# Anisotropic hydrodynamics

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International Conference on the Initial Stages in High-Energy Nuclear Collisions
December 3–7, 2014, Embassy Suites Napa Valley, California

# 1. Introduction

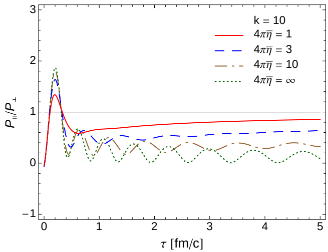
# 1.1 Plan

# 1.2 Motivation

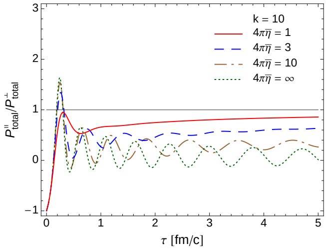
The results of microscopic models suggest that the initial state of matter produced in heavy-ion collisions is highly anisotropic in the momentum space (AdS/CFT, CGC, string models,...).

Color-flux-tube model results for different viscosity to entropy density ratios, Ryblewski+WF, PRD 88, 034028 (2013)

ratio of pressures without the color field




ratio of pressures with the color field



# 1.2 Motivation

The viscous hydrodynamics is based on the gradient expansion around the equilibrium state, this type of the expansion may be questioned in the situation where the space-time gradients are very large.

The goal of the anisotropic hydrodynamics program is to create a dissipative hydrodynamics framework that more accurately describes several features such as: i) the early time dynamics of the QGP created in heavy-ion collisions, ii) dynamics near the transverse edges of the nuclear overlap region, iii) temperature-dependent (and potentially large) $\eta/s$ . The general idea is to start the hydro expansion around the anisotropic state.

# 2. Anisotropic hydrodynamics an attempt of classification

# 2.1 Phenomenological vs. kinetic-theory formulations

**Phenomenological formulation**

R. Ryblewski, WF

PRC 83, 034907 (2011), JPG 38 (2011) 015104

1. energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$
2. ansatz for the entropy source, e.g., $\partial(\sigma U^\mu) \propto (\lambda_\perp - \lambda_\parallel)^2/(\lambda_\perp \lambda_\parallel)$

**Kinetic-theory formulation**

M. Martinez, M. Strickland

NPA 848, 183 (2010), NPA 856, 68 (2011)

1. first moment of the Boltzmann equation = energy-momentum conservation
2. zeroth moment of the Boltzmann equation = specific form of the entropy source

3. Generalized form of the equation of state based on the Romatschke-Strickland (RS) form

generalization of equilibrium/isotropic distributions, frequently used in the studies of anisotropic quark-gluon plasma (here as a modified Boltzmann distribution in the local rest frame)

$$f_{RS} = \exp\left(-\sqrt{\frac{p_\perp^2}{\lambda_\perp^2} + \frac{p_\parallel^2}{\lambda_\parallel^2}}\right) = \exp\left(-\frac{1}{\lambda_\perp}\sqrt{p_\perp^2 + x\, p_\parallel^2}\right) = \exp\left(-\frac{1}{\Lambda}\sqrt{p_\perp^2 + (1+\xi)\, p_\parallel^2}\right)$$

anisotropy parameter $x = 1 + \xi = \left(\frac{\lambda_\perp}{\lambda_\parallel}\right)^2$ and transverse-momentum scale $\lambda_\perp = \Lambda$

# 2.1 Phenomenological vs. kinetic-theory formulations

**4.** Energy-momentum tensor (with single anisotropy parameter)

$$T^{\mu\nu} = (\varepsilon + P_\perp) U^\mu U^\nu - P_\perp g^{\mu\nu} - (P_\perp - P_\parallel) Z^\mu Z^\nu$$

$\varepsilon(\sigma, x)$ — energy density, $P_\perp(\sigma, x)$ — transv. pressure, $P_\parallel(\sigma, x)$ — long. pressure
alternatively one may use: $\varepsilon(\Lambda, \xi)$, $P_\perp(\Lambda, \xi)$, $P_\parallel(\Lambda, \xi)$

$U$ — flow four-vector, $Z$ — beam four-vector, $U^2 = 1$, $Z^2 = -1$, $U \cdot Z = 0$

this form follows from the covariant version of RS

$$f_{RS} = \exp\left(-\frac{1}{\Lambda}\sqrt{(p\cdot U)^2 + \xi\, (p\cdot Z)^2}\right), \quad Z = (z/\tau, 0, 0, t/\tau)$$

**5.** Early applications

**5.1** (0+1)–D expansion with boost-invariance, transversely homogeneous matter
agreement with the Israel-Stewart second-order hydrodynamics has been demonstrated

**5.2** (0+1)–D expansion without boost-invariance, transversely homogeneous matter

**5.3** phenomenological approach applied in arbitrary number of dimensions, we shall come back to this subject in the second part of the talk

