UNIVERSITÀ DEGLI STUDI DI CATANIA
INFN-LNS

Initial state fluctuations and anisotropic flows within an event by event transport approach

S. Plumari, G.L. Guardo, A. Puglisi,
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Outline

- **From kinetic transport theory to ideal hydro:**
  - **extraction of viscous corrections to f(x,p) and $v_n(p_T)$**
- **Transport approach at fixed η/s:**

  fix locally η/s ↔ σ(θ), M, T -> Chapman-Enkog approach.
- **Initial state fluctuations:**
  - **η/s and generation of v_n(pT): from RHIC to LHC**
  - **Correlations between ε_n and v_n**
- **Conclusions**

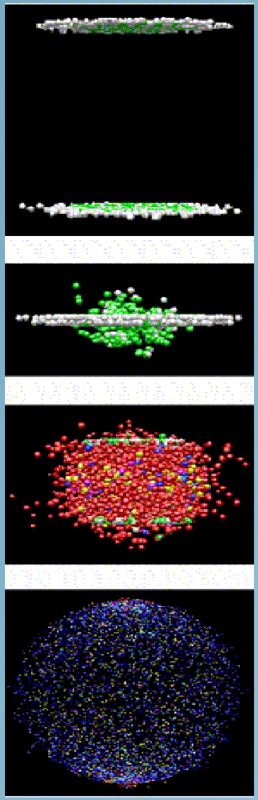
Motivation for a kinetic approach:

$$\{p^{\mu}\partial_{\mu}+[p_{\nu}F^{\mu\nu}+M\,\partial^{\mu}M]\partial_{\mu}^{p}\}f(x,p)=C_{22}+C_{23}+\ldots$$

Free streaming

Field Interaction → ε≠3P

Collisions → η≠0



- Starting from 1-body distribution function and not from $T^{\mu\nu}$:

  - possible to include f(x,p) out of equilibrium.

    M. Ruggieri et.al, PLB 727 (2013) 177

  - extract information about the viscous correction δf to f(x,p)

- It is not a gradient expansion in η/s.
- Valid at intermediate p_T out of equilibrium.
- Valid at high η/s (cross over region): + self consistent kinetic freeze-out

Parton Cascade model

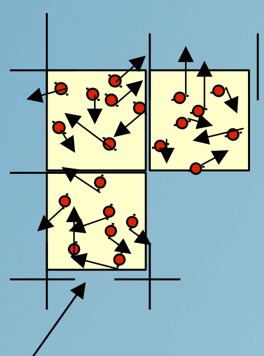
$$p^{\mu}\partial_{\mu} f(X,p) = C = C_{22} + C_{23} + \ldots$$

Collisions → $\varepsilon - 3p = 0,\ \eta \neq 0$

$$C_{22} = \frac{1}{2E_1}\int \frac{d^3 p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3 p'_1}{(2\pi)^3 2E'_1}\frac{d^3 p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 \left|M_{1'2'\to 12}\right|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

For the numerical implementation of the collision integral we use the stochastic algorithm. (Z. Xu and C. Greiner, PRC 71 064901 (2005))

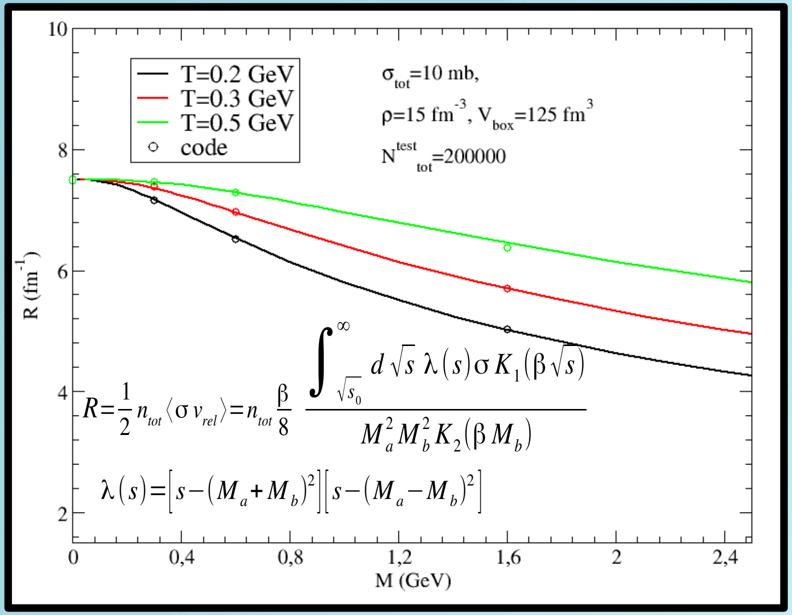
$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\,\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$



$\Delta^3 x$

$\Delta t \to 0$

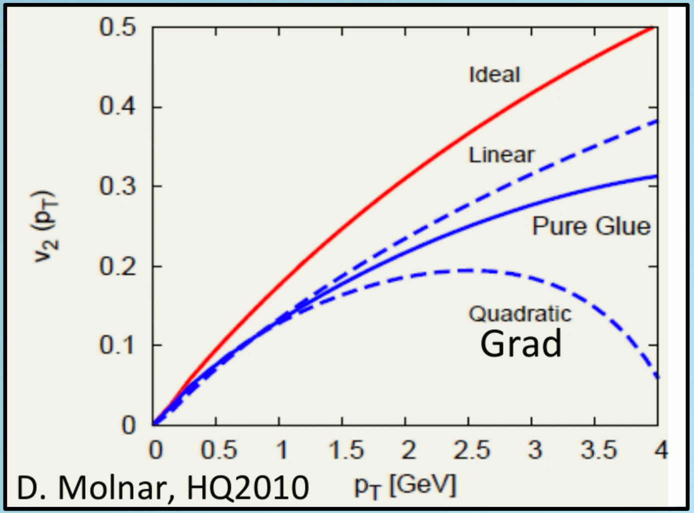
$\Delta^3 x \to 0$ ⇒ **right solution**



From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

$$f(x,p) = f^{(0)}(x,p) + \delta f(x,p)$$

$$T^{\mu\nu} = T^{(0)\mu\nu} + \delta T^{\mu\nu} \leftarrow f^{(0)} + \delta f$$



A common choice for δf – the Grad ansatz

$$\delta f \propto \Gamma_s f^{(0)} p^\alpha p^\beta \langle \nabla_\alpha u_\beta \rangle \propto p_T^2$$

BUT it doesn't care about the microscopic dynamics

In general in the limit σ→∞, f(σ) can be expanded in power of 1/σ.

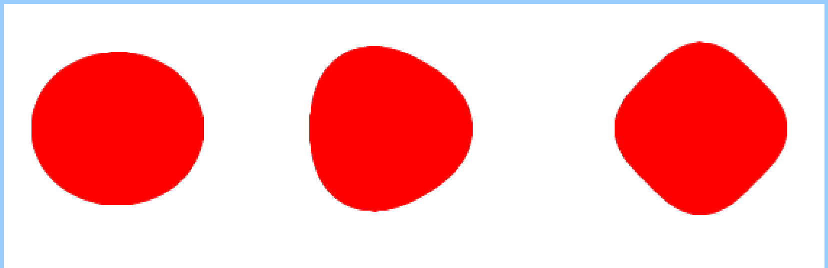
$$f(\sigma) \underset{\sigma\to\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right) \Longrightarrow v_n(p_T) \underset{\sigma\to\infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma}\delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

PURPOSE: evaluate the ideal hydrodynamics limit $f^{(0)}$, $v_n^{(0)}$ and the viscous corrections δf and δv_n solving the Relativistic Boltzmann eq for large values of the cross section σ

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

Coodinate space (x,y)

- We start with an initial azimuthally symmetric profile (optical Glauber model).
- Then we deform the initial distribution to generate a new one with 2π/n symmetry.



We create only one ε_n

Momentum space

- Thermal distribution:

$$dN/d^3p \propto \exp(-p/T)$$

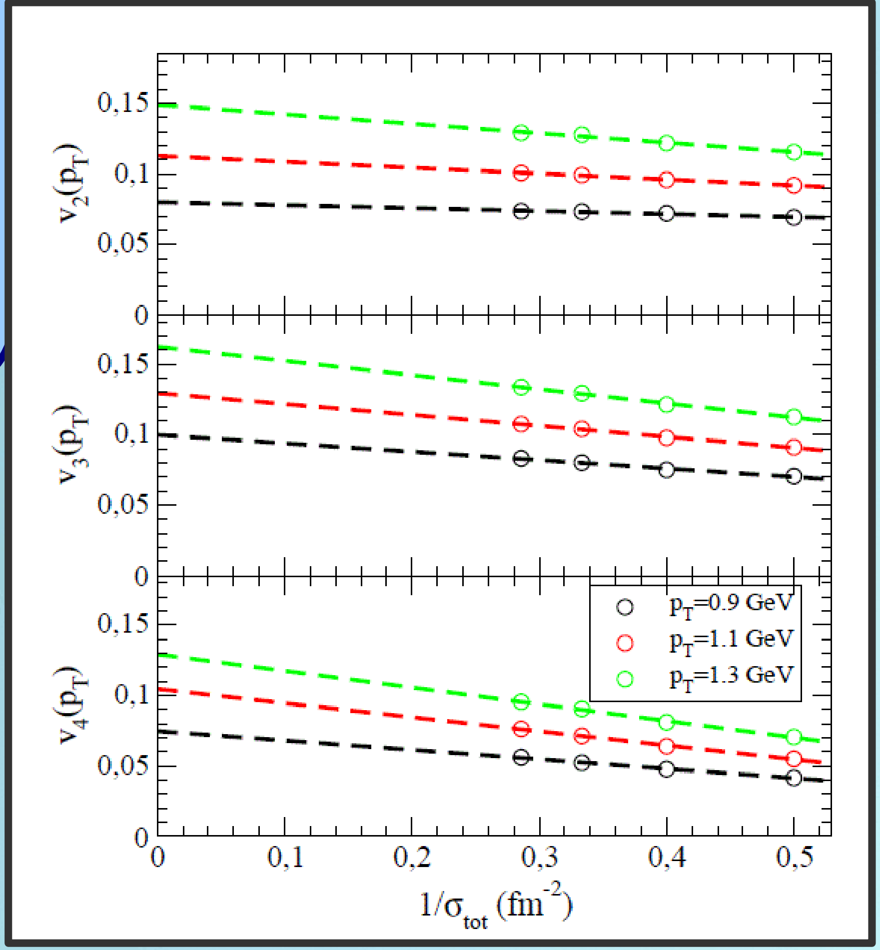
- Constant distribution:

$$dN/d^3p \propto \theta(p_0 - p)$$

We assume initially the same local $T^{\mu\nu}(x)$

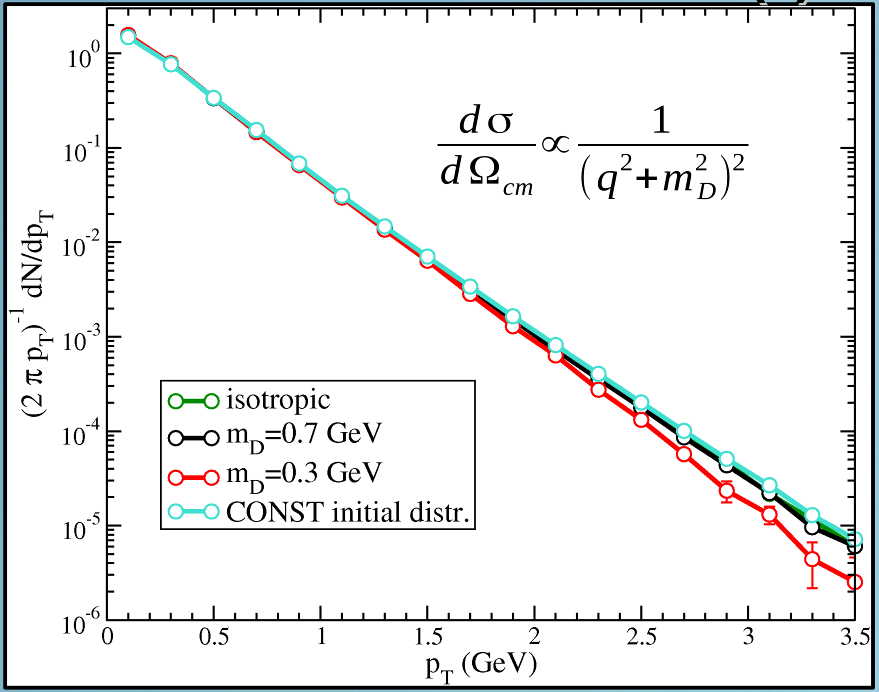
$$f(\sigma) \underset{\sigma\to\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right)$$

