# From Beginning to End

## Learning about the initial state from final-state measurements

**Scott Pratt, Michigan State University**

*MADAI Collaboration*

*Models and Data Analysis Initiative*

*http://madai.us*

Cyber-enabled Discovery and Innovation

MICHIGAN STATE UNIVERSITY Duke UNIVERSITY THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL renci

1st MADAI Collaboration Meeting, SANDIA 2010

# Basic truths

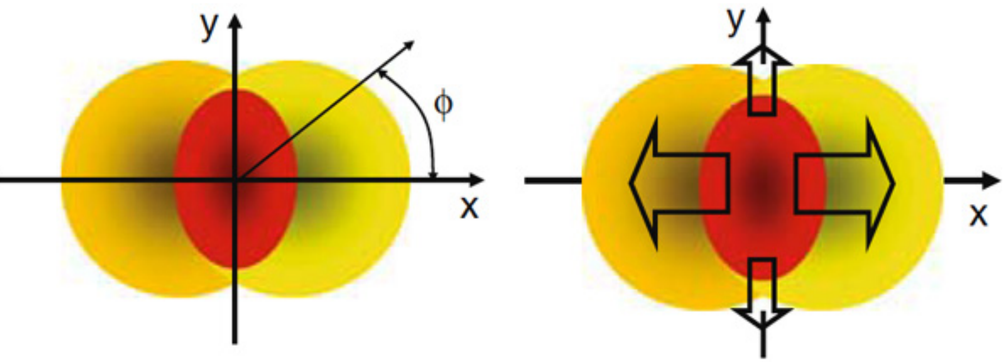
1. Most initial state energy is swallowed by bulk

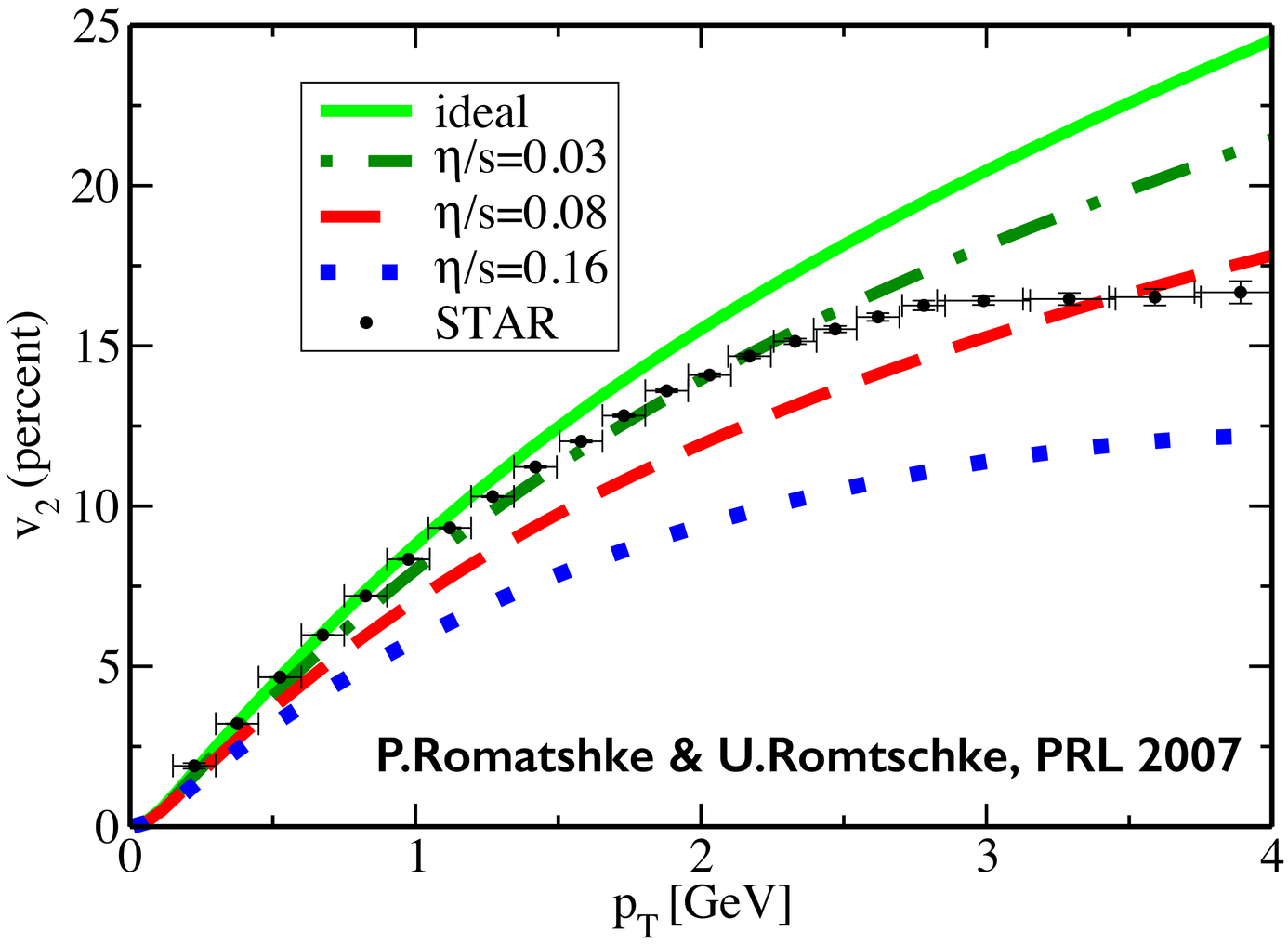
2. All bulk properties are driven by
   numerous unknown properties (model parameters)

# How this was done before (v2 and η/s)

## Study single parameter vs. single observable

$$v_2 \equiv \langle \cos 2\phi \rangle$$



# PROBLEM

v2 depends on ....

- viscosity
- saturation model
- pre-thermal flow
- Eq. of State
- T-dependence of $\eta/s$
- initial $T_{xx}/T_{zz}$
- . . . .

# Correct Way (MCMC)

- Simultaneously vary *N* model parameters $x_i$
- Perform random walk weight by likelihood

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\sum_a \frac{(y_a^{(\mathrm{model})}(\mathbf{x}) - y_a^{(\mathrm{exp})})^2}{2\sigma_a^2}\right\}$$

