Fluid Fluctuations in heavy ion collisions and cosmology

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Report on one aspect (“backreaction”) of our recent work:
S. Flörchinger, N. Tetradis, U.A. Wiedemann, arXiv:1411.3280
N. Brouzakis, S. Flörchinger, N. Tetradis, U.A. Wiedemann, arXiv:1411.2912

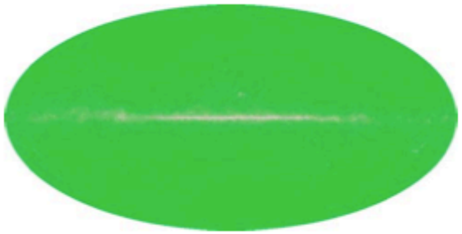
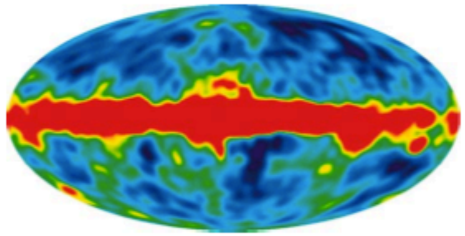
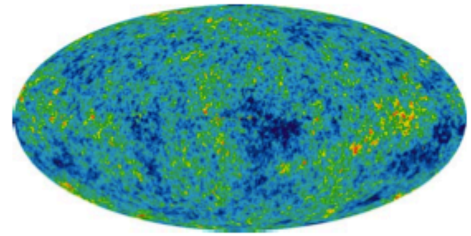
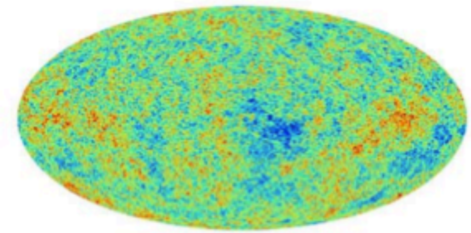
and earlier work:
S. Flörchinger, UAW, A. Beraudo, L. DelZanna, G. Inghirami, V. Rolando, arXiv:1312.5482, Phys. Lett. B735 (2014) 305-310

S. Flörchinger, UAW, JHEP 1408 (2014) 005
Phys.Lett. B728 (2014) 407-411
Phys.Rev. C89 (2014) 034914
Phys.Rev. C88 (2013) 044906

Napa Valley, 4 Dec 2014

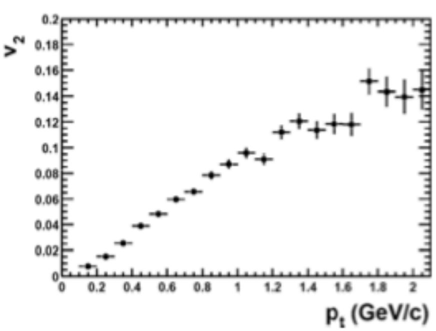
A (valid) analogy

Penzias/Wilson 1965	COBE 2003	WMAP 2007	Planck 2012

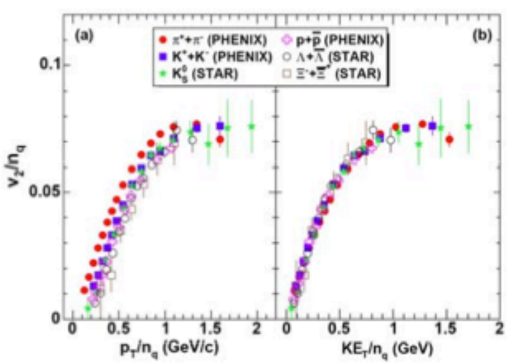





From a signal … via fluctuations ….

