Particle production at NLO in pA collisions: the wave function approach

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[T. A. , N. Armesto, G. Beuf, A. Kovner, M. Lublinsky, arXiv:1411.2869]

Outline

- Motivation
- What is new in this approach?
- A little tasting of the calculation: the quark channel
- Summary

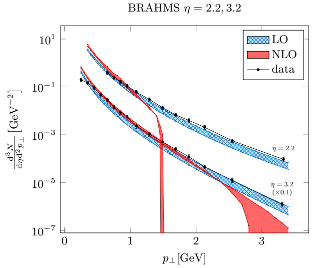
Motivation

Single inclusive hadron production at forward rapidities in pA scattering:

"Hybrid" formalism : Dimitru, Hayashigaki & Jalilian-Marian

- The wave function of the projectile proton is treated in the spirit of collinear factorization. Perturbative corrections to this wave function are provided by the usual QCD perturbative splitting processes.
- Target is treated as distribution of strong color fields which during the scattering event transfer transverse momentum to the propagating partonic configuration (CGC like treatment).

T.A., A. Kovner - 2011 Elastic & Inelastic contributions (part of NLO)
G.A. Chirilli, B.W. Xiao, F. Yuan - 2012 Full NLO calculation...
A.M.Stasto, B.W.Xiao, D. Zaslavsky, - 2013
Numerical analysis...



What are the missing pieces of the puzzle?

1. The Ioffe Time Restriction
   provides a consistent description on what will be resolved by the target and what not!
   - Only the pairs whose coherence time (Ioffe time) is greater than the propagation time through the target can be resolved by the target!
   - Ioffe time is related with the size of the target at initial energy s_0.

2. The rapidity to which eikonal scattering amplitudes have to be evolved??

   $$Y_T \text{ vs } Y_g$$

   - $Y_g = \ln \frac{1}{x_g}$ & $x_g = e^{-\eta} \frac{p_\perp}{\sqrt{2s}}$
     for a dense target projectile parton undergoes multiple scattering.
     the momentum transfer p^- is not from a single gluon but from several.
     x_g is an upper bound on the momentum fraction of the target gluon $\Rightarrow$ Y_g gives a lower bound on the rapidity up to which the target wave function has to be evolved!
   - $Y_T = \ln \frac{s}{s_0}$ ✓

The quark channel

The parton level production cross section at LO :

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = \frac{1}{(2\pi)^2}\int d^2x d^2y\; e^{ip_\perp(x-y)} s_{Y_T}(x,y)$$

fundamental dipole scattering amplitude

$$\boxed{s(x,y) = \frac{1}{N_c} tr\; [S_F(x) S_F^\dagger(y)]}$$

At NLO the quark splits in the projectile wave function with probability of order α_s into a quark-gluon configuration.

The dressed quark state :

$$|(q)\, x_B P^+, k_\perp, \alpha, s\rangle_D = \int_x e^{ik_\perp x}\Big\{ A^q |(q)\, x_B P^+, x, \alpha, s\rangle$$

$$+g\int_{\xi,yz} F_{(qg)}(x_B P^+, \xi, y-x, z-x)_{s\bar{s};j}\; t^a_{\alpha\beta}$$

$$|(q)\;y, p^+ = (1-\xi)x_B P^+, \beta, \bar{s}; (g)\; z, q^+ = \xi x_B P^+, a, j\rangle\Big\}$$

- A^q is of order g^2 and needed to preserve the normalisation of the state at order α_s.
- $F_{(qg)}$ is the function that defines the splitting of a quark into a quark-gluon pair.

The quark channel

The dressed quark scatters on the target and produces final state particles.

Within "hybrid" formalism, the scattering of the qg pair is treated as a completely coherent process $\Rightarrow$ each parton picks an eikonal phase during the interaction with the target.

THIS IS ONLY POSSIBLE if the coherence time (Ioffe Time) of the configuration is greater than the propagation time through the target.

$$\boxed{\text{coherent scattering} \Rightarrow \frac{2(1-\xi)\xi x_B P^+}{k_\perp^2} > \tau}$$

$\tau \equiv$ a fixed time scale determined by the longitudinal size of the target.

It enters to calculation via initial energy $P^+/\tau = s_0/2$.

The Ioffe time restriction is in fact given in terms of initial energy s_0!

The pairs that do not exist long enough are not resolved! Those pairs:

- have large $k_\perp$ and have small transverse size.
- scattering and particle production from those pairs are indistinguishable from single parent quark.