- Use all observables $y_a$
- Obtain representative sample of posterior

# Very Difficult Because…

**I. Too Many Model Runs**
Requires running model ~$10^6$ times

**II. Many Observables**
Could be hundreds of plots,
each with dozens of points
Complicated Error Matrices

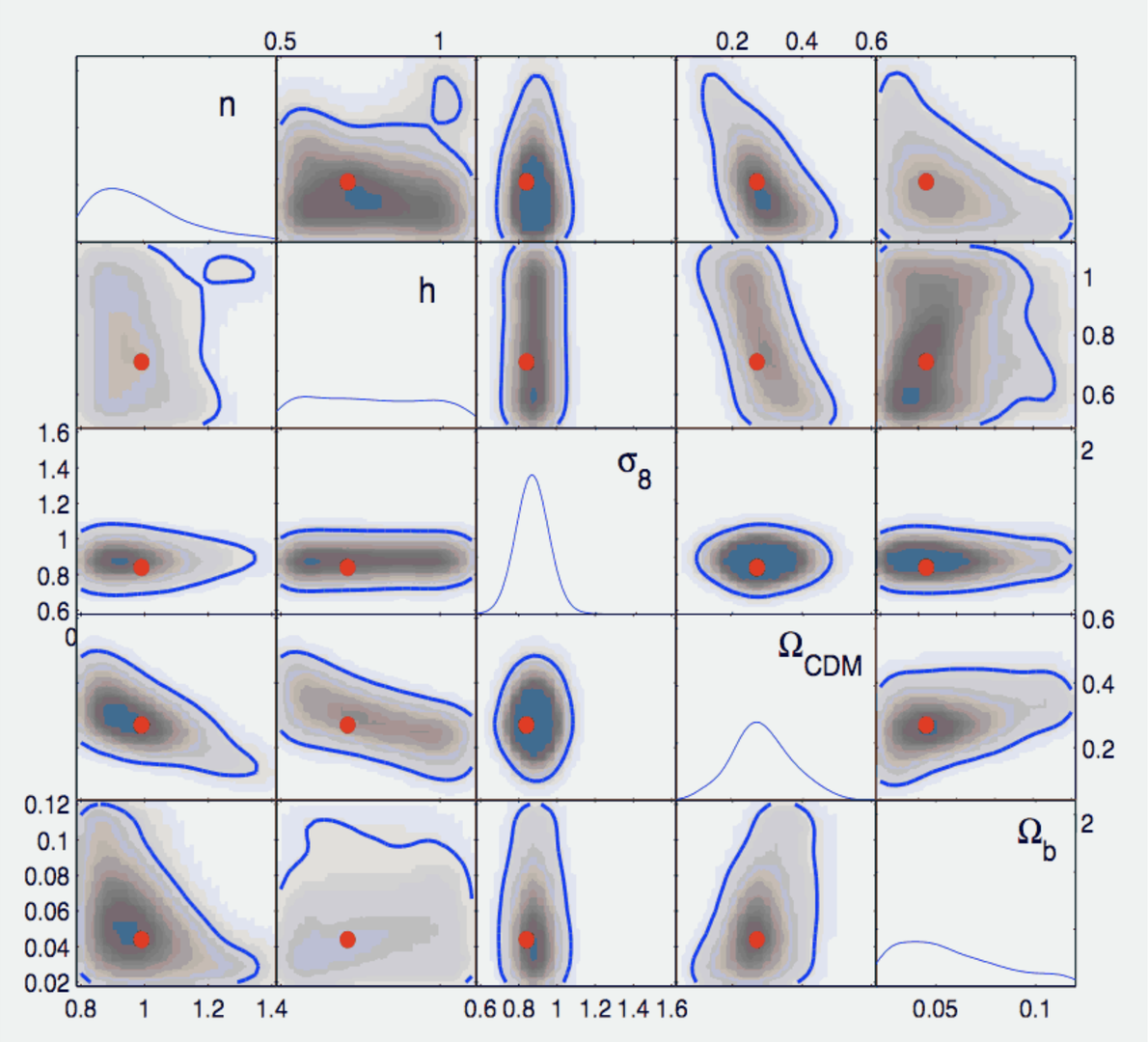
# Model Emulators

1. **Run the model ~1000 times**
   Semi-random points (LHS sampling)

2. **Determine Principal Components**
   $(y_a - \langle y_a \rangle)/\sigma_a \rightarrow z_a$

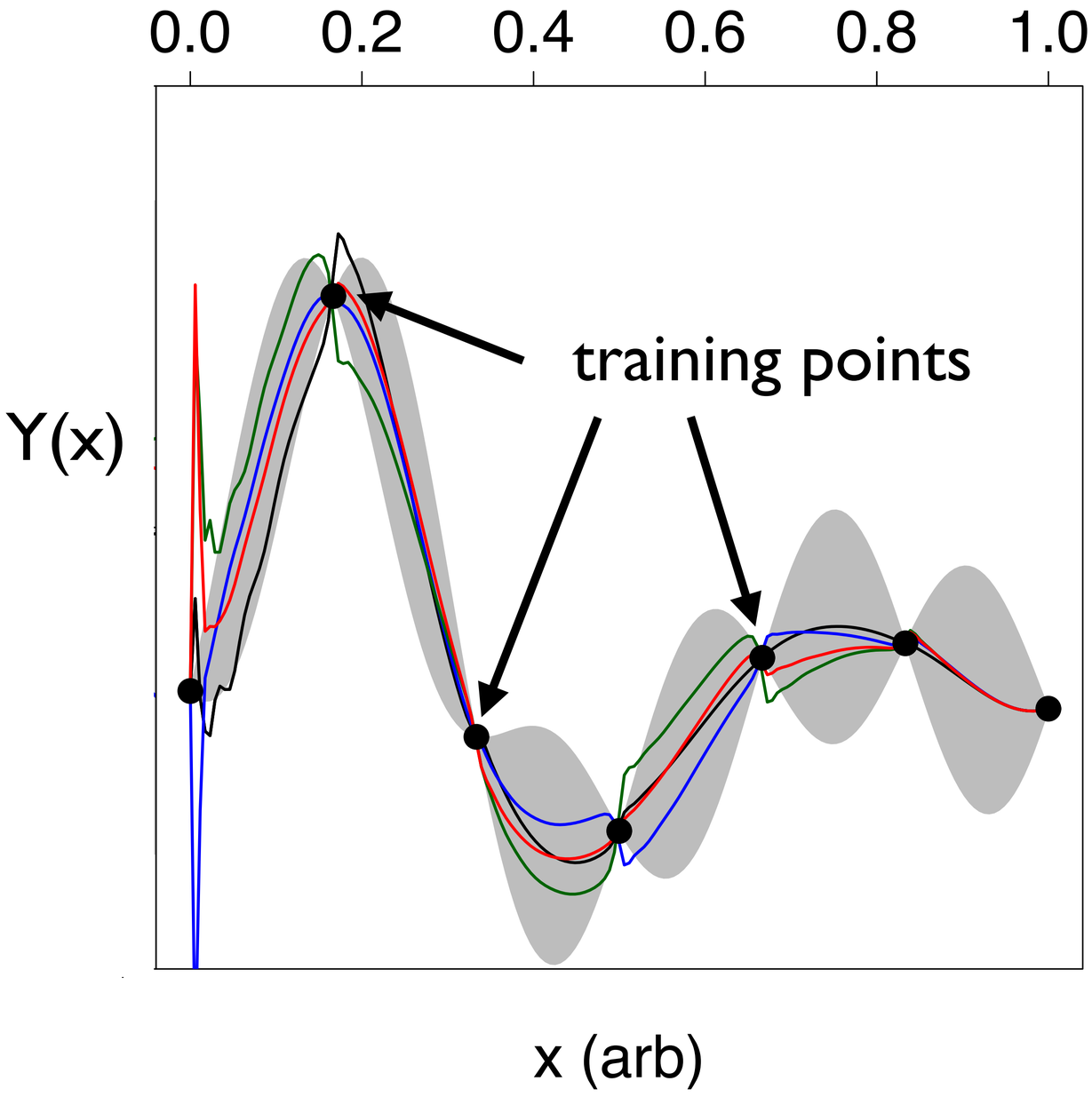
3. **Emulate $z_a$ (Interpolate) for MCMC**
   Gaussian Process…

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\frac{1}{2}\sum_a (z_a^{(\mathrm{emulator})}(\mathbf{x}) - z_a^{(\mathrm{exp})})^2\right\}$$




S. Habib,K.Heitman,D.Higdon,C.Nakhleh&B.Williams, PRD(2007)

# Emulator Algorithms




- Gaussian Process
  - Reproduces training points
  - Assumes localized Gaussian covariance
  - Must be trained, i.e. find "hyper parameters"
- Other methods also work

## 14 Parameters

- 5 for Initial Conditions at RHIC
- 5 for Initial Conditions at LHC
- 2 for Viscosity
- 2 for Eq. of State

## 30 Observables

- $\pi, K, p$ Spectra
  $\langle p_t \rangle$, Yields
- Interferometric Source Sizes
- $v_2$ Weighted by $p_t$