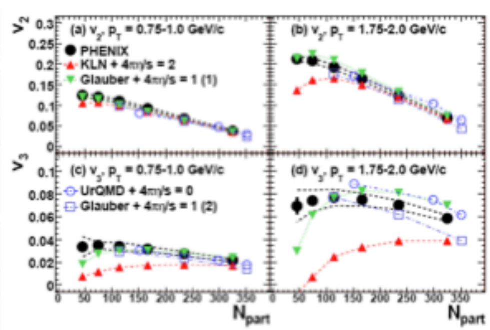
…. to properties of matter



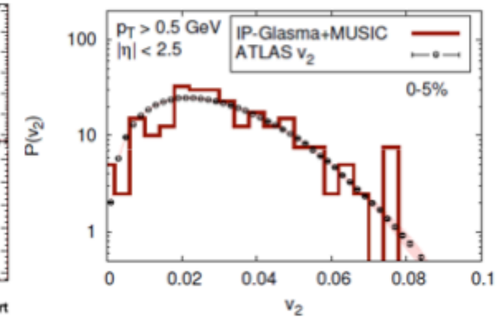
2001



2004



2008



2012

Slide adapted from W. Zajc

How are fluid fluctuations described?

In Cosmology:

Perturbatively, on top of homogeneous background fields.

$$\delta\varepsilon/\varepsilon \sim 10^{-5}$$

(at photon decoupling)

Gravitational collapse

$$\delta\varepsilon/\varepsilon \sim 1$$

(at time of structure formation)

Perturbative methods and N-body simulations are applied.

Early .. => .. Late

In heavy ion physics:

Non-perturbatively, via numerical codes.

$$\delta\varepsilon/\varepsilon \sim 1$$

(at hydrodynamization)

Dissipation in near-ideal fluid

$$\delta\varepsilon/\varepsilon \sim 10^{-1}$$

(at freeze-out)

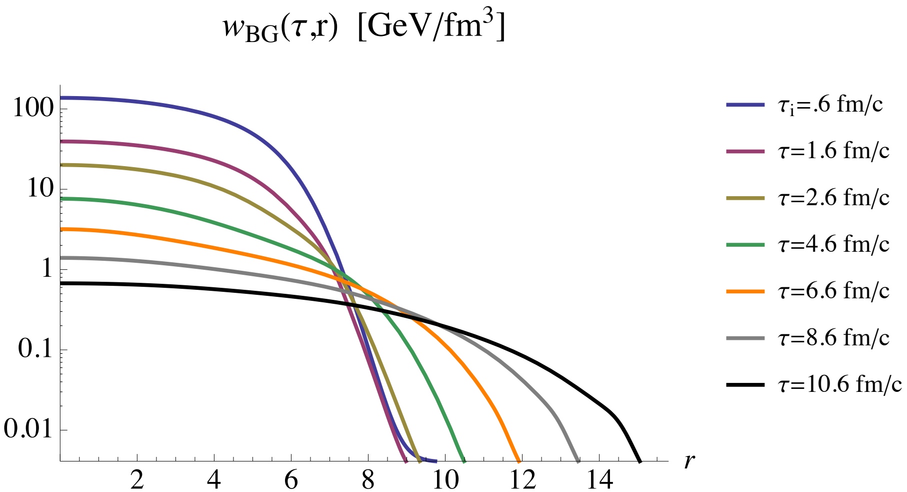
Can perturbative methods apply? P.T.O.

Fluid perturbation theory in HICs

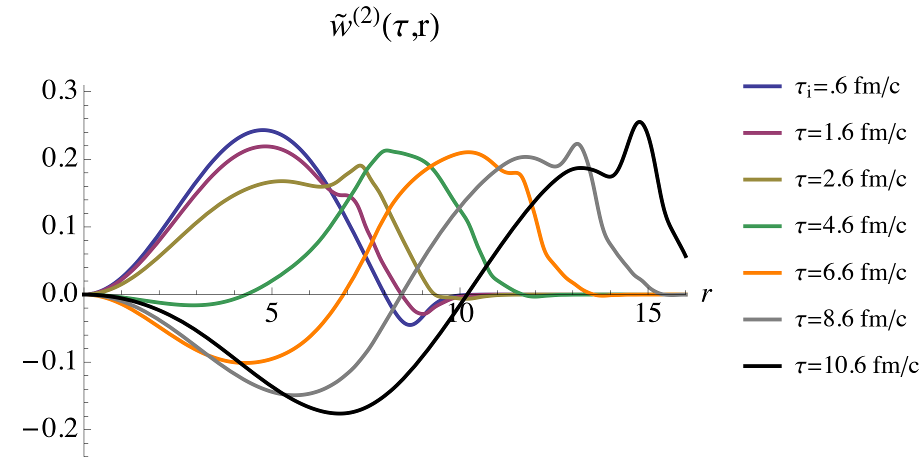
- Background-flucutation splitting of all fluid dynamic fields

