Plasmons in anisotropic quark-gluon plasma

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Outline

- Motivation
- General dispersion equation
- Isotropic system
- Weakly anisotropic system
- Finite prolateness or oblateness
- Extremely prolate system
- Extremely oblate system
- Conclusions

Motivation

- Spectrum of collective excitations is an important characteristics of any many body system.
- Anisotropic plasma is qualitatively different than the isotropic one.
- QGP from relativistic heavy-ion collisions is anisotropic.
- Existing analyses of collective excitations are not complete.

Momentum distribution

The anisotropic momentum distribution is obtained from an isotropic one by rescaling it in one direction

$$f_\xi(\mathbf{p}) = C_\xi \; f_{\text{iso}}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

$$f_\xi(\mathbf{p}) \xrightarrow[\xi\to\infty]{} \delta(p_L)$$

$$\xi \in (-1, \infty)$$

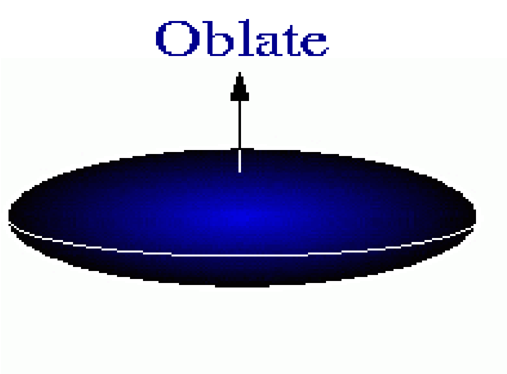
$$f_\sigma(\mathbf{p}) = C_\sigma \; f_{\text{iso}}\left(\sqrt{(\sigma+1)\mathbf{p}^2 - \sigma(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

$$f_\sigma(\mathbf{p}) \xrightarrow[\sigma\to\infty]{} \delta(p_T)$$

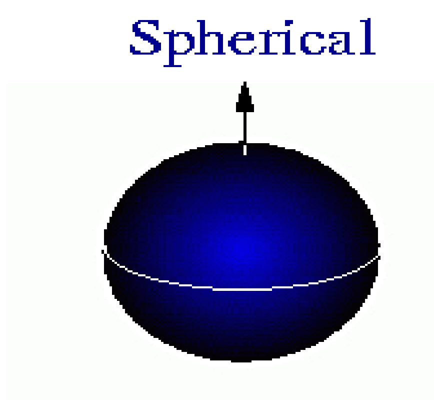
$$\sigma \in (-1, \infty)$$

P.Romatschke and M. Strickland, Phys. Rev. D **68**, 036004 (2003)

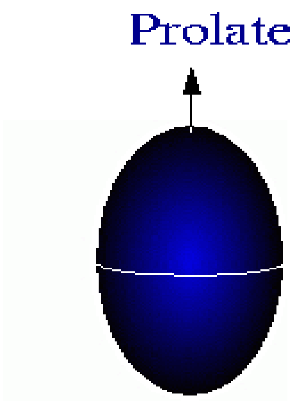
$\xi > 0; \quad \sigma < 0$



$\xi = \sigma = 0$



$\xi < 0; \quad \sigma > 0$



Momentum distribution

There is a freedom in choosing the normalization constants

$$f_\xi(\mathbf{p}) = C_\xi \; f_{\rm iso}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right) \qquad f_\sigma(\mathbf{p}) = C_\sigma \; f_{\rm iso}\left(\sqrt{(\sigma+1)\mathbf{p}^2 - \sigma(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

- $$\int \frac{d^3p}{(2\pi)^3} f_\xi(\mathbf{p}) = \int \frac{d^3p}{(2\pi)^3} f_{\rm iso}(|\mathbf{p}|) = \int \frac{d^3p}{(2\pi)^3} f_\sigma(\mathbf{p})$$

$$C_\xi = \sqrt{1+\xi} \qquad C_\sigma = \sigma + 1$$

- $$m^2 \equiv \int \frac{d^3p}{(2\pi)^3} \frac{f_\xi(\mathbf{p})}{|\mathbf{p}|} = \int \frac{d^3p}{(2\pi)^3} \frac{f_{\rm iso}(|\mathbf{p}|)}{|\mathbf{p}|} = \int \frac{d^3p}{(2\pi)^3} \frac{f_\sigma(\mathbf{p})}{|\mathbf{p}|}$$

$$C_\xi = \frac{\sqrt{\xi}}{\mathrm{Arctanh}\sqrt{\xi}} \qquad C_\sigma = \frac{\sqrt{\sigma(\sigma+1)}}{\mathrm{Arctan}\sqrt{\dfrac{\sigma}{\sigma+1}}}$$

General dispersion equation

The dispersion equation

$$\det\left[(\omega^2-\mathbf{k}^2)\delta^{ij}+k^ik^j-\Pi^{ij}(\omega,\mathbf{k})\right]=0$$

$$\underbrace{\qquad\qquad}_{\Sigma^{ij}(\omega,\mathbf{k})}$$

$\Sigma^{ij}(\omega,\mathbf{k})$ inverse gluon propagator in temporal axial gauge

$\omega(\mathbf{k})$ - collective mode in a plasma system

The dielectric tensor is related to the retarded gluon polarization tensor

$$\varepsilon^{ij}(\omega,\mathbf{k})=\delta^{ij}-\frac{1}{\omega^2}\Pi^{ij}(\omega,\mathbf{k})$$

Instead of looking for zeros of det Σ one looks for poles of Σ^{-1}

How to invert matrix Σ ?

