $\delta f$ - bulk only

JNH, Denicol, Noronha, Andrade, Grassi, PRC88(2013)044916

\[ \delta f_k^{(\pi)} = f_{0k}^{\pi} \prod^* \left[ B_0^{(\pi)} + D_0^{(\pi)} u \cdot k_\pi + E_0^{(\pi)} (u \cdot k_\pi)^2 \right] \]

<table>
<thead>
<tr>
<th></th>
<th>$E_0 \ [fm^4]$</th>
<th>$D_0 \ [fm^4/GeV]$</th>
<th>$B_0 \ [fm^4/GeV^2]$</th>
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<tr>
<td>mo</td>
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<td>-38.96</td>
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Event-by-Event $v_2$
JNH, Noronha, Grassi, PRC90(2014)034907

![Graphs showing $v_2$ as a function of $p_T$ for different pseudorapidity ranges.](image)
Integrated $v_n$’s - Comparing $\delta f$

JNH, Noronha, Grassi, PRC90(2014)034907
Integrated $v_n$’s - Comparing $\zeta/s$

JNH, Noronha, Grassi, PRC90(2014)034907
$\frac{\zeta}{s} = \frac{\eta}{s}$ integrated

JNH, Noronha, Grassi, PRC90(2014)034907
\[ \nu_1 \text{ from } \varepsilon_1 + \varepsilon_2 \varepsilon_3 + \varepsilon_1 \varepsilon_5 \]
Gardim, JNH, Luzum, Grassi, arXiv:1411.2574

- Shear viscosity most strongly correlated to initial conditions
- Initial flow/3+1 dimensions less correlated with initial eccentricities (especially for central/peripheral collisions)
- Higher order eccentricities help correlate peripheral collisions
Conclusions

- Bulk viscosity may compensate the effects of shear viscosity (more relevant for longer hydrodynamical evolution)
- When $\zeta/s = \eta/s$ the effects of bulk may dominate
- Shear viscosity most strongly correlates to the initial eccentricities, shear+bulk is not as strongly correlated.
- $\zeta/s$ must be significantly smaller than $\eta/s$ - otherwise runs into problems with $\delta f$.
- v-USPhydro+UrQMD results coming soon!
Types of Initial Conditions

Energy Density profile


The distribution of particles can be written as a Fourier series (event plane method)

\[
E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} \left[ 1 + \sum_n 2v_n \cos [n(\phi - \psi_n)] \right]
\]

- **Flow Harmonics at mid-rapidity**

\[
v_n(p_T) = \frac{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi} \cos [n(\phi - \psi_n)]}{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi}}
\]

where \(\psi_n = \frac{1}{n} \arctan \frac{\langle \sin[(n\phi)] \rangle}{\langle \cos[(n\phi)] \rangle}\)

\[
n = 2 \quad n = 3 \quad n = 4 \quad n = 5 \quad n = 6
\]
Smoothed Particle Hydrodynamics (SPH) Overview

Motivation

SPH discretizes the fluid into a number of SPH particles whose trajectories \( (r \text{ and } u) \) you observe over time.

Imagine you want to observe the motion of a lake:
- SPH (Lagrangian)- you are in a boat on the lake and move over the coarse of time watching your trajectory.
- Grid (Euler)- you are seated at a dock and observe the rise and fall of water at a set spot 2m away from you.
Event-by-Event $v_2$
JNH PRC90(2014)034907

![Graphs showing $v_2$ vs. $p_T$ for different pT ranges.](image)

- **Ideal**
- $\frac{\eta}{s}$
- $\frac{\eta + \zeta}{s}$ MOM
- MH

**Legend:**
- $0-10\%$
- $10-20\%$
- $20-30\%$
- $30-40\%$
- $40-50\%$
- $50-60\%$

**Axes:**
- $v_2$ on the y-axis
- $p_T$ (GeV) on the x-axis

**Graphs:**
- For each pT range, there are four curves representing different scenarios.
- The curves are labeled with their respective mechanisms and scenarios.

