Jet vs. Jettiness – the event shape in
p + A and e + A collisions

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Based on works by Z. Kang, X. Liu, S. Mantry, J. Qiu, …
D. Kang, C. Lee, I. Stewart, …
1204.5469, 1303.6952, 1303.6954,
1312.0301, 1404.6706, …

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Jets

- **Definition:**
  - Inclusive cross section with limited phase space
  - “footprints” or “trace” of quarks and gluons

Sterman & Weinberg, PRL 1977
Suppression of jets – Jet quenching

- Jets vs. leading hadron:
  - Narrow jet
  - Same suppression as leading hadron

- Similar $R_{AA}$

Graphs showing $R_{AA}$ as a function of $p_T$ and $p_{T,\text{jet}}$ for different centrality and jet types.
Where does the lost energy go?

- **Medium induced radiation:**
  - Small angle in/near cone
  - Thermalize with the medium:
  - Broaden the jet

No suppression if the cone is bigger enough!
Radiation is gone!
Jet cone dependence!

**Where does the lost energy go?**

*We do not know, since we did not keep track of every particles*

**What if we do keep track of every particles?**

*We should know the full event shape!*
Event shapes

- Event shapes are theoretically cleaner (more inclusive!):

- Thrust, as an example:

\[ T = \max_i \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \]

- Two jet configurations obtained in the limit:

\[ T \rightarrow 1 \]

- Resummation of logarithms of \((1-T)\), corresponds to a resummation of the jet veto logs

- Structure of resummation is simpler, *no jet algorithm dependence* (jet algorithm dependence begins at NNLO with two emissions)
N-Jettiness

- Event structure:
  \[ pp \rightarrow \text{leptons plus jets} \]

- N-Jettiness:
  \( \tau_N^{i} = \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\} \)

  The sum include all final-state hadrons excluding more than \( N \) jets

  Allows for an event-shape based analysis of multi-jets events (a generalization of Thrust), and is complementary to jets

- N-infinitely narrow jets – isolated single hadron(s) (jet veto):
  As a limit of N-Jettiness: \( \tau_N \rightarrow 0 \)
N-Jettiness – implementation

Steps for implementation:

- Use a standard jet algorithms to find N-jets
- Initial reference vectors = momenta of the N-jets + hadron beam directions (reference vectors are the only information used from the jet algorithm)
- Calculate value for the N-jettiness global event shape: \( \tau_N \) (new reference directions from the minimization)
- Select events with N narrow well-separated jets and impose veto on additional jets

New “jet” momenta = sum of momenta in jet regions

\[
P_i^\mu = \sum_k p_k^\mu \prod_{j \neq i} \theta(\hat{q}_j \cdot p_k - \hat{q}_i \cdot p_k)
\]

N-jettiness momentum = sum of jettiness from each region:

\[
\tau_N = \sum_i \tau_N^i = \sum_i 2\hat{q}_i \cdot P_i
\]

Dependence on Jet algorithms is power suppressed
1-Jettiness cross section in DIS

Very much "like" the calculation for the "Thrust"
(Minimization vs maximization!)

\[ \tau_1 = \frac{2}{Q^2} \sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\} \]

\[ d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \rightarrow J + X)}{du \ dP_{JT} \ d\tau_1} \]

1-jettiness: global event shape
Event shape with 1-Jettiness

- Configurations of large and small 1-jettiness:

\[ \tau_1 \sim P_{J_T} \]
\[ \tau_1 \ll P_{J_T} \]

- 1-jettiness distributions can be a probe of nuclear structure and dynamics.