The quark channel

The standard eikonal paradigm for propagation of the initial dressed quark with vanishing transverse momentum through the target leads to the final state

$$
\begin{aligned}
|\mathrm{out}, \alpha, s\rangle = \int_x \Big\{ & S^F_{\alpha\beta}(x)\, |(q) x, \beta, s\rangle_D \\
&+ \frac{g^2}{2\pi} \int d\mathrm{LPS} \int_{y,z} \Big[t^a_{\alpha\beta} S^F_{\beta\gamma}(y) S^A_{ab}(z) - S^F_{\alpha\beta}(x) t^b_{\beta\gamma} \Big]\, \bar{F}^2_{(qg)}(\xi, x_p, y-x, z-x) \\
&\qquad t^b_{\gamma\delta}\, |(q)\ x,\ \delta, s\rangle_D \\
&+ \frac{g}{2\pi} \int d\mathrm{LPS} \int_{y,z} F_{(qg)}(\xi, x_p, y-x, z-x)_{s,\bar{s},i} \Big[t^a_{\alpha\beta} S^F_{\beta\gamma}(y) S^A_{ab}(z) - S^F_{\alpha\beta}(x) t^b_{\beta\gamma} \Big] \\
&\qquad |(q)\ y, (1-\xi), \gamma, \bar{s}; (g)\ z, \xi, b, i\rangle_D \Big\}
\end{aligned}
$$

$\mathrm{LPS} \equiv$ **L**ongitudinal **P**hase **S**pace

The function $F_{(qg)}$ is written as

$$
F_{(qg)} = \frac{i}{\sqrt{2\xi x_B P^+}} \Big\{ \delta_{s\bar{s}} \delta_{ij} (2-\xi) - i \epsilon_{ij} \sigma^3_{s\bar{s}} \xi \Big\} \delta^2\Big(x - [(1-\xi) y + \xi z] \Big) A^i_{\xi, x_B}(y-z)
$$

Modified Weiszacker-Williams field

The quark channel

The modified Weizsacker-Williams field is defined as

$$A^i_{\xi,x_B}(y-z) = -i\int_{l_\perp^2<2\xi(1-\xi)x_B\frac{P^+}{\tau}} \frac{d^2l_\perp}{(2\pi)^2}\frac{l^i_\perp}{l^2_\perp}e^{il_\perp(y-z)}$$
$$= -\frac{1}{2\pi}\frac{(y-z)^i}{(y-z)^2}\left[1-\mathrm{J}_0\left(|y-z|\sqrt{2\xi(1-\xi)\frac{x_BP^+}{\tau}}\right)\right]$$

with transverse momentum $l_\perp$ is

$$\boxed{l_\perp = \xi p_\perp - (1-\xi)q_\perp}$$

- The Ioffe time constraint is implemented on the phase space $\{k_\perp, \xi\}$ in the definition of $F_{(qg)}(y-x, z-x)$ rather than in the integral over ξ.
- Neglecting the Ioffe time constraint on $l_\perp$, one gets the Fourier transform of the standard Weizsacker-Williams field.
- With the Ioffe time constraint, the relative contribution of short distances are suppressed. $F_{(qg)}$ at small $z-x$ becomes reduced.

The quark channel

The quark production cross section is given by the expectation value of the dressed quark number in the outgoing state, multiplied by the number of dressed quarks in the incoming wave function:

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = x_p f^D_{\mu^2}(x_p)\langle\mathrm{out}|D^\dagger(k_\perp, x)D(k_\perp, x)|\mathrm{out}\rangle$$

For the quark production we find

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = \underbrace{\frac{1}{(2\pi)^2}\, x_p f^D_{\mu^2}(x_p)\int_{x,y} e^{ip_\perp(x-y)} s_{Y_T}(x,y)}_{\text{LO}} + \underbrace{\frac{d\sigma^q_1}{d^2p_\perp d\eta}}_{\text{NLO}}$$

The quark production cross section at NLO :

$$\frac{d\sigma^q_1}{d^2p_\perp d\eta} = p^+ \frac{d\sigma^q_1}{d^2p_\perp dp^+} = p^+ \frac{d\sigma_1^{q\to q,\mathrm{r}}}{d^2p_\perp dp^+} + p^+ \frac{d\sigma_1^{q\to q,\mathrm{v}}}{d^2p_\perp dp^+}$$

The quark channel

The NLO cross section contain collinear divergences.