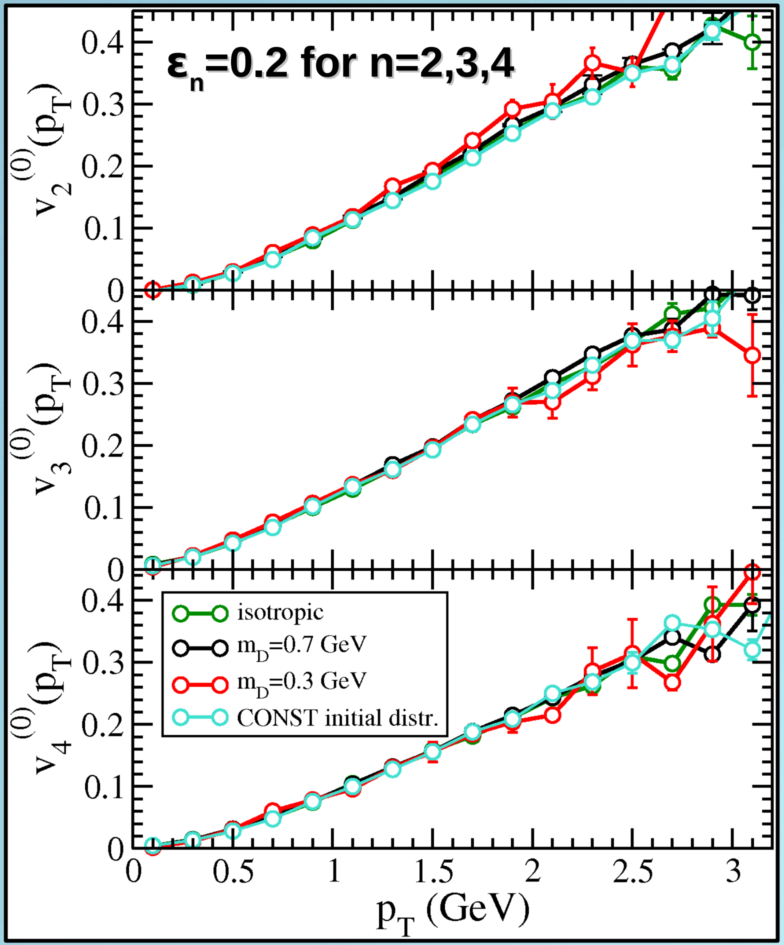
$$v_n(p_T) \underset{\sigma\to\infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma}\delta v_n + O\left(\frac{1}{\sigma^2}\right)$$



From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

For the same initial local $T^{\mu\nu}(x)$:

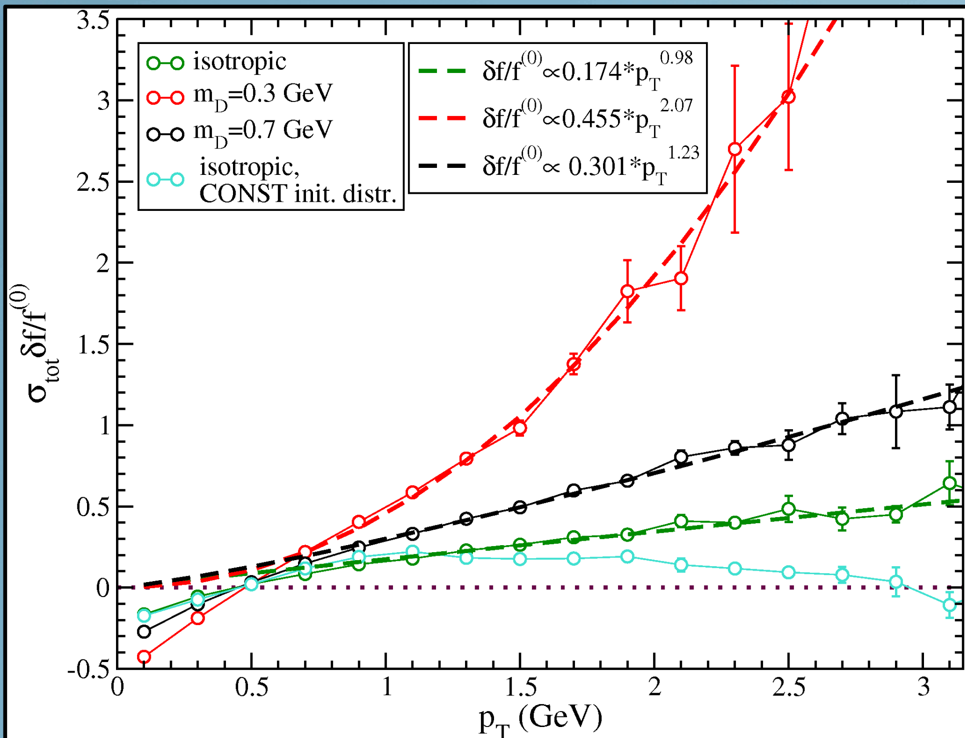


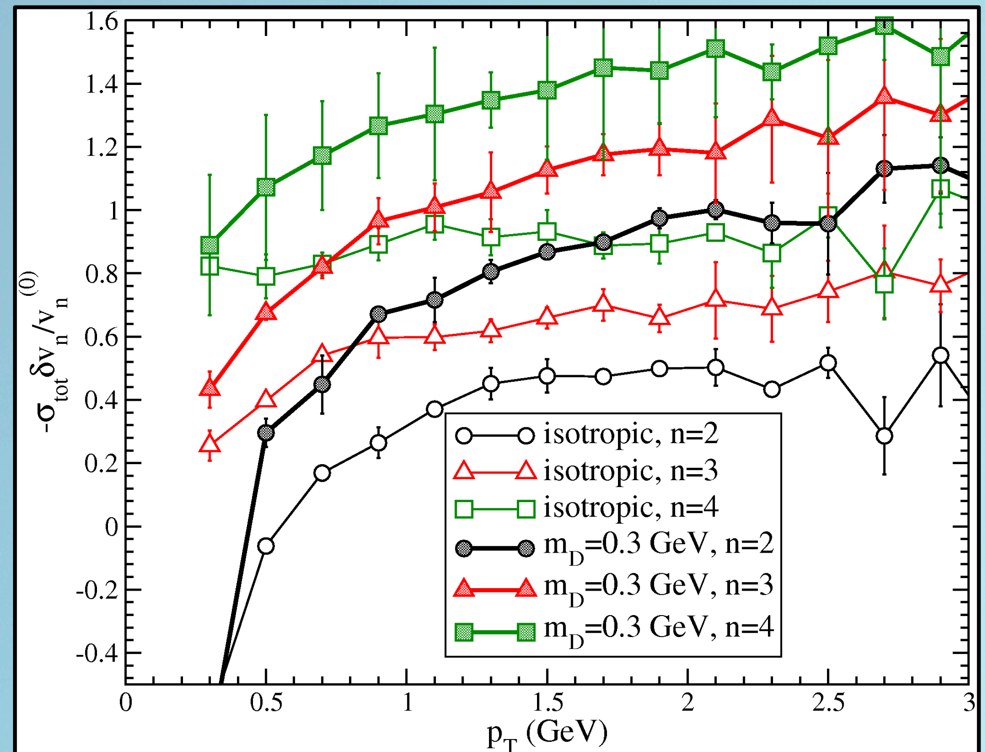


For $\sigma \to \infty$ we find the ideal Hydro limit:

- $f^{(0)}$ doesn't depends on microscopical details (i.e. mD).
- Universal behavior of $v_n^{(0)}(p_T)$
- $v_n^{(0)}(p_T)/\varepsilon_n$ is approximatively the same for all n and p_T.

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)





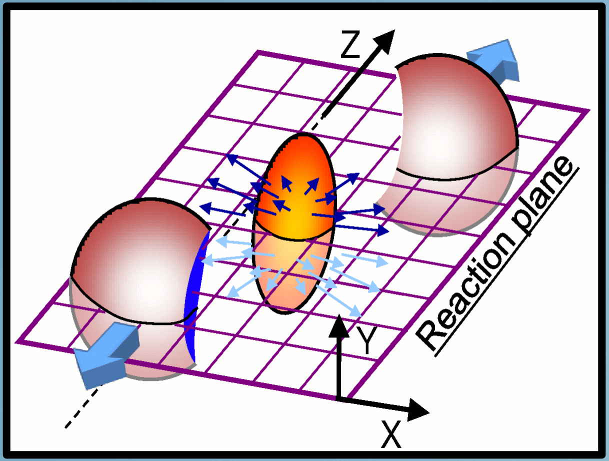
In δf and δv_n it is encoded the information about the microscopical details

- **$\delta f(p_T)/f^{(0)} \propto p_T^{\alpha}$ with α = 1. - 2. and $\alpha(m_D)$.**

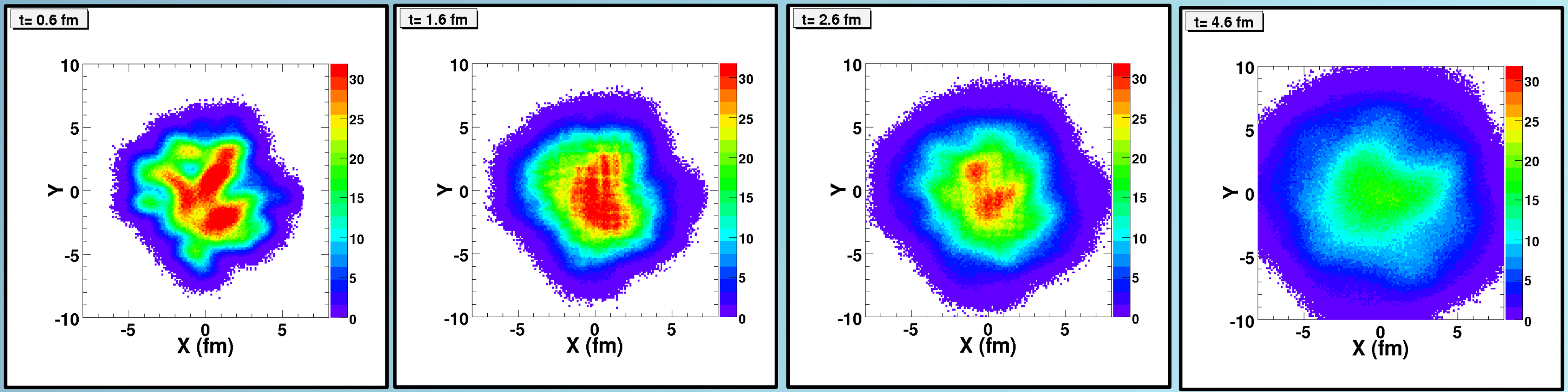
For isotropic σ similar to R.S. Bhalerao et al. PRC 89, 054903 (2014)

- **Larger is n larger is the viscous correction to $v_n(p_T)$**
- **Scaling: for p_T > 1.5 GeV → $-\delta v_n(p_T)/v_n^{(0)} \propto n$**

Applying kinetic theory to A+A Collisions....