# 2.2 Two expansion methods

Kinetic-theory formulation

## Perturbative approach

Bazov, Heinz, Strickland
PRC 90, 044908 (2014)

$$f = f_{RS} + \delta f$$

1. the leading order is still described by the Romatschke-Strickland form (accounting for the difference between the longitudinal and transverse pressures)
2. advanced methods of traditional viscous hydrodynamics are used to restrict the form of the correction $\delta f$ and to derive aHydro equations — non-trivial dynamics included in the transverse plane and, more generally, in (3+1)D case

## Non-perturbative approach

Nopoush, Ryblewski, Strickland,Tinti, WF

$$f = f_{ANISO} + \dots$$

1. all effects due to anisotropy included in the leading order, in the generalised RS form
2. (1+1)D conformal case, two anisotropy parameters
3. (1+1)D non-conformal case, two anisotropy parameters + one bulk parameter
4. full (3+1)D case, five anisotropy parameters + one bulk parameter (shear tensor and bulk pressure)

TALKS BY: Nopoush, Ryblewski and Tinti

# 2.3 Exact solutions of the Boltzmann equation

new technique: aHydro and viscous hydro predictions are checked agains exact solutions of the Boltzmann kinetic equation in the relaxation time approximation, important constraints on the structure of the hydro equations and the form of the kinetic coefficients

One dimensional expansion

Denicol, Maksymiuk, Ryblewski, Strickland, WF

1. conformal case
2. non-conformal case
3. non-conformal case with quantum statistics

TALK BY Ryblewski

(1+1)D flow with Gubser symmetry

Denicol, Heinz, Martinez, Noronha, Strickland

TALK BY Nopoush

important result: there is a significant shear-bulk coupling that should be included in the calculations of the bulk viscous pressure

Denicol, WF, Ryblewski, Strickland, PRC 90, 044905 (2014)

Jaiswal, Ryblewski, Strickland, PRC 90, 044908 (2014)

# 3. aHydro and thermalization

we come back to the phenomenological (3+1)D formulation
series of papers by Radoslaw Ryblewski and WF

WF and Ryblewski, PRC 83, 034907 (2011); Ryblewski and WF, JPG 38, 015104 (2011)
Ryblewski and WF, EPJC 71, 1761 (2011); Ryblewski and WF, PRC 85, 064901 (2012)
review article: Ryblewski, JPG 40, 093101 (2013)

in this formulation, the system approaches the perfect-fluid regime for $\tau > \tau_0 + \tau_{\rm eq}$

# 3. aHydro and thermalization
## 3.1 Initial conditions

- Initial evolution time $\tau_0 = 0.25$ fm, equilibration time $\tau_{\rm eq} = 0.25$ fm and 1 fm
- initial anisotropy choices: $x_0 = 100$ (initial oblate configuration), $x_0 = 1$ (isotropic state), and $x_0 = 0.032$ (prolate configuration)
- initial energy density profile (tilted source by P.Bozek)

$$\varepsilon_0(\tau_0, \eta, \mathbf{x}_\perp) = \varepsilon_{\rm i}\, \tilde{\rho}(b, \eta, \mathbf{x}_\perp) \qquad \tilde{\rho}(b, \eta, \mathbf{x}_\perp) = \frac{\rho(b, \eta, \mathbf{x}_\perp)}{\rho(0,0)}$$

$$\rho(b, \eta, \mathbf{x}_\perp) = (1-\kappa)\left[\rho_W^+(b, \mathbf{x}_\perp)\, f^+(\eta) + \rho_W^-(b, \mathbf{x}_\perp)\, f^-(\eta)\right] + \kappa \rho_B(b, \mathbf{x}_\perp)\, f(\eta)$$

- initial longitudinal profile

$$f(\eta) = \exp\left[-\frac{(\eta - \Delta\eta)^2}{2\sigma_\eta^2}\theta(|\eta| - \Delta\eta)\right] \qquad \Delta\eta = 1, \sigma_\eta^2 = 1.3$$

- mixing factor $\kappa = 0.14$, initial energy density in the center $\varepsilon_{\rm i}$ chosen separately for each pair of $x$ and $\tau_{\rm eq}$

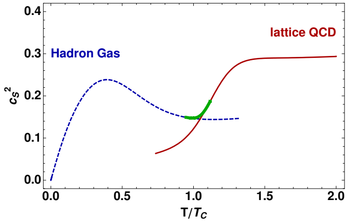
# 3. aHydro and thermalization

## 3.2 Generalized EOS – inclusion of the phase transition

to connect the isotropization with the process of formation of the equilibrated quark-gluon plasma we may consider the following ansatz

$$\begin{aligned} \varepsilon(\sigma, x) &= \varepsilon_{\rm qgp}(\sigma) r(x) \\ P_\perp(\sigma, x) &= P_{\rm qgp}(\sigma) \left[ r(x) + 3x r'(x) \right] \\ P_\parallel(\sigma, x) &= P_{\rm qgp}(\sigma) \left[ r(x) - 6x r'(x) \right] \end{aligned}$$

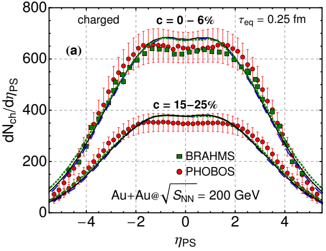
Here, the functions $\varepsilon_{\rm qgp}(\sigma)$ and $P_{\rm qgp}(\sigma)$ describe the realistic equation of state :
M. Chojnacki and WF, Acta Phys. Pol. B38 (2007) 3249.