**Most importantly, the radiation pattern following the additional scattering**
Event shapes for DIS

- Event shapes have been studied before in DIS:
  - Breit Frame
  - Thrust, NLL +NLO
  - Broadening, NLL +NLO
  - Non-Global Event Shapes

- 1-jettiness global event shape for DIS was first introduced about a year ago:
  - 1-jettiness factorization in SCET
  - Proposed as probe of nuclear physics
  - Proton target, NLL results

- More recently:
  - NNLL resummation
  - Variety of nuclear targets: Proton, C, Ca, Fe, Au, Ur

- Most recently:
  - Also, considered 1-jettiness with NNLL resummation
  - Introduced two new variations of 1-jettiness and their factorization
  - Analysis restricted to proton target

- Matching from small $\tau$ to large $\tau
Three ways to define the 1-jettiness

\[ q_B = xP \]
\[ q_J = \text{true jet axis} \]

Kang, Mantry, Qiu (2012)

\[ p_{\text{ISR}} = (\xi - x)P + k_\perp \]
\[ q_J = q + xP - k_\perp \]
\[ k_\perp \sim Q\lambda \]

\[ \tau_1^a \]

CM frame

\[ \mathcal{H}_B \quad \mathcal{H}_J \]

\[ p_J \]

\[ q_J \text{ true jet axis} \]

\[ p_B \]

\[ q \]

\[ \xi P \quad xP - k_\perp \]

\[ q_j \text{ is A}ligned with the jet momentum, with no relative label transverse momentum: find by jet algorithm or minimization} \]

\[ \text{depends on momenta of final-state hadrons} \]
Three ways to define the 1-jettiness

\[ \tau_1^b \]

\[ q_B = xP \]
\[ q_J = q + xP \]

same as DIS thrust by Antonelli, Dasgupta, Salam (1999)

\[ q_J \] no longer exactly aligned with jet, but simpler in that \( q + xP \) is given only by lepton and initial-state proton momenta

**Breit frame:**
\[ q = (Q, 0, 0, Q) \]
\[ q_B = Q\hat{n}_z \quad q_J = Qn_z \]

1-jettiness regions are hemispheres in Breit frame

Direction of scattered quark at lowest order
Three ways to define the 1-jettiness

\[ q_B = P \]
\[ q_J = k \]

(measured momentum)
Kang, Lee, Stewart 2013

measures thrust in back-to-back hemispheres in Center-of-momentum frame

momentum transfer \( q \) itself has a nonzero transverse component:

\[
q = y \sqrt{s \frac{n_z}{2}} - xy \sqrt{s \frac{\hat{n}_z}{2}} + \sqrt{1 - y} Q \hat{n}_\perp
\]

seemingly simplest definition: in practice hardest to calculate!

**Restriction:** \( p_J^\perp \) has to be small for 1-jettiness \( \tau_1^c \) to be small \( \Rightarrow 1 - y \sim \lambda^2 \)
Tree-level 1-jettiness distribution

\[
d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \rightarrow J + X)}{dy
dP_{JT} d\tau_1}
\]

Two scales observables!
- \( P_T \): localized probe
- \( \tau_1 \): sensitive to event shape

Tree-level distribution in 1-jettiness:

\[
\frac{d^3\sigma^{(0)}}{dy dP_{JT} d\tau_1} = \sigma_0 \delta(\tau_1) \sum_q e_q^2 \frac{1}{A} f_{q/A}(x_A, \mu)
\]
Hierarchy of energy scales

\[ d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \rightarrow J + X)}{dy \ dP_{JT} \ d\tau_1} \]

- Hierarchy of scales:
  \[ Q_S, \Lambda_{QCD} \ll \tau_1 \ll P_{JT} \]

- Nuclear scales
- I-jettiness
- Jet transverse momentum

- Jet-veto Sudakov logarithms:
  \[ \sim \alpha_s^n \ln^{2n}(\tau_1 / P_{JT}) \]

- Hard:
  \[ \mu_H \sim P_{JT} \]

- Beam, Jet:
  \[ \mu_B \sim \mu_J \sim \sqrt{\tau_1 P_{JT}} \]

- Soft:
  \[ \mu_S \sim \tau_1 \]