$$h_i(\tau_0,r,\varphi) = \boxed{h_i^{BG}(\tau_0,r)} + \boxed{h_i^{fluct}(\tau_0,r,\varphi)}$$

Background enthalpy



Fluctuating mode on top of bkg



- Perturbative expansion in power of initial fluctuating modes **converges** for fluctuations of realistic size (upon suitable choice of basis function)

S. Flörchinger, UAW, A. Beraudo, L. DelZanna, G. Inghirami, V. Rolando, arXiv:1312.5482
Phys. Lett. B735 (2014) 305-310

$$\tilde{h}_i^{(m)} = \underbrace{\boxed{\sum_{l_1} G_{i;l_1}^{(m)} \tilde{w}_{l_1}^{(m)}}}_{\textbf{linear}} + \frac{1}{4\pi} \underbrace{\boxed{\sum_{l_1 l_2 m_1 m_2} H_{i;l_1 l_2}^{(m_1,m_2)} \tilde{w}_{l_1}^{(m_1)} \tilde{w}_{l_2}^{(m_2)} \delta_{m,m_1+m_2}}}_{\textbf{quadratic}} \underbrace{\boxed{+\, O(\varepsilon^3) \tilde{w}_{l_1}^{(m_1)} \tilde{w}_{l_2}^{(m_2)} \tilde{w}_{l_3}^{(m_3)}}}_{\textbf{cubic}}$$

Backreaction

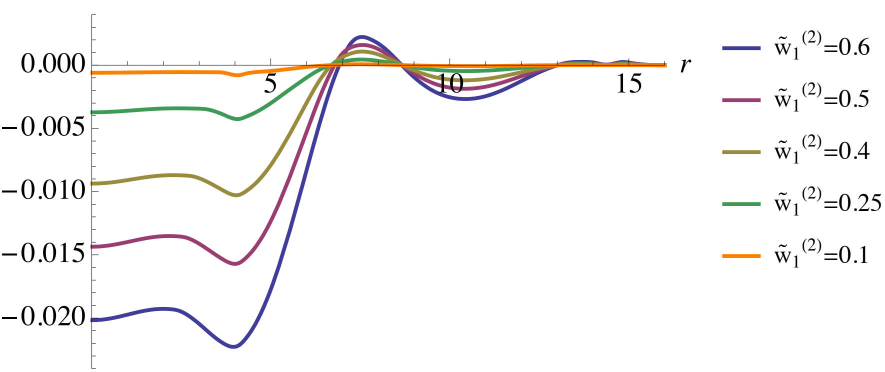
- Seed fluctuation of weight $\tilde{w}_1^{(2)}$ on top of azimuthally symmetric background

$$h_i^{fluct}(\tau_0,r,\varphi)=\delta_{i1}w^{BG}(\tau_0,r)\sum_{m=-\infty}^{\infty}\sum_{l=1}^{\infty}\tilde{w}_l^{(m)}J_m\left(k_l^{(m)}r\right)e^{im\varphi}$$