- Impact of $\eta/s(T)$ on the build-up of $v_n(p_T)$ vs. beam energy.
- To include the Initial state fluctuations.



Simulating a constant η/s

For the general case of anisotropic cross section and massless particles:

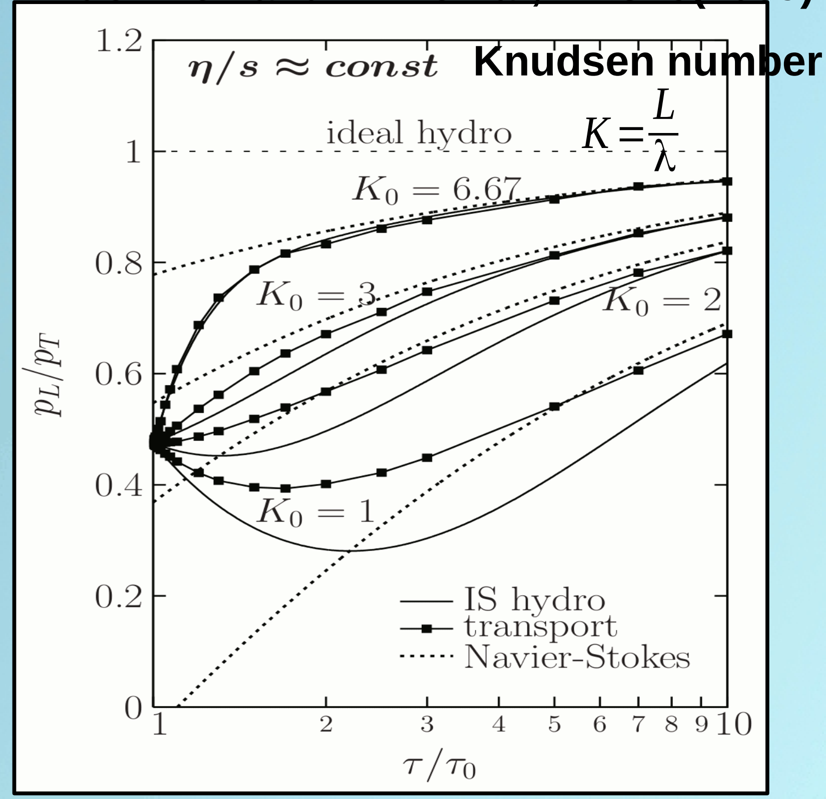
$$\eta(\vec{x},t)/s = \frac{1}{15}\langle p\rangle\,\tau_\eta \quad \Longrightarrow \quad \sigma_{tot}^{\eta/s} = \frac{1}{15}\frac{\langle p\rangle}{g(m_D/2T)\,n}\frac{1}{\eta/s}$$

σ is evaluated in such way to keep fixed the η/s during the dynamics according the Chapman-Enskog equation. (similar to D. Molnar, arXiv:0806.0026[nucl-th] but our approach is local.)

- **We know how to fix locally η/s(T)**
- **We have checked the Chapmann-Enskog (CE):**
  - CE good already at 1st order ≈ *5%*
  - Relaxation Time Approx. severely understimates η

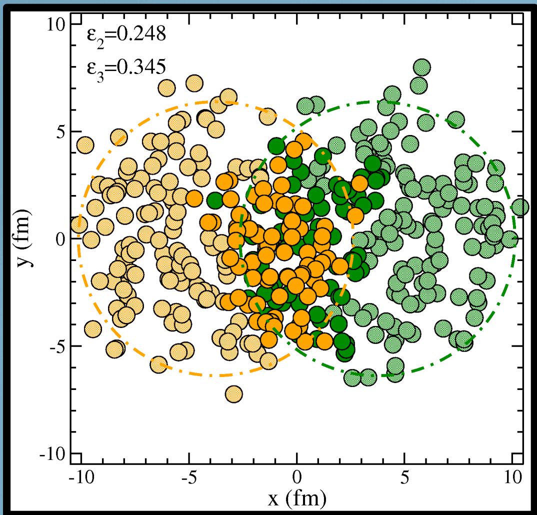
    S. Plumari et al.,PRC86 (2012) 054902.

P. Huovinen and D. Molnar, PRC79(2009)



In the limit of small η/s (<0.16) and for small pT equivalent viscous hydro

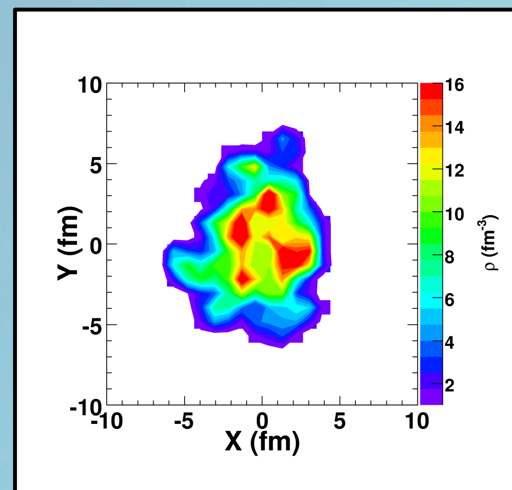
Initial State Fluctuations

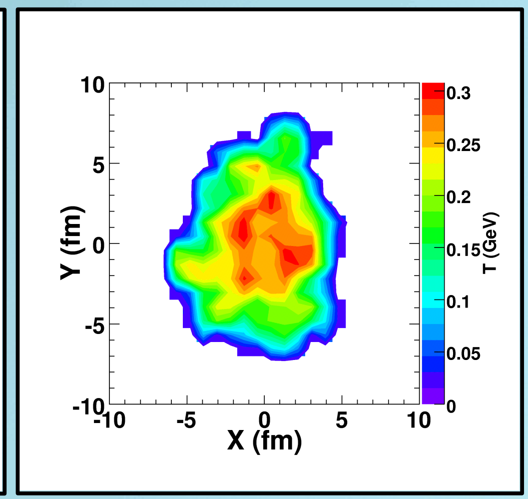


smooth distribution

Monte Carlo Glauber

$$\rho_\perp(x,y) \propto \sum_{i=1}^{N_{part}} \exp\{-[(x-x_i)^2+(y-y_i)^2]/(2\sigma^2)\}$$

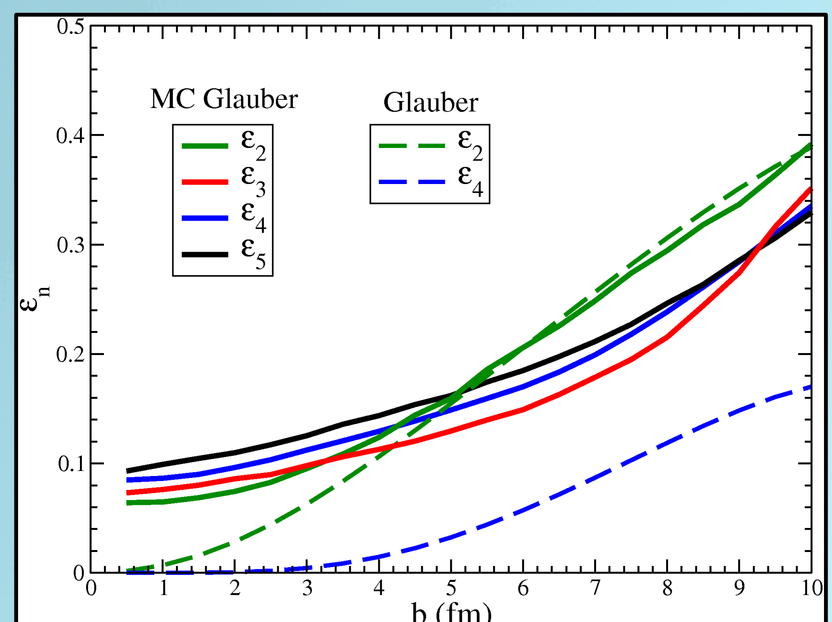




Characterization of the initial profile in terms of Fourier coefficients

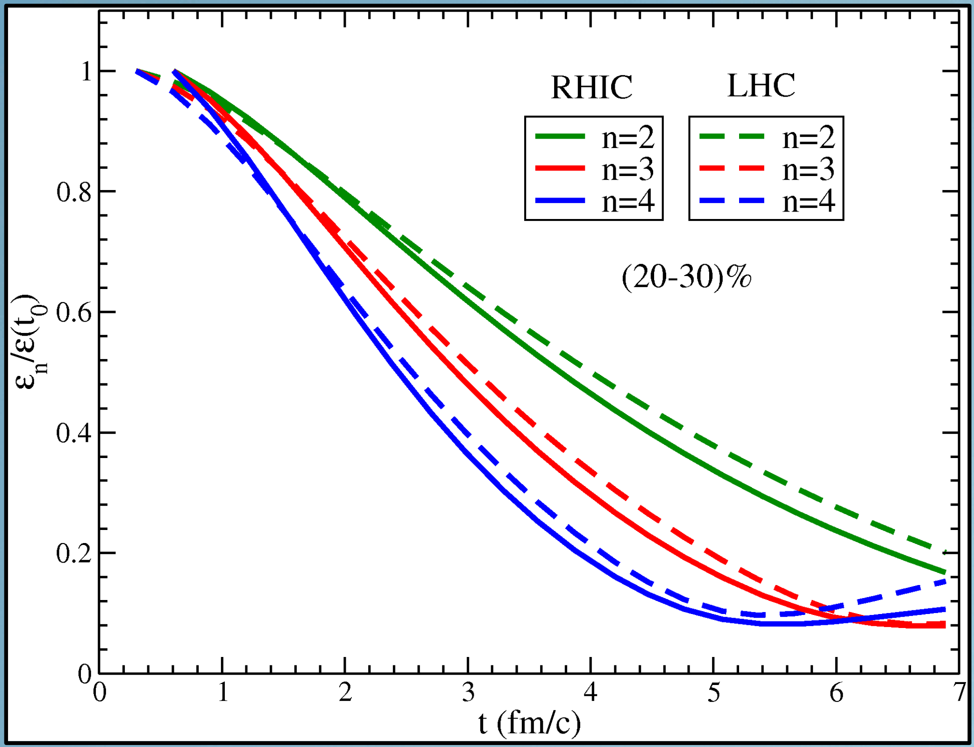
$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)]\rangle}{\langle r_\perp^n \rangle} \qquad \Phi_n = \frac{1}{n}\arctan\frac{\langle r_\perp^n \sin(n\varphi)\rangle}{\langle r_\perp^n \cos(n\varphi)\rangle}$$