- Nuclear:
  \[ Q_s^2(A) \sim A^\alpha \Lambda_{QCD}^2 \]
Factorization – SCET

- Schematic form of factorization:

\[
\frac{d^3\sigma}{dy dP_{JT} d\tau_1} \sim H \otimes B_A \otimes J \otimes S,
\]

- Beam function
- Soft function
- Jet function
- Hard function

- Hard function
- Beam function
- Jet function
- Soft function
Factorized cross section

- Detailed form of factorization:

\[
\frac{d^3\sigma}{dydP_Td\tau_1} = \frac{\sigma_0}{A} \sum_{q,i} e_q^2 \int_0^1 dx \int ds_J \int dt_a \times H(xAQ_e P_T e^{-y}, \mu; \mu_H) \delta \left[ x - \frac{e^y P_T}{A(Q_e - e^{-y} P_T)} \right] \\
\times J^q(s_J, \mu; \mu_J) B^q(x, t_a, \mu; \mu_B) \times S \left( \tau_1 - \frac{t_a}{Q_a} - \frac{s_J}{Q_J}, \mu; \mu_S \right),
\]

- Beam function matching onto the PDF:

(Fleming, Leibovich, Mehen; Jouttenus, Stewart, Tackmann, Waalewijn)

\[
B^q(x, t_a, \mu; \mu_B) = \int_x^1 \frac{dz}{z} T^q_i \left( \frac{x}{z}, t_a, \mu; \mu_B \right) f_{i/A}(z, \mu_B)
\]

- Tree-level matching:

\[
B^q(x, t_a, \mu_B) = \delta(t_a) f_{q/A}(x, \mu_B)
\]

\~\text{“collinear” “perturbative”}
Resummation

- Resummation achieved through renormalization group equations:

\[
\begin{align*}
\mu_H & \sim P_{J_T} \\
\mu_B & \sim \mu_J \sim \sqrt{\tau_1 P_{J_T}} \\
\mu_S & \sim \tau_1 \\
Q_s(A) & \\
\end{align*}
\]

- All objects in factorization formula have well defined evolution equations:

\[
\begin{align*}
\mu \frac{d}{d\mu} H(Q^2, \mu) &= \gamma_H H(Q^2, \mu), \\
\mu \frac{d}{d\mu} B_A^q(x, t, \mu) &= \int dt' \gamma_B(t - t', \mu) B_A^q(x, t', \mu), \\
\mu \frac{d}{d\mu} J(s, \mu) &= \int ds' \gamma_J(s - s', \mu) J(s', \mu), \\
\mu \frac{d}{d\mu} S(k_a, k_J, \mu) &= \int dk_a' \int dk_J' \gamma_S(k_a - k_a', k_J - k_J', \mu) S(k_a', k_J', \mu)
\end{align*}
\]
Differences between the three definitions

\[
\frac{1}{\sigma_0} \frac{d\sigma(x, Q^2)}{d\tau^a_1} = H(Q^2, \mu) \int dt_J dt_B dk_S \delta \left( \tau^a_1 - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q} \right) \\
\times J_q(t_J, \mu) B_q(t_B, x, \mu) S(k_S, \mu)
\]

Z. Kang, Mantry, Qiu, 2012

\[
\frac{1}{\sigma_0} \frac{d\sigma(x, Q^2)}{d\tau^b_1} = H(Q^2, \mu) \int d^2 p_\perp dt_J dt_B dk_S \delta \left( \tau^b_1 - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q} \right) \\
\times J_q(t_J - p_\perp^2, \mu) B_q(t_B, x, p_\perp^2, \mu) S(k_S, \mu)
\]

