The (3+1)-D Structure of Nuclear Collisions

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IS 2014
Napa, December 5, 2014

Overview

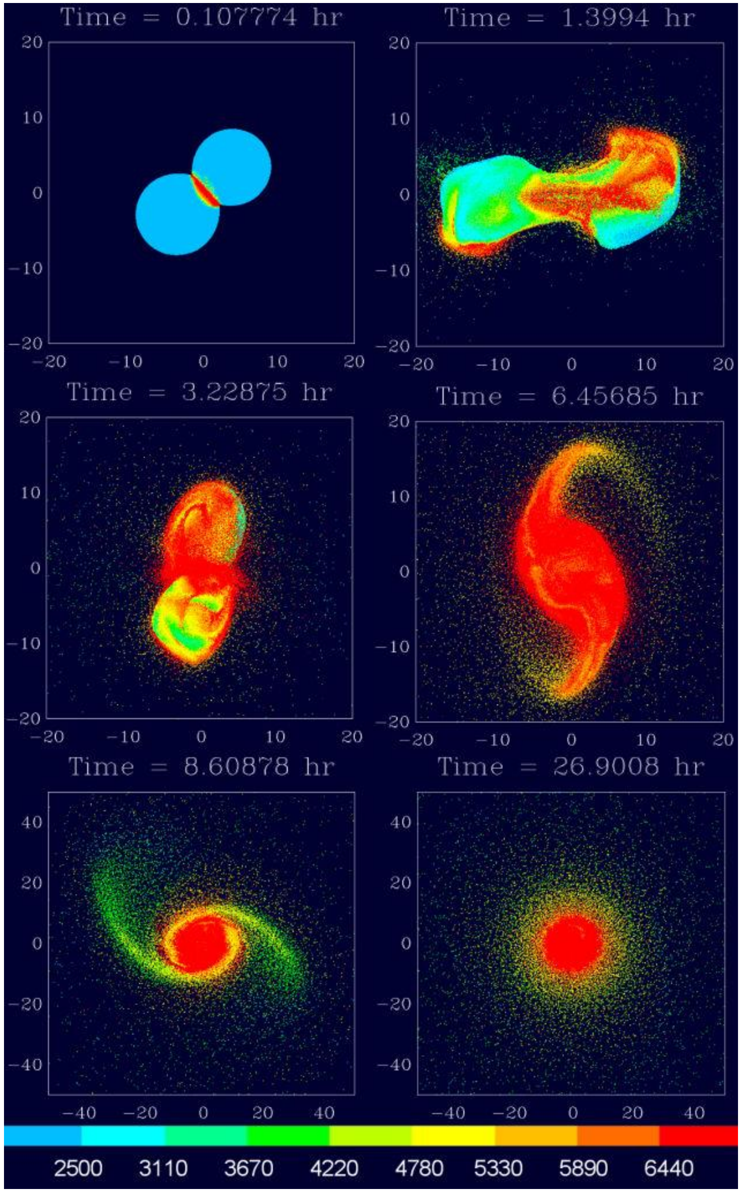
- Motivation
- Classical gluon fields at early times: angular momentum and directed flow
- Matching to hydro: results and interpretation
- Beyond Boost-Invariance

Work in collaboration mainly with

- Guangyao Chen (Texas A&M, now Iowa State University)
- Sidharth Somanathan (Texas A&M)
- Sener Ozonder (INT/Seattle)

Little Bang And Big Splat

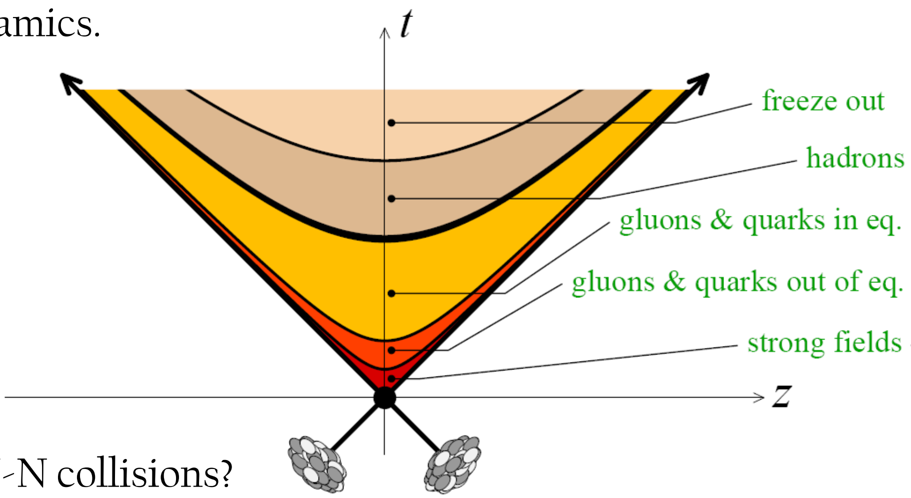
- Colliding spheres of interacting particles.
- Finite impact parameter → interesting spactial structures.
  - Determined by interplay of angular momentum conservation and interactions between particles
- Heavy Ion Physics: state of the art are cylinders with no interesting 3+1 D structure.
  - Exceptions: e.g. Bergen group hydro.
- How does it look for heavy ions?
- Is it important?



[R. M. Canup, Science 338, 1052 (2012)]

Little Bang Evolution

- Thermalized phase
  - Matter close to local thermal equilibrium after time 0.2-1 fm/c.
  - Expansion and cooling via viscous hydrodynamics.
- Transport phase?
  - For matter close to kinetic freeze-out
- Poorly known: pre-equilibrium phase
  - Strings? Strong classical fields? Incoherent N-N collisions?
- Color glass condensate (CGC) candidate theory for asymptotically large energies.



Classical YM Dynamics

- Nuclei/hadrons at asymptotically high energy:
  - Saturated gluon density ~ $Q_s^{-2} \rightarrow$ scale $Q_s \gg \Lambda_{QCD}$, classical fields. [L. McLerran, R. Venugopalan]
- Solve Yang-Mills equations $\left[D_\mu, F^{\mu\nu}\right] = J^\nu$ for gluon field $A^\mu(\rho)$.
  - Source = intersecting light cone current J (given by SU(3) charge distribution ρ).
  - Calculate observables $O(\rho)$ from the gluon field $A^\mu(\rho)$.
  - ρ from color fluctuations of a color-neutral nucleus.

