Bulk viscosity-driven suppression of shear viscosity effects on the flow harmonics at RHIC

Phys.Rev.C**88** (2013) 044916
Phys.Rev.C**90** (2014) 034907
arXiv:1411.2574

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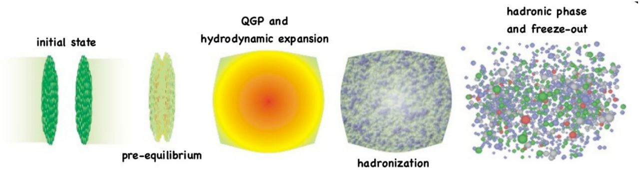
Outline

Evolution of a Heavy-Ion Collision

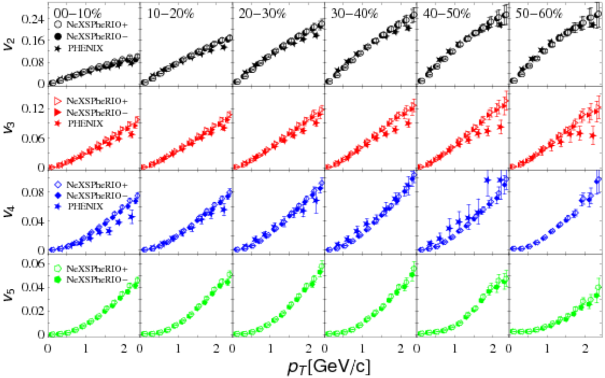


Heavy ion collisions are modeled through

- Initial Condition: Pre-equilibrium state using gluon saturation models/Glauber-like models
- Viscous hydrodynamical evolution/Lattice Equation of State
- Hadronization mechanism: Cooper Frye including viscous corrections
- Hadronic afterburner

Ideal hydrodynamics and Collective Flow

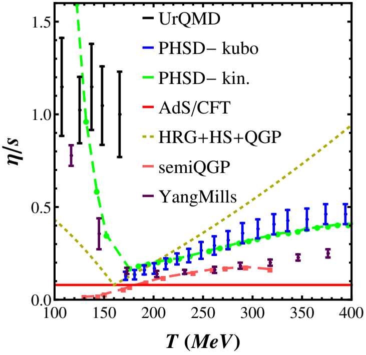
- Event-by-event NeXus initial conditions and 3+1 ideal relativistic hydrodynamics fit the flow harmonics well



Gardim, Grassi, Luzum, Ollitrault, Phys.Rev.Lett. 109 (2012) 202302

Shear Viscosity in Heavy-Ion Collisions

- Resistance against the deformation of a fluid $\Pi^{\mu\nu}_{Navier-Stokes} \sim \eta \partial^{\langle\mu} u^{\nu\rangle}$



- Dyson-Schwinger Yang-Mills (arXiv:1411.7986)

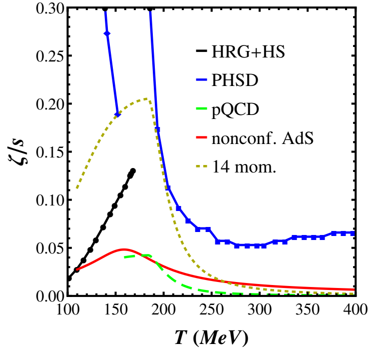
- HRG+HS+QGP(JNH et al PRL103(2009)172302, Niemi et al PRL106(2011)212302)
- PHSD (PRC87(2013)064903)
- AdS/CFT -KSS limit (Kovtun,Son,Stairnets PRL94(2005)111601)
- UrQMD (Demir, Bass PRL(2009)102)
- semi-QGP- $\kappa = 32$ (Hidaka,Pisarski PRD81(2010)076002)
- Also, Csernai,Kapusta,McLerran PRL 97, 152303 (2006) (not shown)

Bulk Viscosity in Heavy-Ion Collisions

- Resistance against the radial expansion or compression of a fluid $\Pi_{Navier-Stokes} \sim -\zeta(\partial_\mu u^\mu)$
- Evolution with a non-zero ζ/s slows down the expansion of the fluid.
- Previous assumption: ζ/s is negligible in hydrodynamics studies of heavy-ion collisions

Bulk Viscosity in Heavy-Ion Collisions

- Resistance against the radial expansion or compression of a fluid $\Pi_{Navier-Stokes} \sim -\zeta(\partial_\mu u^\mu)$
- Peak at T_c?



Peak also seen in:
JNH, PRL 103 (2009) 172302,
Kharzeev JHEP 0809 (2008) 093

- HRG+HS(Kadam and Mishra arXiv:1408.6329)
- PHSD (PRC 87, 064903 (2013))
- non-conformal holographic model (Finazzo, Rougemont, Noronha - to appear shortly)
- pQCD (Arnold, Dogan, Moore Phys.Rev. D74 (2006) 085021)
- 14 mom. (Denicol et al, PRC90(2014)024912)

Second-order Transport Coefficients

Equations of Motion - 2nd order

Denicol et all, PRD85(2012)114047

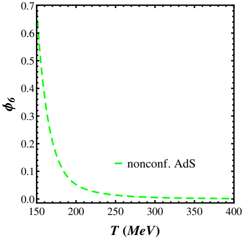
$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\frac{\zeta/s}{\tau_\Pi}\theta + -\delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \phi_1\Pi^2 + \phi_3\pi^{\mu\nu}\pi_{\mu\nu} \quad (1)$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = \frac{2\eta/s}{\tau_\pi}\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} + \phi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma_{\mu\nu} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \phi_6\Pi\pi^{\mu\nu} \quad (2)$$

v-USPhydro - in black
MUSIC non-zero terms - red
MUSIC zero terms - gray

Shear+Bulk Direct Coupling Terms

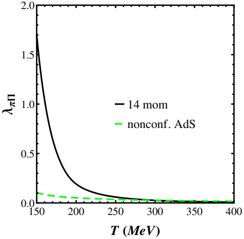
$\phi_6 \Pi \pi^{\mu\nu}$
in $\pi^{\mu\nu}$ evolution



Finazzo, Rougemont, Noronha

to appear shortly

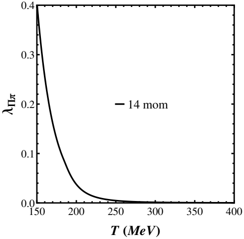
$\lambda_{\pi\Pi} \Pi \sigma_{\mu\nu}$
in $\pi^{\mu\nu}$ evolution



Denicol et al, PRC90(2014)024912

Molnar et al, PRD89(2014)074010

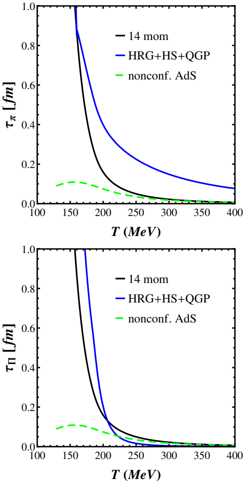
$\lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}$
in Π evolution