D. Kang, Lee, Stewart, 2013

\[
\frac{1}{\sigma_0} \frac{d\sigma(x, Q^2)}{d\tau^c_1} = H(Q^2, \mu) \int d^2 p_\perp dt_J dt_B dk_S \delta \left( \tau^c_1 - \frac{t_J}{Q^2} - \frac{t_B}{xQ^2} - \frac{k_S}{\sqrt{xQ}} \right) \\
\times J_q(t_J - (q_\perp + p_\perp)^2, \mu) B_q(t_B, x, p_\perp^2, \mu) S(k_S, \mu)
\]

D. Kang, Lee, Stewart, 2013
1-jettiness and rapidity distribution

One can study distributions in the space of:

\[ \{ A, Q_e, P_{J_T}, y, \tau_1 \} \]

Proton target: NNLL resummation

- Larger \( \tau \) less "jet" structure
- Smaller \( \tau \) better "jet" structure
1-Jettiness cross section in e+A DIS

View in the center of mass frame

Multiple scattering
Change in radiation pattern

Same definition:

$$\tau_1 = \frac{2}{Q^2} \sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

Additional variable: $A$

1-jettiness: global event shape
1-Jettiness cross section in e+A DIS

Leading power case

Factorization formula:

$$\left. \frac{d^3\sigma}{dydP_{JT}d\tau_1} \right|_{EPS09} = \sigma_0 \sum_{q,i} e_q^2 \int_{x_*}^{1} \frac{dx}{x} \int ds_J \int dt_a \times H(\xi^2, \mu; \mu_H) J^q(s_J, \mu; \mu_J) T^{qi} \left( \frac{x_*}{x}, t_a, \mu; \mu_B \right) \times S \left( \tau_1 - \frac{t_a}{Q_a} - \frac{s_J}{Q_J}, \mu; \mu_S \right) f_{i/A}^{EPS09}(x, \mu_B),$$

Lower limit of Bjorken-x integration:

$$x_* = \frac{e^y P_{JT}}{Q_e - e^{-y} P_{JT}}$$

Determines the Bjorken-x region

See appendices of arXiv: 1303.3063 for details on each functions and their evolutions

Similar leading power factorization formula for p+A collisions

Two beam functions, two soft functions
Nuclear PDFs

- At leading twist, we directly probe nuclear PDFs (Eskola, Paukunnen, Salgado)

\[ f_{u/A}^{EPS09}(x, \mu) = \frac{Z}{A} R_u^A(x, \mu) f_{u/p}(x, \mu) + \frac{A-Z}{A} R_d^A(x, \mu) f_{d/p}(x, \mu), \]
\[ f_{d/A}^{EPS09}(x, \mu) = \frac{Z}{A} R_i^A(x, \mu) f_{d/p}(x, \mu) + \frac{A-Z}{A} R_u^A(x, \mu) f_{u/p}(x, \mu), \]
\[ f_{s,c,b/A}^{EPS09}(x, \mu) = R_{s,c,b}^A(x, \mu) f_{s,c,b/p}(x, \mu), \]
\[ f_{g/A}^{EPS09}(x, \mu) = R_{g}^A(x, \mu) f_{g/p}(x, \mu), \]

- Schematic behavior of nuclear modification factors

[Graph showing the behavior of the nuclear modification factor \( R_L(x, \mu) \)]

\[ R_L(x, \mu) = \frac{\sum_q e_q^2 f_{E}^{EPS09}(x, \mu)}{\sum_q e_q^2 f_{q/p}(x, \mu)} \]
Scale variations – theory uncertainty

- Four independent scale variations employed to estimate perturbative uncertainties: (Stewart, Tackmann, Waalewijn)