# Initial State Parameters

$$\epsilon(\tau = 0.8\mathrm{fm}/c) = f_{\mathrm{wn}}\epsilon_{\mathrm{wn}} + (1 - f_{\mathrm{wn}})\epsilon_{\mathrm{cgc}},$$

$$\epsilon_{\mathrm{wn}} = \epsilon_0 T_A \frac{\sigma_{\mathrm{nn}}}{2\sigma_{\mathrm{sat}}} \{1 - \exp(-\sigma_{\mathrm{sat}} T_B)\} + (A \leftrightarrow B)$$

$$\epsilon_{\mathrm{cgc}} = \epsilon_0 T_{\mathrm{min}} \frac{\sigma_{\mathrm{nn}}}{\sigma_{\mathrm{sat}}} \{1 - \exp(-\sigma_{\mathrm{sat}} T_{\mathrm{max}})\}$$

$$T_{\mathrm{min}} \equiv \frac{T_A T_B}{T_A + T_B},$$

$$T_{\mathrm{max}} \equiv T_A + T_B,$$

$$u_\perp = \alpha\tau \frac{\partial T_{00}}{2T_{00}}$$

$$T_{zz} = \gamma P$$

**5 parameters for RHIC, 5 for LHC**

# Equation of State and Viscosity

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon / \epsilon_h$$

$$\frac{\eta}{s} = \left.\frac{\eta}{s}\right|_{T=165} + \kappa \ln(T/165)$$

**2 parameters for EoS, 2 for η/s**

# DATA Distillation

1. Experiments reduce PBs to 100s of plots

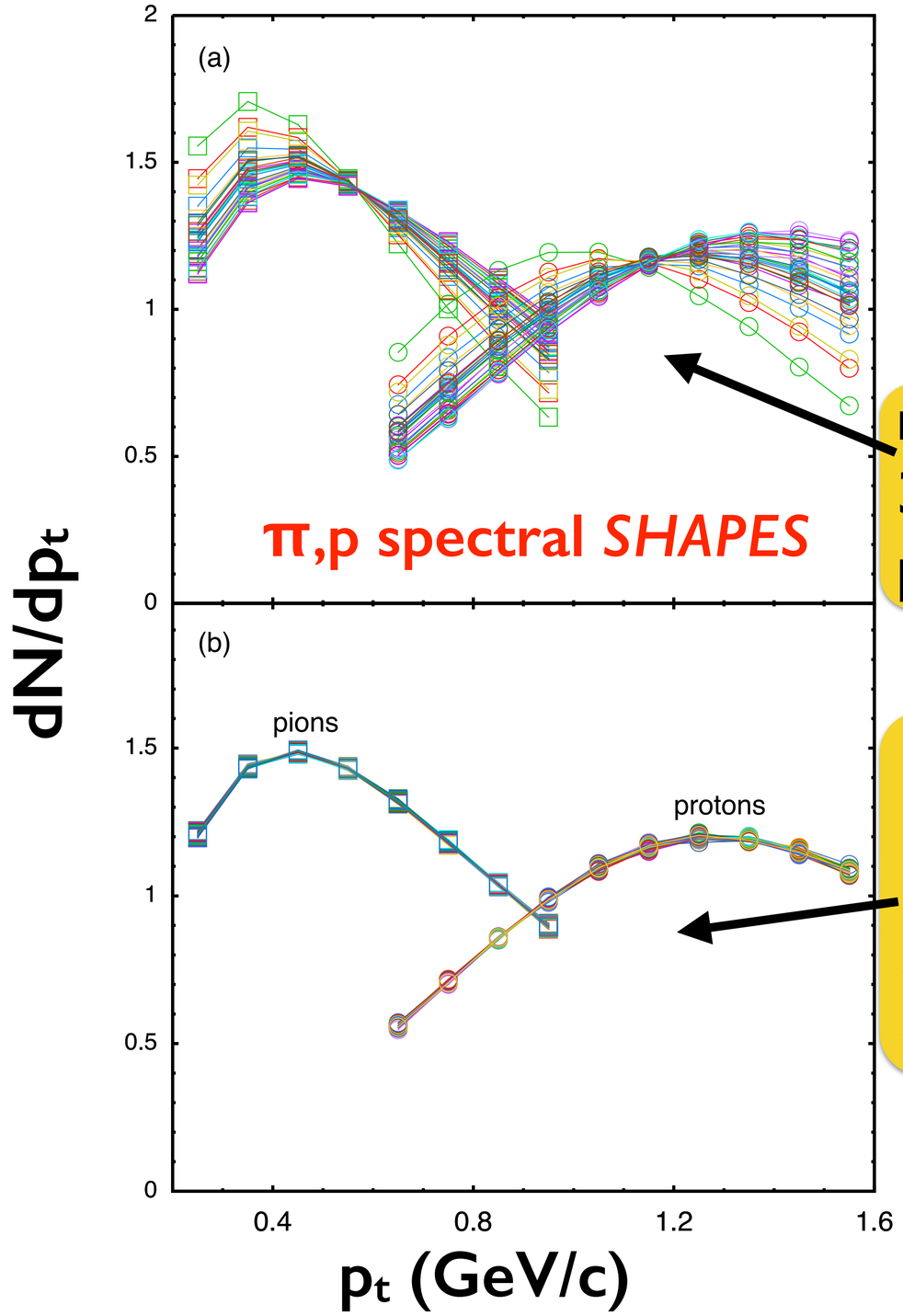
2. Choose which data to analyze
   Does physics *factorize*?

3. Reduce plots to a few representative numbers, $y_a$

4. Transform to principal components

# Checking the Distillation

**Spectral information encapsulated by two numbers, dN/dy & $\langle p_t \rangle$**




**model spectra from 30 random points in parameter prior**

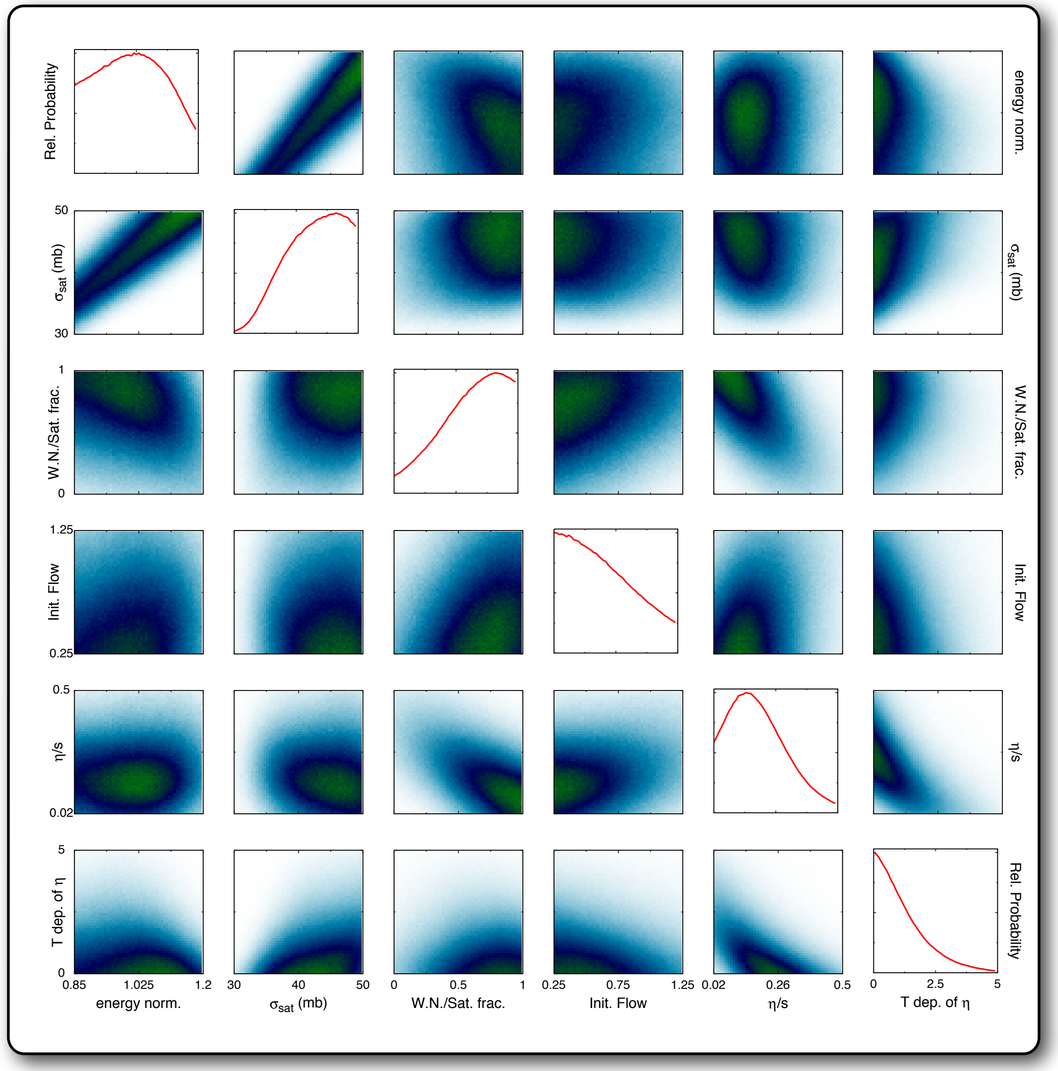
**74 pion spectra: with $573 < \langle p_t \rangle_\pi < 575$ MeV**