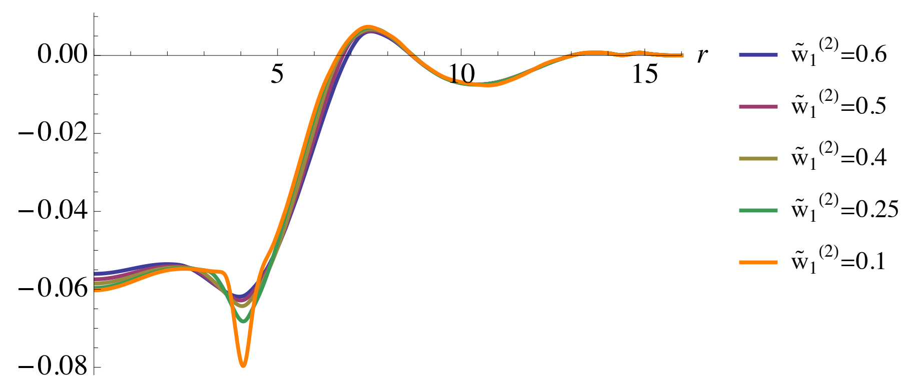
- Observe <u>'heating of background'</u> due to dissipation of this fluctuation.
  i.e. the 0th (and 4th) harmonic of fluid fields have a contribution $\propto\left(\tilde{w}_1^{(2)}\right)^2$

S. Flörchinger, UAW, A. Beraudo, L. DelZanna, G. Inghirami, V. Rolando, arXiv:1312.5482
Phys. Lett. B735 (2014) 305-310

$w_{BG}(\tau,r)\tilde{w}^{(0)}(\tau,r)$ [GeV/fm^3], $\tau=\tau_0+10$ fm/c



$w_{BG}(\tau,r)\tilde{w}^{(0)}(\tau,r)/(\tilde{w}_1{}^{(2)})^2$, $\tau=\tau_0+10$ fm/c



- In principle, this 2nd order backreaction of fluctuations on the homogeneous background implies **EbyE correlations between v_n and T**.

Time-RG Flow appoach – *as cosmologists do it*

M. Pietroni, arXiv:0806.0971, JCAP 0810 (2008) 036

- Study fluid evolution of **spectra** and **bispectra** that quantify fluctuations on top of **homogeneous background**

$$\langle \phi_a(k)\phi_b(p)\rangle = \boxed{P_{ab}(k)}\delta(k+p) + \boxed{\bar{\phi}_a\bar{\phi}_b}\,\delta(k)\delta(p)$$

$$\langle \phi_a(k)\phi_b(p)\phi_c(q)\rangle = \boxed{B_{ab}(k,p,q)}\delta(k+p+q)$$

$$+\,\bar{\phi}_a P_{bc(p)}\,\delta(k)\delta(p+q)[3perm] + \bar{\phi}_a\bar{\phi}_b\bar{\phi}_c\delta(k)\delta(p)\delta(q)$$

- The fluid eqs of motion for $\phi_a(t,k) \equiv \left(\tilde{d}_k, \tilde{f}_k\right)$

$$d \equiv \ln\left[T/T_{Bj}(\tau)\right]$$

$$v_m \equiv \partial_m f$$

$$\dot{\phi}_a(k) = -\Omega_{ab}(k)\phi_b(k) + \int dp\,dq\,\delta(k-p-q)\gamma_{abc}(p,q)\phi_b(p)\phi_c(q)$$

can be written and <u>solved directly for spectrum and bispectrum</u>, e.g.