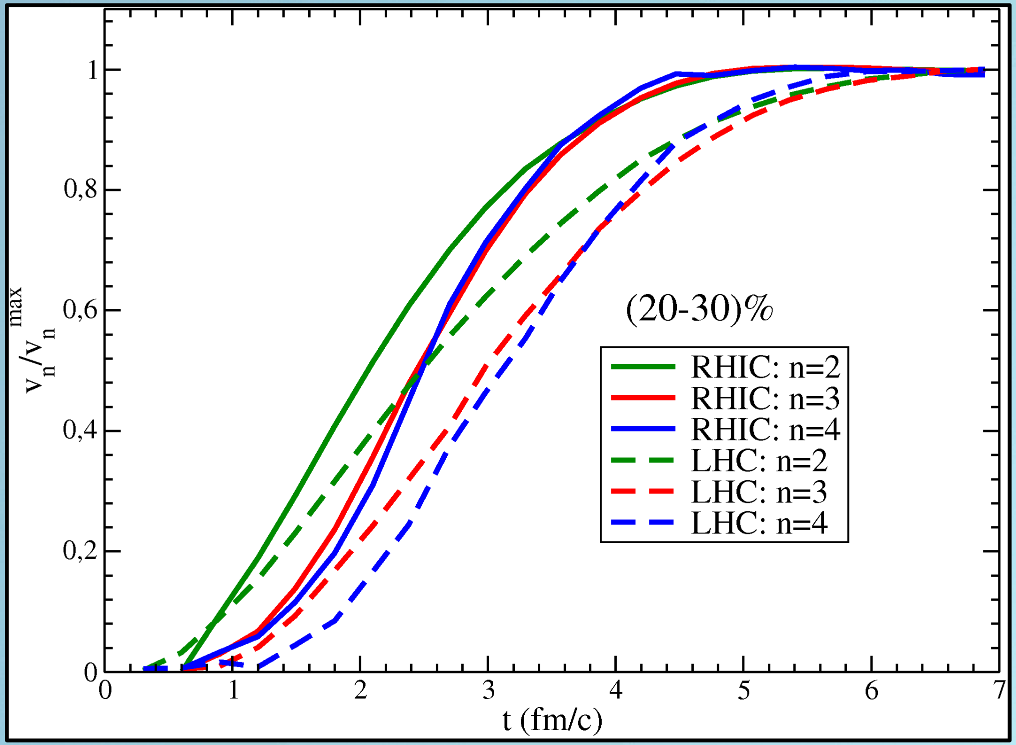
$$r_\perp = \sqrt{x^2+y^2}, \qquad \varphi = \arctan(y/x)$$



G-Y. Qin, H. Petersen, S.A. Bass and B. Muller, PRC82,064903 (2010).
H.Holopainen, H. Niemi and K.J. Eskola, PRC83, 034901 (2011).

Initial State Fluctuations: time evolution of $<v_n>$ and ε_n

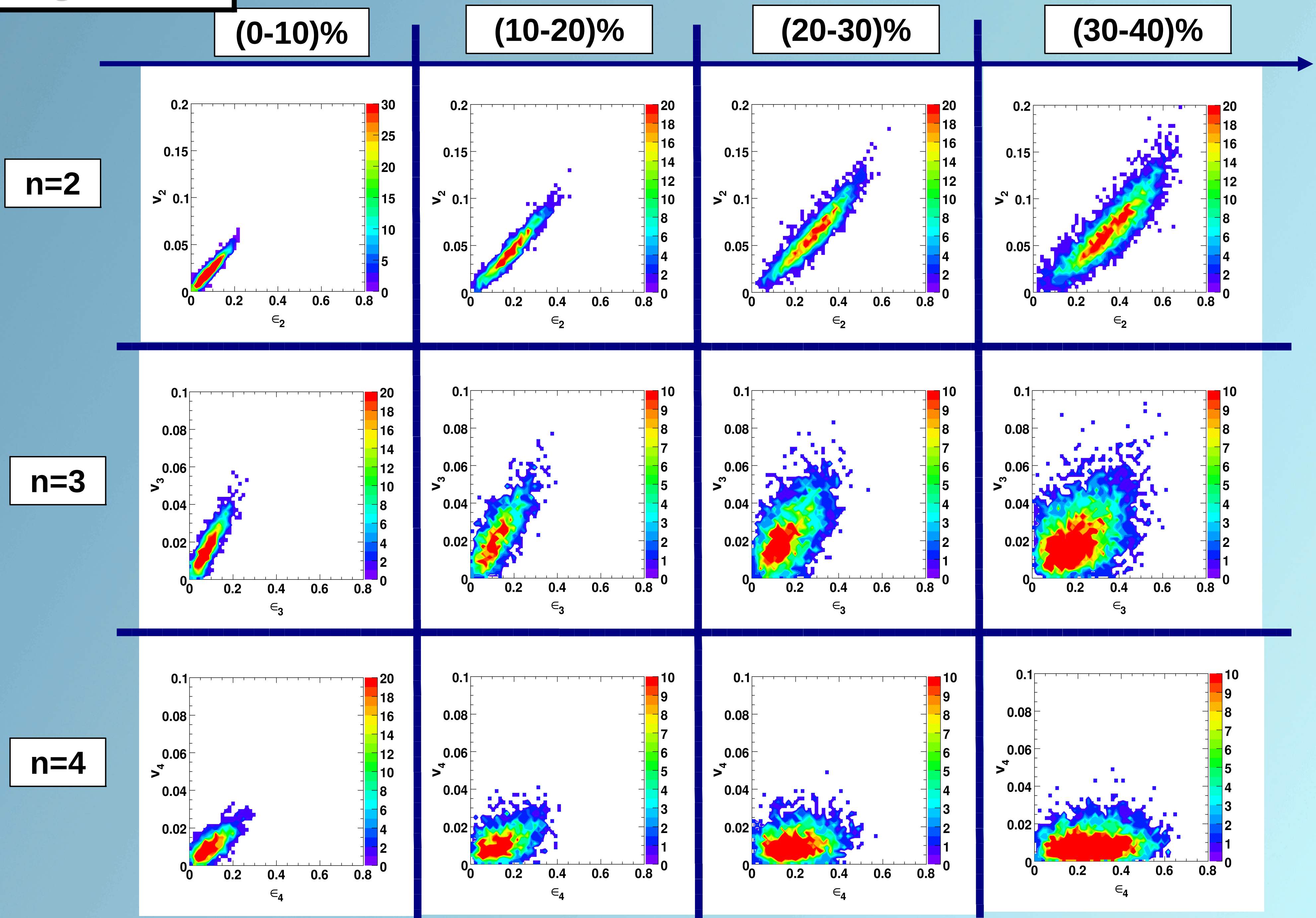




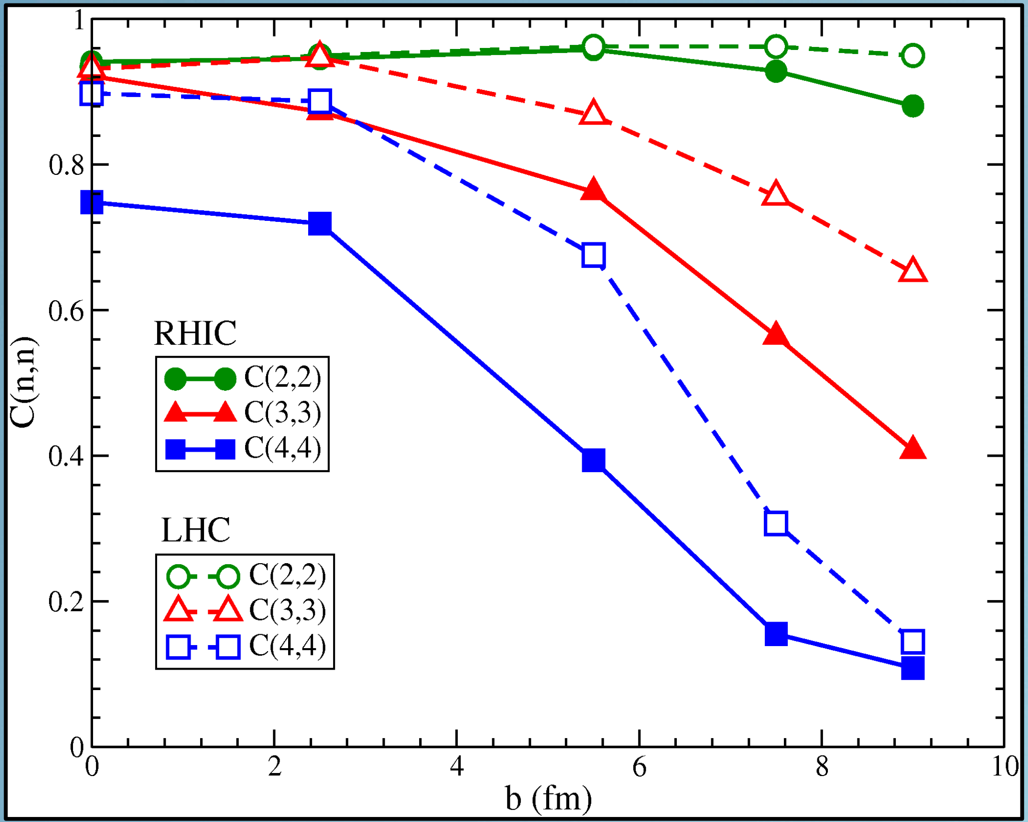
- **The time evolution for ε_n is faster for large n. At very early times $\varepsilon_n(t)=\varepsilon_n(t_0)-\alpha_n t^{n-2}$.**
- **$<v_n>$ shows an opposite behaviour: $<v_n>$ develops later for large n. At very early times $<v_n>\propto t^{n+1}$.**
- **Different v_n can probes different value of $\eta/s(T)$ during the expansion of the fireball.**

Initial State Fluctuations: v_n vs ε_n

RHIC:
Au+Au @ 200 GeV



Initial State Fluctuations: v_n vs ϵ_n



$$C(n,m)=\left\langle \frac{(v_n-\langle v_n\rangle)(\epsilon_m-\langle \epsilon_m\rangle)}{\sigma_{v_n}\sigma_{\epsilon_m}} \right\rangle$$

B.H. Alver, C. Gombeaud, M. Luzum and J.-Y. Ollitrault, Phys.Rev. C82 (2010) 034913.

H. Petersen, G.-Y. Qin, S.A. Bass and B. Muller, Phys.Rev. C82 (2010) 041901.

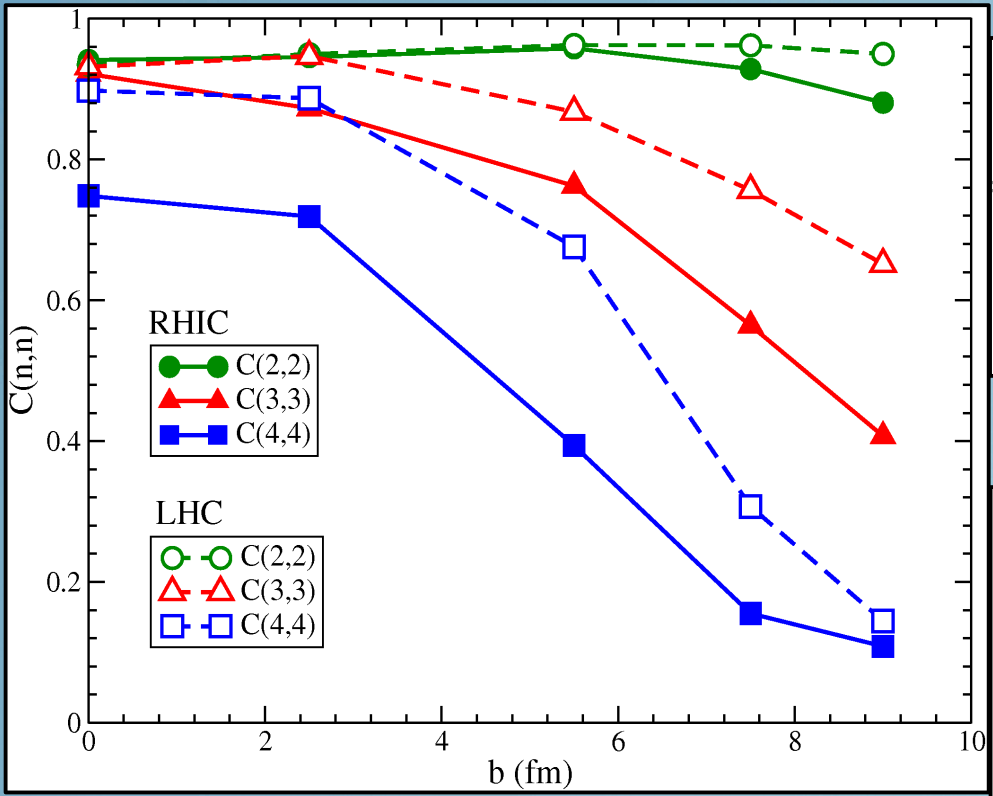
Z. Qiu and U. W. Heinz, Phys.Rev. C84 (2011) 024911.