Method to inverse the matrix Σ

Inversion of the matrix Σ which depends on **k** and **n**

$$\Sigma = \alpha A + \beta B + \gamma C + \delta D$$

basis of matrices

$$A^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}, \quad B^{ij} = \frac{k^i k^j}{\mathbf{k}^2}$$

$$C^{ij} = \frac{n_T^i n_T^j}{\mathbf{n}_T^2}, \quad D^{ij} = n_T^i k^j + k^i n_T^j$$

$$n_T^i = \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right) n^j$$

$$\Sigma^{-1} = \bar{\alpha} A + \bar{\beta} B + \bar{\gamma} C + \bar{\delta} D$$

$$\Sigma \Sigma^{-1} = \mathbf{1} \quad \Rightarrow \quad \bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta}$$

The coefficients α, β, γ, δ are determined by the following contractions:

$$k^i \Sigma^{ij} k^j = k^2 \beta, \qquad n_T^i \Sigma^{ij} k^j = n_T^2 k^2 \delta,$$

$$n_T^i \Sigma^{ij} n_T^j = n_T^2 (\alpha + \gamma), \qquad \mathrm{Tr}\,\Sigma = 2\alpha + \beta + \gamma,$$

P. Romatschke and M. Srickland, Phys. Rev. D **68**, 036004 (2003)

Collective mode in isotropic QGP

In isotropic plasma the matrix Σ is decomposed as: $\Sigma^{ij} = \alpha_{\rm iso} A^{ij} + \beta_{\rm iso} B^{ij}$

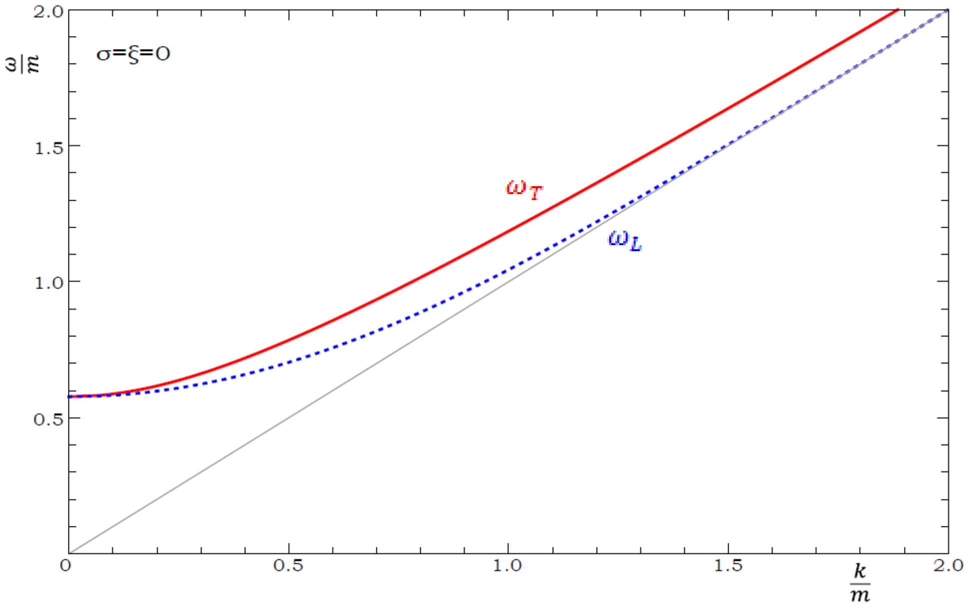
$$\alpha_{iso}(\omega,\mathbf{k}) = \omega^2 - k^2 - \frac{m^2\omega^2}{2k^2}\left[1-\left(\frac{\omega}{2k}-\frac{k}{2\omega}\right)\ln\left(\frac{\omega+k}{\omega-k}\right)\right]$$

and

$$\beta_{iso}(\omega,\mathbf{k}) = \omega^2 + \frac{m^2\omega^2}{k^2}\left[1-\frac{\omega}{2k}\ln\left(\frac{\omega+k}{\omega-k}\right)\right]$$

Only four real solutions
(two positive & two negative)

$$m^2 = g^2\int\frac{d^3p}{(2\pi)^3}\frac{f_\xi(\mathbf{p})}{|\mathbf{p}|}$$ Debye mass



Weakly anisotropic system

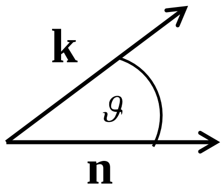
Weakly anisotropic distribution $|\xi| << 1$

$$f_\xi(\mathbf{p}) \approx \left(1+\frac{\xi}{3}\right) f_{\rm iso}(p) + \frac{\xi}{2}\frac{d\ f_{\rm iso}(p)}{d\ p}\, p(\mathbf{v}\cdot\mathbf{n})^2$$

Coefficients α, β, γ, δ are found in analytic form

$$\alpha(\omega,\mathbf{k}) = \left(1+\frac{\xi}{3}\right)\alpha_{iso}(\omega,\mathbf{k}) + \xi\frac{m^2}{8}\left\{\frac{8}{3}\cos^2\vartheta\ +\frac{2}{3}\left(5-19\cos^2\vartheta\right)\frac{\omega^2}{k^2} - 2\left(1-5\cos^2\vartheta\right)\frac{\omega^4}{k^4}\right.$$
$$\left. +\left[1-3\cos^2\vartheta\ - \left(2-8\cos^2\vartheta\right)\frac{\omega^2}{k^2} + \left(1-5\cos^2\vartheta\right)\frac{\omega^4}{k^4}\right]\frac{\omega}{k}\ln\left(\frac{\omega+k}{\omega-k}\right)\right\},$$

$$\beta(\omega,\mathbf{k}) = \ldots,$$
$$\gamma(\omega,\mathbf{k}) = \ldots,$$
$$\delta(\omega,\mathbf{k}) = \ldots$$



Weakly anisotropic system

Dispersion equations:

1) $\omega^2 - k^2 - \alpha(\omega, \mathbf{k}) = 0$

2) $\left(\beta(\omega, \mathbf{k}) - \omega^2\right)\left(\alpha(\omega, \mathbf{k}) + \gamma(\omega, \mathbf{k}) - \omega^2 + k^2\right) - \delta^2(\omega, \mathbf{k}) k^2 n_T^2 = 0$

In the limit of weak anisotropy, we have three dispersion equations because $\delta^2 = O(\xi^2)$

1) $\omega^2 - k^2 - \alpha(\omega, \mathbf{k}) = 0$

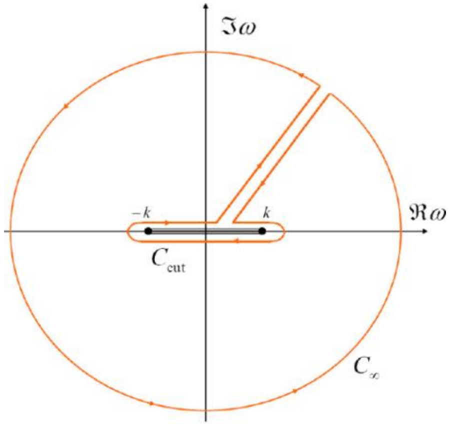
2) $\omega^2 - \beta(\omega, \mathbf{k}) = 0$