$$\left\langle \rho_i^a(x) \right\rangle = 0$$

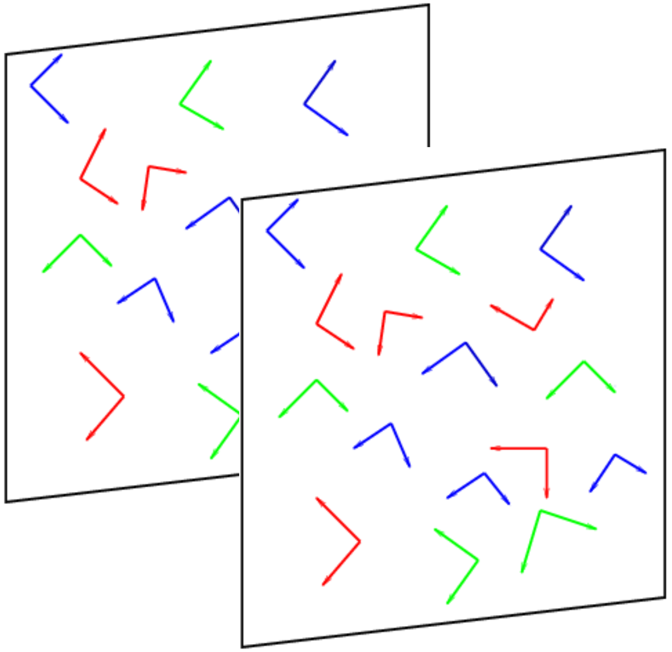
$$\left\langle \rho_i^a(x_1) \rho_j^b(x_2) \right\rangle = \frac{g^2}{N_c^2 - 1} \delta_{ij} \delta^{ab} \lambda_i\left(x_1^\mp\right) \delta\left(x_1^\mp - x_2^\mp\right) \delta^2\left(\mathbf{x}_{1T} - \mathbf{x}_{2T}\right) \qquad \mu_i = \int dx^\mp \lambda_i\left(x^\mp\right)$$

- Boost-invariant setup: sources fixed on the light cone.

Gluon Fields At Collision

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.

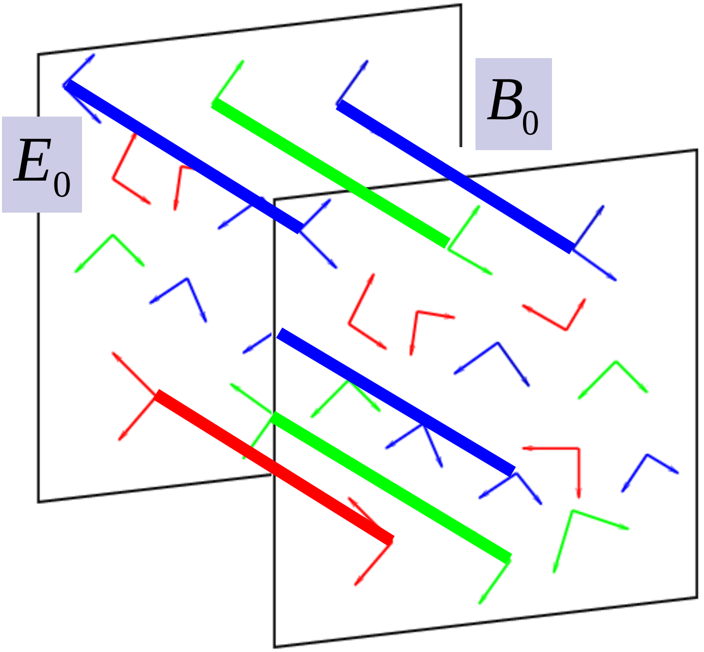
$$F_1^{i+} = \delta(x^-) A_1^i$$



$$F_2^{i-} = \delta(x^+) A_2^i$$

Gluon Fields At Collision

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.
- Immediately after overlap (forward light cone, $\tau \to 0$): strong *longitudinal* electric & magnetic fields. Non-abelian effect.



$$F_{(0)}^{+-} = ig\left[A_1^i, A_2^i\right] \leftarrow E_0$$

$$F_{(0)}^{21} = ig\varepsilon^{ij}\left[A_1^i, A_2^j\right] \leftarrow B_0$$

[L. McLerran, T. Lappi, 2006]
[RJF, J.I. Kapusta, Y. Li, 2006]

Fields: Into the Forward Light Cone

- With *non-abelian* longitudinal fields E_0, B_0 seeded, the next step in time can be understood in terms of the QCD versions of Ampere's, Faraday's and Gauss' Law.
  - Longitudinal fields E_0, B_0 decrease in both *z* and *t* away from the light cone
- Here *abelian* version for simplicity:
- Gauss' Law at fixed time *t*
  - Long. flux imbalance compensated by transverse flux
  - Gauss: *rapidity-odd radial* field
- Ampere/Faraday as function of *t*:
  - Decreasing long. flux induces transverse field
  - Ampere/Faraday: *rapidity-even curling* field
- Full classical QCD:

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\,\varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\,\varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

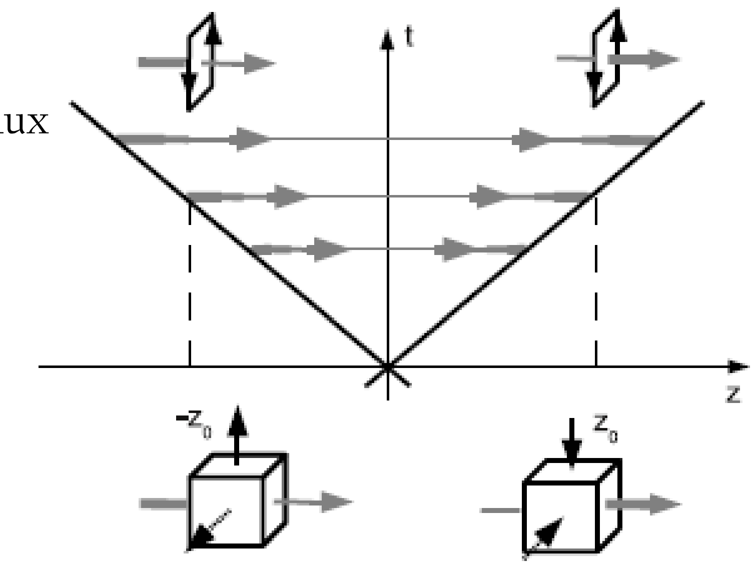


Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.

[G. Chen, RJF, PLB 723 (2013)]

Initial Transverse Field: Visualization

- Transverse fields for randomly seeded A_1, A_2 fields (abelian case).

- $\eta = 0$: Closed field lines around longitudinal flux maxima/minima

- $\eta \neq 0$: Sources/sinks for transverse fields appear

E^i (background B_0) B^i (background E_0)