**44 proton spectra: with $1150 < \langle p_t \rangle_p < 1152$ MeV**

# Two Calculations

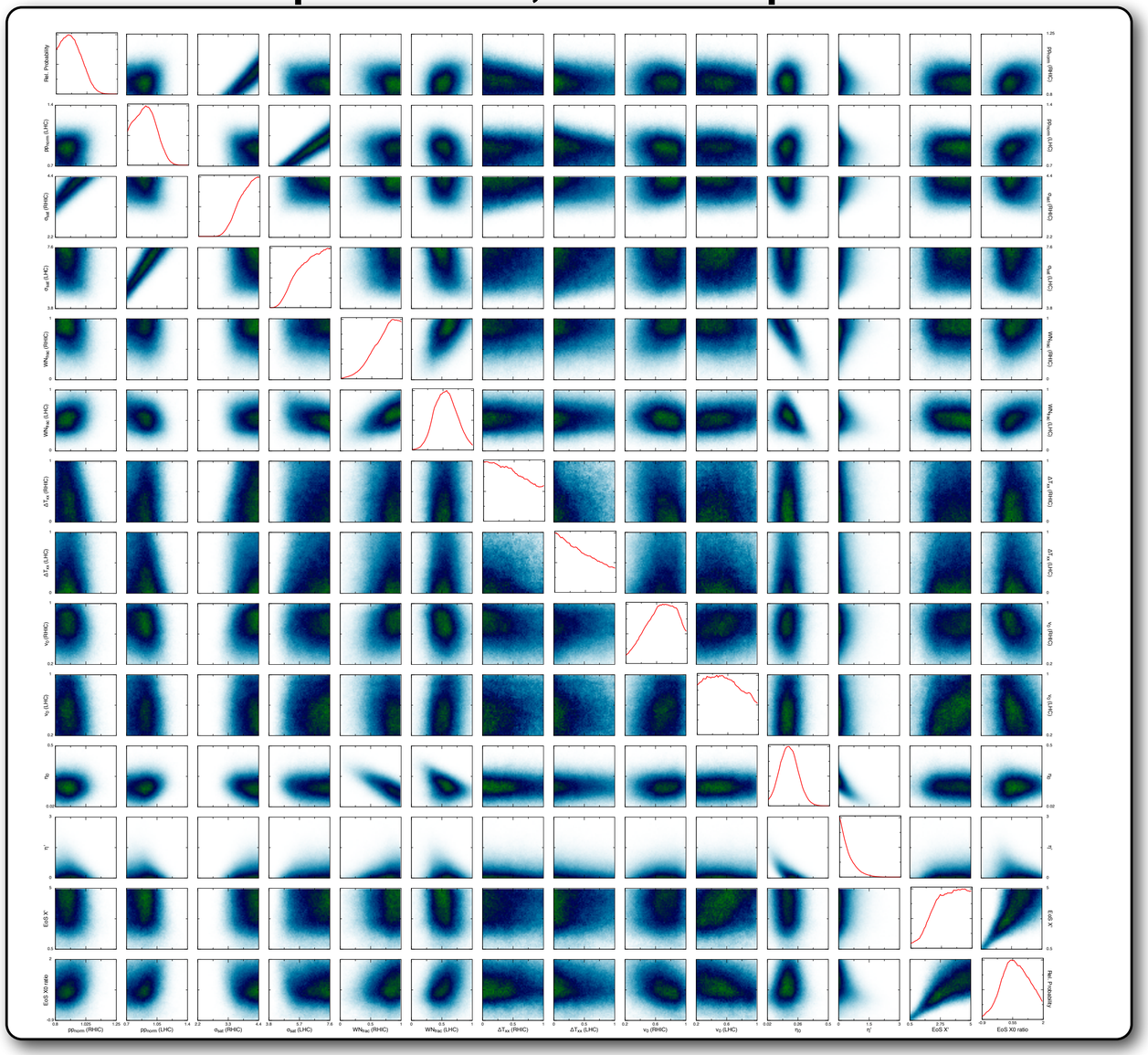
1. J.Novak, K. Novak, S.P., C.Coleman-Smith & R.Wolpert, ArXiv:1303.5769