3) $\alpha(\omega, \mathbf{k}) + \gamma(\omega, \mathbf{k}) - \omega^2 + k^2 = 0$

Nyquist analysis

Nyquist analysis allows one to find the number of solution of the equation

The number of zeros of the function $f(\omega)$: $n_Z = n_P + n_\infty + n_W$



$$\oint_C \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} = n_Z - n_P$$

$$\oint_C \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} = \oint_{C_\infty} \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} + \oint_{C_{cut}} \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)}$$

- C_∞ $$\oint_{C_\infty} \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} = \lim_{|\omega|->\infty} \omega \frac{f'(\omega)}{f(\omega)} \equiv n_\infty$$
- C_{cut} $$\oint_{C_{cut}} \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} = \frac{1}{2\pi i}\oint_{C_{cut}} \frac{d}{d\omega}\ln f(\omega) = \frac{1}{2\pi i}(\ln f(\omega_e) - \ln f(\omega_s)) \equiv n_W$$

Number of solutions

The number of modes for each system

Momentum distribution	Number of real modes	Number of imaginary modes	Total number of modes	Maximal number of modes
extremely prolate	$6 + 2\Theta(k - k_{\rm p})$	$2\Theta(k_{\rm p} - k)$	8	8
weakly prolate	6	$2\Theta(k_{\rm C} - k)$	$6 + 2\Theta(k_{\rm C} - k)$	8
isotropic	6	0	6	6
weakly oblate	6	$2\Theta(k_{\rm A} - k) + 2\Theta(k_{\rm C} - k)$	$6 + 2\Theta(k_{\rm A} - k) + 2\Theta(k_{\rm C} - k)$	10
extremely oblate	6	$2\Theta(k_{\rm oA} - k) + 2\Theta(k_{\rm oG} - k)$	$6 + 2\Theta(k_{\rm oA} - k) + 2\Theta(k_{\rm oG} - k)$	10

Equation $\omega^2 - k^2 - \alpha(\omega, \mathbf{k}) = 0$

$k^2 - \xi \frac{m^2}{3} \cos^2 \vartheta \geq 0$ 2 solutions

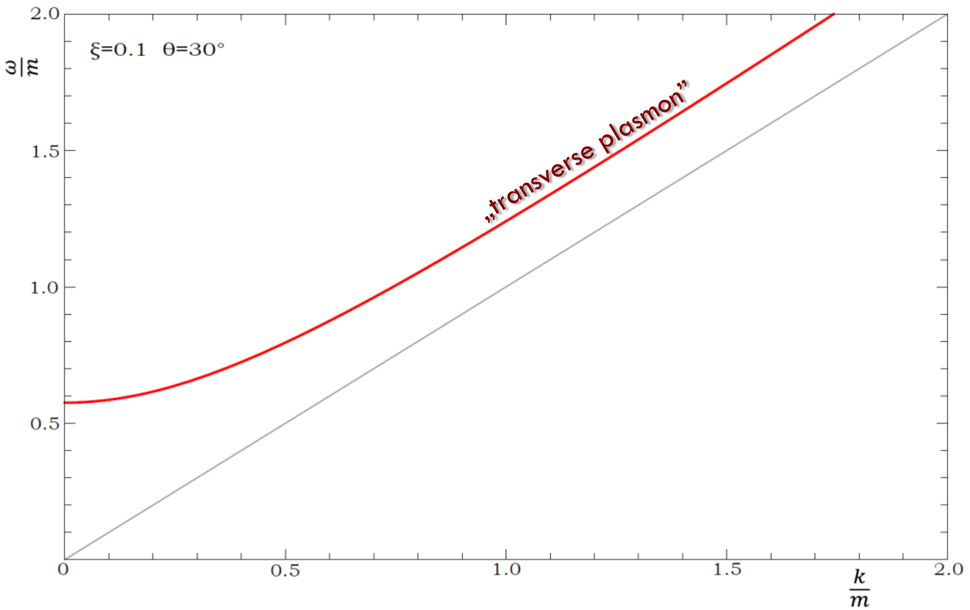
$k^2 - \xi \frac{m^2}{3} \cos^2 \vartheta < 0$ 4 solutions

Equation $\alpha(\omega,\mathbf{k}) - \omega^2 + k^2 = 0$

$k^2 - \xi \frac{m^2}{3}\cos^2\vartheta \geq 0$ **2 solutions**

$k^2 - \xi \frac{m^2}{3}\cos^2\vartheta < 0$ **4 solutions**

$$\omega^2(\mathbf{k}) \underset{k<<m}{\approx} \frac{m^2}{3}\left(1-\frac{\xi}{15}\right) + \frac{6}{5}\left[1+\frac{\xi}{14}\left(\frac{4}{15}+\cos^2\vartheta\right)\right]k^2$$