- **0-10%**
- **10-20%**
- **20-30%**
- **30-40%**
- **40-50%**
- **50-60%**
Event-by-Event $v_5$

JNH PRC90(2014)034907

![Graphs showing the dependence of $v_5$ on $p_T$ for different scenarios, including ideal, $\eta/s$, $\eta/s + \xi$, MOM, and MH, with various centrality classes.](image)
Shear viscosity is most strongly correlated to initial conditions

\( \nu_1 \) requires higher order eccentricities, correlates most strongly to low \( p_T \) for \( \nu_1 \)
$\zeta/s = \eta/s$ $p_T$ dependent

JNH PRC90(2014)034907
### Tables (hydro only)

\[
\begin{align*}
(\Pi)_{ev} &= 100 \frac{(\Pi_{sb})_{ev} - (\Pi_b)_{ev}}{(\Pi_b)_{ev}} \\
(\sigma^2_{\Pi})_{ev} &= 100 \frac{(\sigma^2_{\Pi_{sb}})_{ev} - (\sigma^2_{\Pi_b})_{ev}}{(\sigma^2_{\Pi_b})_{ev}}
\end{align*}
\]
### Tables (hydro only)

<table>
<thead>
<tr>
<th>Centrality</th>
<th>(\langle \pi^{00}\rangle_{\text{early}})</th>
<th>(\sigma^2_{\pi^{00}})_{\text{early}}</th>
<th>(\langle \pi^{12}\rangle_{\text{early}})</th>
<th>(\sigma^2_{\pi^{12}})_{\text{early}}</th>
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<td>-19.90%</td>
<td>-2.87%</td>
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<td>50-60%</td>
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<td>-19.61%</td>
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</table>

### TABLE II. Percentage change of the mean values of the bulk pressure \(\Pi\) and its corresponding variance \(\sigma^2_{\Pi}\) averaged over all events for different centrality classes due to the presence of shear viscosity. \(\langle \Pi\rangle\) and \(\sigma^2_{\Pi}\) take into account the parts of the fluid that have frozen out throughout the whole time evolution, \(\langle \Pi\rangle_{\text{early}}\) and \(\sigma^2_{\Pi}\)_{\text{early}} are computed using only the parts of the fluid that have frozen out between \(\tau_0 = 1 \text{ fm}\) and \(\tau = 2 \text{ fm}\), \(\langle \Pi\rangle_{\text{late}}\) and \(\sigma^2_{\Pi}\)_{\text{late}} are computed using only the parts of the fluid that have frozen out in the last \(\text{fm}\) of the time evolution.

<table>
<thead>
<tr>
<th>Centrality</th>
<th>(\langle \pi^{00}\rangle_{\text{late}})</th>
<th>(\sigma^2_{\pi^{00}})_{\text{late}}</th>
<th>(\langle \pi^{12}\rangle_{\text{late}})</th>
<th>(\sigma^2_{\pi^{12}})_{\text{late}}</th>
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</table>

### TABLE III. The percentage change in the mean values and variance of the \(\pi^{00}\) and \(\pi^{12}\) components of the shear stress tensor \(\pi^{\mu\nu}\) averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account the parts of the fluid that have frozen out for early times (between \(\tau = \tau_0\) and \(\tau = 2 \text{ fm}\)).

<table>
<thead>
<tr>
<th>Centrality</th>
<th>(\langle \pi^{00}\rangle_{\text{late}})</th>
<th>(\sigma^2_{\pi^{00}})_{\text{late}}</th>
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### TABLE IV. The percentage change in the mean values and variance of the \(\pi^{00}\) and \(\pi^{12}\) components of the shear stress tensor \(\pi^{\mu\nu}\) averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account only the parts of the fluid that have already frozen for early times (between \(\tau = \tau_0\) and \(\tau = 2 \text{ fm}\)).

### TABLE V. The percentage change in the mean values and variance of the \(\pi^{00}\) and \(\pi^{12}\) components of the shear stress tensor \(\pi^{\mu\nu}\) averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account only the parts of the fluid that have already frozen for early times (between \(\tau = \tau_0\) and \(\tau = 2 \text{ fm}\)).
Equations of Motion

Conservation of Energy and Momentum

\[ \partial_\mu T^{\mu\nu} = 0 \quad (4) \]

The energy-moment tensor contains a bulk dissipative term \( \Pi \) and the shear stress tensor \( \pi^{\mu\nu} \) is

\[ T^{\mu\nu} = \varepsilon u^\nu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \quad (5) \]

Coordinate System: \( x^\mu = (\tau, x, y, \eta) \) where \( \tau = \sqrt{t^2 - z^2} \) and \( \eta = 0.5 \ln \left( \frac{t+z}{t-z} \right) \)
Cooper-Frye Freeze-out
Derivation of $\delta f_{k}^{(i)}$ 1/2: Denicol et al, PRD85(2012)114047

Particle distribution function computed using a version of Grad’s 14 moment approximation for the Boltzmann equation:

- Factorize $\delta f_{k}^{(i)}$: $\delta f_{k}^{(i)} = f_{0k}^{(i)} \tilde{f}_{0k}^{(i)} \phi_{k}^{(i)}$ where $\tilde{f}_{0k}^{(i)} = 1 + af_{0k}^{(i)}$
- Determine $\phi_{k}^{(i)}$, out of equilibrium contribution, by establishing a basis of