**RHIC Au+Au Data**
**6 parameters**

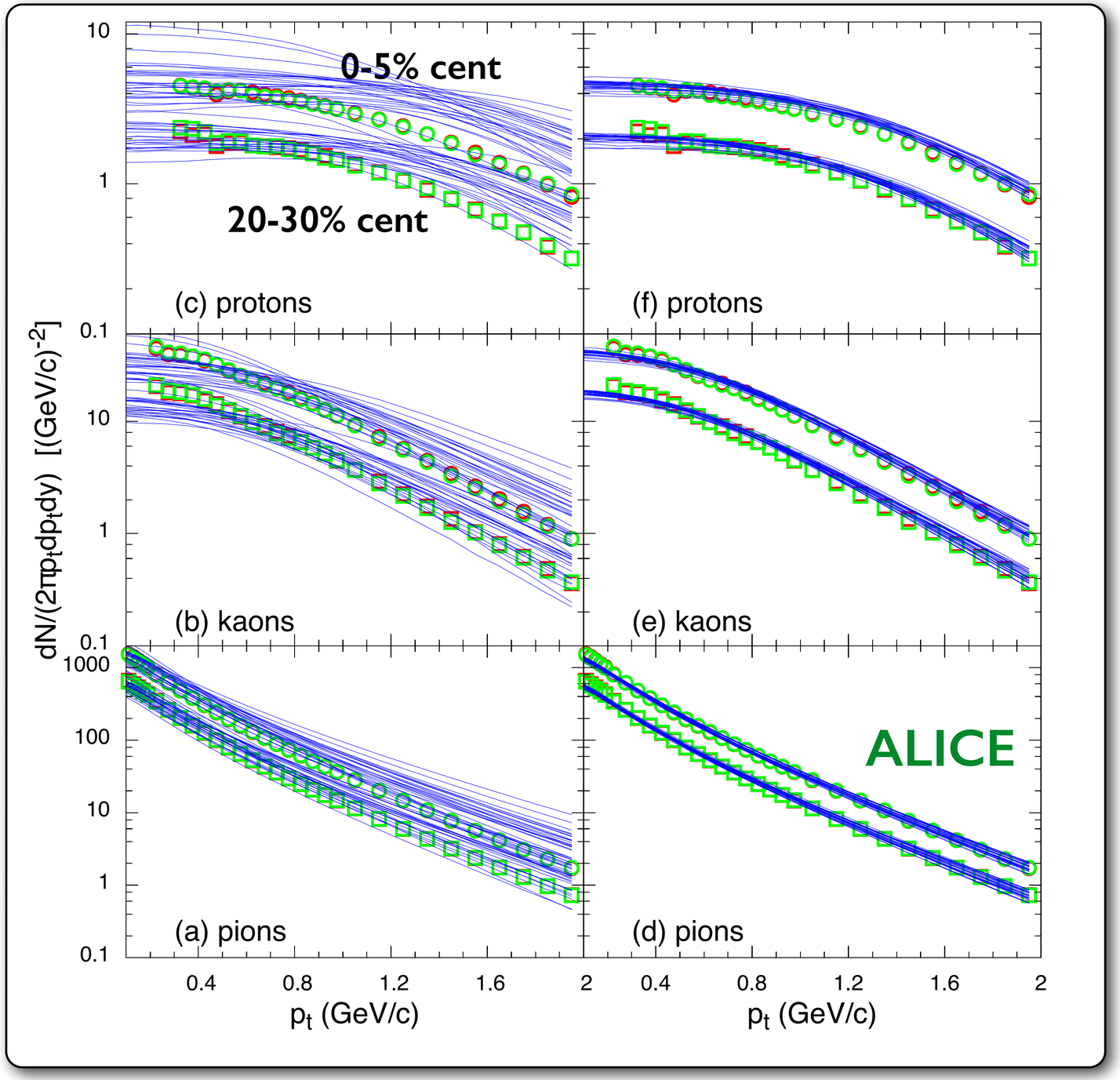



2. S.P., E.Sangaline, P.Sorensen & H.Wang, in progress

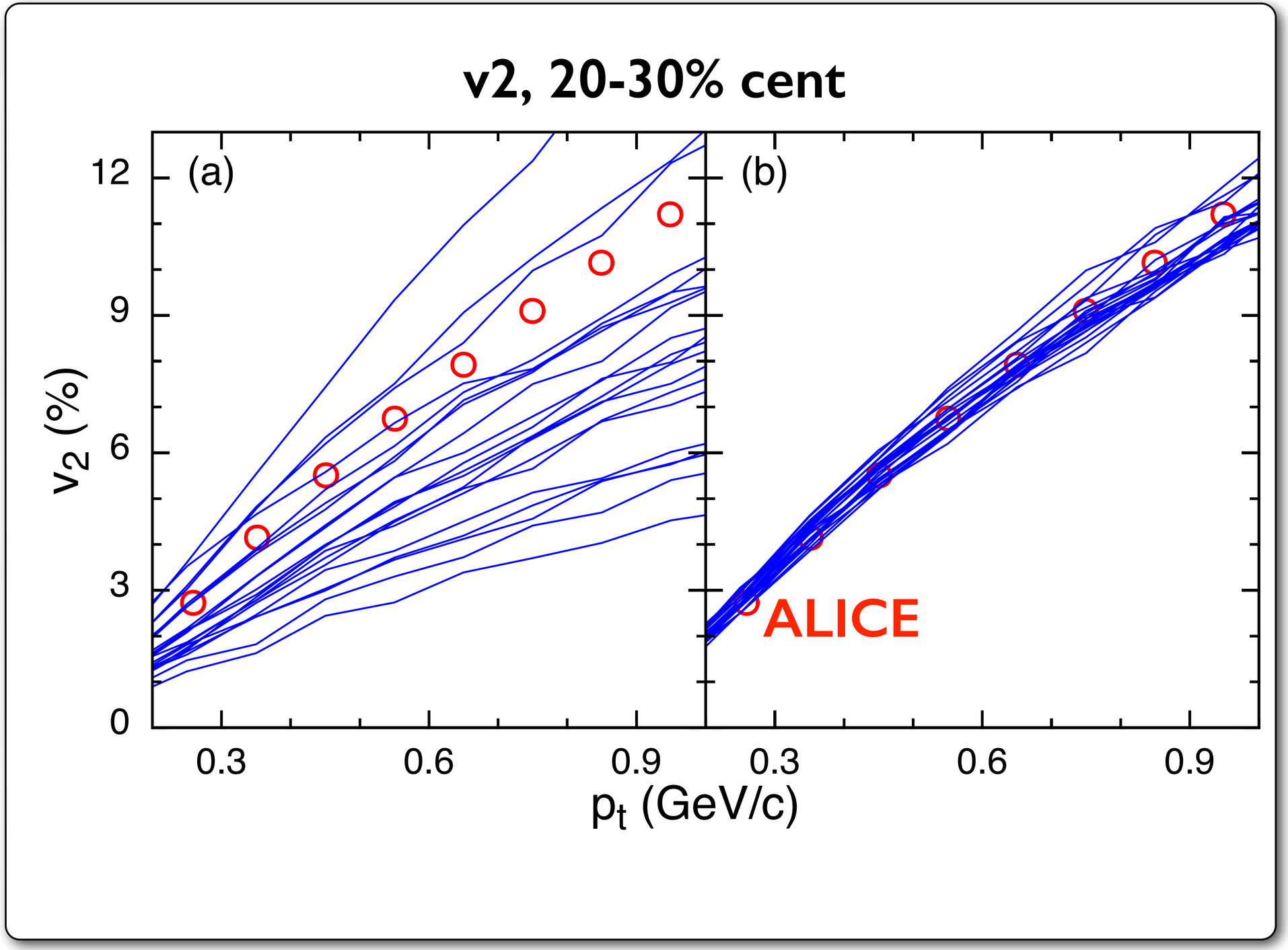
**RHIC Au+Au and LHC Pb+Pb Data**
**14 parameters, include Eq. of State**



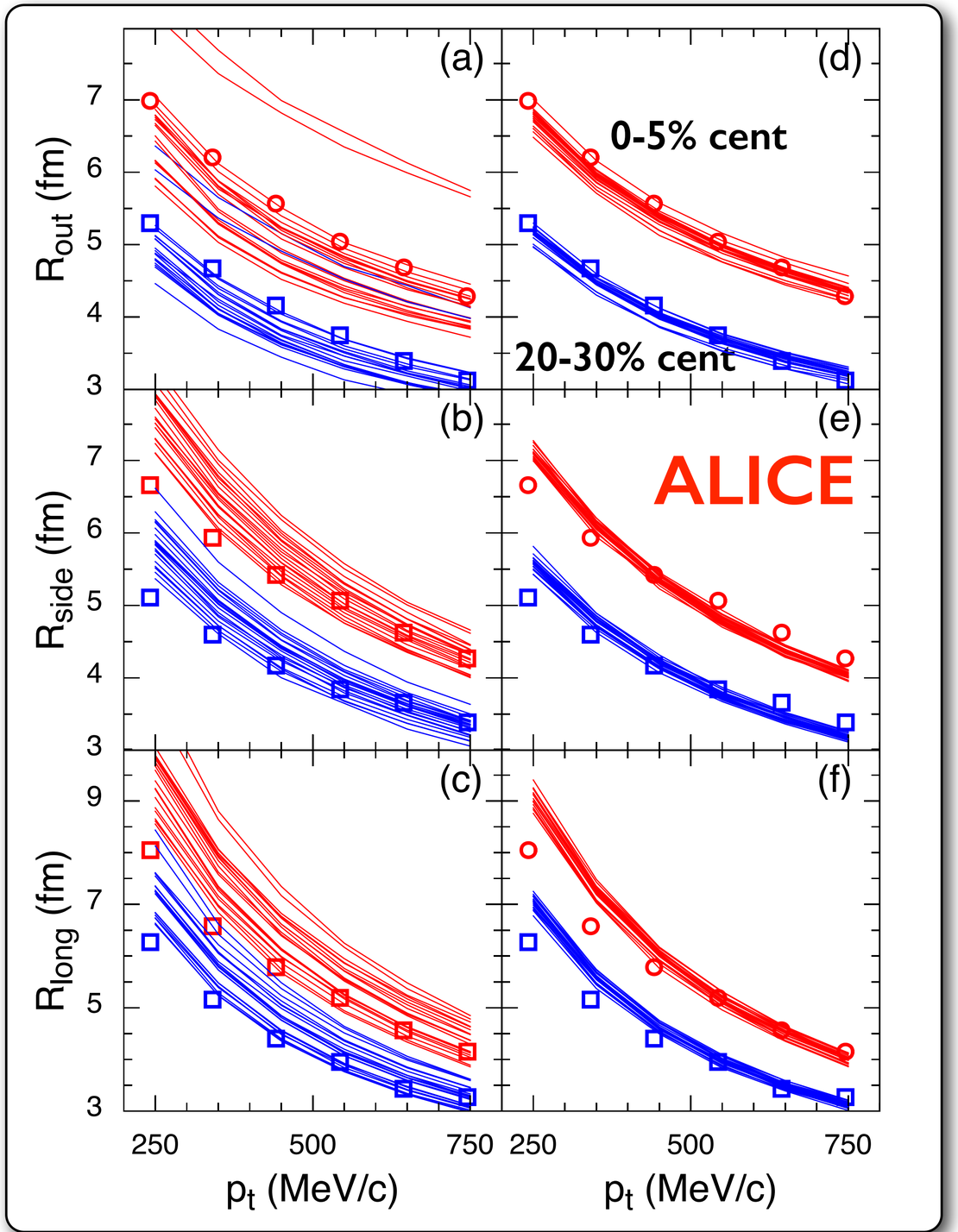
# Sample Spectra from Prior and Posterior