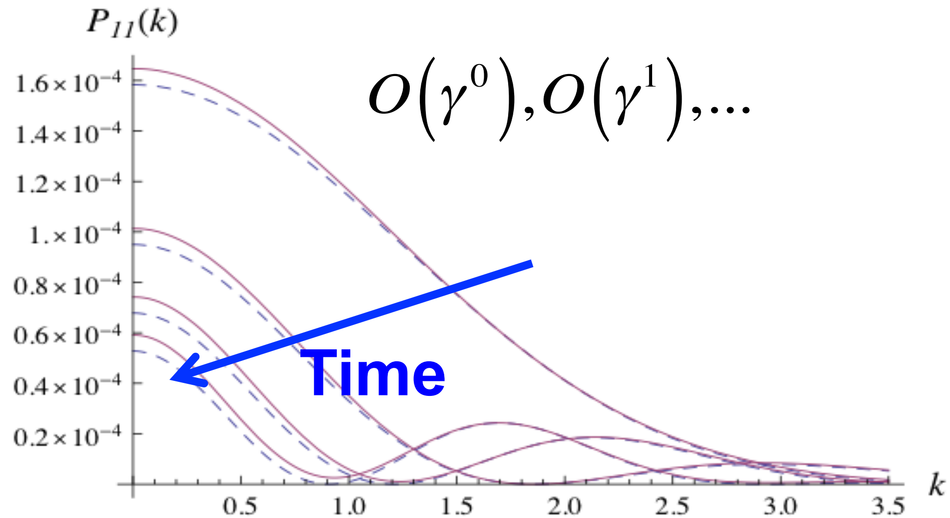
$$\partial_t P_{ab}(k) = -\Omega_{ac}(k)P_{cb}(k) - \Omega_{bc}(-k)P_{ac}(k) \qquad O\left(\gamma^0\right)$$

$$+\int dp\left[\gamma_{abc}(p,k-p)B_{bcd}(-k,p,k-p) + \gamma_{bcd}(p,-k-p)B_{acd}(k,p,-k-p)\right] \qquad O\left(\gamma^1\right)$$

Fluctuation spectra on Bjorken flow – numerical solution

N. Brouzakis, S. Flörchinger, N. Tetradis, U.A. Wiedemann, arXiv:1411.2912

- Solve numerically for spectrum:
  - IPSM Gaussian initialization
  - visualizes evolution of fluctuation spectrum to longer wavelengths.
  - another consistency check for fluid dynamic perturbation theory



- $O(\gamma^2)$ contribution to **backreaction**. One can show

N. Brouzakis, S. Flörchinger, N. Tetradis, U.A. Wiedemann, arXiv:1411.2912

$$\boxed{\bar{\phi}_a(t) = \int_{t_0}^{t} dt' \int dq\, g_{ab}(0,t,t')\gamma_{bcd}(-q,q) P_{cd}(q,t')}$$

With propagator $\partial_t g_{ab}(k,t,t') = -\Omega_{ac}(k,t) g_{ab}(k,t,t'), \qquad g_{ab}(k,t,t) = \delta_{ab}$

The bkg energy density $\bar{\phi}_1$ receives contribution from dissipating fluctuations.

But where is this backreaction in cosmology? …

Standard homogeneous solution for cosmological fluid

$$G_{\mu\nu} = -8\pi G_N T_{\mu\nu}$$

Assume isotropy and homogeneity (neglect curvature & cosmological constant)

$$ds^2 = a^2(\tau)\left[-d\tau^2 + d\vec{x}.d\vec{x}\right] \qquad T^0_{\ 0} = \varepsilon, \quad T^i_{\ j} = \overbrace{\left(p + \pi_{bulk}\right)}^{\equiv p_{eff}}\delta^i_j$$

00 – component of Einstein field equations gives 1st Friedmann equation

$$\boxed{\varepsilon = \frac{3}{8\pi G_N} H^2}, \qquad H \equiv \frac{\dot{a}}{a^2}$$

Recall: 1st Friedmann eq. governs cosmological expansion, e.g.:

- non-relativistic matter $\quad \varepsilon \propto \left(a/a_0\right)^3 \Rightarrow a(t) \propto t^{2/3}$
- radiation $\quad \varepsilon \propto \left(a/a_0\right)^4 \Rightarrow a(t) \propto \sqrt{t}$

ij – component of Einstein field equations gives 2nd Friedmann equation whose only remaining information is **energy 'conservation'**:

$$\boxed{\frac{1}{a}\dot{\varepsilon} + 3H\left(\varepsilon + p_{eff}\right) = 0} \qquad p_{eff} = p - 3\varsigma H$$

($\nabla_\mu T^{\mu\nu} = 0, \quad \nabla_\mu N^\mu = 0$ contained in Einstein field eqs.)