$$
\begin{aligned}
I_1^r &= \frac{g^2}{(2\pi)^3} C_F \int dx_B\, f^{D,q}_{\mu^2}(x_B) \int_0^{1-x_p} d\xi\, \frac{x_p}{1-\xi}\, \delta\left(x_B - \frac{x_p}{1-\xi}\right) \left[\frac{1+(1-\xi)^2}{\xi}\right] \\
&\quad \times C_{\mu^2}(\xi, x_B) \int_{y\bar{y}} e^{ip_\perp(y-\bar{y})} s[y,\bar{y}] \\
I_2^r &= \frac{g^2}{(2\pi)^3} C_F \int dx_B\, f^{D,q}_{\mu^2}(x_B) \int_0^{1-x_p} d\xi\, \frac{x_p}{1-\xi}\, \delta\left(x_B - \frac{x_p}{1-\xi}\right) \left[\frac{1+(1-\xi)^2}{\xi}\right] \\
&\quad \times (1-\xi)^2 C_{\mu^2}(\xi, x_B) \int_{y\bar{y}} e^{ip_\perp(y-\bar{y})} s\big[(1-\xi)y, (1-\xi)\bar{y}\big] \\
I^v &= -(1+1) \frac{g^2}{(2\pi)^3} C_F \int dx_B\, f^{D,q}_{\mu^2}(x_B)\, x_p\, \delta(x_B - x_p) \int_0^1 d\xi \left[\frac{1+(1-\xi)^2}{\xi}\right] C_{\mu^2}(\xi, x_B) \\
&\quad \times \int_{y\bar{y}} e^{ip_\perp(y-\bar{y})} s[y,\bar{y}]
\end{aligned}
$$

where the integral over z up to “factorization scale” μ can be defined for example as

$$
C_{\mu^2}(\xi, x_B) = \int_z A^i_{\xi,x_B}(z)\, A^i_{\xi,x_B}(z)\, \theta(z^2\mu^2 - 1)
$$

PDF's and Fragmentation functions

f^D that appears in the LO term is the number of "dressed quarks" in the proton. Part of the $O(\alpha_s)$ terms complete it to the NLO pdf of bare quarks (DGLAP of PDFs and FFs).

$$\begin{aligned} f^q_{\mu^2}(x_p) &= f^D_{\mu^2}(x_p) + \frac{g^2 C_F}{2\pi}\int_0^{1-x_p}\frac{d\xi}{1-\xi} f^D_{\mu^2}\left(\frac{x_p}{1-\xi}\right)\frac{1+(1-\xi)^2}{\xi} C_{\mu^2}\left(\xi, \frac{x_p}{1-\xi}\right) \\ &- \frac{g^2 C_F}{2\pi} f^D_{\mu^2}(x_p)\int_0^1 d\xi \frac{1+(1-\xi)^2}{\xi} C_{\mu^2}(\xi, x_p) \end{aligned}$$

The fragmentation function of the "dressed quark":

$$\begin{aligned} D^{D,q}_{H,\mu^2}(\zeta) = D^q_{H,\mu^2}(\zeta) &+ \frac{g^2}{2\pi} C_F D^q_{H,\mu^2}(\zeta)\int_0^1 d\xi \frac{1+(1-\xi)^2}{\xi} C_{\mu^2}\left(\xi, \frac{x_p}{\zeta}\right) \\ &- \frac{g^2}{2\pi} C_F \int_0^{1-\zeta}\frac{d\xi}{1-\xi} D^q_{H,\mu^2}\left(\frac{\zeta}{1-\xi}\right)\frac{1+(1-\xi)^2}{\xi} C_{\mu^2}\left(\xi, \frac{x_p}{\zeta}\right) \end{aligned}$$

Relating our factorization scheme to $\overline{MS}$

The collinear factorization scheme that we use, does not coincide with the standard $\overline{MS}$ scheme.

In order to find the relation between the two schemes:

- we have calculated the d-dimensional generalisation of our collinear subtraction term C_{μ^2}.
- use the fact that single inclusive cross section is scheme independent.
- compare our result (scheme X) with the $\overline{MS}$ one.

$$f_X^q\big(x_B;\mu_F^2\big) = f_{\overline{MS}}^q\big(x_B;R^2\mu_F^2\big)$$

$$D_{H,X}^q\big(\zeta;\mu_{frag}^2\big) = D_{H,\overline{MS}}^q\big(\zeta;R^2\mu_{frag}^2\big)$$

with the rescaling factor $R = 2e^{\psi(1)} \approx 1.1229.$

The Final Result

Adding all the pieces together, we have the final expression for the quark channel:

$$p^+ \frac{d\sigma^{q\to H}}{d^2p_\perp dp^+} = \frac{1}{(2\pi)^2}\int \frac{d\zeta}{\zeta^2} D_H^q(\zeta)\frac{x_p}{\zeta} f_{p_\perp}^q\left(\frac{x_p}{\zeta}\right)\int_{y\bar{y}} e^{i\frac{p_\perp}{\zeta}(y-\bar{y})} s_{Y_T}[y,\bar{y}]$$
$$+ \int \frac{d\zeta}{\zeta^2} D_H^q(\zeta)\frac{d\bar{\sigma}^q}{d^2p_\perp d\eta}\left(\frac{p_\perp}{\zeta}, \frac{x_p}{\zeta}\right)$$