Denicol et al, PRC90(2014)024912

Molnar et al, PRD89(2014)074010

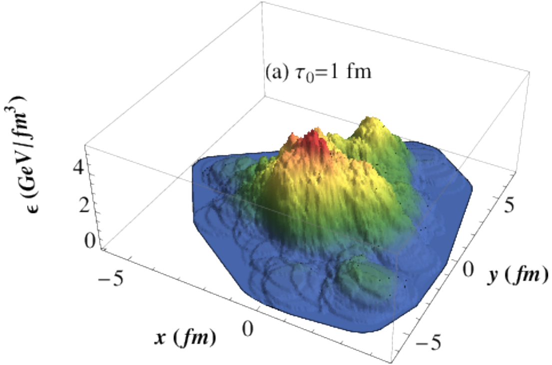
Relaxation Times



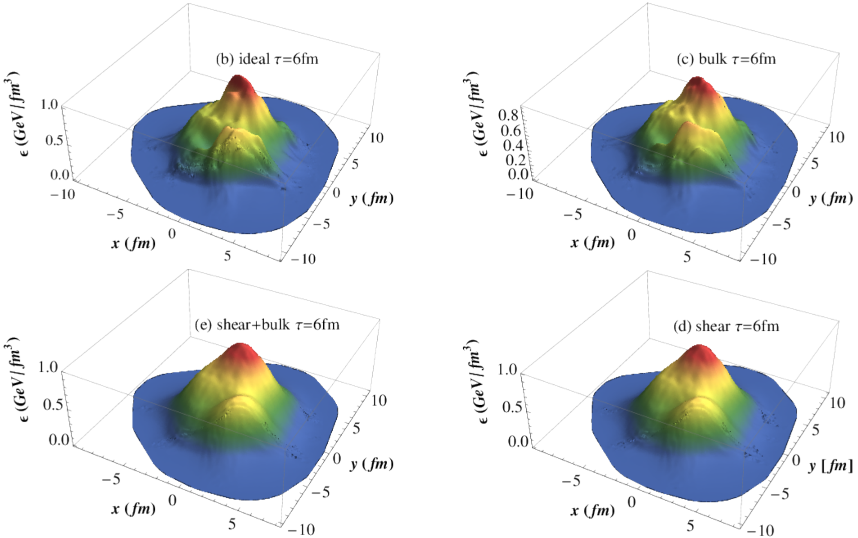
- τ_π for HRG+HS+QGP
  PRL105(2010)162501
- τ_Π HRG+HS+QGP
  Huang et al,PRC83(2011)024906
- 14 moment
  (Denicol et al, PRC90(2014)024912,Molnar et al, PRD89(2014)074010)
- nonconformal AdS
  (Finazzo, Rougemont, Noronha to appear shortly)

Viscosity in Heavy-Ion Collisions

Given the Glauber Initial Condition $\tau = 1\,fm$



Viscosity in Heavy-Ion Collisions



v-USPhydro - viscous Ultrarelativistic Smootheded Particle hydrodynamics

JNH et al, PRC90(2014)034907,PRC88(2013)044916

$$T^{\mu\nu} = \varepsilon u^{\nu} u^{\nu} - (p + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu} \tag{3}$$

Motivation

Write a modular event-by-event 2+1 hydrodynamical code that runs ideal & viscous hydro with nonzero ζ/s and η/s

- Initial conditions easily implemented from other sources.
- Equations of motion solved using Smoothed Particle Hydrodynamics (SPH)- quick comp. time and avoids numerical viscosity/grid size issues
- Coupled to UrQMD- results shown here without decays.

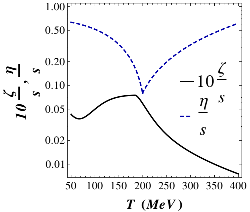
Description of Shear and Bulk Viscosity

$$\frac{\eta}{s}(T > T_{tr}) = -0.289 + 0.288\left(\frac{T}{T_{tr}}\right) + 0.0818\left(\frac{T}{T_{tr}}\right)^2$$

$$\frac{\eta}{s}(T < T_{tr}) = 0.681 - 0.0594\left(\frac{T}{T_{tr}}\right) - 0.544\left(\frac{T}{T_{tr}}\right)^2$$

JNH

PRL**103**(2009)172302,PRC**86**(2012)014909&PRL**106**(2011)212302

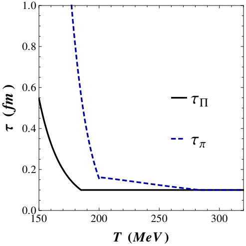


$$\tau_\pi = 5\eta/(\varepsilon + p)$$

PRL**105**, 162501 (2010)

$$\left(\frac{\zeta}{s}\right) = 0.5\,\frac{\eta}{s}\left(\frac{1}{3} - c_s^2\right), \qquad \tau_\Pi = 9\,\frac{\zeta}{\epsilon - 3p}$$

BuchelPLB663(2008)286
Huang,Kodama,Koide,RischkePRC83(2011)024906



Effects of viscosity with hydrodynamics (hydro only)

Compare percentage change of mean and variance in the presence of shear+bulk vs. bulk only (or shear only)

Effects of shear on Π

- The mean has almost no variation
- Shear increases the variation in bulk (at late times)
- Variation decreases significantly at early times

Effects of bulk on π^{00} and π^{12}

- Bulk suppresses the $\pi^{\mu\nu}$
- Largest effect at late times.
- Variation decreases across the board

Cooper-Frye Freeze-out
Overview

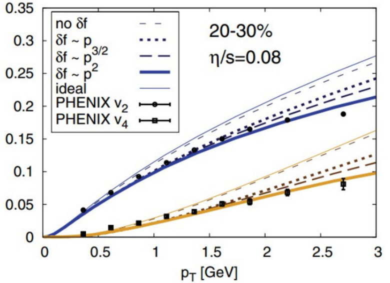
$$\left(E_p \frac{dN}{d^3p}\right)_i = g_i \int_\Sigma d\Sigma_\mu p^\mu f_i$$

Particle distribution function:

$$f_{\mathbf{k}}^{(i)} = f_{0\mathbf{k}}^{(i)} + \delta f_{\mathbf{k}}^{(i)}$$
$$f_{0\mathbf{k}}^{(i)} = (\exp[E_i/T] + a_i)^{-1}$$

Fermions: $a_i = 1$, Bosons: $a_i = -1$
Boltzmann gas: $a_i = 0$

Note that majority of viscous effects come from δf.



Schenke, Jeon, Gale,PRC85(2012)024901

Cooper-Frye Freeze-out

Derivation of $\delta f_{\mathbf{k}}^{(i)}$ 2/2: Denicol et al, PRD85(2012)114047

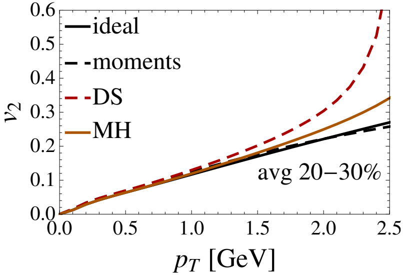
Truncating in momentum space up to the 2nd order and using the orthogonality relations from the basis:

$$
\begin{aligned}
f_{\mathbf{k}}^{(i)} &= f_{0\mathbf{k}}^{(i)} + \delta f_{\mathbf{k}}^{(i)Bulk} + \delta f_{\mathbf{k}}^{(i)Shear} \;, \\
\delta f_{\mathbf{k}}^{(i)Shear} &= \frac{f_{0\mathbf{k}}^{(i)}}{2\left(\varepsilon_i + P_i\right) T^2} \frac{\eta_i}{\eta} \pi^{\mu\nu} k_{i,\mu} k_{i,\nu} \;, \\
\delta f_{\mathbf{k}}^{(i)Bulk} &= f_{0\mathbf{k}}^{(i)} \Pi \left[B_0^{(i)} + D_0^{(i)} u \cdot k_i + E_0^{(i)} \left(u \cdot k_i\right)^2 \right]
\end{aligned}
$$

