Eigenmode Analysis of Anisotropic Flow — Principal Component Analysis of Event-by-Event Fluctuations

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Outline

- Introduction
- Principal Component Analysis (PCA)
- Application to two-particle correlation matrix
- Results – AMPT $(\eta,\ p_T)$, ALICE (p_T)
- Discussion

Analysis of anisotropic flow v_n

- Methods currently in use (event-plane, cumulant): devised before the importance of flow fluctuations was recognized

- New method: extraction of flow fluct. directly from expt. data on 2-particle correlations

- Based on Principal Component Analysis (PCA) — applied to the 2-particle correlation matrix, $\langle \cos n\Delta\phi \rangle$

- Leading eigenmode $\longleftrightarrow$ usual $v_2, \ v_3$
  Subleading modes of $v_2, \ v_3$ revealed for the 1st time

Principal Component Analysis (PCA)

- Statistical procedure to elucidate the underlying covariance structure in the multi-dimensional data
- To identify the directions (PC) where there is the most variance, and possibly reduce the dimension of data
- Diagonalize the covariance matrix: Eigenvector with the largest eigenvalue is the direction of greatest variance; that with the 2nd largest eigenvalue ...
- PCA can be thought of as fitting a hyper-ellipsoid to the cloud of data points — each of its axes representing a PC

Flow Picture

- Single-particle distribution in an event:
  $\frac{dN}{d\mathbf{p}} = \sum_{n=-\infty}^{\infty} V_n(p) \exp(in\phi)$
- Pair distribution averaged over events:
  $\left\langle \frac{dN_{pairs}}{d\mathbf{p_1} d\mathbf{p_2}} \right\rangle = \left\langle \frac{dN}{d\mathbf{p_1}} \frac{dN}{d\mathbf{p_2}} \right\rangle + \mathcal{O}(N) \leftarrow$ nonflow correl.
  $= \sum_{n=-\infty}^{\infty} V_{n\Delta}(p_1, p_2) \exp(in(\phi_1 - \phi_2))$
- Two-particle correlation matrix:
  $V_{n\Delta}(p_1, p_2) = \langle V_n(p_1) V_n^*(p_2) \rangle$,
  neglecting non-flow correlations
- N.B. $n = 0$ multiplicity; $n \neq 0$: anisotropic flow

Covariance Matrix

- $V_{n\Delta}(p_1, p_2) = \langle V_n(p_1) V_n^*(p_2) \rangle$ is a covariance matrix.
- Covariance matrix is symmetric, positive semidefinite, and its eigenvalues are non-negative.
- PCA yields $V_{n\Delta}(p_1, p_2) \simeq \sum_{\alpha=1}^{k} V_n^{(\alpha)}(p_1) V_n^{(\alpha)*}(p_2)$ $=$ sum over modes of flow fluct. Here $(k \leq N_b)$.
- If no flow fluctuations, then factorization occurs: $V_{n\Delta}(p_1, p_2) \simeq V_n^{(1)}(p_1) V_n^{(1)*}(p_2)$

Method

- Divide the detector acceptance into several bins in p_T and/or η. Let p: bin index.
- Flow vector in an event: $Q_n(p) \equiv \sum_{j=1}^{M(p)} \exp(in\phi_j)$
- Pair distribution $V_{n\Delta}(p_1, p_2)$
  $\equiv \langle Q_n(p_1) Q_n^*(p_2) \rangle - \langle M(p_1) \rangle \, \delta_{p_1 p_2} - \langle Q_n(p_1) \rangle \, \langle Q_n^*(p_2) \rangle$
  RHS_2: subtracts self-correl. RHS_3: singles out fluct.
- PC are obtained by diagonalizing $V_{n\Delta}(p_1, p_2)$:
  Eigenvalues: $\lambda^{(1)} > \lambda^{(2)} > \lambda^{(3)} \ldots$
  Eigenvectors: $\psi^{(\alpha)}(p) = V_n^{(\alpha)}(p) / \sqrt{\lambda^{(\alpha)}}$

Method (contd.)