Backreaction on homogeneous solution

S. Flörchinger, N. Tetradis, U.A. Wiedemann, arXiv:1411.3280

E.o.m. for energy density in presence of fluctuations

(*) $$u^\mu \partial_\mu \varepsilon + (\varepsilon + p)\nabla_\mu u^\mu - \varsigma \Theta^2 - 2\eta \sigma_{\mu\nu}\sigma^{\mu\nu} = 0$$

This contains fluctuations in fluid fields and in the metric

$$ds^2 = a^2\left[-(1+2\psi)d\tau^2 + (1+2\phi)d\vec{x}.d\vec{x}\right] \qquad u^\mu = \tfrac{1}{a}\left((1-\psi)\gamma, (1+\phi)\gamma\vec{v}\right), \quad \vec{v}^2 << 1$$

$$\nabla_\mu u^\mu = \tfrac{1}{a}\left[\vec{\nabla}.\vec{v} + 3H + \phi\vec{\nabla}.\vec{v} - 3H\psi - 3\dot{\phi} + \vec{v}.\vec{\nabla}(\phi - 2\psi)\right]$$

The spatial average of (*) yields to leading $O(\phi), O(\psi)$

$$\tfrac{1}{a}\dot{\bar{\varepsilon}} + 3H\left(\bar{\varepsilon} + \bar{p} - 3\bar{\varsigma}H\right) = D$$

$$D = \tfrac{1}{a^2}\left\langle \eta\left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \tfrac{2}{3}\partial_i v_i \partial_j v_j\right]\right\rangle + \tfrac{1}{a^2}\left\langle \varsigma\left[\vec{\nabla}.\vec{v}\right]^2\right\rangle + \tfrac{1}{a}\left\langle \vec{v}.\vec{\nabla}(p - 6\varsigma H)\right\rangle$$

Backreaction:

Average energy density increases due to

- Shear viscous dissipation
- Bulk viscous dissipation
- Work done against pressure gradients

acts like negative pressure

$$D = -\tfrac{1}{a}\int d\vec{q}\, P_{\Theta p}(\vec{q}) + \frac{(\bar{\varsigma} + 4\bar{\eta}/3)}{a^2}\int d\vec{q}\, P_{\Theta\Theta}(\vec{q}) + \frac{\bar{\eta}}{a^2}\int d\vec{q}\, P_{vorticity}(\vec{q})$$

Effect on cosmological expansion

S. Flörchinger, N. Tetradis, U.A. Wiedemann, arXiv:1411.3280

To study acceleration parameter $q(t) \equiv \frac{1}{aH^2}\frac{d^2 a(t)}{dt^2} = 1 + \dot{H}\big/\left(aH^2\right)$

supplement $\boxed{\tfrac{1}{a}\dot{\bar{\varepsilon}} + 3H\left(\bar{\varepsilon} + \bar{p} - 3\bar{\varsigma}H\right) = D}$

with equation for scale parameter. $\left\langle G^{\mu}_{\mu}\right\rangle = -8\pi G_N \left\langle T^{\mu}_{\mu}\right\rangle$ yields

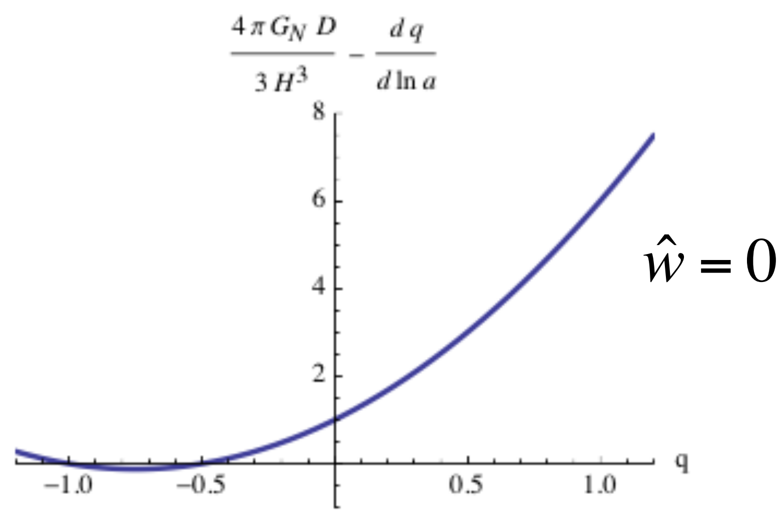
$$\ddot{a}\big/a^3 = \tfrac{4\pi G_N}{3}\left(\bar{\varepsilon} - 3\bar{p} - 3\bar{\pi}_{bulk}\right)$$