- $E_{0,i}$, $D_{0,i}$,$B_{0,i}$: functions of mass m_i and T- determined through basis

Dependence on δf - bulk only

JNH, Denicol, Noronha, Andrade, Grassi, PRC88(2013)044916



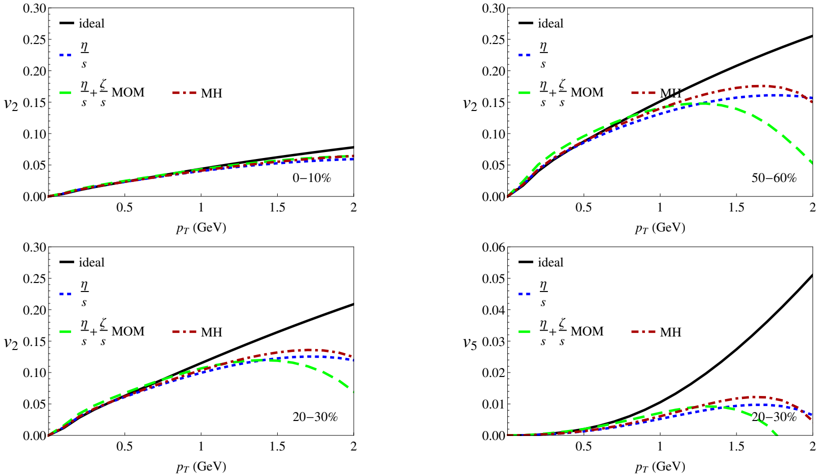
Averaged Glauber

$$\delta f^{(\pi)}_{\mathbf{k}} = f^{\pi}_{0\mathbf{k}}\, \Pi * \left[B_0^{(\pi)} + D_0^{(\pi)} u\cdot k_\pi + E_0^{(\pi)} (u\cdot k_\pi)^2 \right]$$

	E_0 $[fm^4]$	D_0 $\left[\frac{fm^4}{GeV}\right]$	B_0 $\left[\frac{fm^4}{GeV^2}\right]$	
mo	-65.85	171.27	-63.05	PRC88(2013)044916
DS	-71.96	121.50	0	PRC85(2012)044909
MH	-0.69	-38.96	49.69	PRC80(2009)054906

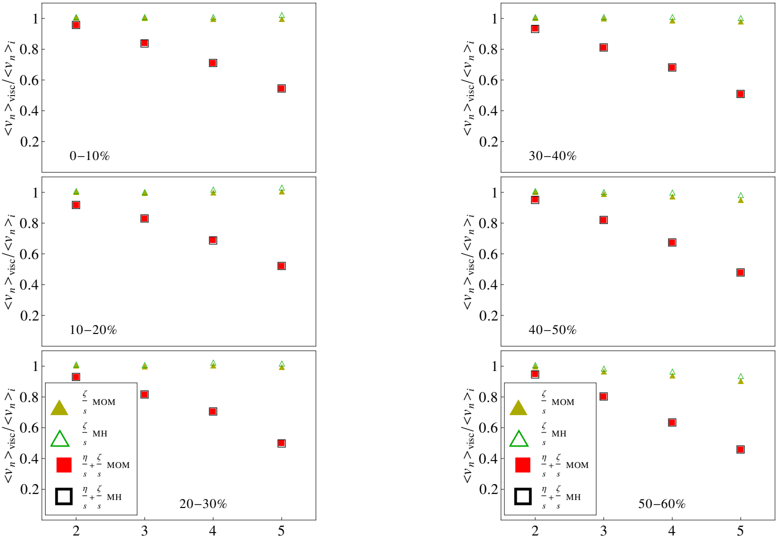
Event-by-Event v_2

JNH, Noronha, Grassi, PRC90(2014)034907



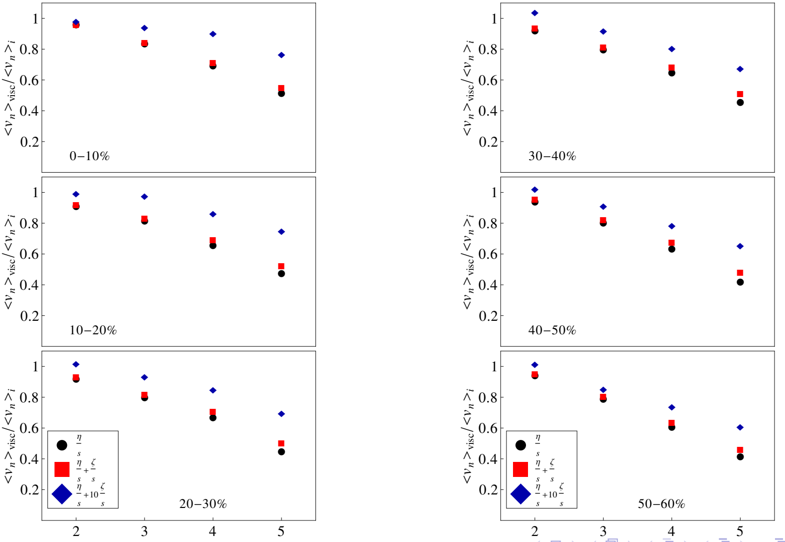
Integrated v_n's - Comparing δf

JNH, Noronha, Grassi, PRC90(2014)034907



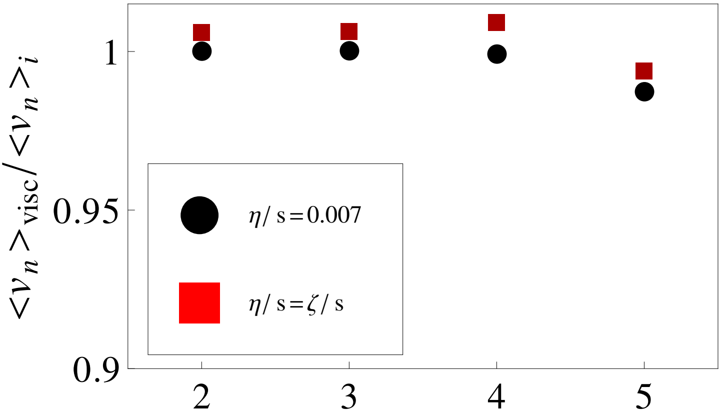
Integrated v_n's - Comparing ζ/s

JNH, Noronha, Grassi, PRC90(2014)034907



$\zeta/s = \eta/s$ integrated

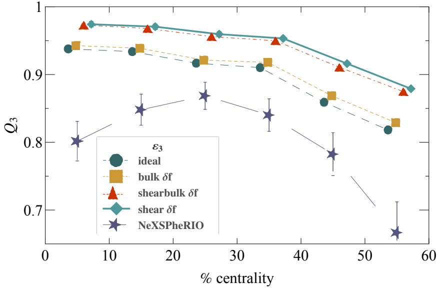
JNH, Noronha, Grassi, PRC90(2014)034907

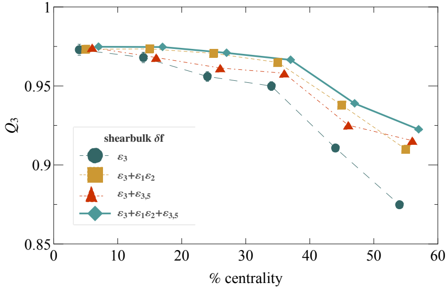


v_1 from $\varepsilon_1 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_5$

Gardim, JNH, Luzum, Grassi, arXiv:1411.2574

- Shear viscosity most strongly correlated to initial conditions
- Initial flow/3+1 dimensions less correlated with initial eccentricities (especially for central/peripheral collisions)
- Higher order eccentricities help correlate peripheral collisions