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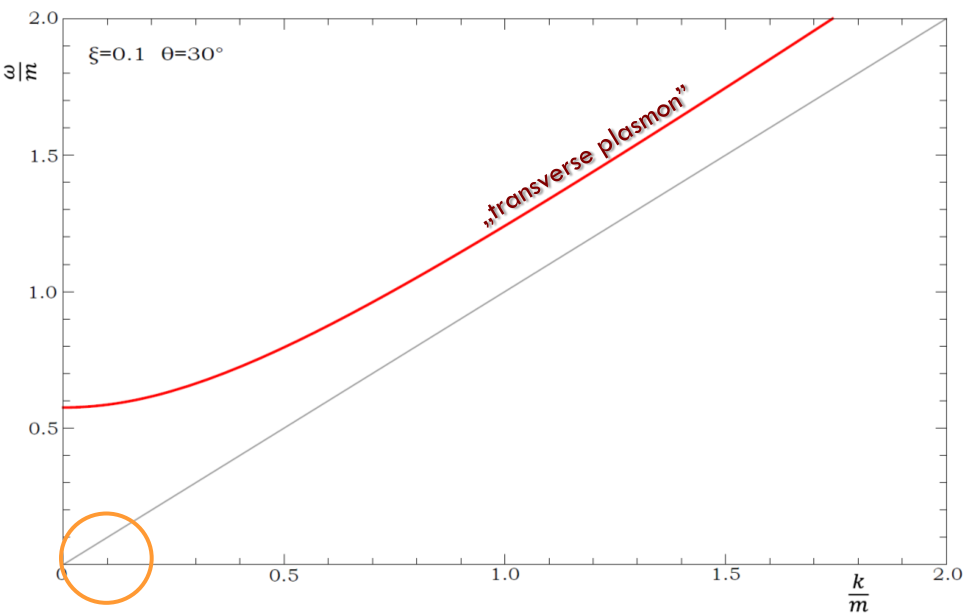
$$\omega^2(\mathbf{k}) \underset{k<<m}{\approx} \frac{m^2}{3}\left(1 - \frac{\xi}{15}\right) + \frac{6}{5}\left[1 + \frac{\xi}{14}\left(\frac{4}{15} + \cos^2 \vartheta\right)\right] k^2$$

$$\boxed{\omega(\mathbf{k}) = \pm i \gamma(\mathbf{k})}$$

$$\gamma(\mathbf{k}) \underset{k>>\gamma}{\approx} \frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2} + 4\left(k_A^2 - k^2\right)} - \frac{\lambda}{k}\right)$$



Equation $\alpha(\omega,\mathbf{k})-\omega^2+k^2=0$

$k^2-\xi\frac{m^2}{3}\cos^2\vartheta\geq 0$ **2 solutions**

$k^2-\xi\frac{m^2}{3}\cos^2\vartheta<0$ **4 solutions**

$$\omega^2(\mathbf{k})\underset{k<<m}{\approx}\frac{m^2}{3}\left(1-\frac{\xi}{15}\right)+\frac{6}{5}\left[1+\frac{\xi}{14}\left(\frac{4}{15}+\cos^2\vartheta\right)\right]k^2$$

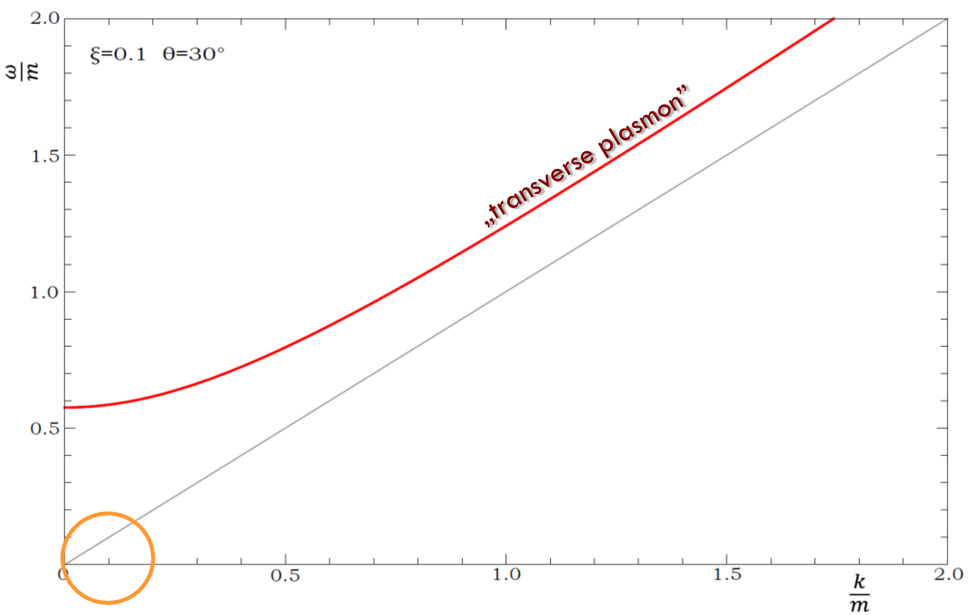
$$\boxed{\omega(\mathbf{k})=\pm i\gamma(\mathbf{k})}$$

$$\gamma(\mathbf{k})\underset{k>>\gamma}{\approx}\frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2}+4\left(k_A^2-k^2\right)}-\frac{\lambda}{k}\right)$$

where

$$\lambda\equiv\frac{\pi}{4}\left[1-\frac{\xi}{2}\left(\frac{1}{3}-3\cos^2\vartheta\right)\right]m^2$$

$$k_A\equiv\sqrt{\frac{\xi}{3}}\left|\cos\vartheta\right|$$



Equation $\alpha(\omega,\mathbf{k})-\omega^2+k^2=0$

$$k^2-\xi\frac{m^2}{3}\cos^2\vartheta\geq 0$$ **2 solutions**

$$k^2-\xi\frac{m^2}{3}\cos^2\vartheta<0$$ **4 solutions**

$$\omega^2(\mathbf{k})\underset{k<<m}{\approx}\frac{m^2}{3}\left(1-\frac{\xi}{15}\right)+\frac{6}{5}\left[1+\frac{\xi}{14}\left(\frac{4}{15}+\cos^2\vartheta\right)\right]k^2$$