Sample V2 from Prior and Posterior

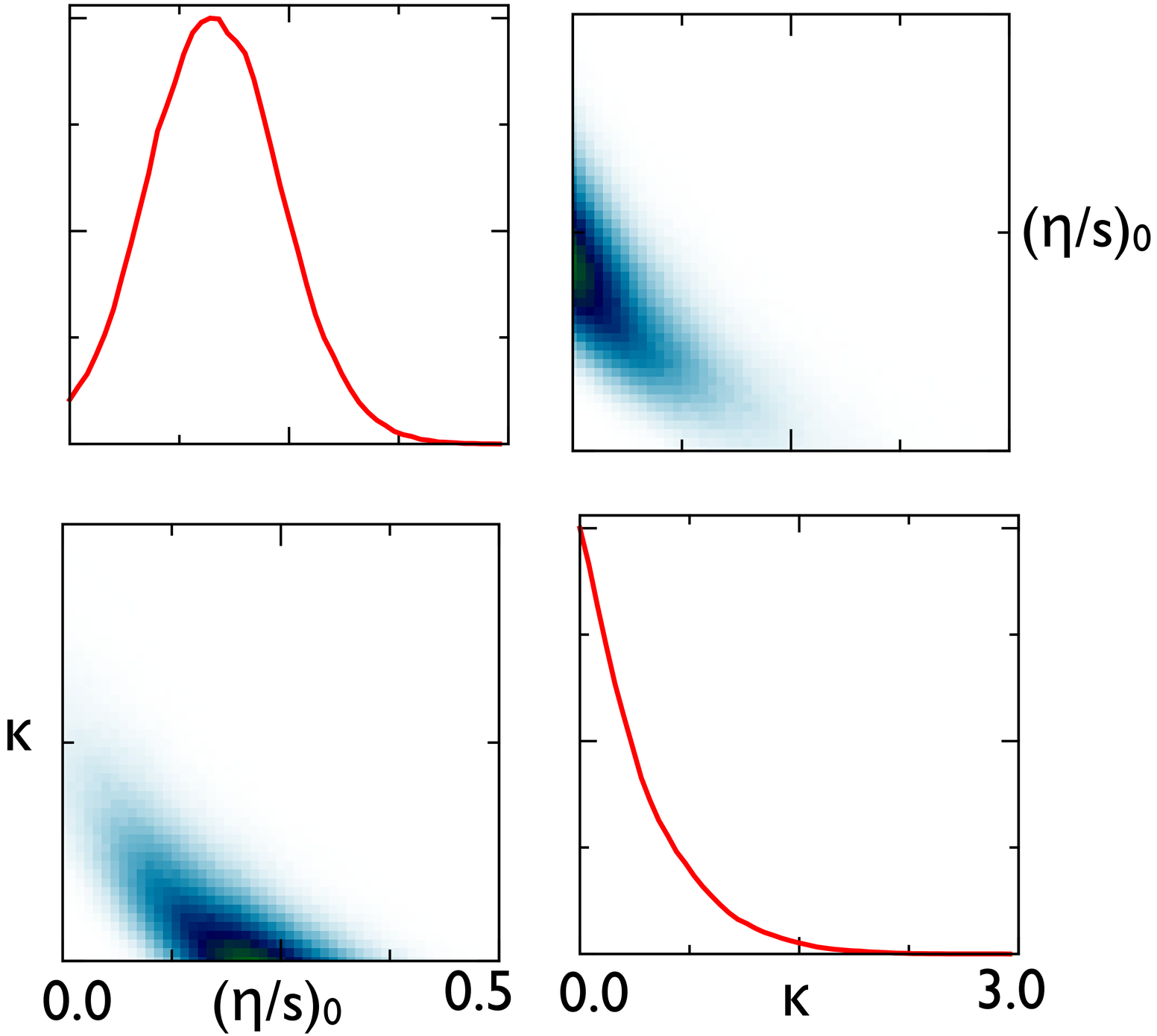


# Sample HBT from Prior and Posterior

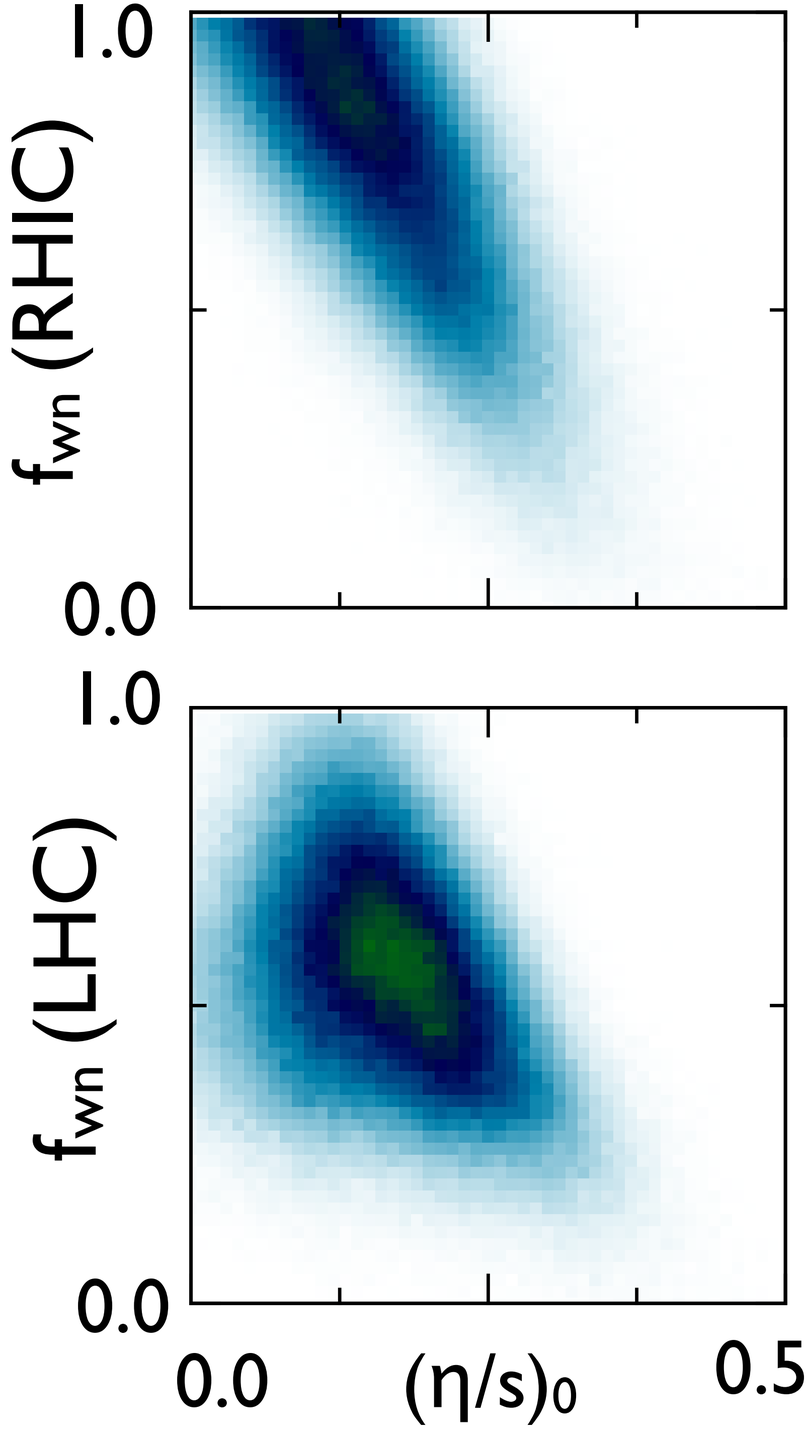


# η/s(T)

$$\eta/s = (\eta/s)_0 + \kappa \ln(T/165)$$



# η/s vs saturation picture



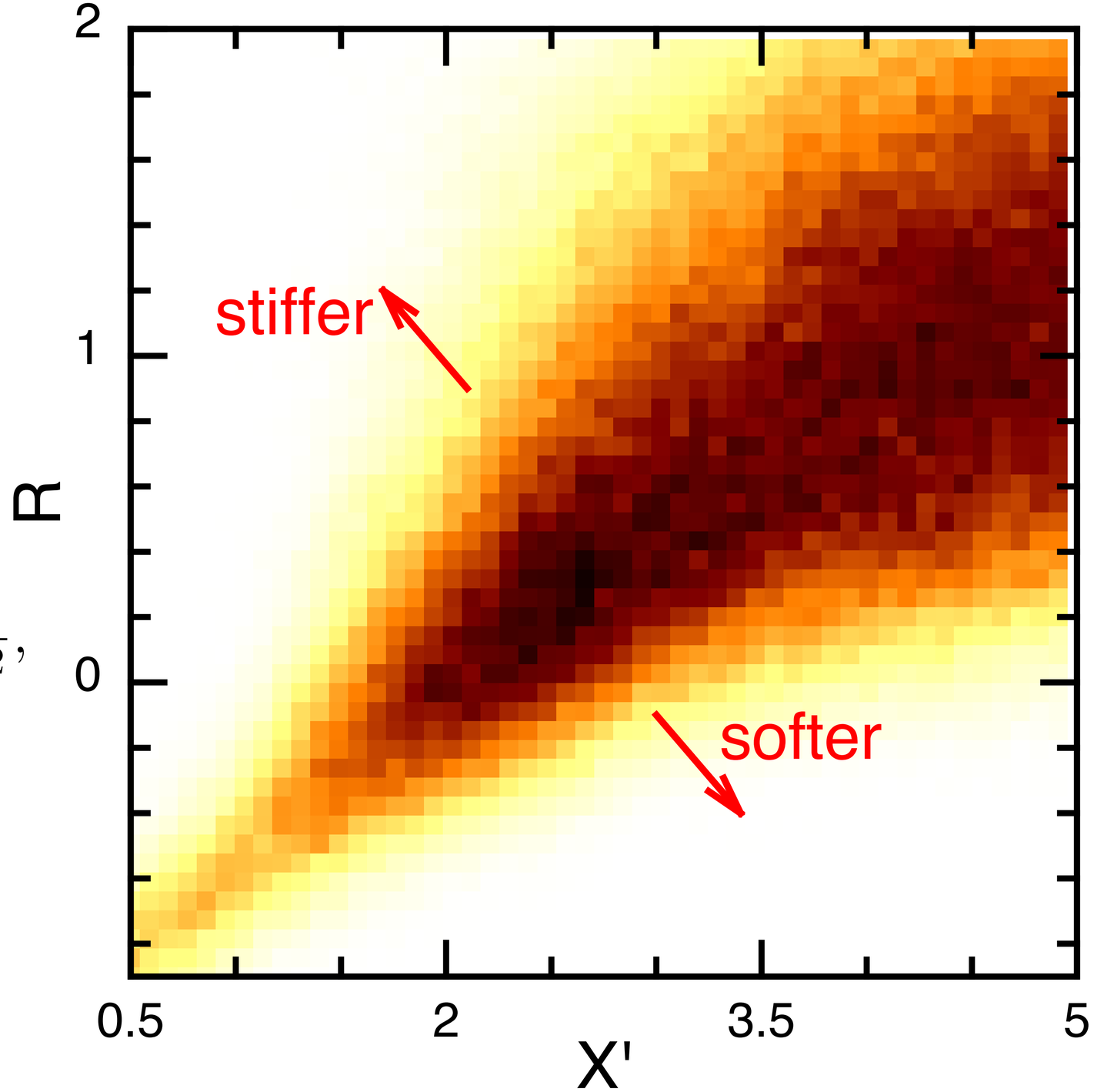

See Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007

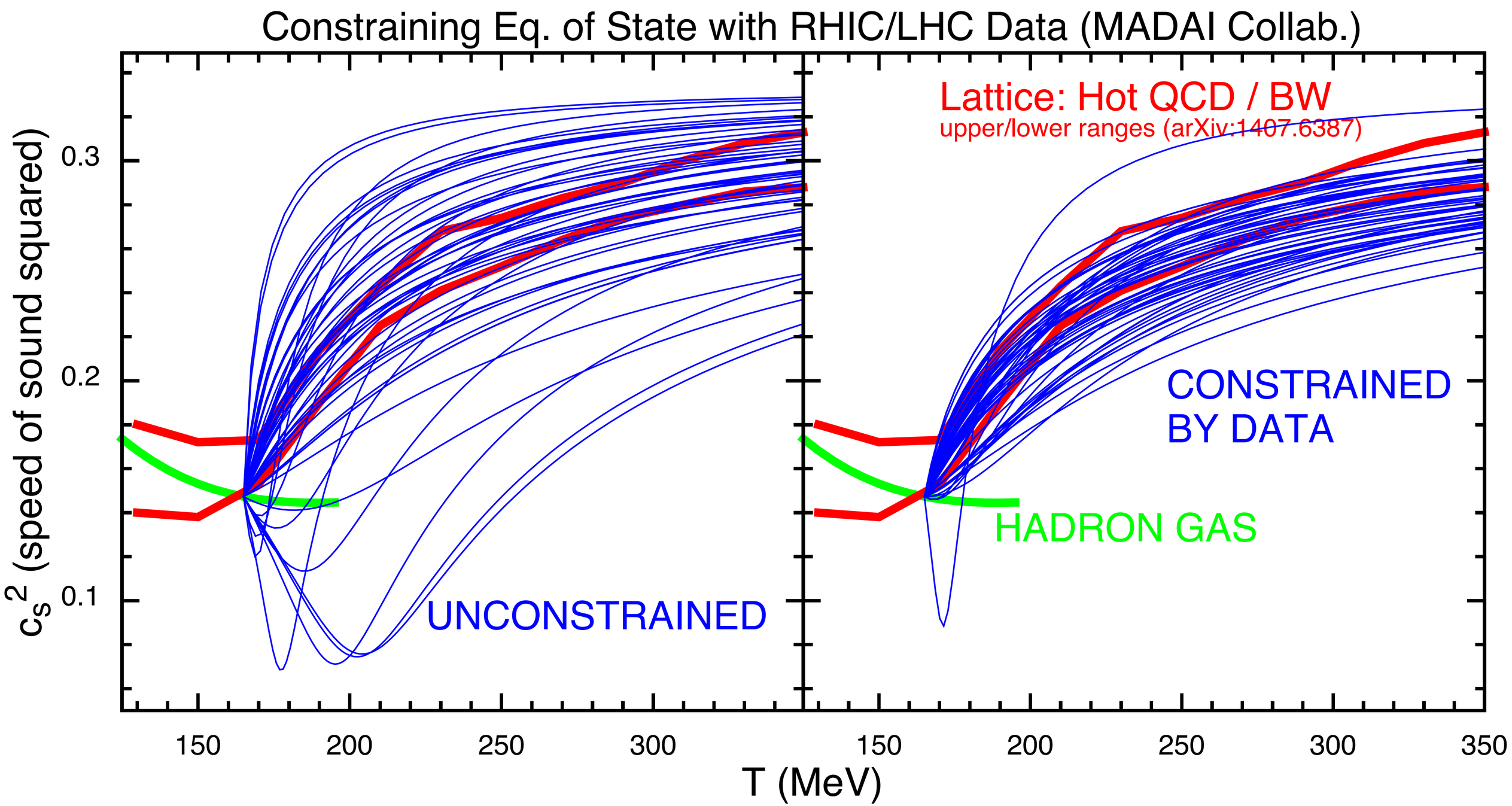
# Eq. of State

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

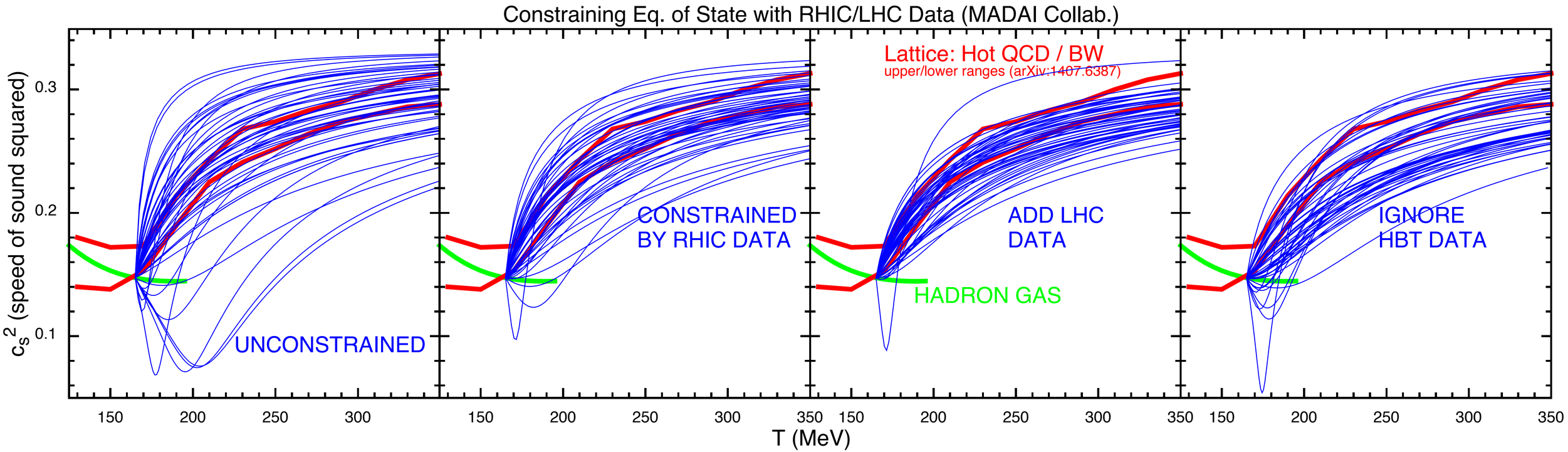
$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

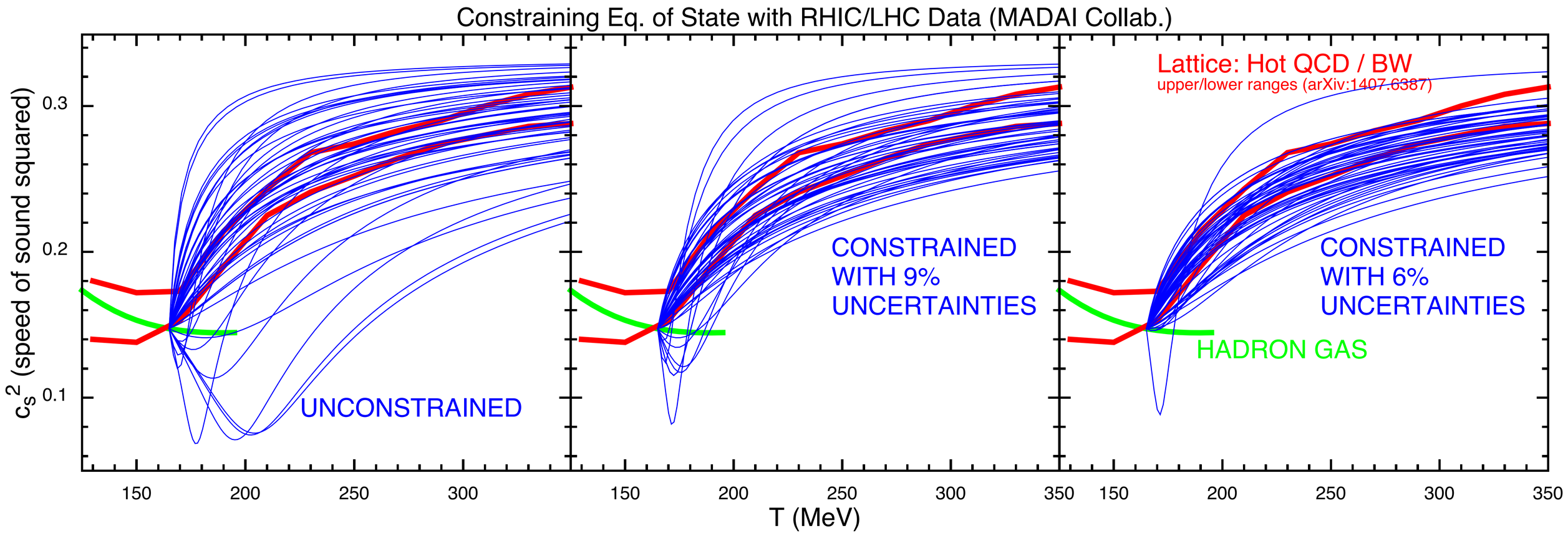



Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)