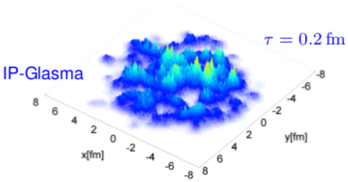




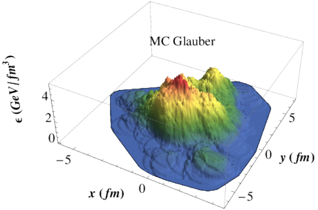
Conclusions

- Bulk viscosity may compensate the effects of shear viscosity (more relevant for longer hydrodynamical evolution)
- When $\zeta/s = \eta/s$ the effects of bulk may dominate
- Shear viscosity most strongly correlates to the initial eccentricities, shear+bulk is not as strongly correlated.
- ζ/s must be significantly smaller than η/s- otherwise runs into problems with δf.
- v-USPhydro+UrQMD results coming soon!

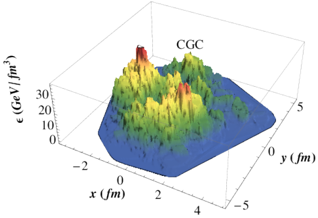
Types of Initial Conditions
Energy Density profile



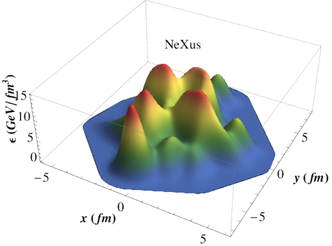
Schenke, Tribedy, Venugopalan,Phys.Rev.Lett. 108 (2012) 252301



Drescher,Nara, Phys. Rev. C 75, 034905 (2007); ,76, 041903 (2007).



Drescher,Nara, Phys. Rev. C 75, 034905 (2007); ,76, 041903 (2007).



Drescher,Hladik,Ostapchenko,Pierog, Werner, Phys.Rept. 350, 93 (2001)

Initial Conditions effects on Collective Flow

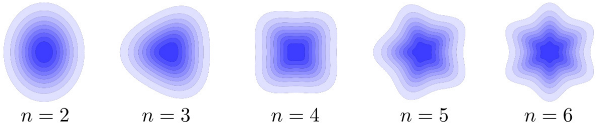
The distribution of particles can be written as a Fourier series (event plane method)

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left[1 + \sum_n 2v_n \cos\left[n\left(\phi - \psi_n\right)\right]\right]$$

- Flow Harmonics at mid-rapidity

$$v_n(p_T) = \frac{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi} cos\left[n\left(\phi - \Psi_n\right)\right]}{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi}}$$

where $\Psi_n = \frac{1}{n} \arctan \frac{\langle sin[(n\phi)]\rangle}{\langle cos[(n\phi)]\rangle}$



Smoothed Particle Hydrodynamics (SPH) Overview

Motivation

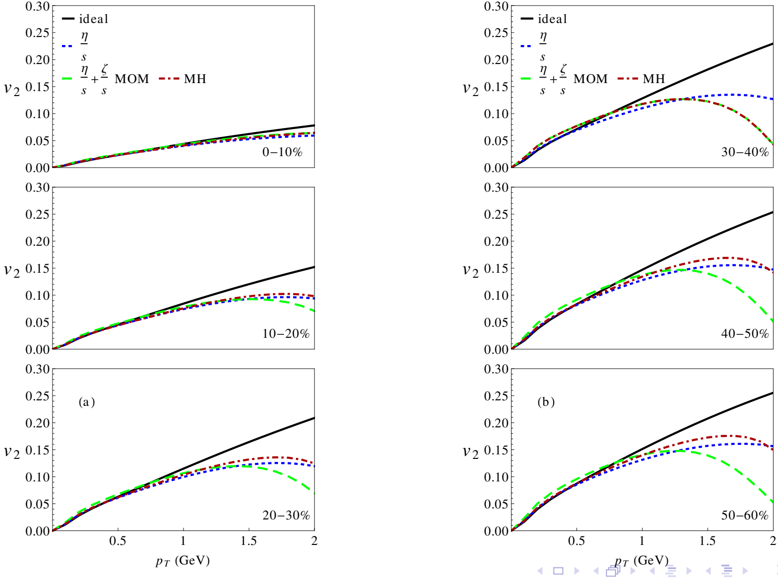
SPH discretizes the fluid into a number of SPH particles whose trajectories (**r** and **u**) you observe over time

Imagine you want to observe the motion of a lake

- SPH (Lagrangian)- you are in a boat on the lake and move over the coarse of time watching your trajectory.
- Grid (Euler)- you are seated at a dock and observe the rise and fall of water at a set spot 2m away from you.