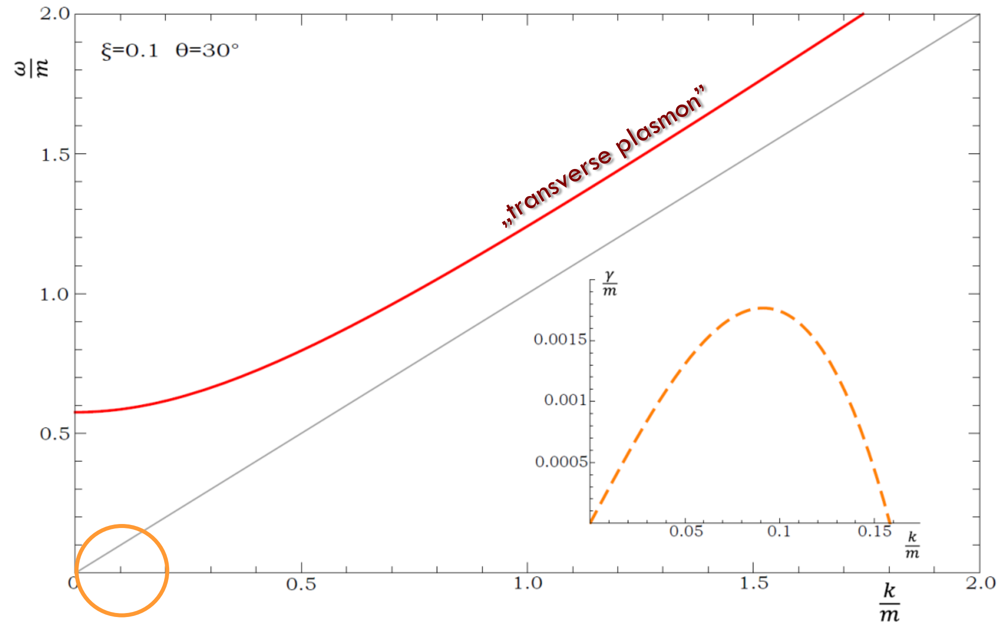
$$\omega(\mathbf{k})=\pm i\gamma(\mathbf{k})$$

$$\gamma(\mathbf{k})\underset{k>>\gamma}{\approx}\frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2}+4\left(k_A^2-k^2\right)}-\frac{\lambda}{k}\right)$$

where

$$\lambda\equiv\frac{\pi}{4}\left[1-\frac{\xi}{2}\left(\frac{1}{3}-3\cos^2\vartheta\right)\right]m^2$$

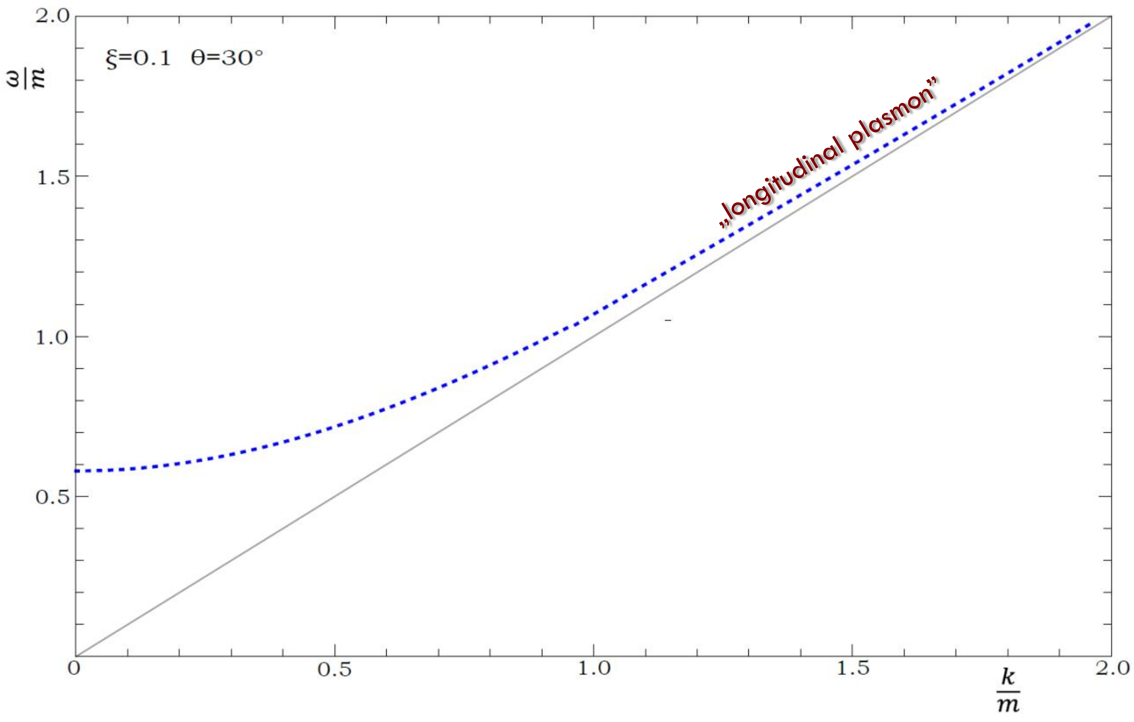
$$k_A\equiv\sqrt{\frac{\xi}{3}}\left|\cos\vartheta\right|$$



Equation $\omega^2 - \beta(\omega, \mathbf{k}) = 0$

There are always two solutions

$$\omega^2(\mathbf{k}) \underset{k<<m}{\approx} \frac{m^2}{3}\left(1+\frac{\xi}{5}\left(-\frac{1}{3}+\cos^2\vartheta\right)\right)+\frac{3}{5}\left[1+\frac{4\xi}{35}\left(1-3\cos^2\vartheta\right)\right]k^2$$



Equation $\omega^2 - k^2 - \alpha(\omega,\mathbf{k}) - \gamma(\omega,\mathbf{k}) = 0$

$k^2 + \xi \frac{m^2}{3}\left(1 - 2\cos^2\vartheta\right) \geq 0$ **2 solutions**

$k^2 + \xi \frac{m^2}{3}\left(1 - 2\cos^2\vartheta\right) < 0$ **4 solutions**

$$\omega^2(\mathbf{k}) \underset{k<<m}{\approx} \frac{m^2}{3}\left(1 + \frac{\xi}{5}\left(\frac{2}{3} - \cos^2\vartheta\right)\right) + \frac{6}{5}\left[1 + \frac{\xi}{5}\left(\frac{23}{42} - \cos^2\vartheta\right)\right]k^2$$