(a) $\mu = \mu_H = r \sqrt{\xi^2}$, $\mu_B = r \sqrt{Q_a \tau_1}$, $\mu_J = r \sqrt{Q_J \tau_1}$, $\mu_S = r \tau_1$,
(b) $\mu = \mu_H = \sqrt{\xi^2}$, $\mu_B = \sqrt{Q_a \tau_1}$, $\mu_J = \sqrt{Q_J \tau_1}$, $\mu_S = r^{-\frac{1}{4} \ln \frac{\tau_1}{\xi}} \tau_1$,
(c) $\mu = \mu_H = \sqrt{\xi^2}$, $\mu_B = r^{-\frac{1}{4} \ln \frac{\tau_1}{\xi}} \sqrt{Q_a \tau_1}$, $\mu_J = \sqrt{Q_J \tau_1}$, $\mu_S = \tau_1$,
(d) $\mu = \mu_H = \sqrt{\xi^2}$, $\mu_B = \sqrt{Q_a \tau_1}$, $\mu_J = r^{-\frac{1}{4} \ln \frac{\tau_1}{\xi}} \sqrt{Q_J \tau_1}$, $\mu_S = \tau_1$,

$\xi^2 \equiv \frac{P_{JT}^2}{1 - e^{-y} P_{JT} / Q_e}$

$r = \{1/2, 2\}$
1-jettiness distribution in e+\(A\) for various nuclei

NNLL resummation

\[ Q_e = 90 \text{ GeV} \]
\[ P_{JT} = 20 \text{ GeV} \]
\[ y = 0 \]

Effect of nPDFs and smearing
Jet rapidity distributions in $e+\Lambda$ for various nuclei

Effect of nPDFs and smearing

NNLL resummation

\[ Q_e = 90 \text{ GeV}, \]
\[ P_{J_T} = 20 \text{ GeV}, \]
\[ \tau_1 = 1.5 \text{ GeV}. \]
Jet rapidity: Nuclei over Proton

\[ R_A(\tau_1, P_{J_T}, y) = \frac{d\sigma_A(\tau_1, P_{J_T}, y)}{d\sigma_p(\tau_1, P_{J_T}, y)} \]

NNLL resummation

\[ Q_e = 90 \text{ GeV} \]
\[ P_{J_T} = 20 \text{ GeV} \]
\[ \tau_1 = 1.5 \text{ GeV} \]

\[ x_* = \frac{e^y P_{J_T}}{Q_e - e^{-y} P_{J_T}} \]

\[ x_* \in [0.2, 0.7] \]

Effect of nPDFs and smearing
Matching from low $\tau$ to high $\tau$

- **Low $\tau$ vs high $\tau$:**
  - Low $\tau$: Resummation by using SCET
  - High $\tau$: Fix order perturbative calculation

- **Matching:**
  \[
  d\sigma = \left[ d\sigma_{\text{resum}} - d\sigma_{\text{resum}}^{FO} \right] + d\sigma^{FO}
  \]

- **Three regions:**
  \[
  \tau_1 \sim \Lambda_{QCD}, \\
  \Lambda_{QCD} \ll \tau_1 \ll P_{JT}, \\
  \tau_1 \sim P_{JT},
  \]

- **Beam function:**
  \[
  B \sim I \otimes f
  \]

- **Eqs.:**
  \[
  d\sigma_{\text{resum}} = \frac{d^3\sigma_{\text{resum}}}{dydP_{JT}d\tau_1} \sim H \otimes B \otimes J \otimes S
  \]
  \[
  \mu_H \sim P_{JT}, \quad \mu_J \sim \mu_B \sim \sqrt{\tau_1 P_{JT}}, \quad \mu_S \sim \tau_1
  \]

  \[
  d\sigma^{FO}_{\text{resum}} = d\sigma_{\text{resum}}(\mu = \mu_H = \mu_J = \mu_B = \mu_S)
  \]

  \[
  d\sigma^{FO} \sim \int dPS \hat{F}_{\text{meas.}}([PS]) |M|^2 \otimes f
  \]
Matching from low $\tau$ to high $\tau$

- Full spectrum in $\tau_1$ on proton:

Kang, Liu, Mantry, 1312.0301

Graph showing $\Delta \sigma/\Delta \tau_1$ (fb/GeV) vs $\tau_1$ (GeV) with different color curves labeled as Expanded SCET, NNLL SCET, NLO QCD.
Event shapes based analysis can be a useful tool to probe gluon shower and induced radiation.