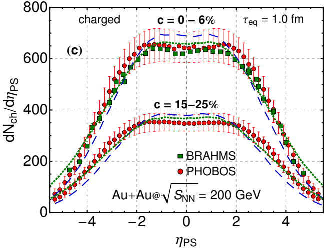


# 3. aHydro and thermalization

## 3.3 $dN/d\eta$ of charged particles

Initial anisotropy : $x_0 = 1$ (black), $x_0 = 100$ (blue), and $x_0 = 0.032$ (green), freeze-out at constant entropy density corresponding to $T = 150$ MeV, first 1 fm/c of the freeze-out hypersurface excluded

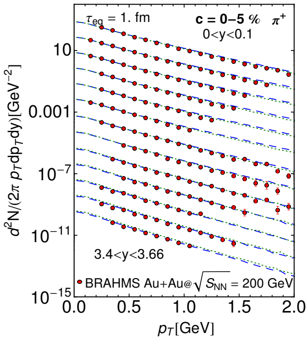


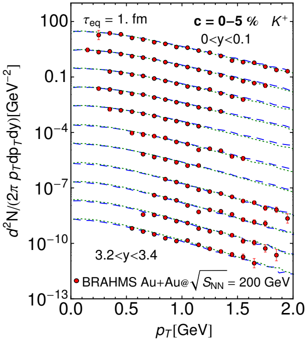



# 3. aHydro and thermalization

## 3.4 $p_\perp$ spectra in different $y$ windows

Initial anisotropy : $x_0 = 1$ (black), $x_0 = 100$ (blue), and $x_0 = 0.032$ (green)

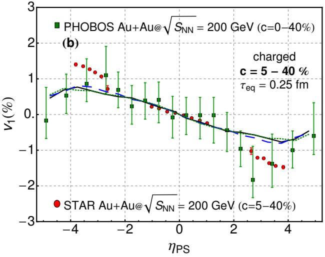


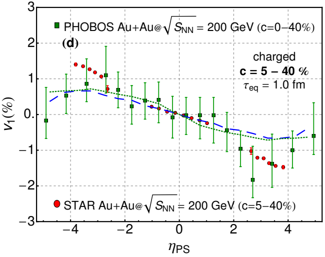



# 3. aHydro and thermalization

## 3.5 pseudorapidity dependence of $v_1$ for charged particles

Initial anisotropy : $x_0$ = 1 (black), $x_0$ = 100 (blue), and $x_0$ = 0.032 (green)

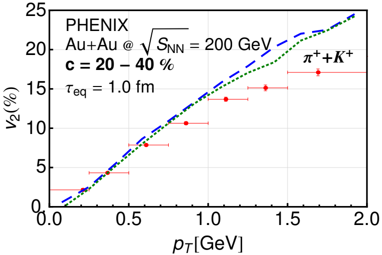


# 3. aHydro and thermalization

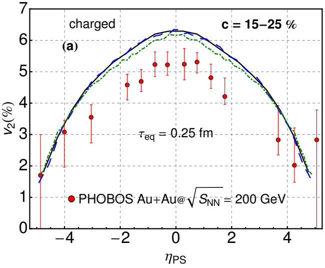
## 3.6 $v_2(p_T)$ in midrapidity

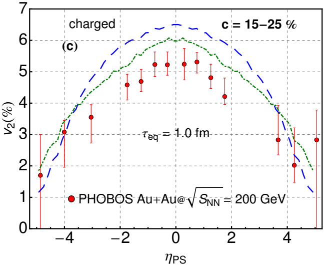
Initial anisotropy : $x_0$ = 1 (black), $x_0$ = 100 (blue), and $x_0$ = 0.032 (green)



# 3. aHydro and thermalization
## 3.7 $v_2$ of charged particles

Initial anisotropy : $x_0 = 1$ (black), $x_0 = 100$ (blue), and $x_0 = 0.032$ (green)



# 4. Summary and conclusions

# 4. Summary and conclusions

- A new framework of ANISOTROPIC HYDRODYNAMICS (aHydro) has been introduced. At the moment there exist two formulations of aHydro based on the kinetic theory and one phenomenological approach.
- Comparisons of aHydro and viscous hydrodynamics with the exact solutions revealed many interesting features and gave quantitative results about the structure of the hydrodynamic equations and the form of the kinetic coefficients (SHEAR-BULK COUPLING)
- Applications of aHydro indicate that a complete THERMALIZATION of the system MAY BE DELAYED to easily acceptable times of about 1–2 fm/c. The early-thermalization puzzle may be circumvented.
- aHydro may be matched with the microscopic models of early stages, as soon as the two pressures become positive