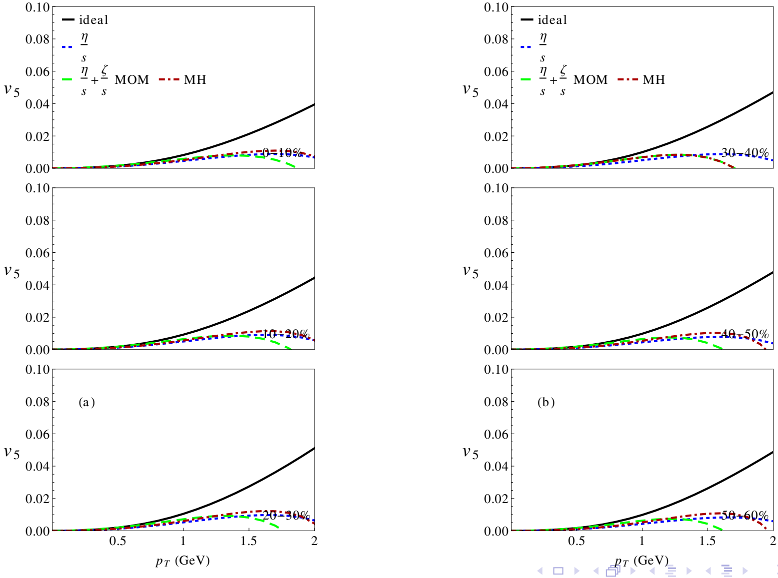
Event-by-Event v_2

JNH PRC90(2014)034907



Event-by-Event v_5

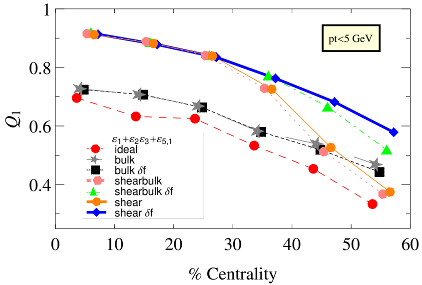
JNH PRC90(2014)034907

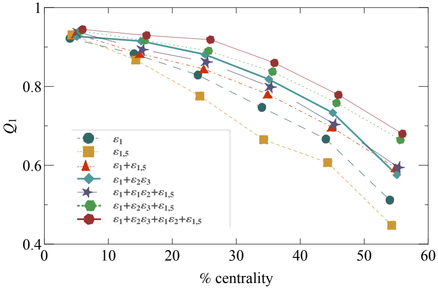


v_1 from $\varepsilon_1 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_5$

JNH arXiv:1411.2574

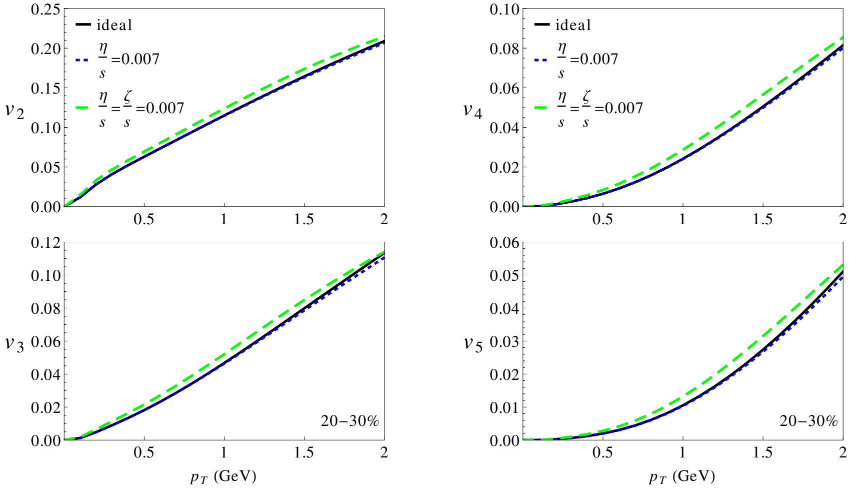
- Shear viscosity is most strongly correlated to initial conditions
- v_1 requires higher order eccentricities, correlates most strongly to low p_T for v_1





$\zeta/s = \eta/s$ p_T dependent

JNH PRC90(2014)034907



Tables (hydro only)

$$(\Pi)_{ev} = 100\,\frac{(\Pi_{sb})_{ev} - (\Pi_b)_{ev}}{(\Pi_b)_{ev}}$$

$$(\sigma^2_\Pi)_{ev} = 100\,\frac{\left(\sigma^2_{\Pi_{sb}}\right)_{ev} - \left(\sigma^2_{\Pi_b}\right)_{ev}}{\left(\sigma^2_{\Pi_b}\right)_{ev}}$$

Tables (hydro only)

	$\langle\Pi\rangle$	σ^2_Π	$\langle\Pi\rangle_{early}$	$(\sigma^2_\Pi)_{early}$	$\langle\Pi\rangle_{late}$	$(\sigma^2_\Pi)_{late}$
0-10%	1.79%	8.59%	1.14%	-59.72%	2.03%	20.50%
10-20%	2.48%	8.95%	2.89%	-52.37%	2.19%	20.59%
20-30%	2.87%	8.96%	4.07%	-40.70%	2.02%	20.66%
30-40%	3.49%	9.15%	3.47%	-36.96%	2.15%	19.97%
40-50%	4.14%	9.11%	3.52%	-37.23%	2.00%	20.86%
50-60%	4.98%	9.23%	6.27%	-22.55%	2.28%	19.73%

TABLE II. Percentage change of the mean values of the bulk pressure Π and its corresponding variance σ^2_Π averaged over all events for different centrality classes due to the presence of shear viscosity. $\langle\Pi\rangle$ and σ^2_Π takes into account the parts of the fluid that have frozen out throughout the whole time evolution, $\langle\Pi\rangle_{early}$ and $(\sigma^2_\Pi)_{early}$ are computed using only the parts of the fluid that have frozen out between $\tau_0 = 1$ fm and $\tau = 2$ fm, $\langle\Pi\rangle_{late}$ and $(\sigma^2_\Pi)_{late}$ are computed using only the parts of the fluid that have frozen out in the last fm of the time evolution.

Centrality	$\langle\pi^{00}\rangle$	$\sigma^2_{\pi^{00}}$	$\langle\pi^{12}\rangle$	$\sigma^2_{\pi^{12}}$
0-10%	-17.61%	-19.09%	-2.87%	-8.50%
10-20%	-17.77%	-18.53%	-2.25%	-8.45%
20-30%	-19.22%	-18.56%	-3.48%	-8.44%
30-40%	-22.98%	-18.53%	-3.26%	-8.35%
40-50%	-38.11%	-19.37%	-2.81%	-8.01%
50-60%	-44.63%	-19.61%	-5.05%	-7.68%

TABLE III. The percentage change in the mean values and variance of the π^{00} and π^{12} components of the shear stress tensor $\pi^{\mu\nu}$ averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account the parts of the

Centrality	$\langle\pi^{00}\rangle_{early}$	$(\sigma^2_{\pi^{00}})_{early}$	$\langle\pi^{12}\rangle_{early}$	$(\sigma^2_{\pi^{12}})_{early}$
0-10%	-6.66%	-12.79%	-5.94%	-10.66%
10-20%	-5.32%	-11.31%	-4.87%	-9.46%
20-30%	-6.07%	-12.72%	-4.81%	-9.15%
30-40%	-7.01%	-14.08%	-4.80%	-9.19%
40-50%	-4.75%	-9.00%	-4.75%	-8.99%
50-60%	-6.83%	-15.02%	-4.63%	-8.76%

TABLE IV. The percentage change in the mean values and variance of the π^{00} and π^{12} components of the shear stress tensor $\pi^{\mu\nu}$ averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account only the parts of the fluid that have already frozen for early times (between $\tau = \tau_0$ and $\tau = 2$ fm).

Centrality	$\langle\pi^{00}\rangle_{late}$	$(\sigma^2_{\pi^{00}})_{late}$	$\langle\pi^{12}\rangle_{late}$	$(\sigma^2_{\pi^{12}})_{late}$
0-10%	-17.68%	-29.13%	-5.94%	-10.80%
10-20%	-15.98%	-29.09%	-4.80%	-9.38%
20-30%	-15.45%	-28.56%	-4.77%	-9.06%
30-40%	-14.97%	-28.28%	-4.88%	-9.34%
40-50%	-13.83%	-27.91%	-4.80%	-9.20%
50-60%	-12.75%	-26.18%	-4.50%	-8.51%

TABLE V. The percentage change in the mean values and variance of the π^{00} and π^{12} components of the shear stress tensor $\pi^{\mu\nu}$ averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account only the parts

Equations of Motion

Conservation of Energy and Momentum

$$\partial_\mu T^{\mu\nu} = 0 \tag{4}$$

The energy-moment tensor contains a bulk dissipative term Π and the shear stress tensor $\pi^{\mu\nu}$ is

$$T^{\mu\nu} = \varepsilon u^\nu u^\nu - (p + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu} \tag{5}$$

Coordinate System: $x^\mu = (\tau, x, y, \eta)$ where $\tau = \sqrt{t^2 - z^2}$ and $\eta = 0.5 \ln\left(\frac{t+z}{t-z}\right)$

Cooper-Frye Freeze-out

Derivation of $\delta f_{\mathbf{k}}^{(i)}$ 1/2: Denicol et al, PRD85(2012)114047

Particle distribution function computed using a version of Grad's 14 moment approximation for the Boltzmann equation:

- Factorize $\delta f_{\mathbf{k}}^{(i)}$: $\delta f_{\mathbf{k}}^{(i)} = f_{0\mathbf{k}}^{(i)} \tilde{f}_{0\mathbf{k}}^{(i)} \phi_{\mathbf{k}}^{(i)}$ where $\tilde{f}_{0\mathbf{k}}^{(i)} = 1 + a f_{0\mathbf{k}}^{(i)}$
- Determine $\phi_{\mathbf{k}}^{(i)}$, out of equilibrium contribution, by establishing a basis of