$\eta = 0$

$\eta = 1$

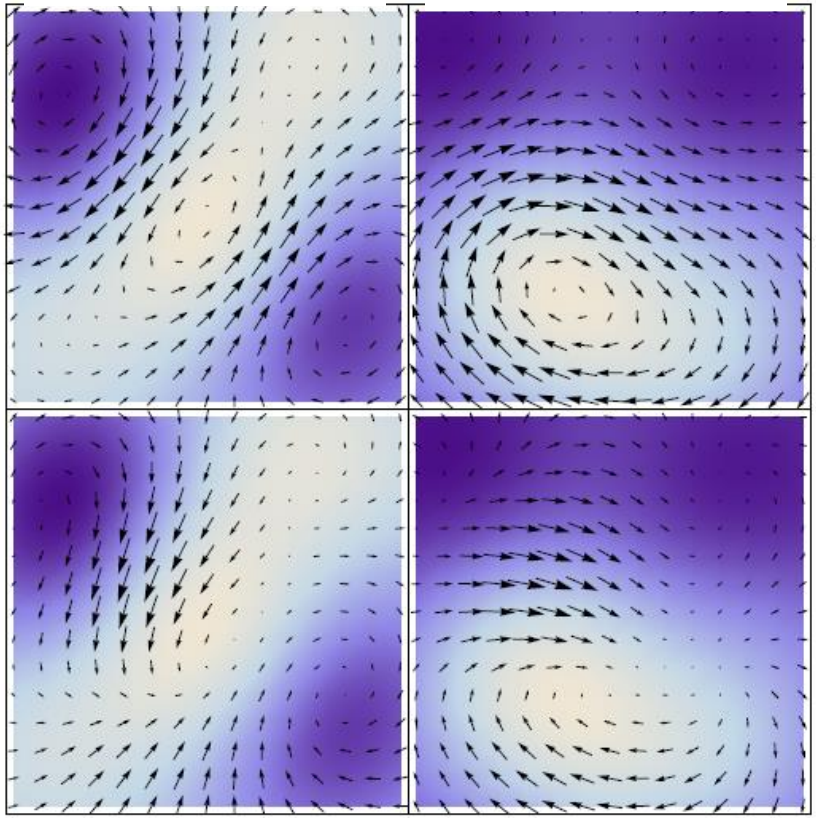


Figure 2: Transverse electric fields (left panels) and magnetic fields (right panels) at $\eta = 0$ (upper panels) and $\eta = 1$ (lower panels) in an abelian example for a random distribution of fields A_1^i, A_2^i. The initial longitudinal fields B_0 (left panels) and E_0 (right panels) are indicated through the density of the background (lighter color = larger values). At $\eta = 0$ the fields are divergence-free and clearly following Ampére's and Faraday's Laws, respectively.

Energy Momentum Tensor

- Initial ($\tau = 0$) structure of the energy-momentum tensor from purely londitudinal fields

$$T^{\mu\nu}_{f(0)} = \begin{pmatrix} \varepsilon_0 & & & \\ & \varepsilon_0 & & \\ & & \varepsilon_0 & \\ & & & -\varepsilon_0 \end{pmatrix}$$

Transverse pressure
$P_T = \varepsilon_0$

Longitudinal pressure
$P_L = -\varepsilon_0$

$$\varepsilon_0 = \frac{1}{2}(E_0^2 + B_0^2)$$

Energy Momentum Tensor

- Flow emerges from pressure at order τ^1:

$$\vec{S} = \vec{E} \times \vec{B}$$

$$T_{\mathrm{f}}^{\mu\nu} = \begin{pmatrix} \varepsilon_0 + O(\tau^2) & \boxed{\alpha^1 \cosh\eta + \beta^1 \sinh\eta} & \boxed{\alpha^2 \cosh\eta + \beta^2 \sinh\eta} & \boxed{O(\tau^2)} \\ \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \varepsilon_0 + O(\tau^2) & O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta \\ \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) & \varepsilon_0 + O(\tau^2) & \alpha^2 \sinh\eta + \beta^2 \cosh\eta \\ O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta & \alpha^2 \sinh\eta + \beta^2 \cosh\eta & -\varepsilon_0 + O(\tau^2) \end{pmatrix}$$

- Transverse Poynting vector gives transverse flow.

[RJF, J.I. Kapusta, Y. Li, (2006)]
[G. Chen, RJF, PLB 723 (2013)]

$$S^i_{\mathrm{even}} = \frac{\tau}{2} \cosh\eta \left(E_0 \left[D^i, E_0 \right] + B_0 \left[D^i, B_0 \right] \right) = \alpha^i \cosh\eta$$

$$S^i_{\mathrm{odd}} = \frac{\tau}{2} \sinh\eta \, \varepsilon^{ij} \left(E_0 \left[D^j, B_0 \right] - B_0 \left[D^j, E_0 \right] \right) = \beta^i \sinh\eta$$

$$\alpha^i = -\frac{\tau}{2} \nabla^i \varepsilon_0$$

Like hydrodynamic flow, determined by gradient of transverse pressure $P_T = \varepsilon_0$; even in rapidity.

$$\beta^i = \frac{\tau}{2} \varepsilon^{ij} \left(\left[D^j, B_0 \right] E_0 - \left[D^j, E_0 \right] B_0 \right)$$

Non-hydro like; odd in rapidity; from field dynamics

Transverse Flow: Visualization

- Transverse Poynting vector for randomly seeded A_1, A_2 fields (abelian case).
- $\eta = 0$: "Hydro-like" flow from large to small energy density
- $\eta \neq 0$: Quenching/amplification of flow due to the underlying field structure.

$(\text{background} = \varepsilon_0)$

$\eta = 0$
(no odd flow)

$\eta = 1$

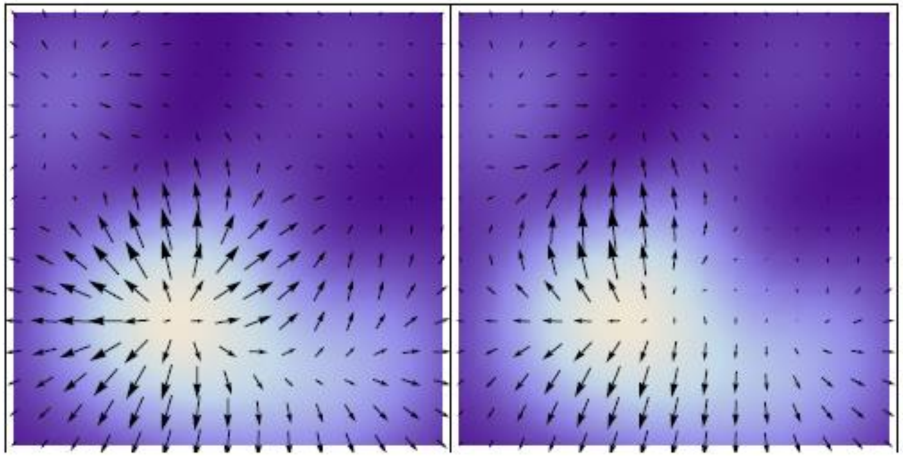


Figure 3: Example for transverse flow of energy for $\eta = 0$ (left panel) and $\eta = 1$ (right panel) in the abelian example for the same random distribution of fields A_1^i, A_2^i as in Fig. 2. The initial energy density T^{00} is shown through the density of the background (lighter color = larger values). At $\eta = 0$ the flow follows the gradient in the energy density in a hydro-like way while away from mid-rapidity energy flow gets quenched in some directions and amplified in others.