- We define

  $v_n^{(\alpha)}(p) \equiv \frac{V_n^{(\alpha)}(p)}{\langle M(p) \rangle}$

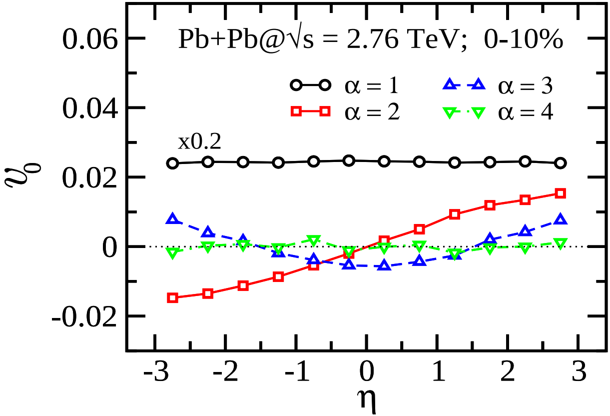
  Thus $v_0^{(\alpha)}(p)$: relative multiplicity fluctuations,

  $v_n^{(\alpha)}(p)$ for $n \neq 0$: fluctuations of anisotropic flow

AMPT (Lin, Ko, Li, Zhang, Pal 2005)

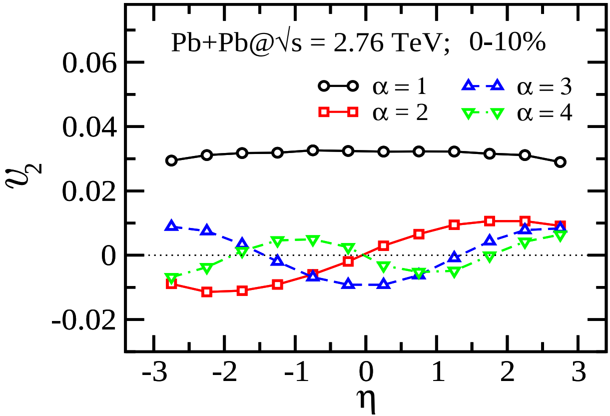
- Initial conditions from HIJING 2.0 (Deng, Wang, Xu 2011)
- These contain nontrivial e-by-e fluctuations
- Flow generated by elastic scatterings of partons
- Resonance decays $\longrightarrow$ nonflow effects
- In agreement with v_2 to v_6 versus p_T for all centralities at 2.76 TeV
- 0-10% centrality, $\sim 10^4$ events, $-3 < \eta < 3$

Relative Multiplicity Fluctuations vs Pseudorapidity



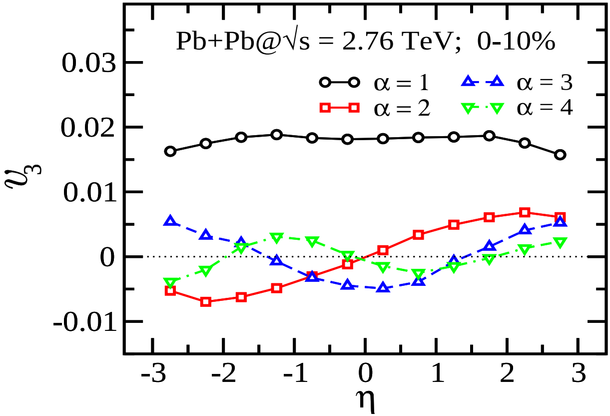
- PCA results for the leading and 3 subleading modes
- PC: Alternating parities; Mutually orthogonal
- Subleading modes $\ll$ Leading modes; Eigenvalues $\lambda^{(3)} \ll \lambda^{(2)} \ll \lambda^{(1)}$

Elliptic flow fluctuations vs Pseudorapidity



- PCA results for the leading and 3 subleading modes
- PC: Alternating parities; Mutually orthogonal
- Subleading modes $\ll$ Leading modes; Eigenvalues $\lambda^{(3)} \ll \lambda^{(2)} \ll \lambda^{(1)}$

Triangular flow fluctuations vs Pseudorapidity



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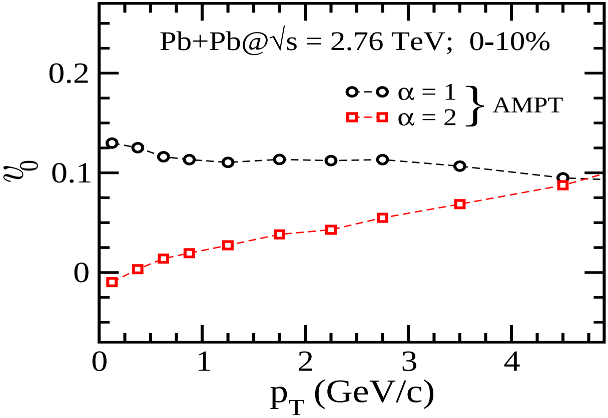
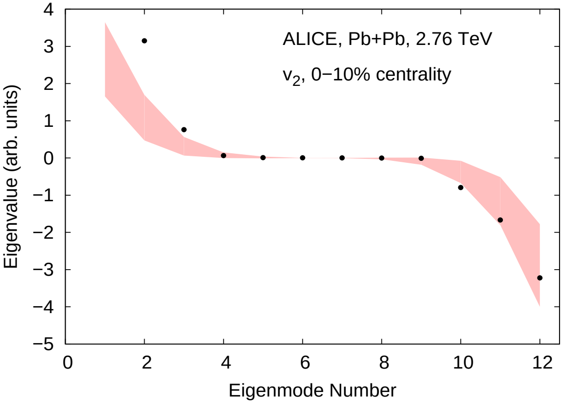
Relative Multiplicity Fluctuations vs Transverse Momentum