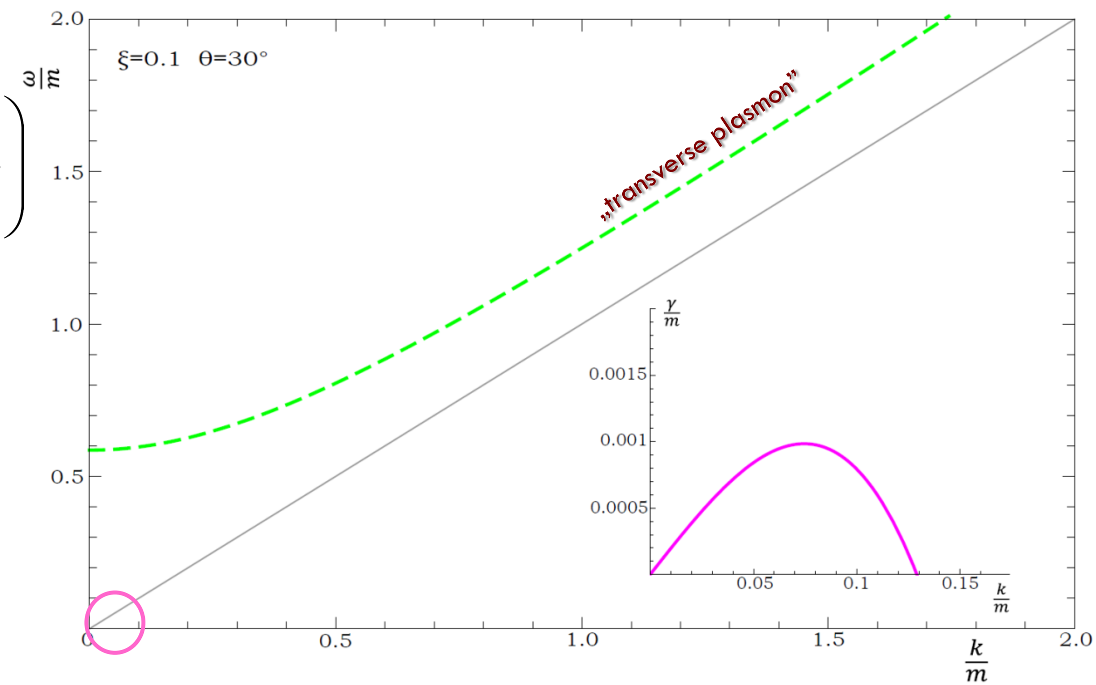
$$\omega(\mathbf{k}) = \pm i\gamma(\mathbf{k})$$

$$\gamma(\mathbf{k}) \underset{k>>\gamma}{\approx} \frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2} + 4\left(k_A^2 - k^2\right)} - \frac{\lambda}{k}\right)$$

where

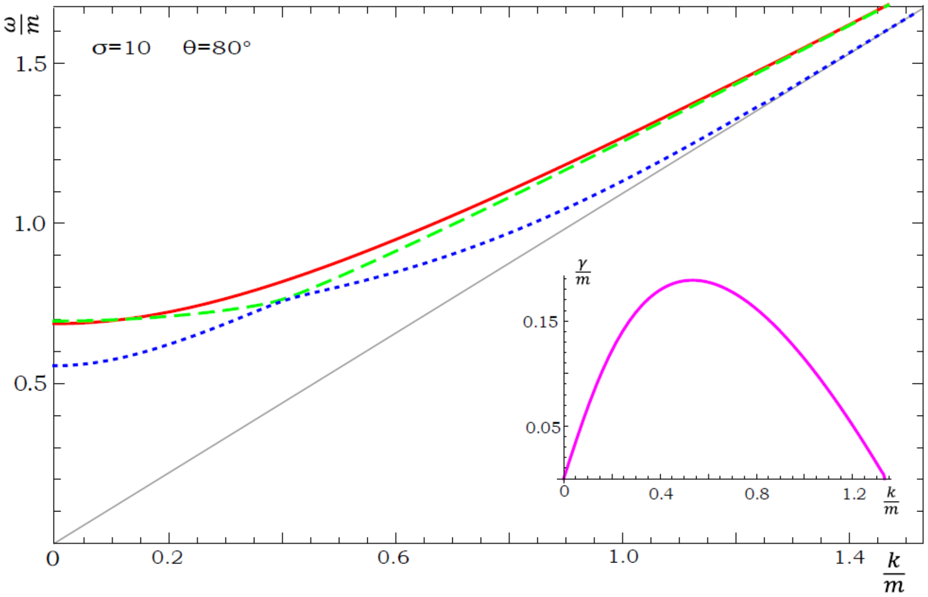
$$\lambda = \frac{\pi}{4}\left[1 - \frac{\xi}{2}\left(\frac{7}{3} - 5\cos^2\vartheta\right)\right]m^2$$