Irreducible Tensors: $k_i^{\langle\mu\rangle}, k_i^{\langle\mu} k_i^{\nu\rangle}, k_i^{\langle\mu} k_i^{\nu} k_i^{\lambda\rangle}, \cdots$,

Orthonormal Polynomials: $P_{i\mathbf{k}}^{(n\ell)} = \sum_{r=0}^{n} a_{nr}^{(\ell)i} \left(u_\mu k_i^\mu\right)^r$,

- Then, $f_{\mathbf{k}}^{(i)} = f_{0\mathbf{k}}^{(i)} + f_{0\mathbf{k}}^{(i)} \tilde{f}_{0\mathbf{k}}^{(i)} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} \mathcal{H}_{i\mathbf{k}}^{(n\ell)} \rho_{i,n}^{\mu_1 \cdots \mu_\ell} k_{i,\mu_1} \cdots k_{i,\mu_\ell}$
  where $\mathcal{H}_{i\mathbf{p}}^{(n\ell)} \equiv \left[N_i^{(\ell)}/\ell!\right] \sum_{m=n}^{\infty} a_{mn}^{(\ell)i} P_{i\mathbf{k}}^{(m\ell)} \left(u_\mu k_i^\mu\right)$

Cooper-Frye Freezeout

Major Assumptions

- We assume Navier-Stokes scaling to relate the moments $\rho_{i,0}$, $\rho_{i,2}$, $\rho_{i,0}^{\mu\nu}$ to Π and $\pi^{\mu\nu}$- neglect effects from τ_Π.

$$\begin{aligned} \Pi &= -\zeta\partial_\mu u^\mu, \rho_{i,m} = -\alpha_{i,m}\partial_\mu u^\mu \Longrightarrow \rho_{i,m} = \frac{\alpha_{i,m}}{\zeta}\Pi, \\ \pi_i^{\mu\nu} &= 2\eta_i\partial^{\langle\mu}u^{\nu\rangle}, \pi_i^{\mu\nu} = 2\eta\partial^{\langle\mu}u^{\nu\rangle} \Longrightarrow \pi_i^{\mu\nu} = \frac{\eta_i}{\eta}\pi^{\mu\nu}. \end{aligned}$$

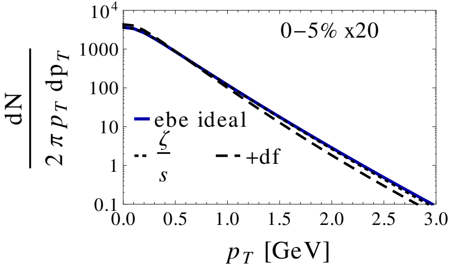
- All hadrons have the same cross-section of 30 mb
- Only hadrons up to a mass of $M = 1.2$ GeV are considered (every additional hadron increases the matrix rank needed for the calculation of transport coefficients, which becomes very costly)
- Freeze-out temperature $T_{FO} = 150$ MeV.

π^+ Spectrum (Direct π^+'s Only)

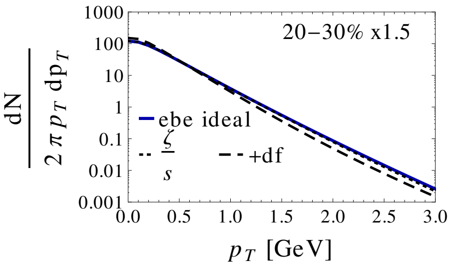
JNH PRC88(2013)044916

At $T = 150$ MeV about 41% of pions are direct pions. For most central collisions there are about 300 π^+'s, so 123 direct π^+'s.

$\pi^+ \approx 123$

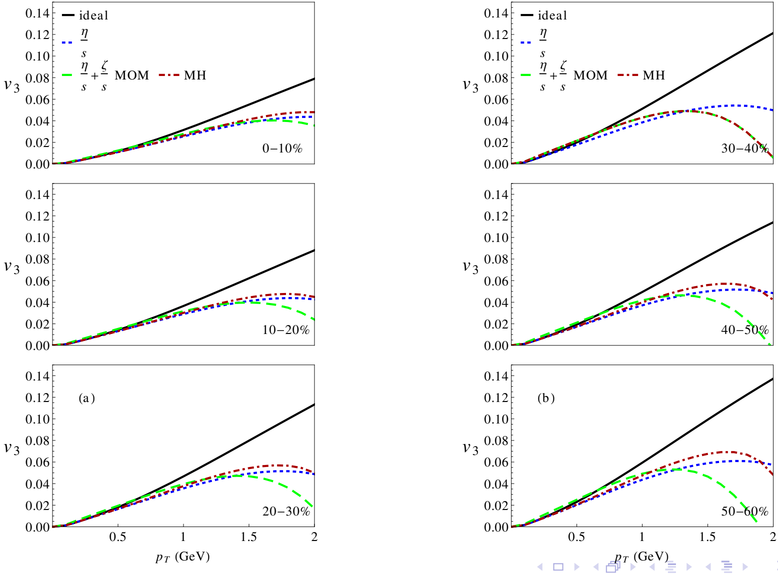


$\pi^+ \approx 54$



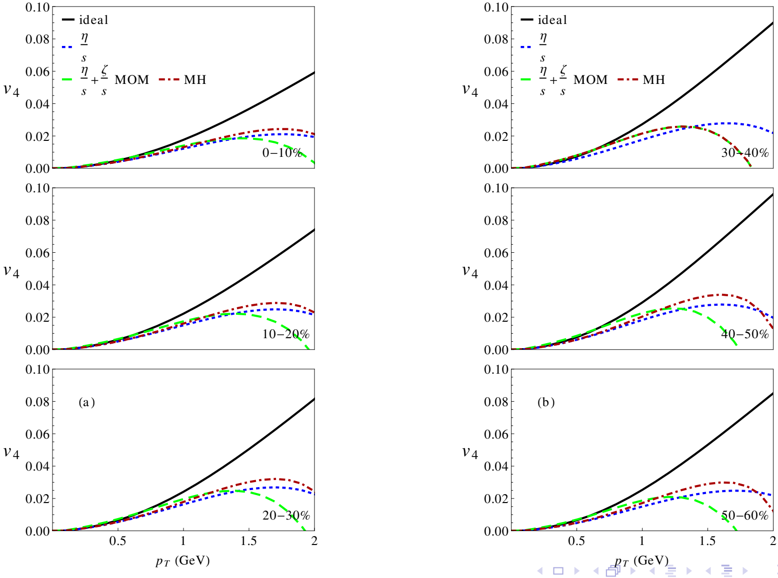
Event-by-Event v_3

JNH PRC90(2014)034907



Event-by-Event v_4

JNH PRC90(2014)034907



Parameters

- Isothermal freeze-out temperature: $T_{FO} = 150$ MeV
- Initial time to start hydrodynamic simulation: $t_0 = 1$ fm
- Lattice-based equation of state from Huovinen&Petreczky, NPA**837**, 26(2010)
  Currently testing HotQCD PRD90(2014)9,094503
- SPH scale $h = 0.3$ fm
- Energy conservation for event-by-event Glauber initial conditions: Ideal case $\sim 0.001\%$, Viscous case $\sim 0.1\%$