Averaged Density and Flow

- Averaging color charge densities. Here no fluctuations!
- Energy density

$$\varepsilon_0 = \frac{g^6 N_c \left(N_c^2 - 1\right)}{8\pi} \mu_1 \mu_2 \ln^2 \frac{Q^2}{m^2}$$

- "Hydro" flow:

[T. Lappi, 2006]
[RJF, Kapusta, Li, 2006]
[Fujii, Fukushima, Hidaka, 2009]

$$\alpha^i = -\tau \frac{g^6 N_c \left(N_c^2 - 1\right)}{64\pi^2} \nabla^i \left(\mu_1 \mu_2\right) \ln^2 \frac{Q^2}{m^2}$$

- "Odd" flow term:

[G. Chen, RJF, PLB 723 (2013)]

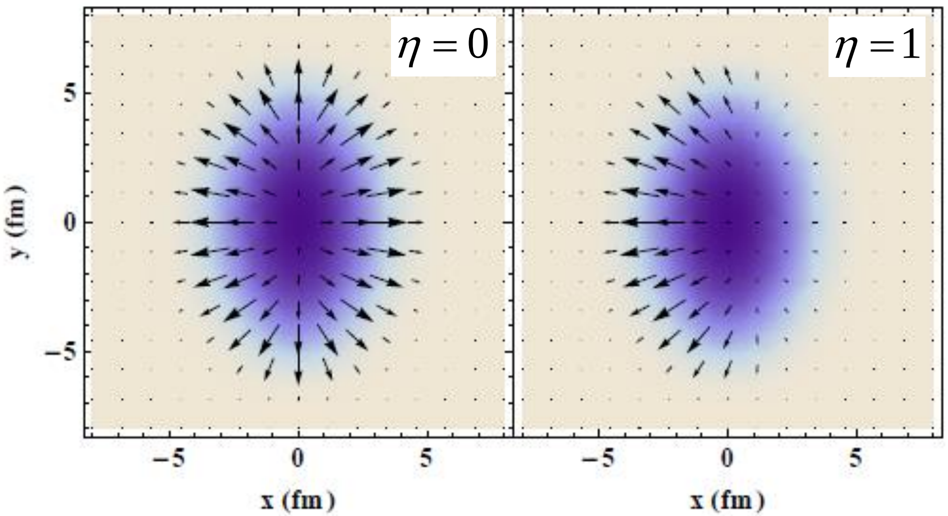
$$\beta^i = -\tau \frac{g^6 N_c \left(N_c^2 - 1\right)}{64\pi^2} \left(\mu_2 \nabla^i \mu_1 - \mu_1 \nabla^i \mu_2\right) \ln^2 \frac{Q^2}{m^2}$$

  - With $E_0 = ig\left[A_1^i, A_2^i\right], \quad B_0 = ig\varepsilon^{ij}\left[A_1^i, A_2^j\right]$ we have $\left\langle E_0 \nabla^i B_0 \right\rangle = -\left\langle B_0 \nabla^i E_0 \right\rangle$

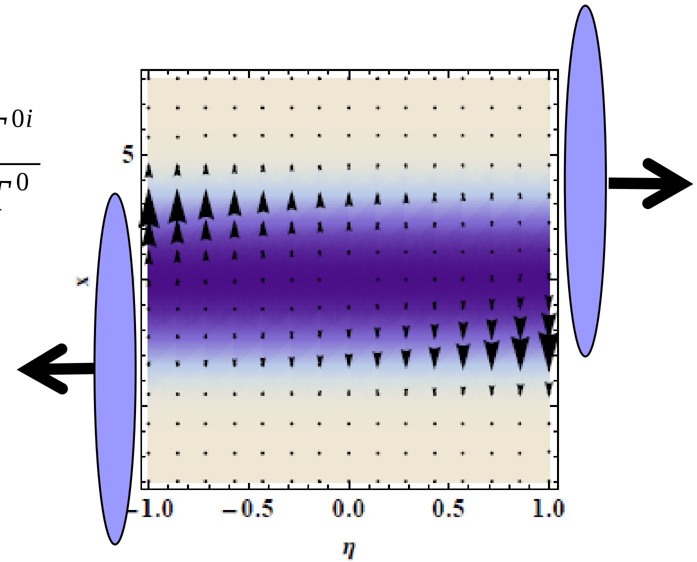
- Order τ^2 terms …

Flow Phenomenology: $b \neq 0$

- Odd flow needs asymmetry between sources. Here: finite impact parameter
- Flow field for Au+Au collision, b = 4 fm.



$$V^i = \frac{T^{0i}}{T^0}$$



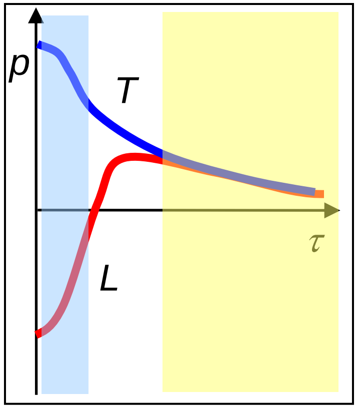
- *Radial* flow following gradients in the fireball at η = 0.
- In addition: *directed* flow away from η = 0.
- Fireball is rotating, exhibits angular momentum.
- $|V|$ ~ 0.1 at the surface @ τ ~ 0.1

Fireball Time Evolution: Field Phase

- Evolution of T^{00} in the reaction plane for b = 10 fm Pb+Pb collisions up to 0.5 fm.

Matching to Hydrodynamics

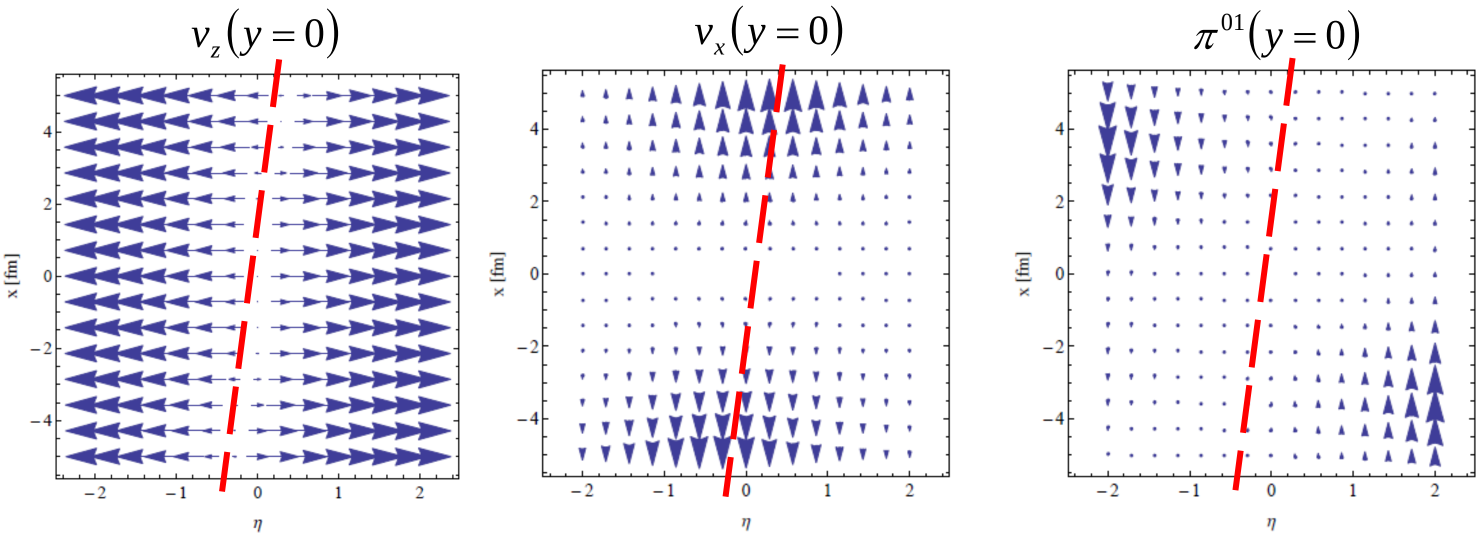
- No dynamic equilibration here; see other talks at this conference.
- Pragmatic solution: extrapolate from both sides ($r(\tau)$ = interpolating fct.)
  $$T^{\mu\nu} = T_{\mathrm{f}}^{\mu\nu} r(\tau) + T_{\mathrm{pl}}^{\mu\nu}\left(1 - r(\tau)\right)$$
- Here: fast equilibration assumption: $r(\tau) = \Theta(\tau_0 - \tau)$



- Matching: enforce $\partial_\mu T^{\mu\nu} = 0$ and
  $\partial_\mu M^{\mu\nu\lambda} = 0 \qquad M^{\mu\nu\lambda} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}$
  (and other conservation laws).
- Mathematically equivalent to imposing smoothness condition on all components of $T_{\mu\nu}$.
- Leads to the same procedure used by Schenke et al.