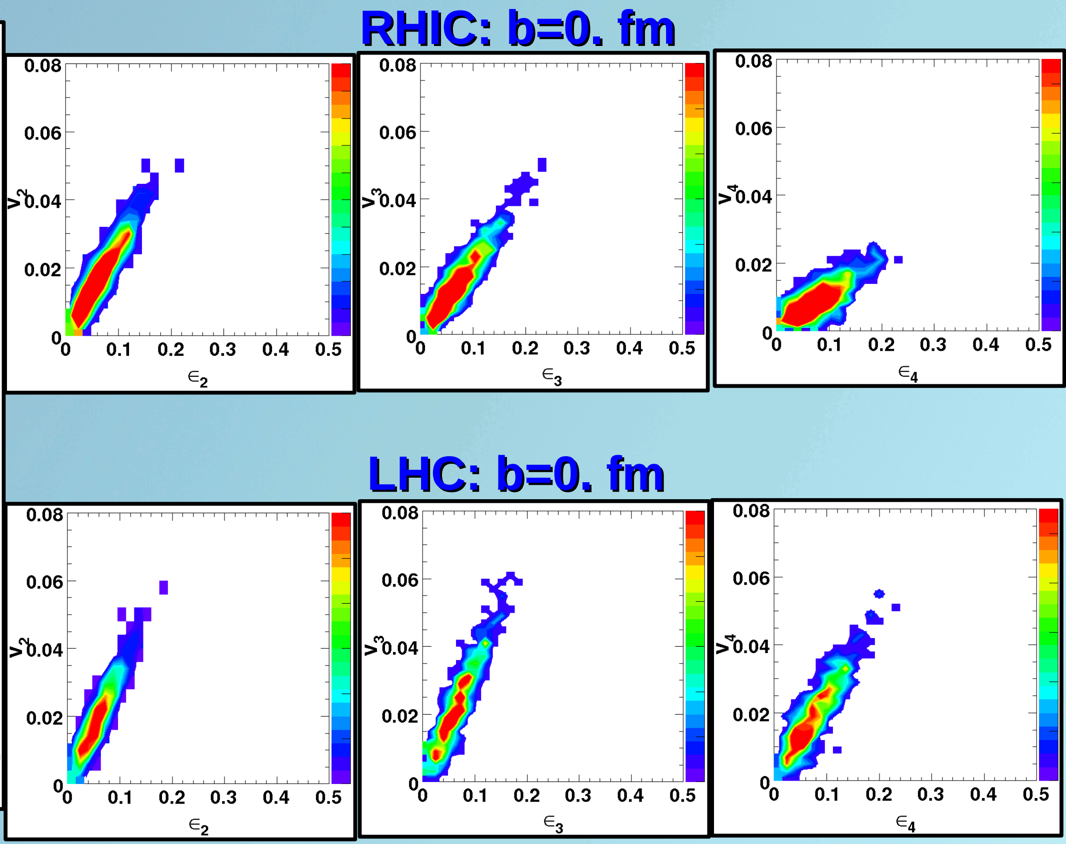
H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen, Phys.Rev. C87 (2013) 5, 054901.

- **At LHC v_n are more correlated correlated to ϵ_n than at RHIC.**
- **v_2 and v_3 linearly correlated to the corresponding eccentricities ϵ_2 and ϵ_3 rispectively.**
- **C(4,4) < C(2,2) for all centralities. v_4 and ϵ_4 weak correlated similar to hydro calculations:**

  F.G. Gardim, F. Grassi, M. Luzum and J.Y. Ollitrault NPA904 (2013) 503.
  H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen PRC87(2013) 054901.
- **For central collisions v_n are strongly correlated to ϵ_n: $v_n \propto \epsilon_n$ for n=2,3,4.**

Initial State Fluctuations: v_n vs ε_n





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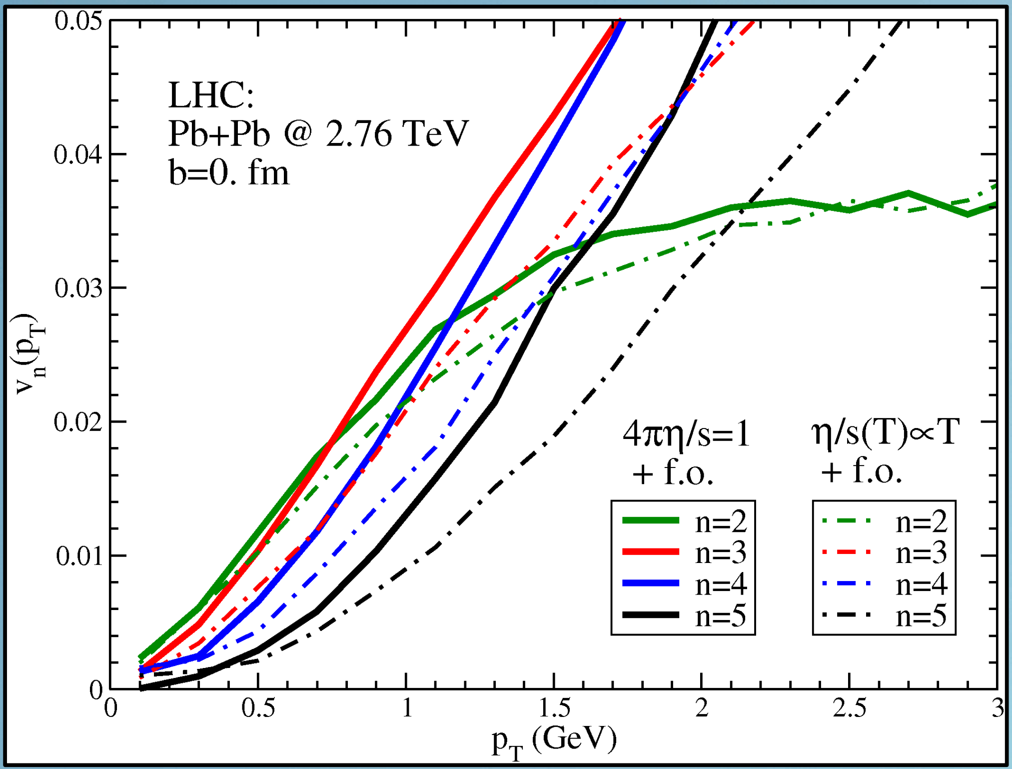
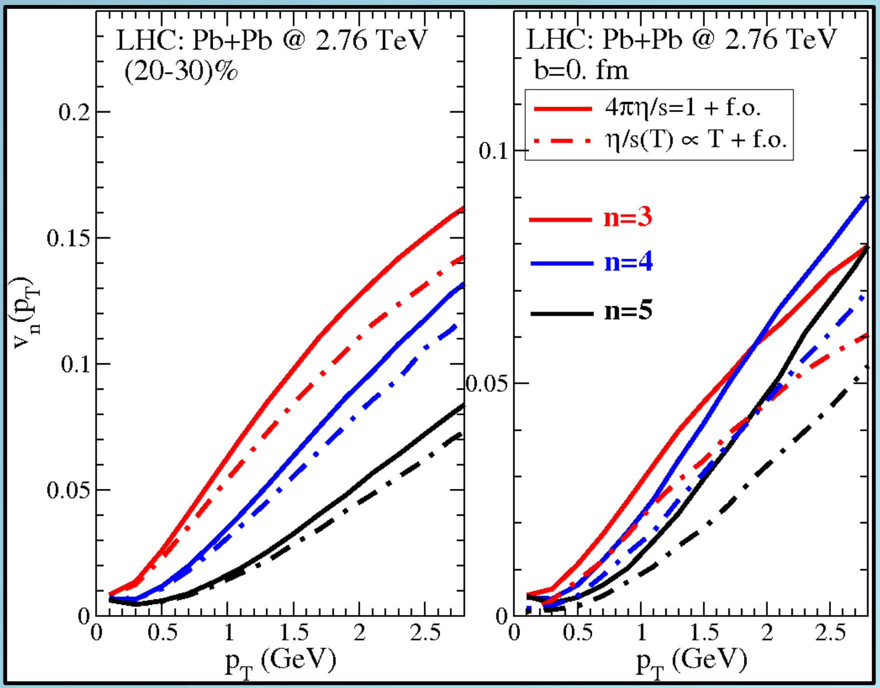
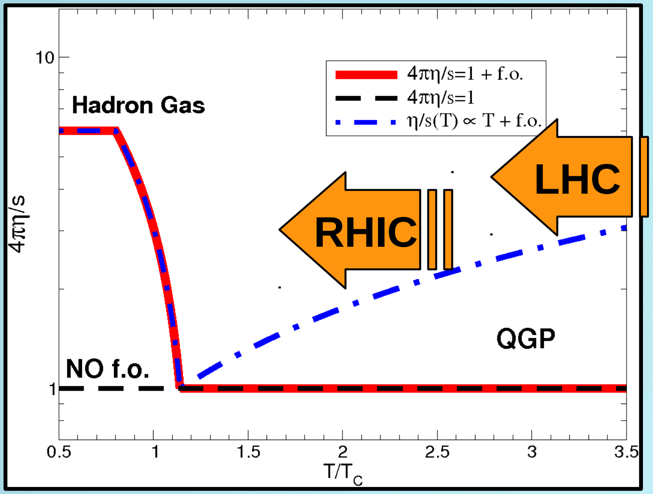
Initial State Fluctuations: $v_n(p_T)$ for central collisions