- Allows for jet shape analysis, and also gives information on wide-angle soft radiation – complementary to jet x-section.
- Probe nuclear dynamics through distributions in multiple dimensional space on various nuclear targets:
  \[ \{ A, Q_e, P_{JT}, y, \tau_1 \} \]
- SCET can be a natural tool to take care of the resummation of large logarithms from various scales.
- Many directions can be pursued with event shape analysis ...

Thank you!
BACKUP SLIDES
DIS Kinematics

\[ s = (k + P)^2 \]

\[ Q^2 = -q^2 \]

\[ x = \frac{Q^2}{2P \cdot q} \]

\[ y = \frac{P \cdot q}{P \cdot k} \]

\[ Q^2 = xys \]

\[ p_X = q + P \]

\[ p_X^2 = \frac{1 - x}{x} Q^2 \]

Limit \( x \rightarrow 1 \) corresponds to single collimated jet in final state.

We will look away from \( x = 1 \) at two-jet like final states.
Kinematics

- **Electron momentum:**
  \[ p_{e}^{\mu} = (p_{e}^{0}, \vec{p}_{e}) \]

- **Nucleus momentum:**
  \[ P_{A}^{\mu} = A(p_{e}^{0}, -\vec{p}_{e}) \]

- **Electron energy:**
  \[ p_{e}^{0} = |\vec{p}_{e}| = \frac{Q_{e}}{2} \]

- **Center of mass energy squared:**
  \[ s = (p_{e} + P_{A})^{2} = A \frac{Q_{e}^{2}}{2} \]

- **Center of mass energy per nucleon:**
  \[ Q_{e} \]
A clean calibration and a lot more:

- Study jet distributions in e-A collisions.
- Probe of nuclear PDFs at leading twist.
- Higher twist correlations.
- Parton propagation through cold nuclear matter.
- Energy loss mechanisms.
- Nuclear medium effects.
- ...
Power corrections

- Many different sources of power corrections.

- Dominant nuclear-dependent power corrections come from the OPE of the beam function

\[ B^q(x, t_a, \mu; \mu_B) = \int_x^1 \frac{dz}{z} I^{qi} \left( \frac{x}{z}, t_a, \mu; \mu_B \right) f_i/A(z, \mu_B) + O\left( \frac{Q_s^2(A)}{t_a} \right) \]

- Size of power corrections controlled by

\[ \frac{Q_s^2(A)}{t_a} \sim \frac{A^\alpha \Lambda^2_{QCD}}{\tau_1 P_{JT}} \]

- Higher twist correlations

- Nuclear medium effects: energy loss, multiple scattering, ...

- Power corrections can be studied as a function of: \( \{ A, \tau_1, P_{JT} \} \)
Non-perturbative region

- Hierarchy of scales:

\[
\frac{d^3 \sigma}{dy dP_{JT} d\tau_1} \sim H \otimes B_A \otimes J \otimes S, \quad P_{JT} \gg \sqrt{\tau_1 P_{JT}} \gg \tau_1
\]

- Soft function becomes non-perturbative for:

\[
\tau_1 \sim \Lambda_{QCD}
\]

- Soft function model

\[
S(k_a, k_J, \mu_S) = \int dk'_a \int dk'_J S_{\text{part.}}(k_a - k'_a, k_J - k'_J, \mu_S) S_{\text{mod.}}(k'_a, k'_J).
\]

\[\text{perturbative soft function}\]

\[\text{model}\]
Role of Jet’s cone size

- Cone size dependence of Jet quenching:

  - Multiple scattering \( \rightarrow \) radiation \( \rightarrow \) energy loss \( \rightarrow \) cone size \( \rightarrow \) …

Ratio is consistent with vacuum jets for peripheral and central collisions