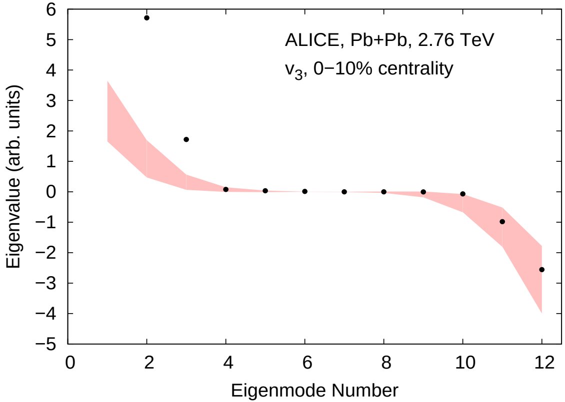
- PCA results for the leading and subleading modes
- Subleading modes ≪ Leading modes; Eigenvalue $\lambda^{(2)} \ll \lambda^{(1)}$
- No experimental results available for comparison

PCA – Eigenvalues for $v_2(p_T)$



- Band: PCA applied to purely statistical fluctuations
- Negative eigenvalues of $V_{n\Delta}(p_1, p_2)$ are compatible with those of large random matrices. Can be attributed to stat. fluct.
- Note the few leading eigenmodes which clearly stand out

PCA – Eigenvalues for $v_3(p_T)$



- Band: PCA applied to purely statistical fluctuations
- Negative eigenvalues of $V_{n\Delta}(p_1, p_2)$ are compatible with those of large random matrices. Can be attributed to stat. fluct.
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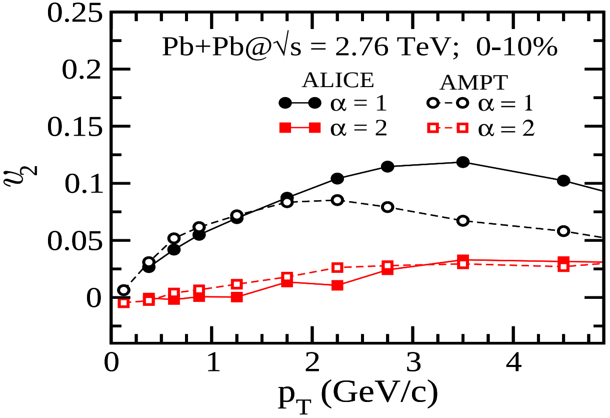
Elliptic Flow Fluctuations vs Transverse Momentum



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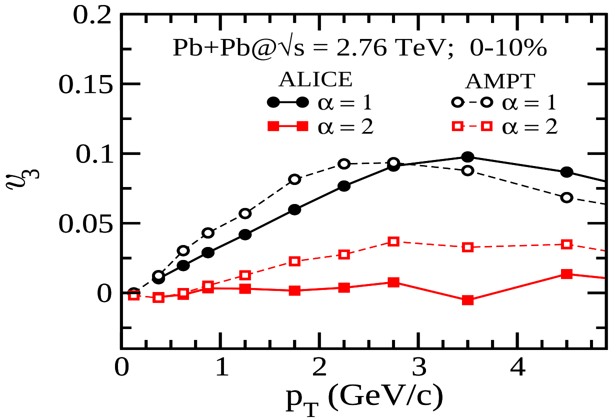
Triangular flow fluctuations vs Transverse Momentum



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Discussion

- Two-particle azimuthal correlations depend on momenta of both particles. Traditional methods: one of the momenta integrated over. New method: Makes use of both the momenta.
- PCA has revealed subleading modes in multiplicity, elliptic flow, and triangular flow fluctuations.
- We anticipate a rich experimental and theoretical program studying the dynamics of subleading flow vectors, which can be used to further constrain the plasma response to the initial geometry.

THANK YOU