Initial Hydro Fields

- Rotation and odd flow terms readily translate into hydrodynamics fields.



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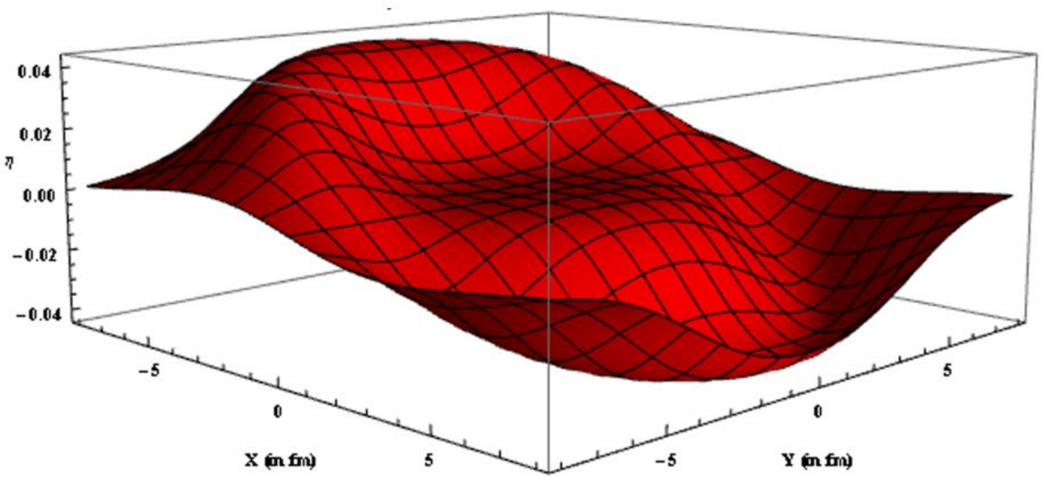
Initial Hydro Fields: Nodal Plane

- Non-trivial (3+1)-D global structure emerges
- Example: nodal plane of longitudinal flow, i.e. $v_z = 0$ in x-y-η space
- Here: initial nodal plane at the start of hydro

SURFACE FOR Vz = 0 (in η units)

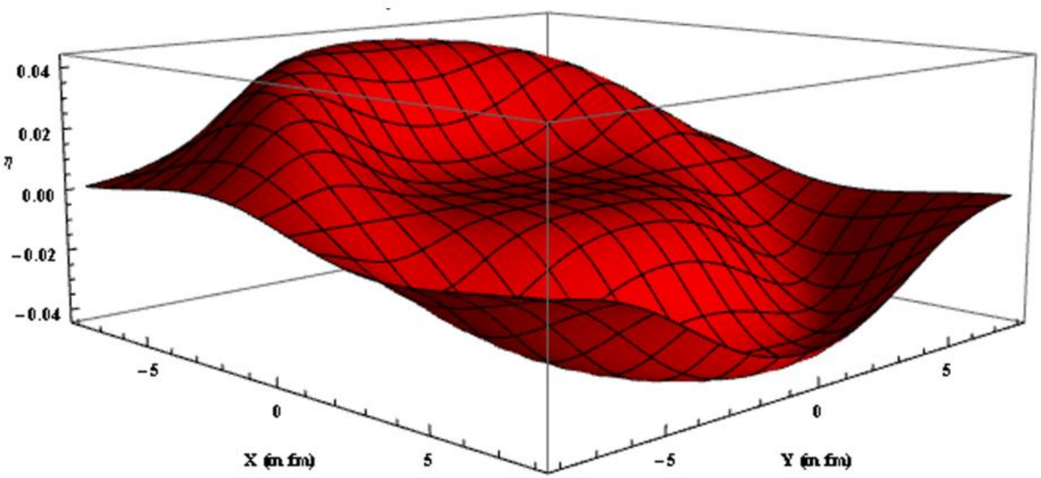


G. Chen and R. J. Fries, Phys. Lett. B 723, 417 (2013)

Evolution of Hydro Fields: Nodal Plane

- Viscous hydro evolution: reversal of distortion of the nodal plane → backlash → relaxation to "naïve" boost invariant configuration

SURFACE FOR Vz = 0 (in η units)



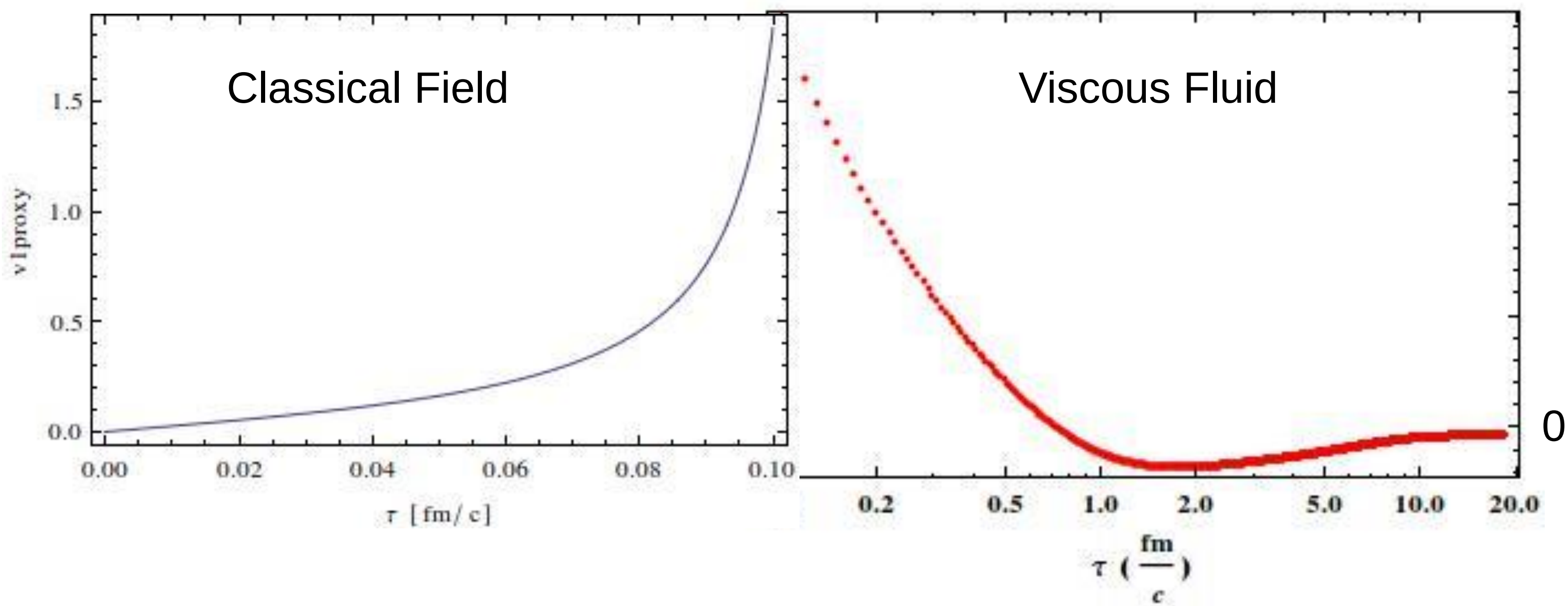
G. Chen and R. J. Fries, Phys. Lett. B 723, 417 (2013)

- Further evolution with viscouse hydro: relaxation to "naïve" boost invariance.