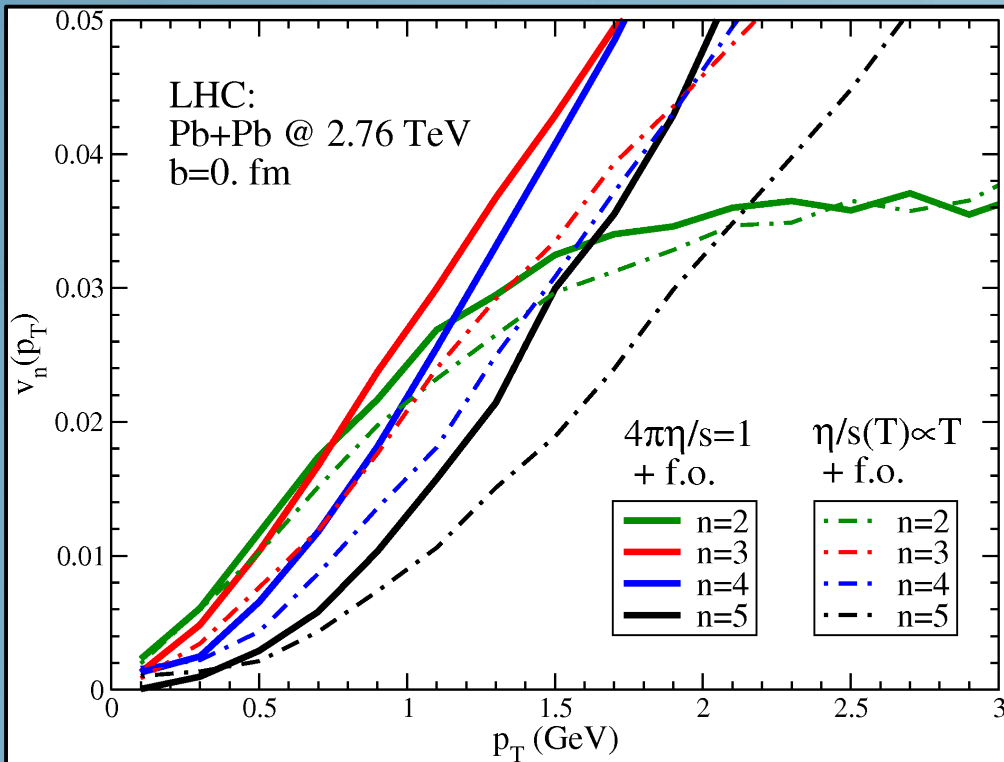


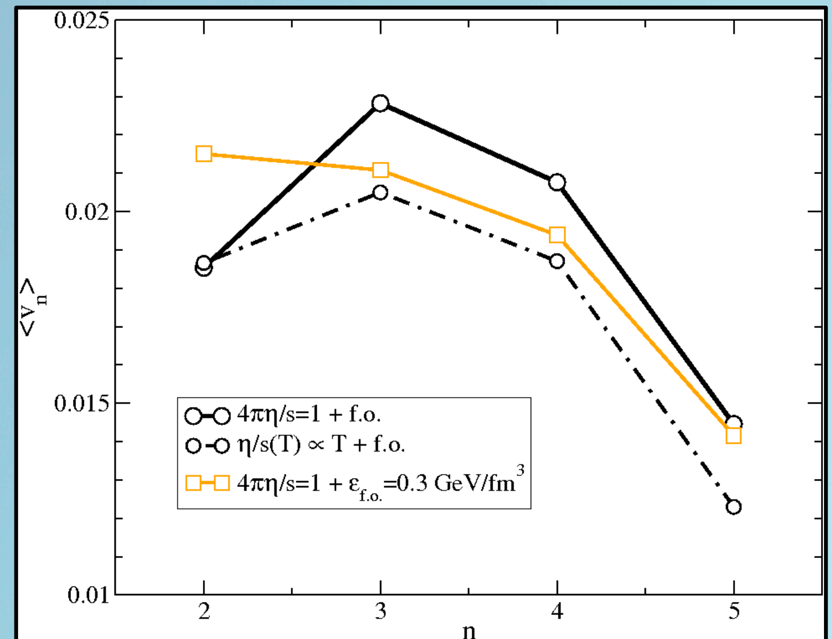


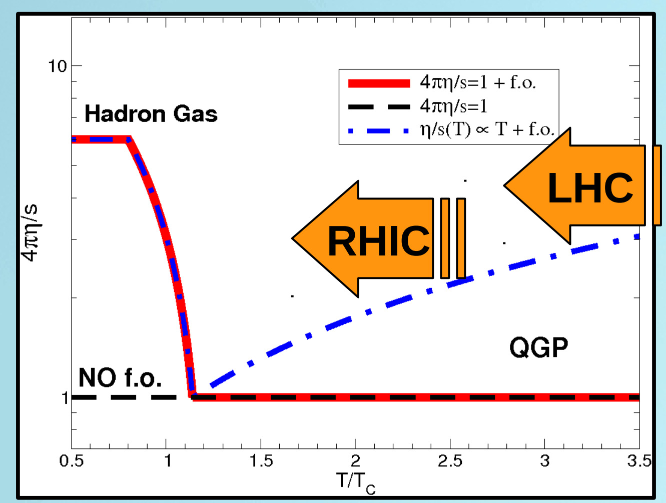


- At low p_T $v_n(p_T) \propto p_T^n$. v_2 for higher p_T saturates while v_n for n>3 increase linearly with p_T.
- For central collisions viscous effect are more relevant. For n>2 the $v_n(p_T)$ are more sensitivity to η/s in the QGP phase.

Initial State Fluctuations: $v_n(p_T)$ for central collisions







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Conclusions

From kinetic transport theory to ideal hydro:

- For $\sigma \to \infty$ we find the ideal hydro limit: $f^{(0)}$ and $v_n^{(0)}$ don't depend on microscopical details. $v_n^{(0)}/\varepsilon_n$ depends little on n.
- 1^{st} order viscous corrections δf and δv_n depend on microscopical details $\delta f/f^{(0)} \propto p_T^{\alpha}$ with α=1-2 and $\delta v_n/v_n^{(0)} \propto n$

Transport at fixed η/s:

- Enhancement of η/s(T) in the cross-over region affect differently the expanding QGP from RHIC to LHC. LHC nearly all the v_n from the QGP phase.

- At LHC there is a stronger correlation between v_n and ε_n than at RHIC for all n.
- Ultra central collisions:
  - $v_n \propto \varepsilon_n$ for n=2,3,4 strong correlation $C(n,n) \approx 1$
  - $v_n(p_T)$ much more sensitive to η/s(T)

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

Coodinate space (x,y)

- We start with an initial azimuthally symmetric profile (optical Glauber model).
- Then we deform the initial distribution (α<<1)

$$z = x + iy \rightarrow z + \delta z \equiv z - \alpha \bar{z}^{n-1}$$

2π/n symmetry

This Creates only

$$\epsilon_n \equiv \frac{-\sum_j (z_j + \delta z_j)^n}{\sum_j |z_j + \delta z_j|^n} \simeq n\alpha \frac{\langle r^{2(n-1)} \rangle}{\langle r^n \rangle}$$

Momentum space

- Thermal distribution:

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- Constant distribution:

$$dN/d^3p \propto \theta(p_0 - p)$$

We assume initially the same local $T^{\mu\nu}(x)$

$$f(\sigma) \underset{\sigma\rightarrow\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right)$$

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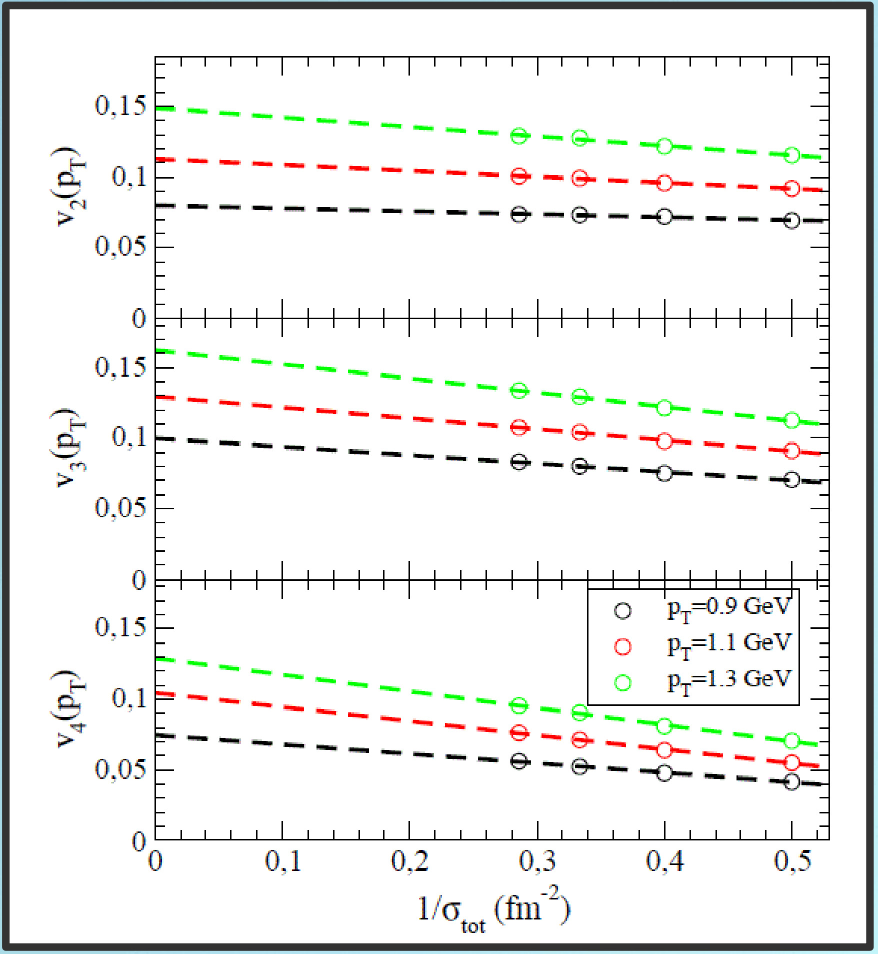
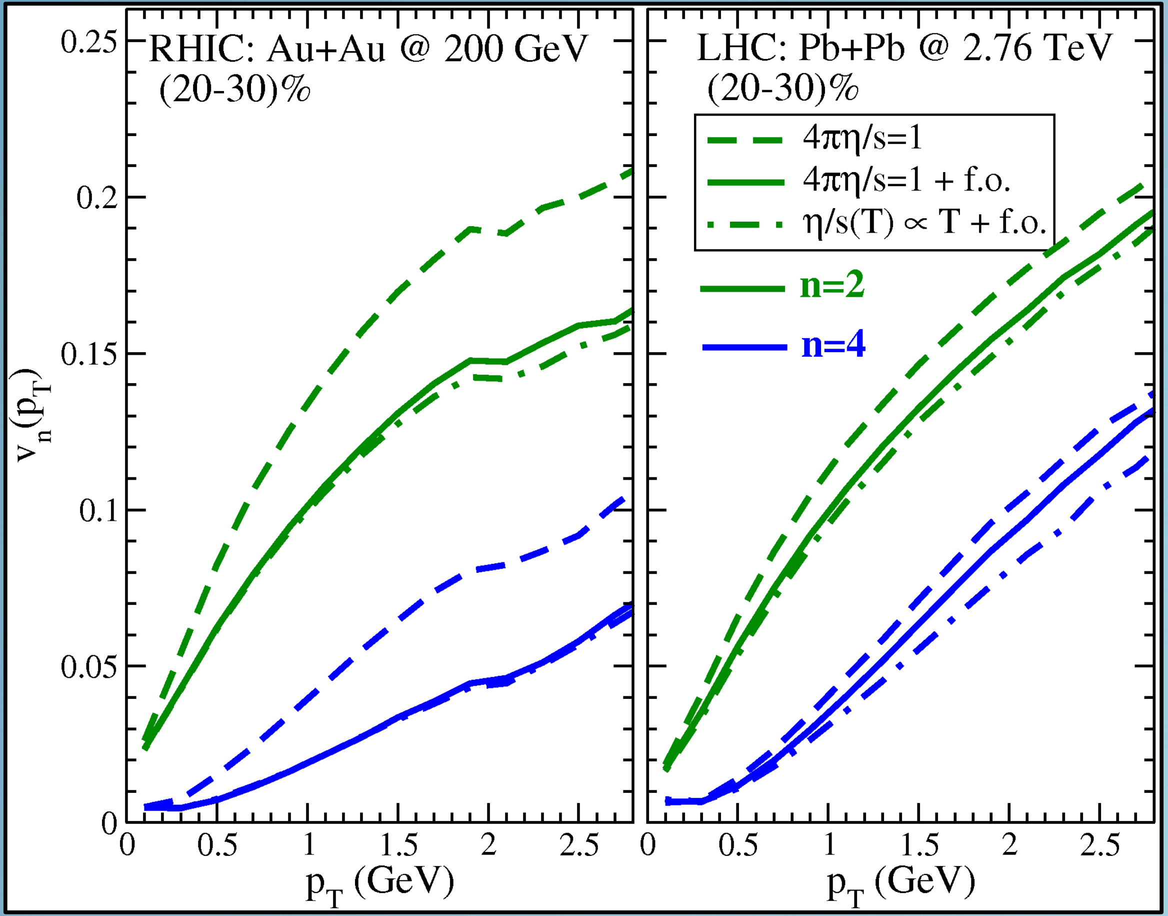
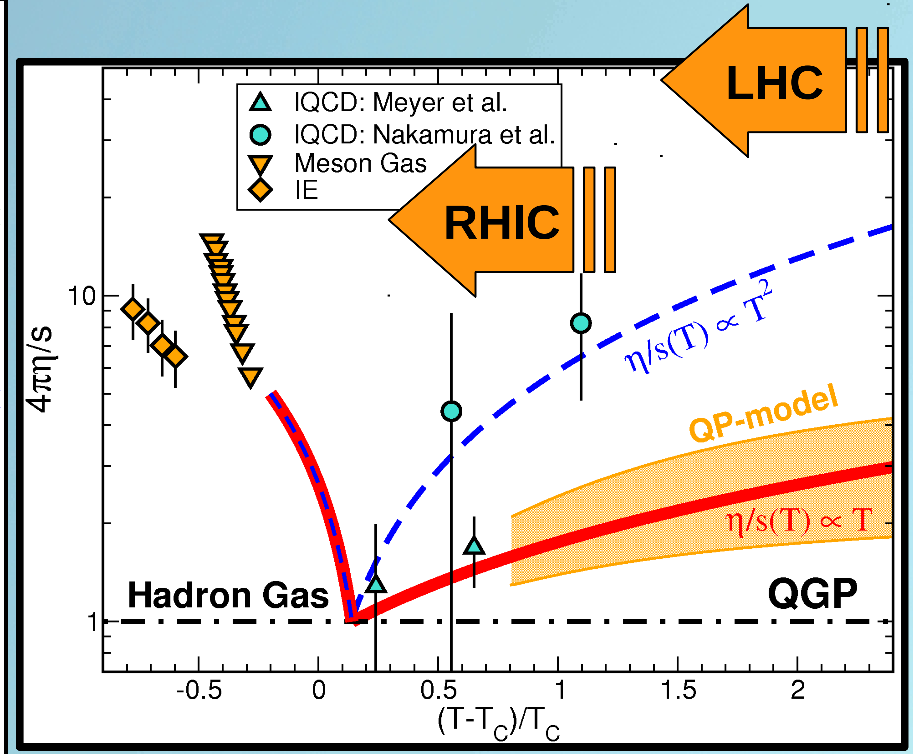