Equations of Motion

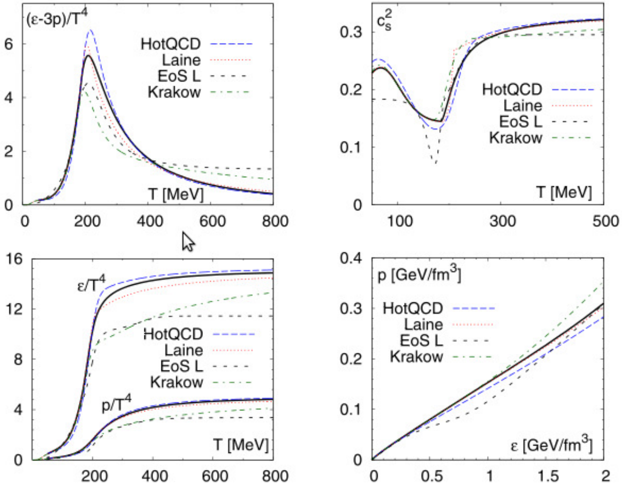
- SPH conserves reference density current: $J^\mu = \sigma u^\mu$ where σ is the local density of a fluid element in its rest frame
- Density obeys $\partial_\mu(\tau\sigma u^\mu) = 0$ in hyperbolic coordinates (in Cartesian $D\sigma + \sigma\theta = 0$) where $D = u^\mu\partial_\mu$ and $\theta = \tau^{-1}\partial_\mu(\tau u^\mu)$
- We use this set of IS equations, which provides the simplest equations for viscous hydrodynamics.

$$\begin{aligned} \tau_\Pi\left(D\Pi + \Pi\theta\right) + \Pi + \zeta\theta &= 0, \\ \tau_\pi\left(\Delta_{\mu\nu\alpha\beta}D\pi^{\alpha\beta} + \frac{4}{3}\pi_{\mu\nu}\theta\right) + \pi_{\mu\nu} &= 2\eta\sigma_{\mu\nu} \end{aligned}$$

PRC75(2007) 034909

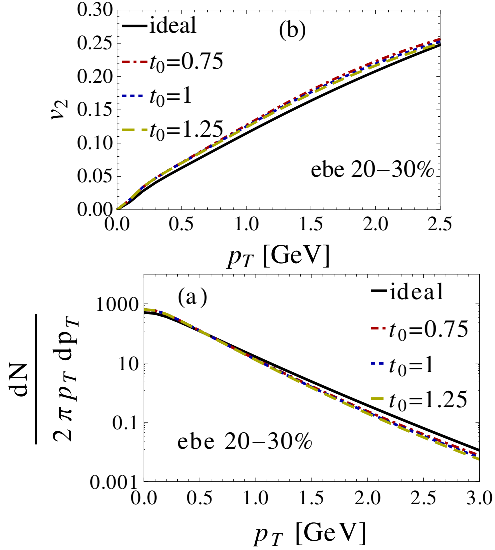
- There are four transport coefficients: η/s, ζ/s, τ_π, and τ_Π

Equation of State



Huovinen&Petreczky, NPA**837**, 26(2010)

Dependence on τ_0 (bulk only)

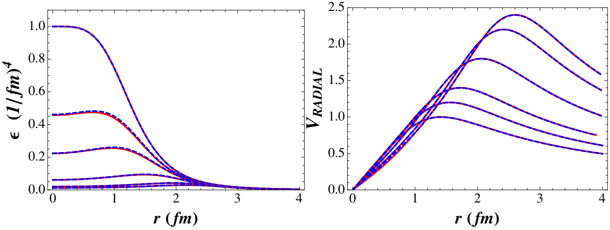


Checks- Gubser Test

- Reproduce analytical sol. from 2+1 conformal ideal hydro

$$\epsilon = \frac{\epsilon_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{\left[1 + 2q^2\left(\tau^2 + x_\perp^2\right) + q^4\left(\tau^2 - x_\perp^2\right)\right]^{4/3}}$$

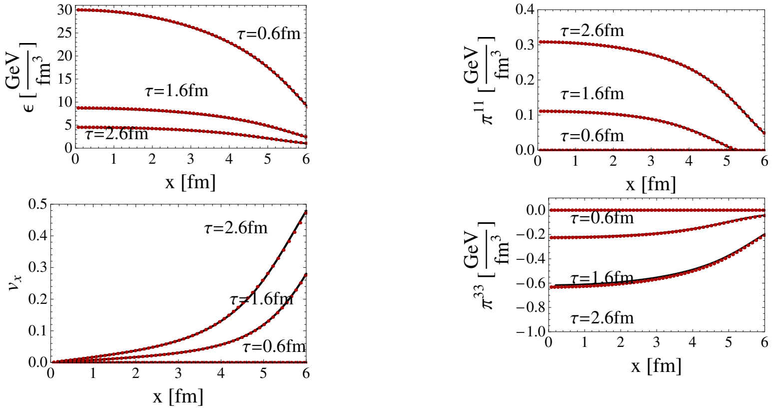
Gubser,PRD**82**,085027(2010), Marrochio et. al. 1307.6130 [nucl-th] (first analytical solution of Israel-Stewart hydro)



- The viscous bulk evolution converges to that computed within ideal hydrodynamics for sufficiently small ζ/s.

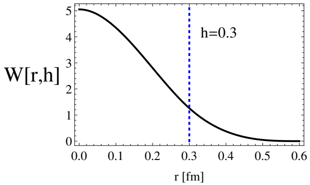
Checks- TECHQM (for shear)

Au+Au, $b = 0$ fm, EOS I ($\epsilon = 3p$), $\tau_0 = 0.6$ fm/c,



SPH Equations of Motion

- Reconstruct all hydrodynamical fields using a discrete set of Lagrangian coordinates $\{\mathbf{r}_\alpha(\tau), \alpha = 1, ..., N_{SPH}\}$ and a normalized piece-wise distribution function $W[\mathbf{r}; h]$



- h is a length scale, determines structure
- Reference density in the lab frame

$$\tau\gamma\sigma \to \sigma^*(\mathbf{r},\tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha W[\mathbf{r} - \mathbf{r}_\alpha(\tau); h] \qquad (6)$$

where ν_α are constants $\to \int d^2\mathbf{r}\, \sigma^*(\mathbf{r},\tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha$

SPH Equations of Motion

- Vector current becomes

$$\mathbf{j}^*(\mathbf{r},\tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \frac{d\mathbf{r}_\alpha(\tau)}{d\tau} W[\mathbf{r}-\mathbf{r}_\alpha(\tau); h], \tag{7}$$

  that satisfies $\partial_\tau \sigma^*(\mathbf{r},\tau) + \nabla_\mathbf{r} \cdot \mathbf{j}^*(\mathbf{r},\tau) = 0$
- Each "SPH particle", α, has $\mathbf{r}_\alpha(\tau)$, $\mathbf{u}_\alpha(\tau) = \gamma_\alpha(\tau)\mathbf{v}_\alpha(\tau)$, where $\mathbf{v}_\alpha(\tau) = d\mathbf{r}_\alpha(\tau)/d\tau$ and $\gamma_\alpha = 1/\sqrt{1-\mathbf{v}_\alpha^2}$, and it carries a quantity ν_α for the reference density σ^*

SPH Variables

- For any density assiociated with some exenstive quantity- $a(\mathbf{r},\tau)$

$$a(\mathbf{r},\tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \frac{a(\mathbf{r}_\alpha(\tau))}{\sigma^*(\mathbf{r}_\alpha(\tau))} W\left[\mathbf{r} - \mathbf{r}_\alpha(\tau); h\right] . \qquad (8)$$