Evolution of Hydro Fields: v_1

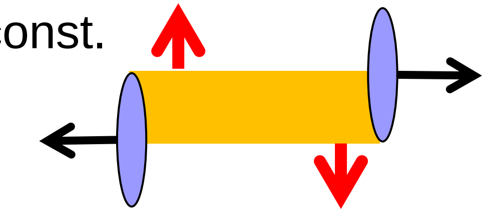
- Measure the "directed flow" T^{01}/T^{00} away from mid-rapidity
- Driven by classical field, then reduction and "backlash" and relaxation to small values.

$$\frac{\int T^{0x}\, d^2 r}{\int T^{00}\, d^2 r} \; @ \; \eta = -1$$

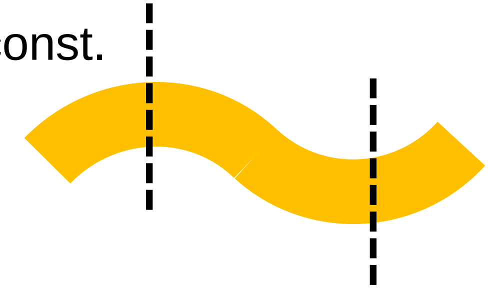


Interpretation

- Boost-invariance wins, fluid dynamics relaxes to the naive boost-invariant solution for $\tau \to \infty$.



- Angular momentum is not conserved globally, need to trace $M^{\alpha\beta\gamma}$ to get the full picture.

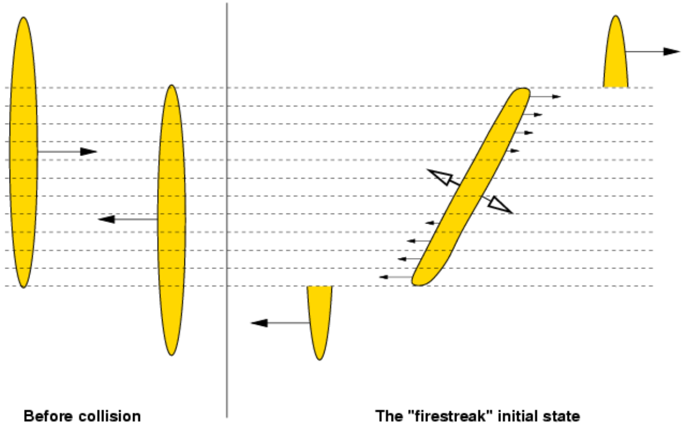


- Obviously we miss the effects of finite beam rapidity. End caps of the fireball are free.
- For example: fire streak model and applications

  [Gosset, Kapusta, Westfall (1978)]

  [Liang, Wang (2005)]

  [L. Csernai et al. PRC 84 (2011)] …



Beyond Boost Invariance

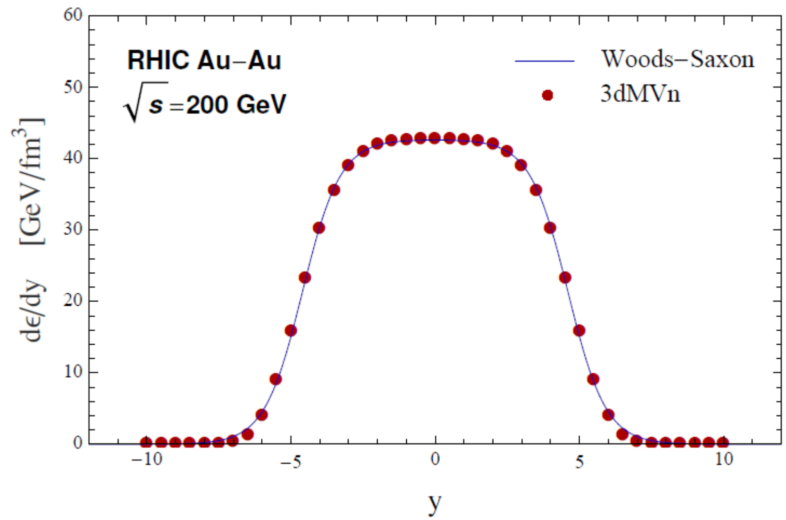
- Real nuclei are slightly off the light cone.
- Classical gluon distributions calculated by Lam and Mahlon.

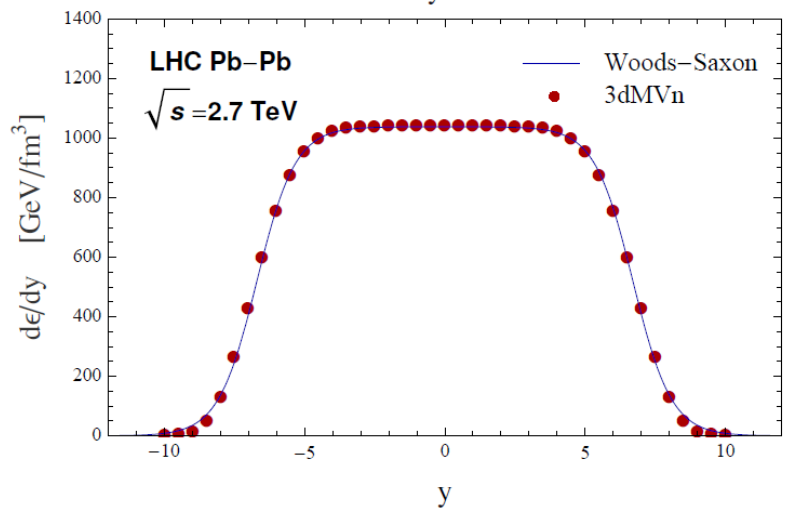
  [C.S. Lam, G. Mahlon, PRD 62 (2000)]

- Using two approximations valid for $R_A/\gamma \ll 1/Q_s$ we estimated the rapidity dependence of the initial energy density ε_0.