Lattice: Hot QCD / BW
upper/lower ranges (arXiv:1407.6387)
CONSTRAINED
BY DATA
HADRON GAS
UNCONSTRAINED
$c_s^2$ (speed of sound squared)
0.1
0.2
0.3
150
200
250
300
350
T (MeV)

# Which observables constrain the EoS?



# Sensitivity to Uncertainty



# CONCLUSIONS

- Robust
- Emulation works splendidly
- Scales well to more parameters & more data
- Eq. of State and Viscosity can be extracted from RHIC & LHC data
- Other parameters not as well constrained
- **Heavy-Ion Physics can be a Quantitative Science!!!!**

# FUTURE

- Improve statement of uncertainties
- Add parameters (many related to hadronization)
- Consider more data
  - more observables
  - data from different beams/energies
- Improve models
  - Lumpy Glauber initial conditions
  - 3D calculations for lower energies
  - Fill in missing physics

## If you're interested…

1. **Tools are easily extended**
2. **Download software and tutorial from http://madai.us**
3. **Talk to me (prattsc@msu.edu)**
   **or Evan Sangaline (esangaline@gmail.com)**

*Made possible by contributions from DOE, NSF, Chris Coleman-Smith, John Novak, Kevin Novak, Evan Sangaline, Paul Sorensen, Joshua Vredevoogd , Hui Wang, Robert Wolpert, and viewers like you.*

# Additional slide: Charge BFs and charge susceptibilities

S.P., C. Ratti and W.McCormack, arXiv:1409.2164