$$k_A \equiv m\Re\sqrt{\frac{\xi}{3}\left(2\cos^2\vartheta - 1\right)}$$



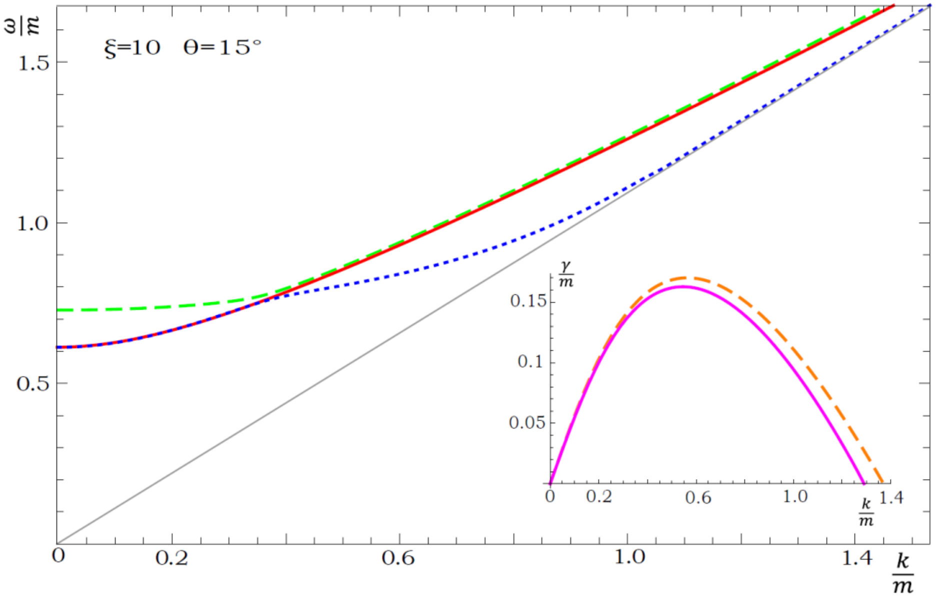
Finite prolateness or oblateness

$\sigma = 10$ and $\theta = 80^0$



prolate

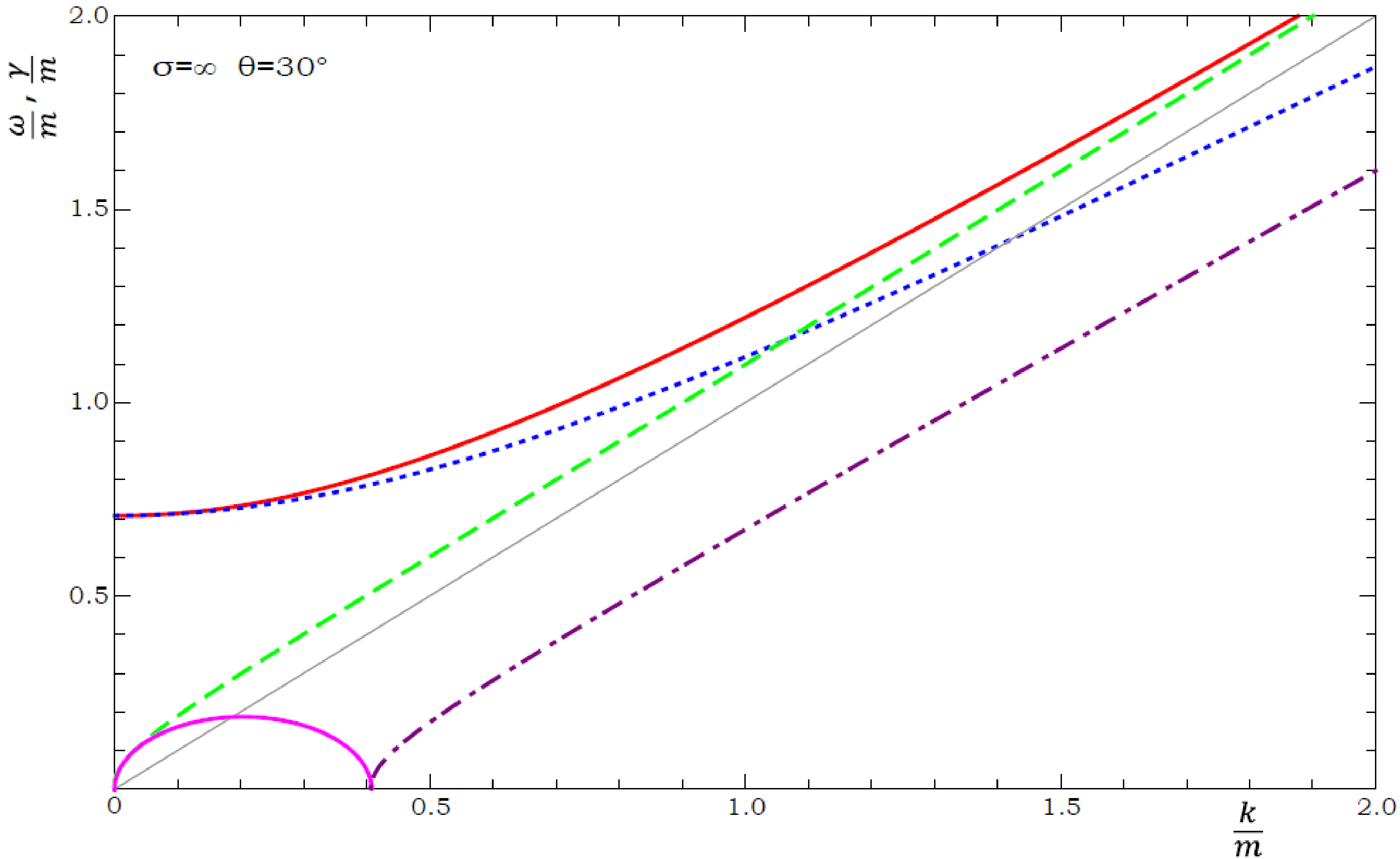
$\xi = 10$ and $\theta = 15^0$



oblate

Extremly prolate QGP $\sigma \to \infty$

Extremly prolate distribution: $f(\mathbf{p}) \sim \delta(p_T)$ **8 solutions** (6 real and 2 imaginary modes)