  [S. Ozonder, RJF, Phys. Rev. C 89, 034902 (2014)]

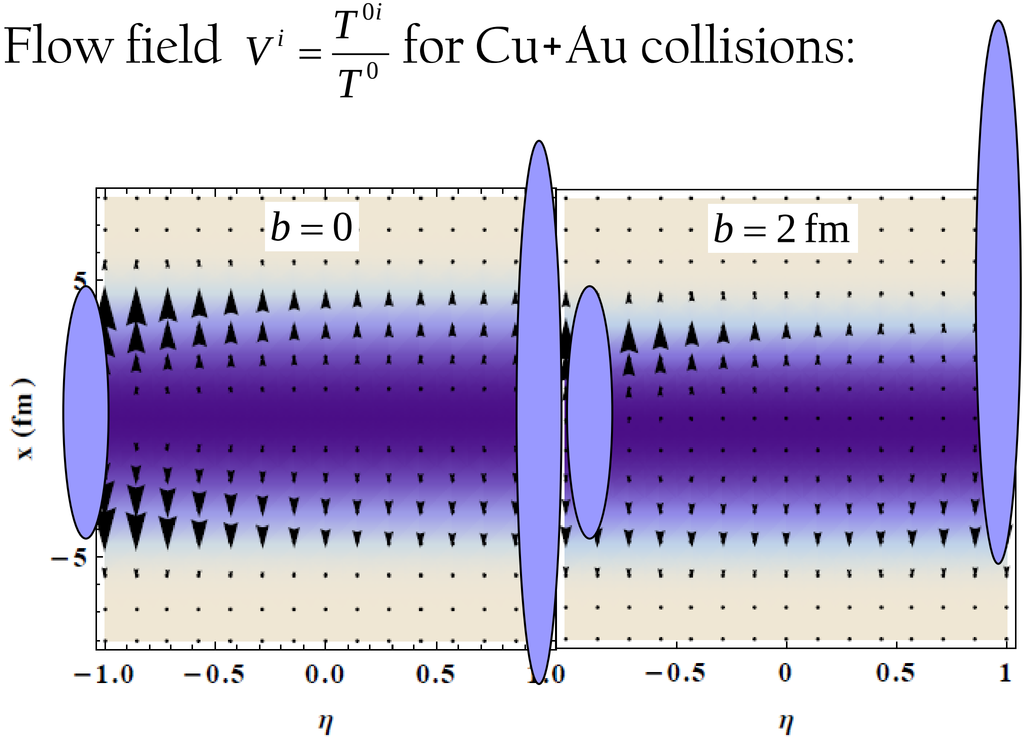
- Parameterizations for RHIC and LHc energies available in the paper.
- Combine with previous result and run hydro: work in progress.

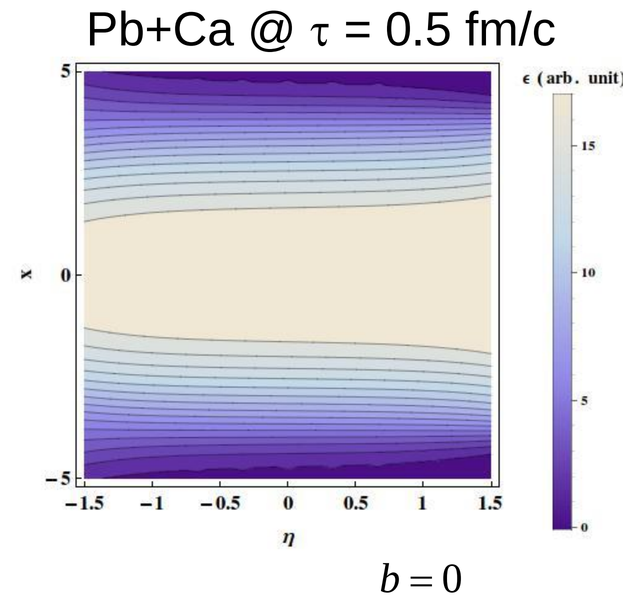




Phenomenology: A ≠ B

- Odd flow needs asymmetry between sources. Here: asymmetric nuclei.
- Flow field $v^i = \frac{T^{0i}}{T^0}$ for Cu+Au collisions:

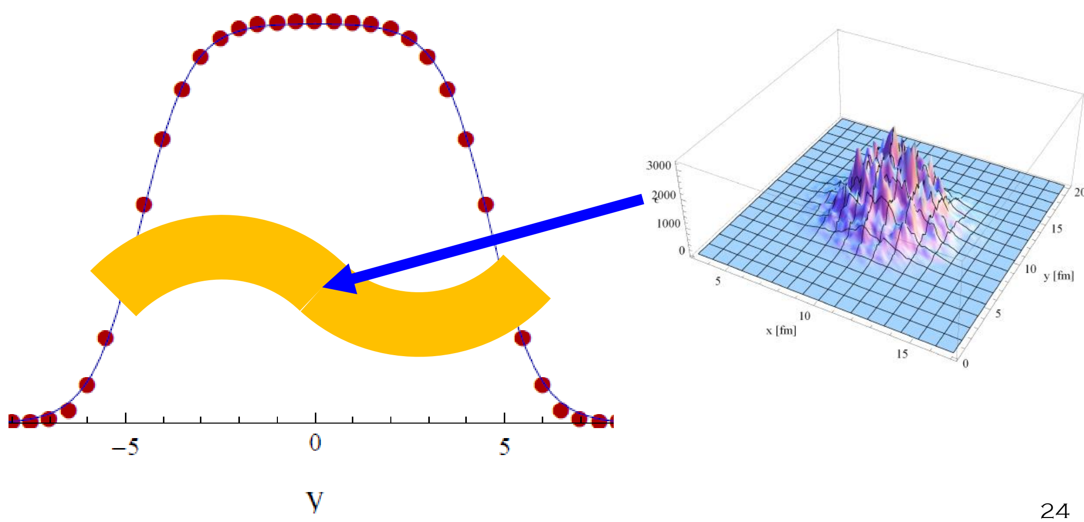




- Odd flow increases expansion in the wake of the larger nucleus, suppresses expansion on the other side.
- $b \neq 0$ & A ≠B: non-trivial flow pattern ⇒ characteristic signatures from classical fields?

Summary

- Early energy momentum tensor from CGC shows interesting and unique (?) features: angular momentum, directed flow, A+B asymmetries, etc.
- Features of gluon energy flow are naturally translated into their counterparts in hydrodynamic fields in a simple matching procedure.
- Fluid dynamics relaxes fireball to the naïve boost-invariant structure.
- Outlook:
  - Trace angular momentum.
  - Use realistic rapidity cutoff (shutoff supply of energy and angular momentum).
  - Use sampling of color charge distributions to incorporate transverse fluctuations.



Backup

MV Model: Classical YM Dynamics

- Two nuclei: intersecting currents J_1, J_2 (given by ρ_1, ρ_2), calculate gluon field $A^\mu(\rho_1, \rho_2)$ from YM.
- Equations of motion

$$\frac{1}{\tau^3}\partial_\tau \tau^3 \partial_\tau A - \left[D^i, \left[D^i, A\right]\right] = 0$$

$$\frac{1}{\tau}\left[D^i, \partial_\tau A_\perp^i\right] - ig\tau\left[A, \partial_\tau A\right] = 0$$

$$\frac{1}{\tau}\partial_\tau \tau \partial_\tau A_\perp^i - ig\tau^2\left[A, \left[D^i, A\right]\right] - \left[D^j, F^{ji}\right] = 0$$