For $p_{eff} = \hat{w}\,\varepsilon$ $\boxed{\frac{dq}{d\ln a} + 2\left(q+1\right)\left(q + \tfrac{1}{2}\left(1+3\hat{w}\right)\right) = \frac{4\pi G_N D\left(1-3\hat{w}\right)}{3H^3}}$

Accelerating expansion (q>0) occurs *in this case* for

$$\frac{4\pi G_N D}{3H^3} > \frac{1+3\hat{w}}{1-3\hat{w}}$$

In general, ac/deceleration depends on e.o.s. and size of dissipative effects.



Cosmological expansion depends on material properties of dark sector

S. Flörchinger, N. Tetradis, U.A. Wiedemann, arXiv:1411.3280

Acceleration parameter positive if $D \sim H^3 / G_N$

- Bulk viscosity: for D=0, ς large, known to lead to accel. expansion,
  but $p_{eff} < 0$ e.g. J. Gagnon, J. Lesgourgues, JCAP 1109 (2011) 026
  for finite D, thdyn. stability problems do not arise
- Shear viscosity: $D \sim \eta O\left(\left(\partial_i v_j\right)^2\right) \sim \sigma\eta H^2$

for very weakly interacting relativistic particle (graviton), shear viscosity given by scattering time times energy density of radiative component S. Weinberg 1971

$$\boxed{\eta = c_\eta \varepsilon_R \tau_R}$$

For gravitons mean free time is $\boxed{\tau_G = \dfrac{1}{16\pi G_N \eta}}$ S. Hawking 1966

$$\Rightarrow \quad \frac{4\pi G_N D}{3H^3} = \frac{4\pi G_N \eta\sigma}{3H} = \sigma\sqrt{\frac{c_\eta \varepsilon_G}{24\rho_{crit}}} \overset{!!!}{\sim} O(1)$$

Coupling DM to graviton field not meant as a ‘realistic model’, but these estimate illustrate that $D \sim H^3 / G_N$ may be obtained for conceivable material properties.

Instead of Conclusions: a plead for knowledge transfer

- In cosmology and heavy ion physics, material properties are inferred by analysis of fluid dynamical fluctuations. Given the successes of modern precision cosmology, the question arises whether we (= heavy ion physicists) can adopt some of the techniques used.

- Likewise, our experience in heavy ion physics with the formulation dissipative phenomena and non-schematic equation of states may contribute to precision cosmology.

- In particular, if an explanation for the current accelerated cosmological expansion could be given in terms of dissipative phenomena, this would
  - explain the coincidence problem of why cosmic acceleration occurs only at late times when structures form
  - most likely replace dark energy by a dissipative dark material

- Our treatment of dissipative effects allows for further quantitative exploration

E.g. 1. $D = -\frac{1}{a}\int d\vec{q}\, P_{\Theta p}\left(\vec{q}\right) + \frac{\left(\bar{\varsigma}+4\bar{\eta}/3\right)}{a^2}\int d\vec{q}\, P_{\Theta\Theta}\left(\vec{q}\right) + \frac{\bar{\eta}}{a^2}\int d\vec{q}\, P_{vorticity}\left(\vec{q}\right)$

is calculable in simulations of gravitational collapse

2. Formulation for realistic eos $s\left(\varepsilon, n\right)$ is richer than the discussion given here. Expansion history depends on eos.