Irreducible Tensors: $k_{i}^{(\mu)}$, $k_{i}^{(\mu) k_{i}^{(\nu)}}$, $k_{i}^{(\mu) k_{i}^{(\nu) k_{i}^{(\lambda)}}}$, $\cdots$,

Orthonormal Polynomials: $P_{ik}^{(n\ell)} = \sum_{r=0}^{n} a_{nr}^{(\ell) i} (u_{\mu} k_{i}^{(\mu)})^{r}$

- Then, $f_{k}^{(i)} = f_{0k}^{(i)} + f_{0k}^{(i)} \tilde{f}_{0k}^{(i)} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} H_{ik}^{(n\ell)} \rho_{i,n}^{(\mu_{1} \cdots \mu_{\ell})} k_{i,\mu_{1}} \cdots k_{i,\mu_{\ell}}$

where $H_{ip}^{(n\ell)} \equiv \left[ N_{i}^{(\ell)} / \ell ! \right] \sum_{m=n}^{\infty} a_{mn}^{(\ell) i} P_{ik}^{(m\ell)} (u_{\mu} k_{i}^{(\mu)})$
Cooper-Frye Freezeout

Major Assumptions

- We assume Navier-Stokes scaling to relate the moments \( \rho_{i,0}, \rho_{i,2}, \rho_{i,0}^{\mu\nu} \) to \( \Pi \) and \( \pi^{\mu\nu} \)-neglect effects from \( \tau_{\Pi} \).

\[
\Pi = -\zeta \partial_{\mu} u^{\mu}, \rho_{i,m} = -\alpha_{i,m} \partial_{\mu} u^{\mu} \implies \rho_{i,m} = \frac{\alpha_{i,m}}{\zeta} \Pi,
\]

\[
\pi_{i}^{\mu\nu} = 2\eta_{i} \partial^{(\mu} u^{\nu)}, \pi_{i}^{\mu\nu} = 2\eta \partial^{(\mu} u^{\nu)} \implies \pi_{i}^{\mu\nu} = \frac{\eta_{i}}{\eta} \pi^{\mu\nu}.
\]

- All hadrons have the same cross-section of 30 mb.

- Only hadrons up to a mass of \( M = 1.2 \) GeV are considered (every additional hadron increases the matrix rank needed for the calculation of transport coefficients, which becomes very costly).

- Freeze-out temperature \( T_{FO} = 150 \) MeV.
At $T = 150$ MeV about 41% of pions are direct pions. For most central collisions there are about 300 $\pi^+$’s, so 123 direct $\pi^+$’s.

\[ \pi^+ \approx 123 \]

\[ \pi^+ \approx 54 \]
Event-by-Event $v_3$

JNH PRC90(2014)034907

```plaintext
<table>
<thead>
<tr>
<th>$v_3$</th>
<th>Pseudorapidity $p_T$ (GeV)</th>
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<tr>
<td>0.00</td>
<td>0.5</td>
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<td>1</td>
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<tr>
<td>0.04</td>
<td>1.5</td>
</tr>
<tr>
<td>0.06</td>
<td>2</td>
</tr>
</tbody>
</table>
```

- $v_3$: third-order harmonic coefficient
- $p_T$: transverse momentum
- $\eta$, $\zeta$, $s$: viscosity coefficients
- Ideal, $\eta/s$, $\eta/s + \zeta$, MOM, MH: different fluid models
Event-by-Event $v_4$
JNH PRC90(2014)034907
Parameters

- Isothermal freeze-out temperature: $T_{FO} = 150$ MeV
- Initial time to start hydrodynamic simulation: $t_0 = 1$ fm
- Lattice-based equation of state from Huovinen & Petreczky, NPA837, 26(2010)
  Currently testing HotQCD PRD90(2014)9,094503
- SPH scale $h = 0.3$ fm
- Energy conservation for event-by-event Glauber initial conditions: Ideal case $\sim 0.001\%$, Viscous case $\sim 0.1\%$
Equations of Motion

- SPH conserves reference density current: \( J^\mu = \sigma u^\mu \) where \( \sigma \) is the local density of a fluid element in its rest frame.

- Density obeys \( \partial_\mu (\tau \sigma u^\mu) = 0 \) in hyperbolic coordinates (in Cartesian \( D\sigma + \sigma \theta = 0 \)) where \( D = u^\mu \partial_\mu \) and \( \theta = \tau^{-1} \partial_\mu (\tau u^\mu) \).