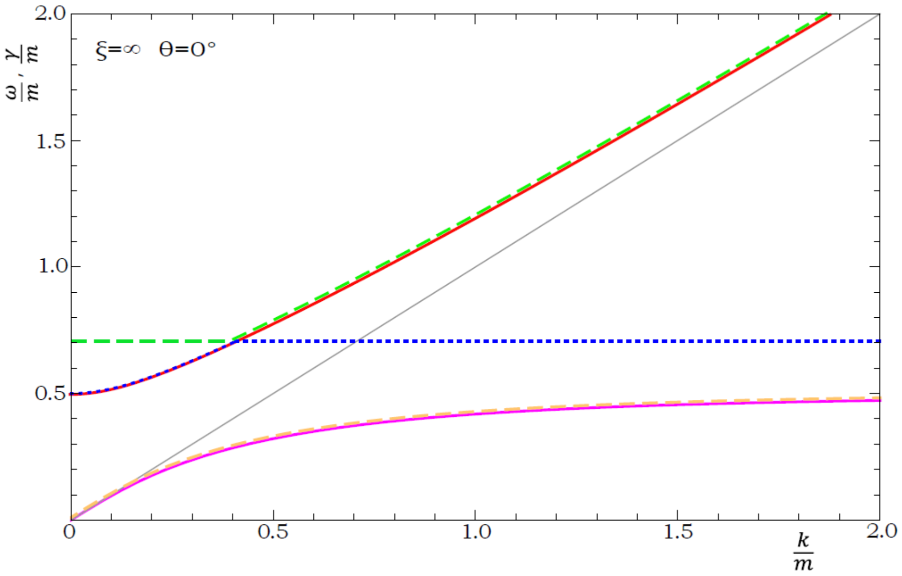


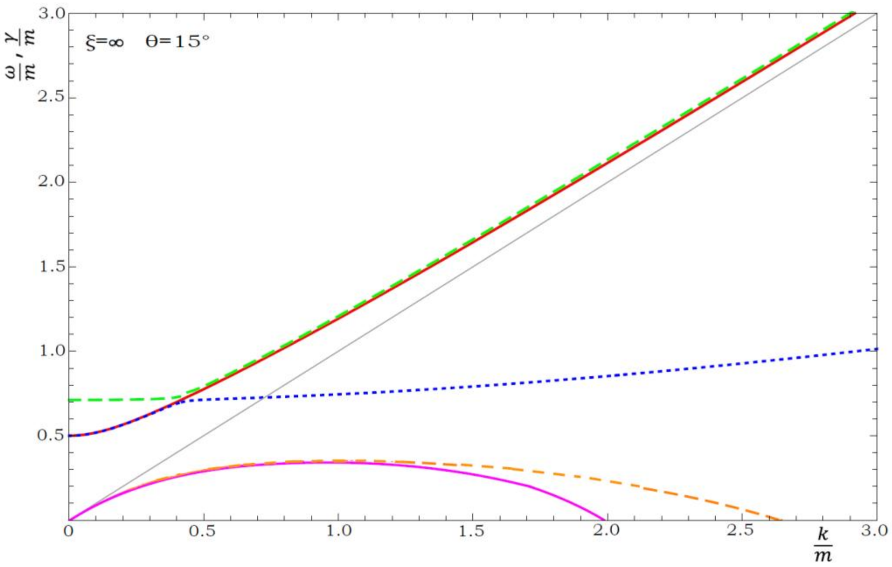
Extremly oblate QGP $\xi \to \infty$

Extremly oblate distribution: $f(\mathbf{p}) \sim \delta(p_L)$

8 or 10 solutions

(6 real and 4 or 2 imaginary modes)

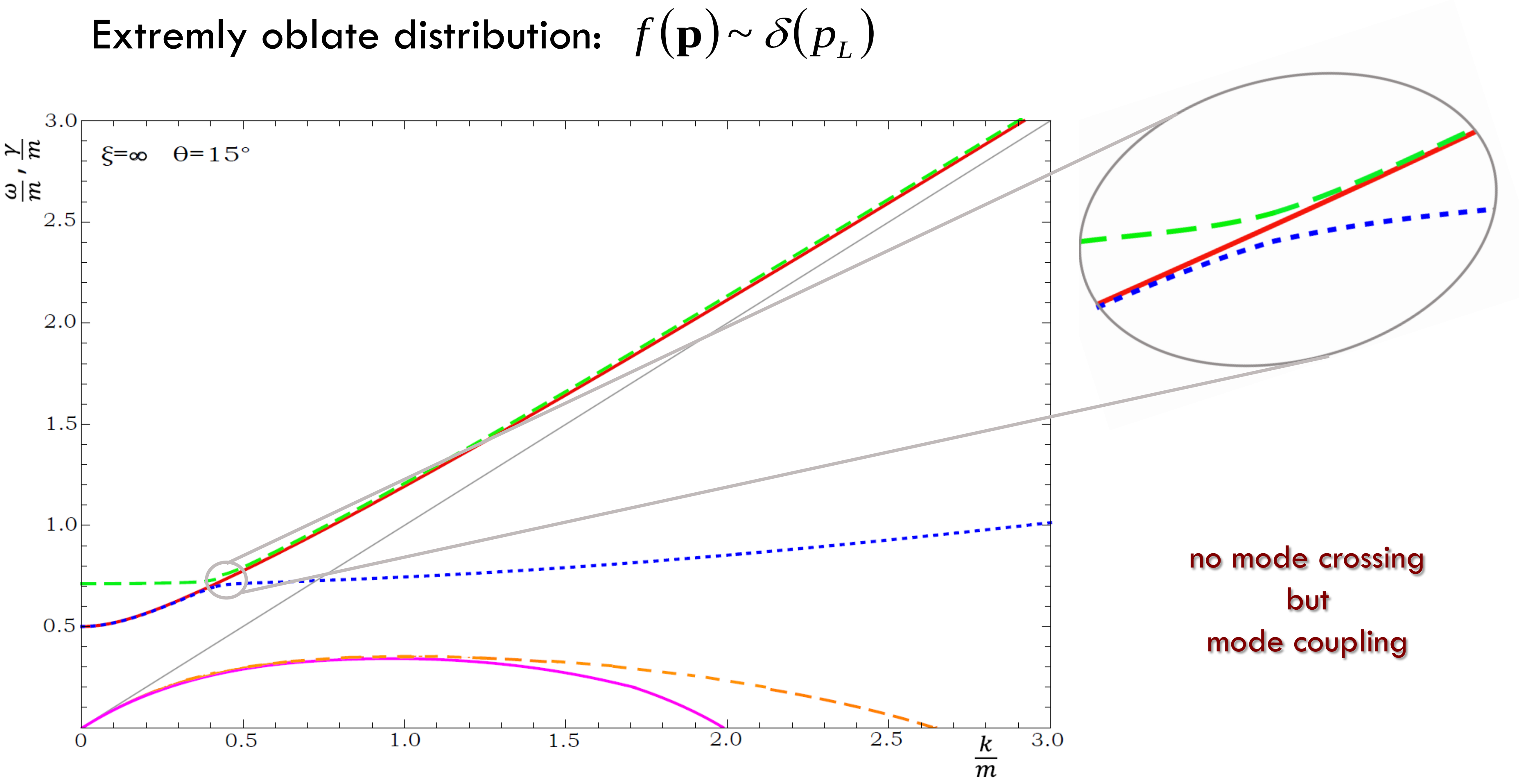




mode crossing ?

Extremly oblate QGP $\xi \to \infty$

Extremly oblate distribution: $f(\mathbf{p}) \sim \delta(p_L)$



no mode crossing
but
mode coupling

Conclusions

- Systematical analysis of the complete mode spectrum is performed.
- The number of modes is found in every case.
- Analytical and numerical solutions are found.
- Complete spectrum of modes is needed to compute various plasma characteristics e.g. the energy loss in anisotropic QGP.

for more details see:
M. Carrington, K. Deja and St. Mrówczyński, Phys. Rev. C **90** (2014) 034913