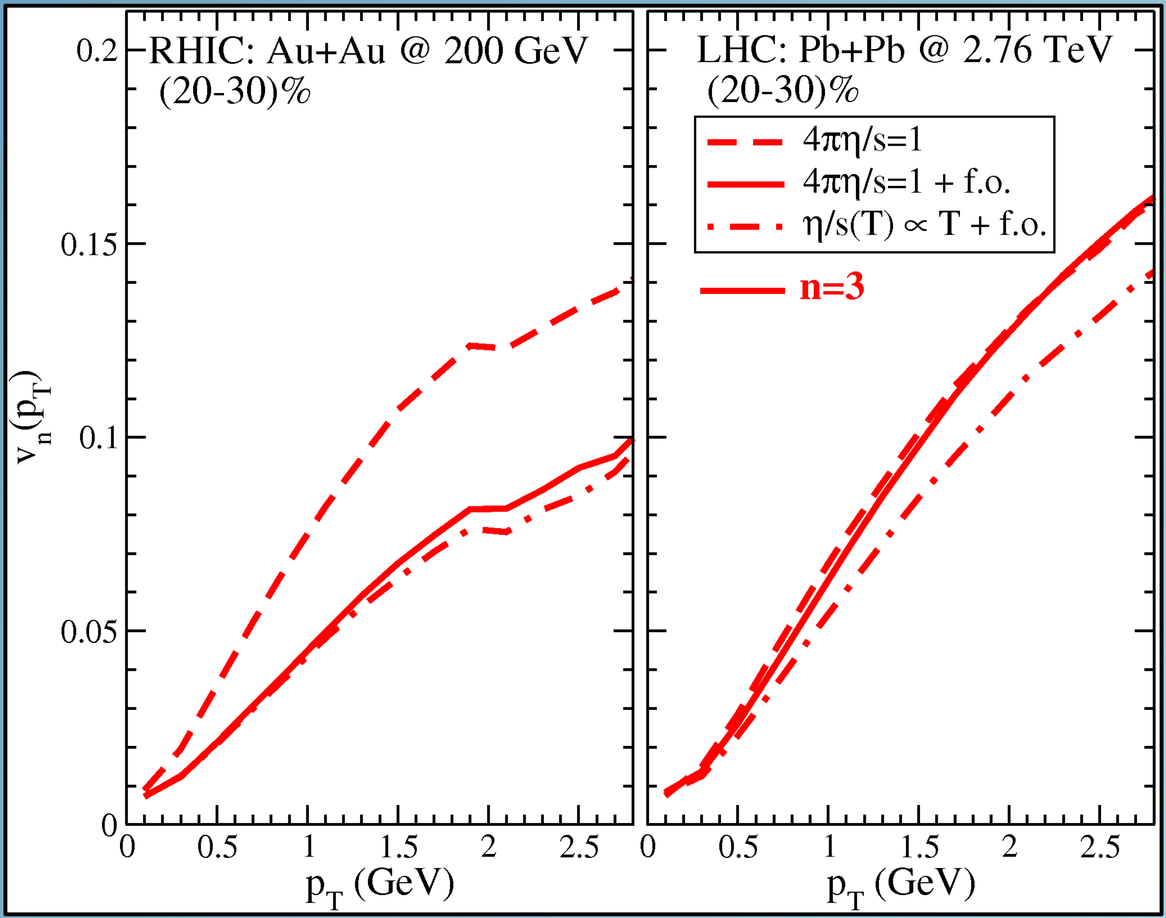
Initial State Fluctuations: $v_n(p_T)$ and η/s

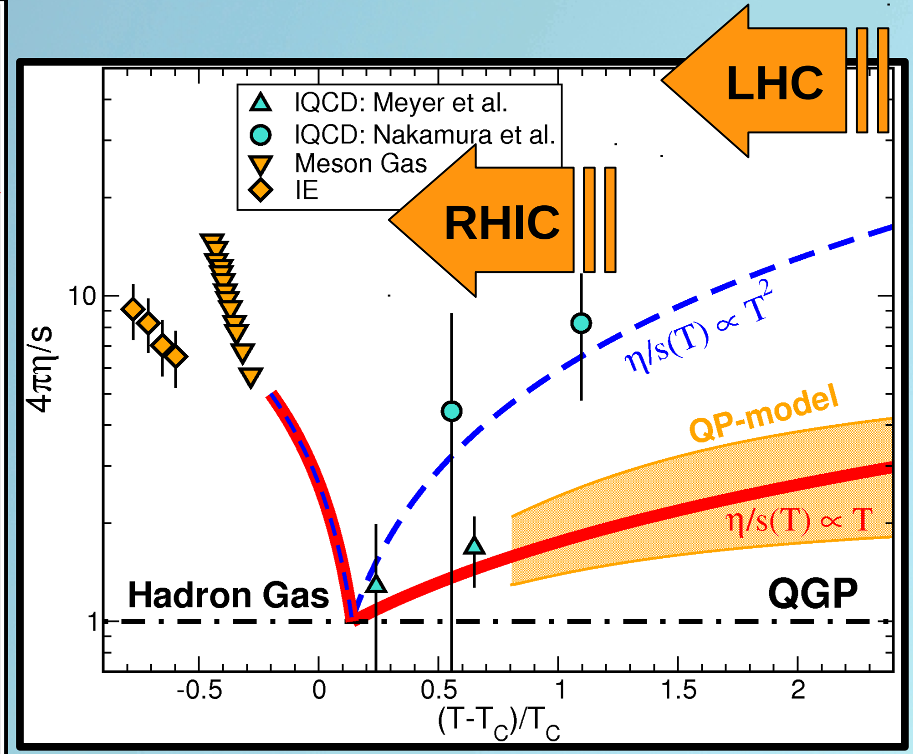




- $v_n(p_T)$ at RHIC is more sensitive to the value of the η/s at low temperature. $v_4(p_T)$ and $v_3(p_T)$ are more sensitive to the value of η/s than the $v_2(p_T)$.
- At LHC energies $v_n(p_T)$ is more sensitive to the value of η/s in the QGP phase (compare solid and dot-dashed lines).

Initial State Fluctuations: $v_n(p_T)$ and η/s





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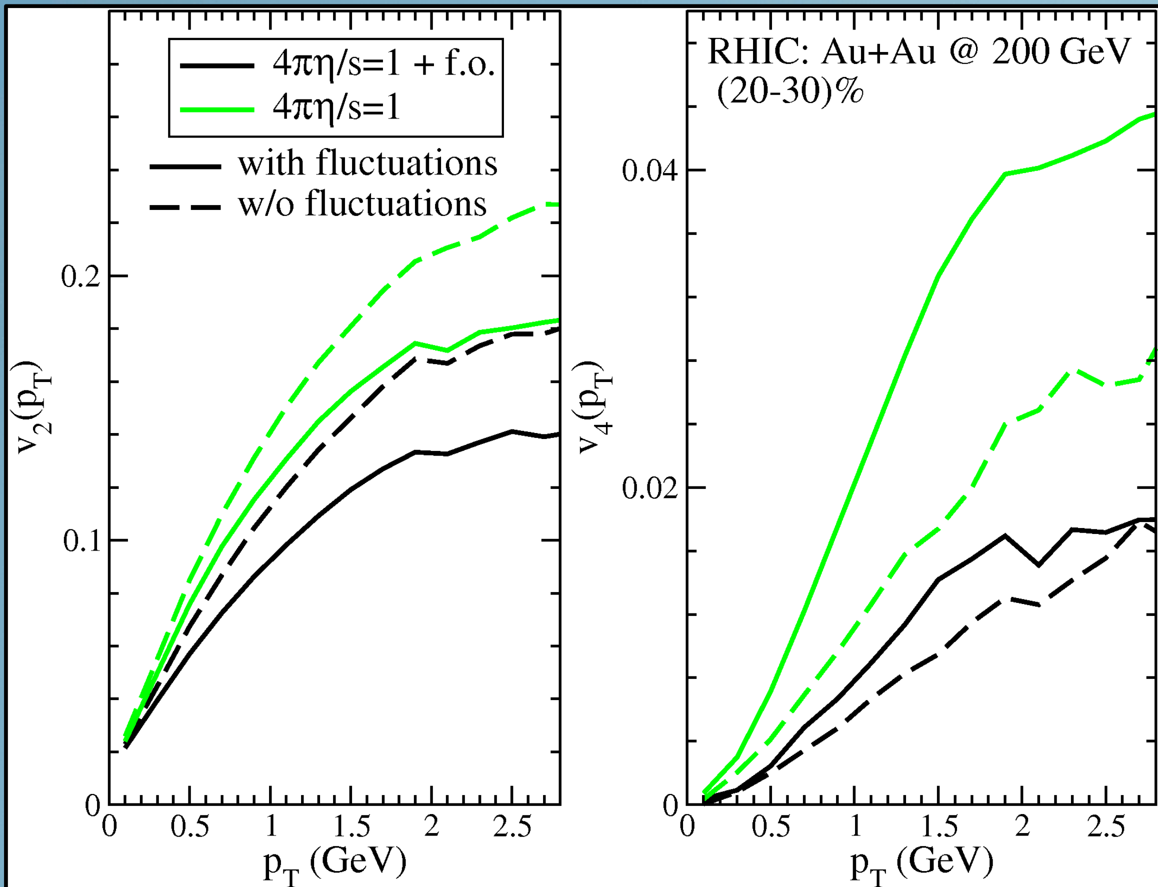
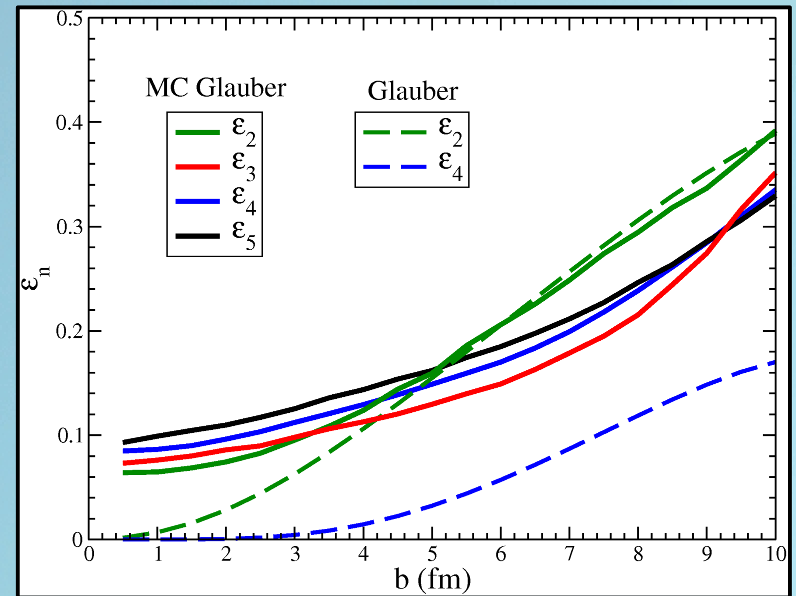
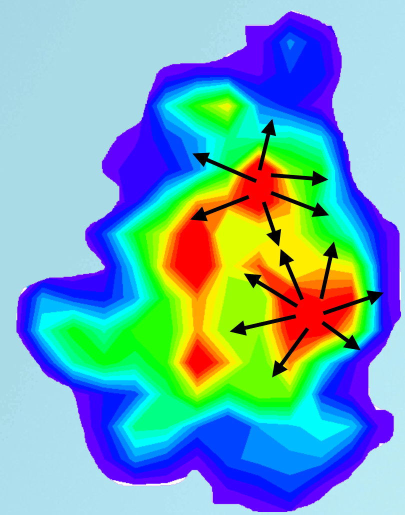
Initial State Fluctuations: $v_n(p_T)$ and η/s