- We use this set of IS equations, which provides the simplest equations for viscous hydrodynamics.

\[
\tau_\Pi (D\Pi + \Pi \theta) + \Pi + \zeta \theta = 0,
\quad \tau_\pi \left( \Delta_{\mu \nu \alpha \beta} D\pi^{\alpha \beta} + \frac{4}{3} \pi_{\mu \nu} \theta \right) + \pi_{\mu \nu} = 2\eta \sigma_{\mu \nu}
\]

PRC75(2007) 034909

- There are four transport coefficients: \( \eta/s, \zeta/s, \tau_\pi, \) and \( \tau_\Pi \).
Equation of State

Huovinen & Petreczky, NPA 837, 26(2010)
Dependence on $\tau_0$ (bulk only)

(a) $dN \propto 2\pi p_T \, dp_T$
- $t_0=0.75$
- $t_0=1$
- $t_0=1.25$
- ebe 20–30%

(b) $v_2$
- ideal
- $t_0=0.75$
- $t_0=1$
- $t_0=1.25$
- ebe 20–30%

$p_T$ [GeV]

$0.00$ $0.05$ $0.10$ $0.15$ $0.20$ $0.25$ $0.30$

$0.0$ $0.5$ $1.0$ $1.5$ $2.0$ $2.5$

$p_T$ [GeV]
Checks- Gubser Test

- Reproduce analytical sol. from 2+1 conformal ideal hydro

\[ \epsilon = \frac{\epsilon_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{1 + 2q^2 (\tau^2 + x_\perp^2) + q^4 (\tau^2 - x_\perp^2)} [4/3] \]

Gubser, PRD 82, 085027(2010), Marrochio et. al. 1307.6130 [nucl-th] (first analytical solution of Israel-Stewart hydro)

- The viscous bulk evolution converges to that computed within ideal hydrodynamics for sufficiently small \( \zeta/s \).
Checks - TECHQM (for shear)

Au+Au, \( b = 0 \text{ fm} \), EOS I (\( \epsilon = 3\rho \)), \( \tau_0 = 0.6 \text{ fm/c} \),

![Graphs showing \( \epsilon \), \( \pi^{11} \), \( v_x \), and \( \pi^{33} \) as functions of \( x \) for different values of \( \tau \).]
SPH Equations of Motion

- Reconstruct all hydrodynamical fields using a discrete set of Lagrangian coordinates \( \{ \mathbf{r}_\alpha(\tau), \alpha = 1, \ldots, N_{SPH} \} \) and a normalized piece-wise distribution function \( W[\mathbf{r}; h] \)

- \( h \) is a length scale, determines structure
- Reference density in the lab frame

\[
\tau \gamma \sigma \rightarrow \sigma^*(\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \ W[\mathbf{r} - \mathbf{r}_\alpha(\tau); h] \quad (6)
\]

where \( \nu_\alpha \) are constants \( \rightarrow \int d^2 \mathbf{r} \ \sigma^*(\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \)
Vector current becomes

\[ \mathbf{j}^* (\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{\text{SPH}}} \nu_\alpha \frac{d\mathbf{r}_\alpha(\tau)}{d\tau} W[\mathbf{r} - \mathbf{r}_\alpha(\tau); h], \]

that satisfies

\[ \partial_\tau \sigma^* (\mathbf{r}, \tau) + \nabla \mathbf{r} \cdot \mathbf{j}^* (\mathbf{r}, \tau) = 0 \]

Each "SPH particle", \( \alpha \), has \( \mathbf{r}_\alpha(\tau) \), \( \mathbf{u}_\alpha(\tau) = \gamma_\alpha(\tau) \mathbf{v}_\alpha(\tau) \), where \( \mathbf{v}_\alpha(\tau) = \frac{d\mathbf{r}_\alpha(\tau)}{d\tau} \) and \( \gamma_\alpha = 1/\sqrt{1 - \mathbf{v}^2_\alpha} \), and it carries a quantity \( \nu_\alpha \) for the reference density \( \sigma^* \).
SPH Variables

For any density associated with some extensive quantity -
\( a(r, \tau) \)

\[
a(r, \tau) = \sum_{\alpha=1}^{N_{\text{SPH}}} \nu_\alpha \frac{a(r_\alpha(\tau))}{\sigma^*(r_\alpha(\tau))} W[ r - r_\alpha(\tau); h] . \tag{8}
\]

