From Beginning to End
Learning about the initial state from final-state measurements

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Basic truths

1. Most initial state energy is swallowed by bulk

2. All bulk properties are driven by numerous unknown properties (model parameters)
How this was done before ($v_2$ and $\eta/s$)

Study single parameter vs. single observable

$v_2 \equiv \langle \cos 2\phi \rangle$

![Diagram of $v_2$](image)

P.Romatshke & U.Romtschke, PRL 2007

![Graph showing $v_2$ vs. $p_T$](image)
PROBLEM

v2 depends on ....

• viscosity
• saturation model
• pre-thermal flow
• Eq. of State
• T-dependence of \( \eta/s \)
• initial \( T_{xx}/T_{zz} \)
• . . . . .
Correct Way (MCMC)

- Simultaneously vary $N$ model parameters $x_i$
- Perform random walk weight by likelihood

$$L(x|y) \sim \exp \left\{ - \sum_a \frac{(y_a^{(\text{model})}(x) - y_a^{(\text{exp})})^2}{2\sigma_a^2} \right\}$$

- Use all observables $y_a$
- Obtain representative sample of posterior
Very Difficult Because…

I. Too Many Model Runs
Requires running model \( \sim 10^6 \) times

II. Many Observables
Could be hundreds of plots, each with dozens of points
Complicated Error Matrices
Model Emulators

1. Run the model ~1000 times
   Semi-random points (LHS sampling)

2. Determine Principal Components
   \[
   (y_a - \langle y_a \rangle) / \sigma_a \rightarrow z_a
   \]

3. Emulate \( z_a \) (Interpolate) for MCMC
   Gaussian Process…

\[
\mathcal{L}(x | y) \sim \exp \left\{ -\frac{1}{2} \sum_a (z_a^{\text{emulator}}(x) - z_a^{\text{exp}})^2 \right\}
\]

Emulator Algorithms

- **Gaussian Process**
  - Reproduces training points
  - Assumes localized Gaussian covariance
  - Must be trained, i.e. find “hyper parameters”

- Other methods also work
14 Parameters

- 5 for Initial Conditions at RHIC
- 5 for Initial Conditions at LHC
- 2 for Viscosity
- 2 for Eq. of State

30 Observables

- $\pi, K, \rho$ Spectra
- $\langle p_t \rangle$, Yields
- Interferometric Source Sizes
- $v_2$ Weighted by $p_t$
Initial State Parameters

\[ \epsilon(\tau = 0.8 \text{fm}/c) = f_{\text{wn}} \epsilon_{\text{wn}} + (1 - f_{\text{wn}}) \epsilon_{\text{cgc}}, \]

\[ \epsilon_{\text{wn}} = \epsilon_0 T_A \frac{\sigma_{\text{nn}}}{2 \sigma_{\text{sat}}} \{1 - \exp(-\sigma_{\text{sat}} T_B)\} + (A \leftrightarrow B) \]

\[ \epsilon_{\text{cgc}} = \epsilon_0 T_{\text{min}} \frac{\sigma_{\text{nn}}}{\sigma_{\text{sat}}} \{1 - \exp(-\sigma_{\text{sat}} T_{\text{max}})\} \]

\[ T_{\text{min}} \equiv \frac{T_A T_B}{T_A + T_B}, \]

\[ T_{\text{max}} \equiv T_A + T_B, \]

\[ u_\perp = \alpha \tau \frac{\partial T_{00}}{2 T_{00}} \]

\[ T_{zz} = \gamma P \]

5 parameters for RHIC, 5 for LHC
Equation of State and Viscosity

\[ c_s^2(\epsilon) = c_s^2(\epsilon_h) \]
\[ \quad + \left( \frac{1}{3} - c_s^2(\epsilon_h) \right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2}, \]
\[ X_0 = \sqrt{12} R c_s(\epsilon), \]
\[ x \equiv \ln \frac{\epsilon}{\epsilon_h} \]
\[ \frac{\eta}{s} = \left. \frac{\eta}{s} \right|_{T=165} + \kappa \ln \left( \frac{T}{165} \right) \]

2 parameters for EoS, 2 for \( \eta/s \)
1. Experiments reduce PBs to 100s of plots

2. Choose which data to analyze
   Does physics *factorize*?

3. Reduce plots to a few representative numbers, $y_a$

4. Transform to principal components
Checking the Distillation

Spectral information encapsulated by two numbers, $dN/dy$ & $\langle p_t \rangle$

- **π, p spectral SHAPES**
  - 30 random points in parameter prior
  - 74 pion spectra: with $573 < \langle p_t \rangle_\pi < 575$ MeV
  - 44 proton spectra: with $1150 < \langle p_t \rangle_p < 1152$ MeV
Two Calculations

1. J. Novak, K. Novak, S.P., C. Coleman-Smith & R. Wolpert,
   ArXiv:1303.5769
   RHIC Au+Au Data
   6 parameters

2. S.P., E. Sangaline, P. Sorensen & H. Wang, in progress
   RHIC Au+Au and LHC Pb+Pb Data
   14 parameters, include Eq. of State
Sample Spectra from Prior and Posterior

ALICE

- 0-5% cent
- 20-30% cent

(a) protons
(b) kaons
(c) pions
(d) kaons
(e) kaons
(f) protons

$\frac{dN}{2N dpd\phi} \ [\text{GeV/c}]^2$ vs $p_t \ [\text{GeV/c}]$
Sample V2 from Prior and Posterior

![Graph showing v2, 20-30% cent with ALICE data points.](image)
Sample HBT from Prior and Posterior
\[ \frac{\eta}{s}(T) \]

\[
\frac{\eta}{s} = \left(\frac{\eta}{s}\right)_0 + \kappa \ln\left(\frac{T}{165}\right)
\]
$\eta/s$ vs saturation picture

See Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007
Eq. of State

\[
c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left( \frac{1}{3} - c_s^2(\epsilon_h) \right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},
\]

\[
X_0 = X' R c_s(\epsilon) \sqrt{12},
\]

\[
x \equiv \ln \frac{\epsilon}{\epsilon_h}
\]
Which observables constrain the EoS?
Sensitivity to Uncertainty

Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)

- **UNCONSTRAINED**
- **CONSTRAINED WITH 9% UNCERTAINTIES**
- **CONSTRAINED WITH 6% UNCERTAINTIES**
- **Lattice: Hot QCD / BW upper/lower ranges (arXiv:1407.6387)**

$C_s^2$ (speed of sound squared)

$T$ (MeV)
CONCLUSIONS

♦ Robust
♦ Emulation works splendidly
♦ Scales well to more parameters & more data
♦ Eq. of State and Viscosity can be extracted from RHIC & LHC data
♦ Other parameters not as well constrained
♦ Heavy-Ion Physics can be a Quantitative Science!!!!
FUTURE

- Improve statement of uncertainties
- Add parameters (many related to hadronization)
- Consider more data
  - more observables
  - data from different beams/energies
- Improve models
  - Lumpy Glauber initial conditions
  - 3D calculations for lower energies
  - Fill in missing physics
If you’re interested…

1. Tools are easily extended
2. Download software and tutorial from http://madai.us
3. Talk to me (prattsc@msu.edu)
   or Evan Sangaline (esangaline@gmail.com)

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Additional slide: Charge BFIs and charge susceptibilities