$$A^\pm = \pm x^\pm A(\tau, x_\perp)$$

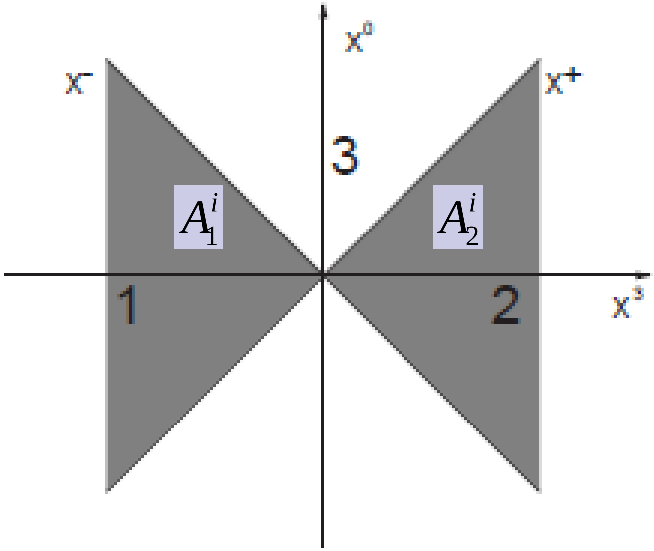
$$A^i = A_\perp^i(\tau, x_\perp)$$

- Boundary conditions

$$A_\perp^i(\tau = 0, x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$

$$A(\tau = 0, x_\perp) = -\frac{ig}{2}\left[A_1^i(x_\perp), A_2^i(x_\perp)\right]$$

[A. Kovner, L. McLerran, H. Weigert]



Analytic Solution: Small Time Expansion

- Here: analytic solution using small-time expansion for gauge field

$$A(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(x_\perp)$$

$$A_\perp^i(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{\perp(n)}^i(x_\perp)$$

- Recursive solution for gluon field:

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D_{(k)}^i, \left[D_{(l)}^i, A_{(m)}\right]\right]$$

$$A_{\perp(n)}^i = \frac{1}{n^2}\left(\sum_{k+l=n-2} \left[D_{(k)}^j, F_{(l)}^{ji}\right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D_{(l)}^i, A_{(m)}\right]\right]\right)$$

  - 0th order = boundary conditions

$$A_{\perp(0)}^i(x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A_1^i(x_\perp), A_2^i(x_\perp)\right]$$

[RJF, J. Kapusta, Y. Li, 2006]
[Fujii, Fukushima, Hidaka, 2009]

Energy Momentum Tensor

- Corrections to energy density and pressure due to flow at second order in time

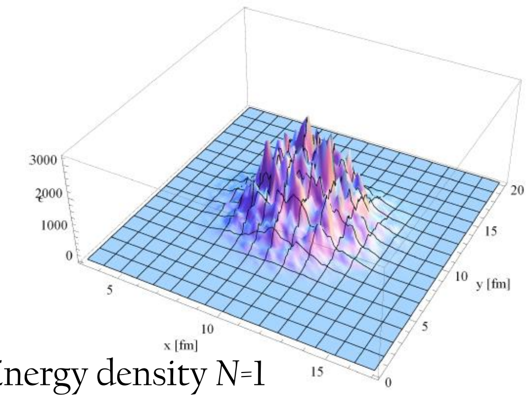
- Example: energy density

Depletion/increase of energy density due to transverse flow

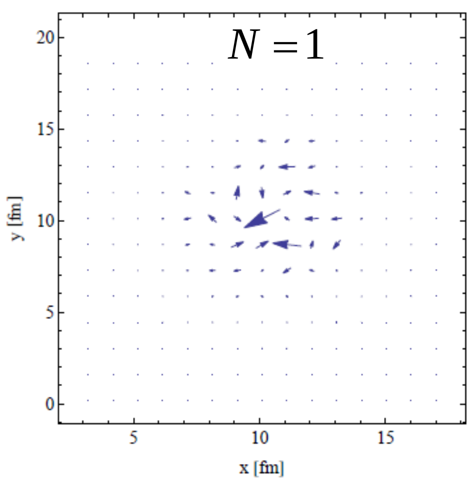
$$T^{00} = \varepsilon_0 - \frac{\tau^2}{8}\left[2\nabla^i \alpha^i + \sinh 2\eta\, \nabla^i \beta^i + (2 - \cosh 2\eta)\delta\right] + O(\tau^4)$$

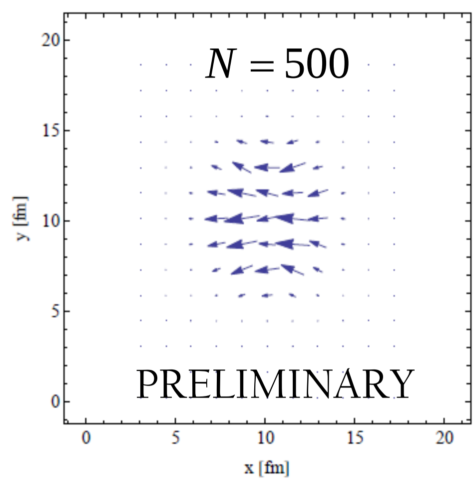
Check: Event-By-Event Picture

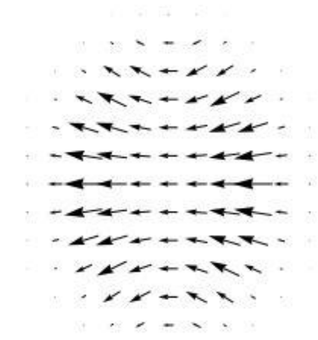
- Example: numerical sampling of charges for "odd" vector β^i in Au+Au (b=4 fm).



Energy density N=1





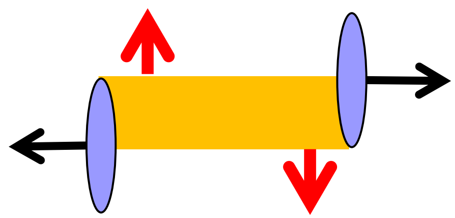


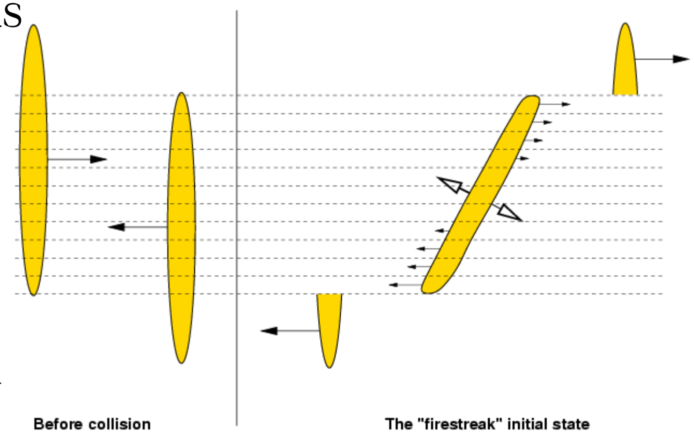
Analytic expectation value

- Averaging over events: recover analytic result.

Phenomenology: $b \neq 0$

- Angular momentum is natural: some old models have it, most modern hydro calculations don't.
  - Some exceptions
    [L. Csernai et al. PRC 84 (2011)] …
  - Do we determine flow incorrectly when we miss the rotation?





[Gosset, Kapusta, Westfall (1978)]

[Liang, Wang (2005)]

- Directed flow v_1:
  - Compatible with hydro with suitable initial conditions.

MV only, integrated over transverse plane, no hydro