- Thus, entropy

$$s^*(\mathbf{r},\tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \frac{s(\mathbf{r}_\alpha(\tau))}{\sigma(\mathbf{r}_\alpha(\tau))} W\left[\mathbf{r} - \mathbf{r}_\alpha(\tau); h\right] \qquad (9)$$

the bulk term

$$\Pi(\mathbf{r},\tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \frac{1}{\gamma_\alpha \tau} \left(\frac{\Pi}{\sigma}\right)_\alpha W[\mathbf{r} - \mathbf{r}_\alpha(\tau); h] . \qquad (10)$$

SPH Variables

- Dynamical variables: $\left\{\mathbf{r}_\alpha, \mathbf{u}_\alpha, \left(\frac{s}{\sigma}\right)_\alpha, \left(\frac{\Pi}{\sigma}\right)_\alpha ;\ \alpha = 1, .., N_{SPH}\right\}$
- Equations of Motion can then be rewritten as

$$\begin{aligned}
M^{ij}_\alpha \frac{du^j_\alpha}{d\tau} &= F_\alpha u^i_\alpha + \partial^i \left(p_\alpha + \Pi_\alpha\right) \\
\gamma_\alpha (\tau_\Pi)_\alpha \frac{d}{d\tau}\left(\frac{\Pi}{\sigma}\right)_\alpha + \left(\frac{\Pi}{\sigma}\right)_\alpha &= -\left(\frac{\zeta}{\sigma}\right)_\alpha (D_\mu u^\mu)_\alpha \\
\gamma_\alpha \frac{d}{d\tau}\left(\frac{s}{\sigma}\right)_\alpha &= -\frac{1}{T_\alpha}\frac{\Pi_\alpha}{\sigma_\alpha}(D_\mu u^\mu)_\alpha
\end{aligned}$$

SPH Equations of Motion

SPH discretizes the fluid into a number of SPH particles whose trajectories (**r** and **u**) you observe over time

Entropy

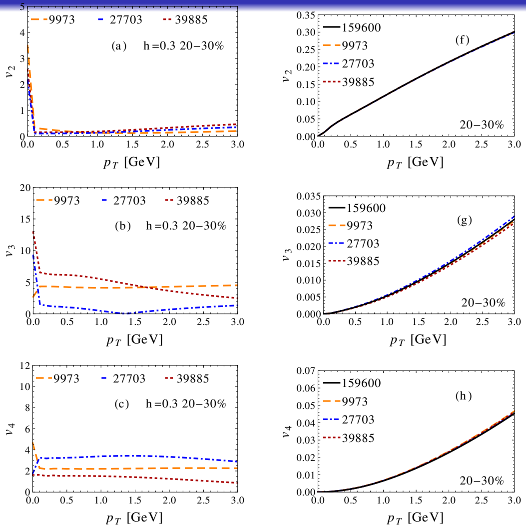
$s^* = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \left(\frac{s}{\sigma}\right)_\alpha W(|\mathbf{r} - \mathbf{r}_\alpha(t)|; h)$

PDE $\rightarrow$ ODE

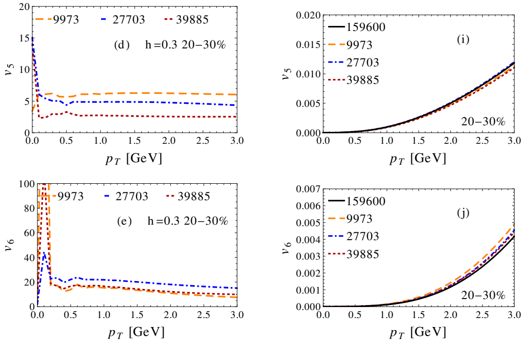
$$
\begin{aligned}
M_\alpha^{ij}\frac{du_\alpha^j}{d\tau} &= Btot_\alpha u_\alpha^i + F^i + \partial^i\left(p_\alpha + \Pi_\alpha\right) + v^j \partial^j \pi^{0i} - \partial^j \pi^{ij} \\
-\left(\frac{\zeta}{\sigma}\right)_\alpha \left(D_\mu u^\mu\right)_\alpha &= \gamma_\alpha (\tau_\Pi)_\alpha \frac{d}{d\tau}\left(\frac{\Pi}{\sigma}\right)_\alpha + \left(\frac{\Pi}{\sigma}\right)_\alpha \\
\gamma_\alpha \frac{d}{d\tau}\left(\frac{s}{\sigma}\right)_\alpha &= -\frac{1}{T_\alpha}\frac{\Pi_\alpha}{\sigma_\alpha}\left(D_\mu u^\mu\right)_\alpha + \frac{1}{T_\alpha}\frac{\pi_\alpha^{\mu\nu}}{\sigma_\alpha}\left(D_\mu u_\nu\right)_\alpha
\end{aligned}
$$

shear is much longer (not shown)

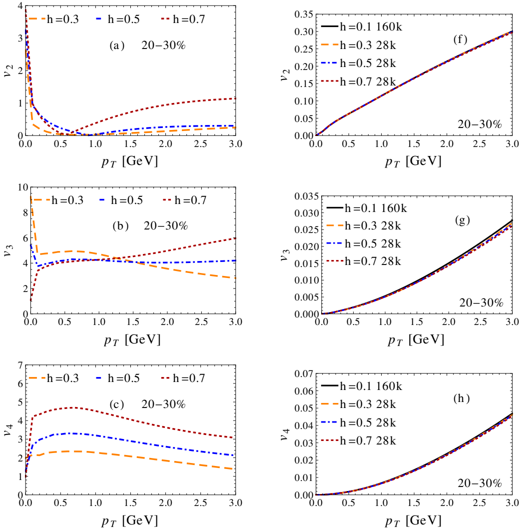
Testing of N_{SPH} with $h = 0.3$



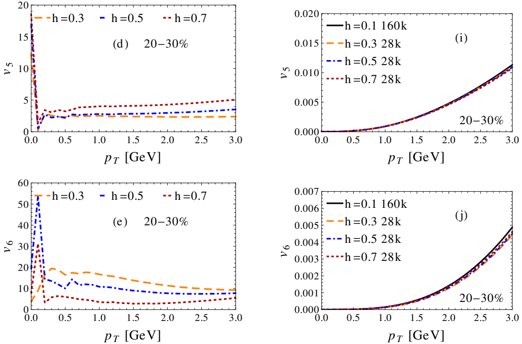
Testing of N_{SPH} with $h = 0.3$



Testing of h


h =0.3
h =0.5
h =0.7
(a) 20–30%
v_2
p_T [GeV]
(b) 20–30%
v_3
(c) 20–30%
v_4
h =0.1 160k
h =0.3 28k
h =0.5 28k
h =0.7 28k
(f)
(g)
(h)
20–30%

Testing of h


h =0.3
h =0.5
h =0.7
(d) 20−30%
v_5
p_T [GeV]
h =0.1 160k
h =0.3 28k
h =0.5 28k
h =0.7 28k
(i)
20−30%
v_5
p_T [GeV]
h =0.3
h =0.5
h =0.7
(e) 20−30%
v_6
p_T [GeV]
h =0.1 160k
h =0.3 28k
h =0.5 28k
h =0.7 28k
(j)
20−30%
v_6
p_T [GeV]

$v_n(p_T)$'s from bulk only