- **The initial state fluctuations reduce the $v_2(p_T)$.**
- **$v_4(p_T)$ increase by the initial state fluctuations and it becomes more sensitive to the viscosity of the QGP. Larger ε_4 gives larger v_4.**

Extraction of the Shear Viscosity: Box calculation

$$\eta^{IS}_{relax}/s = \frac{1}{15}\langle p\rangle \tau_r = \frac{1}{15}\frac{\langle p\rangle}{\sigma_{tot}\langle f(a)\, v_{rel}\rangle \rho}$$

$$\sigma_{tr} = \int d\Omega \sin^2(\theta_{cm}) \frac{d\sigma}{d\Omega_{cm}} = \sigma_{tot} f(a) \leqslant \frac{2}{3}\sigma_{tot}$$

Employed also for non-isotropic cross section:

G.Ferini, PLB(2009); D. Molnar, JPG35(2008); V.Greco, PPNP(2009);

For the standard pQCD-like cross section

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{9\pi\alpha_S^2}{2}\frac{1}{(q^2+m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$

- **m_D regulates the anisotropy of collision**
- **$m_D \rightarrow \infty$ we recover the isotropic limit**

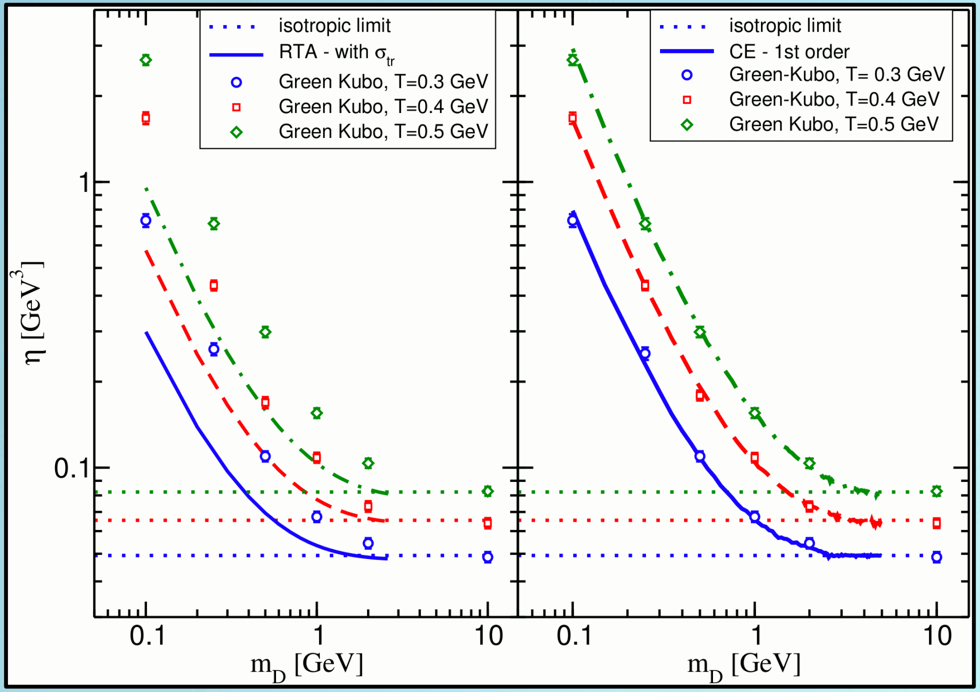
$$f(a) = 4a(1+a)[(2a+1)\ln(1+a^{-1})-2], \quad a = m_D^2/s$$

1st Chapman-Enskog approximation

$$[\eta]_{1\,st}/s = \frac{1}{15}\langle p\rangle \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{\sigma_{tot}\, g(a)\, \rho}$$

$$g(a) = \frac{1}{50}\int_0^\infty dy\, y^6 [(y^2+\frac{1}{3})K_3(2y) - y K_2(2y)] f(a), \quad a = \frac{m_D}{2T}$$

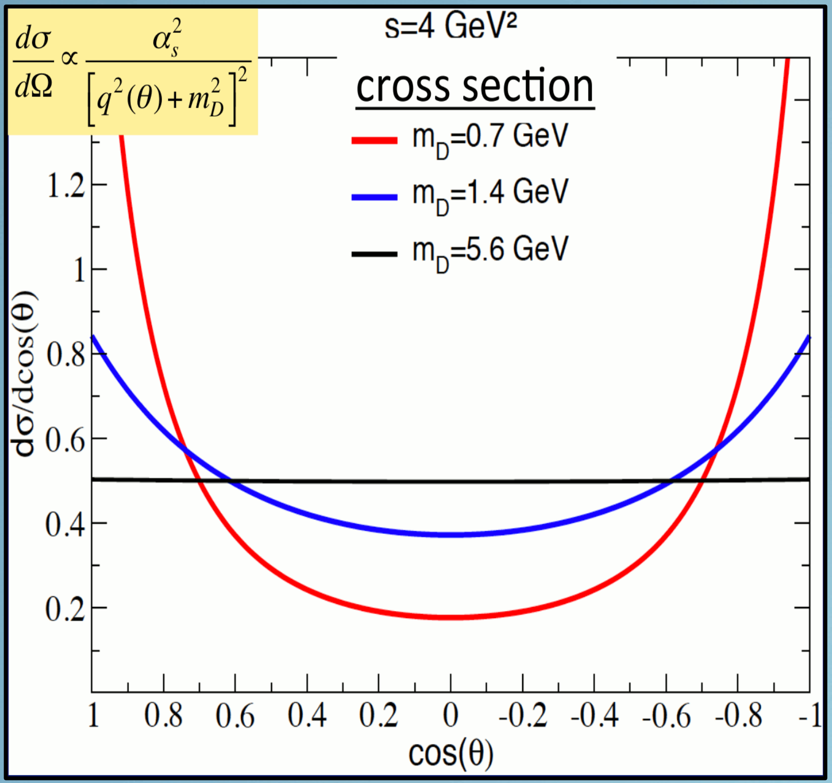
S. Plumari et al.,PRC86 (2012) 054902.

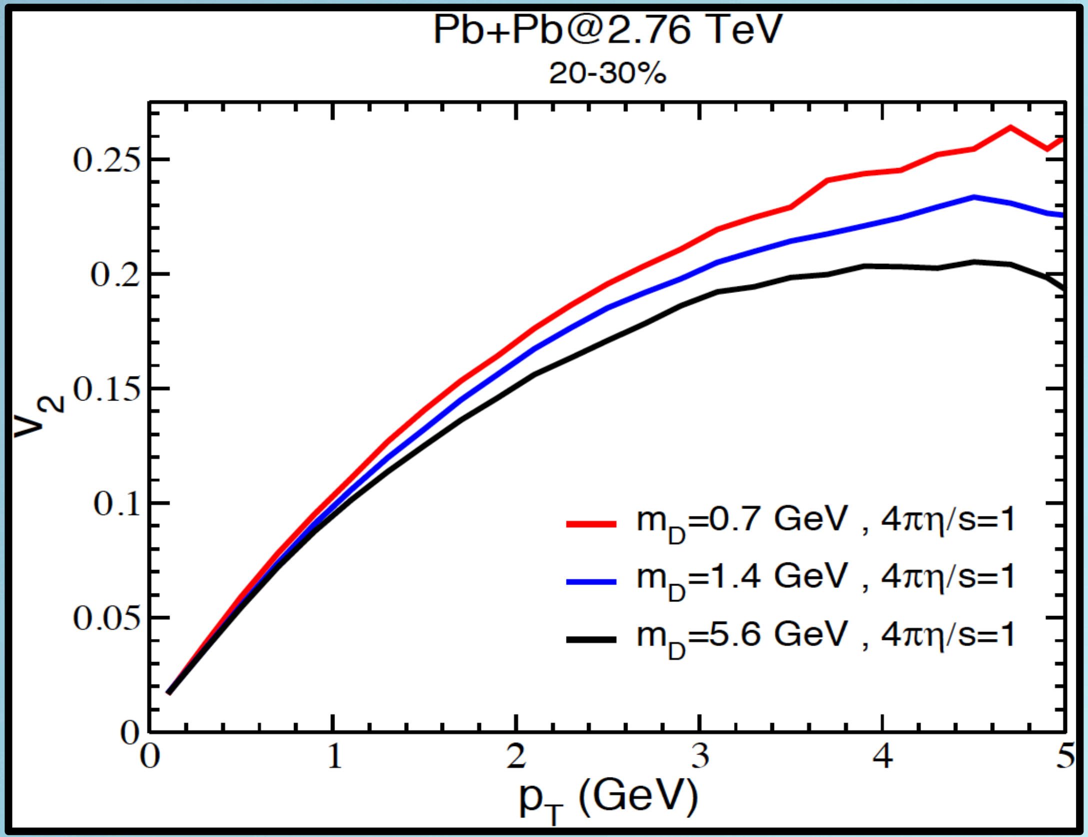


- **CE and RTA can differ by a factor of 2**
- **Green-Kubo agree with CE (< 5%)**

A. Wiranata, M. Prakash, PRC85 (2012) 054908.
O. N. Moroz, arXiv:1112.0277 [hep-ph].

η/s or detail of the corss section





$$\frac{\eta}{s} = \frac{1}{15} \langle p \rangle \, \tau_\eta$$

$$\tau_\eta = \frac{1}{\sigma_{tot} \, g(a) \rho}$$

- **η/s is the physical parameter determining the v_2 at least up to p_T 1.5 -2 GeV.**
- **microscopic details becomes important at higher p_T.**