The quark production cross section has two parts:

- a piece that is independent of Ioffe time restriction and coincides with the existing results in the literature.
- a piece that carries the Ioffe time restriction:

$$\frac{g^2}{(2\pi)^3} N_c x_p f_{\mu^2}^q(x_p)\int_0^1 \frac{d\xi}{\xi}\int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})}\left[A_\xi^i(y-z) - A_\xi^i(\bar{y}-z)\right]^2$$
$$\times\left[s(y,z)s(z,\bar{y}) - s(y,\bar{y})\right]$$

What about the evolution?

- The way we set up the problem, the dipole scattering amplitude is evolved up to rapidity $Y_T = \ln \frac{s}{s_0}$ starting with an initial condition provided at Y_T^0.
- The final result should not care which s_0 we choose if we evolve the dipole cross section appropriately.
- The dependence on s_0 enters explicitly through the cutoff on the phase space and through the dependence of the scattering amplitude on the amount of evolution Y_T. Therefore

$$s_0 \frac{d}{ds_0}\left[\frac{d\sigma}{d^2p_\perp d\eta}\right] = \left[s_0 \frac{\partial}{\partial s_0} - \frac{ds_{Y_T}}{dY_T}\frac{\delta}{\delta s_{Y_T}}\right]\frac{d\sigma}{d^2p_\perp d\eta} = 0$$

and

$$s_0 \frac{\partial}{\partial s_0}\left[\frac{d\sigma}{d^2p_\perp d\eta}\right] = -\frac{\alpha_s N_c}{\pi} x_p f(x_p)\int_{y,\bar{y},z}\frac{1}{(2\pi)^3}e^{ip_\perp(y-\bar{y})}\frac{(y-\bar{y})^2}{(y-z)^2(\bar{y}-z)^2} \times\left[s(y,\bar{y}) - s(y,z)s(z,\bar{y})\right]$$

$\Rightarrow$ the dipole amplitude evolves according to the BK equation...

Summary

- By introducing the *Ioffe Time Restriction*, we have defined clearly the limits of coherent scattering and distinguish what will be resolved by the target and what not.
- We have defined the rapidity up to which the scattering amplitude has to be evolved.
- We have shown that how the Balitsky-Kovchegov evolution equation arises as the appropriate tool to evolve the leading order amplitude in this setup.
- Need numerical analysis to make sure that we have cured the original problem!!!

EXTRA CONTRIBUTION TO THE EVOLUTION??

The finite term that appears in the quark production cross section looks like an extra contribution to the evolution:

$$\frac{g^2}{(2\pi)^3} N_c x_p f^q_{\mu^2}(x_p) \int_0^1 \frac{d\xi}{\xi} \int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})} \left[A^i_\xi(y-z) - A^i_\xi(\bar{y}-z)\right]^2 \\ \times \left[s(y,z)s(z,\bar{y}) - s(y,\bar{y})\right]$$

extra evolution if the $A^i_\xi \to A^i$. This substitution IS NOT POSSIBLE, since the Ioffe time regulates the pole at $\xi = 0$ in this term.

If we try to write it as an extra contribution to evolution:

- change the order of integration: ξ and F.T. WW field:

$$\int d^2 l_\perp \int d^2 m_\perp \ln\left(\frac{1}{\xi_{min}}\right) \frac{d}{dY} s(l_\perp + p_\perp, m_\perp - p_\perp)$$

  with

$$\xi_{min} = \max\left\{\frac{l^2_\perp}{x_p s_0}, \frac{m^2_\perp}{x_p s_0}\right\}$$

Back-up slide

Then, one can forget about this term and evolve the leading order term to $Y_T + \ln \frac{1}{\xi_{min}}$:

$$
\begin{aligned}
Y_{l_\perp} &= Y_T + \ln \frac{x_p s_0}{l_\perp^2} \\
&= \ln \frac{s}{s_0} + ln \frac{x_p s_0}{l_\perp^2} \\
&= \ln \frac{s x_p}{p_\perp^2} + \ln \frac{p_\perp^2}{l_\perp^2}
\end{aligned}
$$

$$
x_p = \frac{|p_\perp|}{\sqrt{s}} e^{\eta} \;\;\&\;\; x_g = \frac{|p_\perp|}{\sqrt{s}} e^{-\eta}
$$

$$
\Rightarrow Y_{l_\perp} = \ln \tfrac{1}{x_g} + \ln \tfrac{p_\perp^2}{l_\perp^2}
$$