Thus, entropy

\[
s^*(r, \tau) = \sum_{\alpha=1}^{N_{\text{SPH}}} \nu_\alpha \frac{s(r_\alpha(\tau))}{\sigma(r_\alpha(\tau))} W[ r - r_\alpha(\tau); h] \tag{9}
\]

the bulk term

\[
\Pi(r, \tau) = \sum_{\alpha=1}^{N_{\text{SPH}}} \nu_\alpha \frac{1}{\gamma_\alpha \tau} \left( \frac{\Pi}{\sigma} \right)_\alpha W[ r - r_\alpha(\tau); h] . \tag{10}
\]
SPH Variables

- Dynamical variables: \( \{ \mathbf{r}_\alpha, \mathbf{u}_\alpha, \left( \frac{s}{\sigma} \right)_\alpha, \left( \frac{\Pi}{\sigma} \right)_\alpha ; \alpha = 1, \ldots, N_{\text{SPH}} \} \)
- Equations of Motion can then be rewritten as

\[
M_{ij}^\alpha \frac{d\mathbf{u}_\alpha^i}{dT} = F_\alpha \mathbf{u}_\alpha^i + \partial^i \left( p_\alpha + \Pi_\alpha \right)
\]

\[
\gamma_\alpha (\tau_\Pi)_\alpha \frac{d}{dT} \left( \frac{\Pi}{\sigma} \right)_\alpha + \left( \frac{\Pi}{\sigma} \right)_\alpha = - \left( \frac{\zeta}{\sigma} \right)_\alpha (D_\mu u^\mu)_\alpha
\]

\[
\gamma_\alpha \frac{d}{dT} \left( \frac{s}{\sigma} \right)_\alpha = - \frac{1}{T_\alpha} \frac{\Pi_\alpha}{\sigma_\alpha} (D_\mu u^\mu)_\alpha
\]
SPH Equations of Motion

SPH discretizes the fluid into a number of SPH particles whose trajectories ($r$ and $u$) you observe over time.

**Entropy**

$$s^* = \sum_{\alpha=1}^{N_{SPH}} \nu_{\alpha} \left( \frac{s}{\sigma} \right)_{\alpha} W(|r - r_{\alpha}(t)|; h)$$

**PDE → ODE**

$$M_{ij}^{\alpha} \frac{du_{\alpha}^i}{d\tau} = B_{tot} u_{\alpha}^i + F^i + \partial^i (p_{\alpha} + \Pi_{\alpha}) + v^j \partial^i \pi_{0i} - \partial^i \pi_{ij}$$

$$- \left( \frac{\zeta}{\sigma} \right)_{\alpha} (D_{\mu} u_{\mu})_{\alpha} = \gamma_{\alpha}(\tau\Pi)_{\alpha} \frac{d}{d\tau} \left( \frac{\Pi}{\sigma} \right)_{\alpha} + \left( \frac{\Pi}{\sigma} \right)_{\alpha}$$

$$\gamma_{\alpha} \frac{d}{d\tau} \left( \frac{s}{\sigma} \right)_{\alpha} = -\frac{1}{T_{\alpha}} \frac{\Pi_{\alpha}}{\sigma_{\alpha}} (D_{\mu} u_{\mu})_{\alpha} + \frac{1}{T_{\alpha}} \frac{\pi_{\mu\nu}^{\alpha}}{\sigma_{\alpha}} (D_{\mu} u_{\nu})_{\alpha}$$

shear is much longer (not shown)
Testing of $N_{SPH}$ with $h = 0.3$
Testing of $N_{\text{SPH}}$ with $h = 0.3$
Testing of $h$

(a) $20-30\%$

(b) $20-30\%$

(c) $20-30\%$

(d) $20-30\%$

(e) $20-30\%$

(f) $20-30\%$

(g) $20-30\%$

(h) $20-30\%$
Testing of $h$

![Graphs showing the effect of viscosity on $h$ for different $p_T$ ranges.](image)

- Graph (d) for $20-30\%$ with $h = 0.3$, $h = 0.5$, and $h = 0.7$.
- Graph (e) for $20-30\%$ with $h = 0.1$, $h = 0.3$, $h = 0.5$, and $h = 0.7$.

These graphs illustrate the changes in $V_s$ with varying $h$ values for different $p_T$ ranges.
$v_n(p_T)$'s from bulk only

\[ v_2(p_T) \]
\[ \frac{\zeta}{s} - + \delta \]
\[ 20-30\% \]

\[ v_3(p_T) \]
\[ \frac{\zeta}{s} - + \delta \]
\[ 20-30\% \]

\[ v_4(p_T) \]
\[ \frac{\zeta}{s} - + \delta \]
\[ 20-30\% \]

\[ v_5(p_T) \]
\[ \frac{\zeta}{s} - + \delta \]
\[ 20-30\% \]