Quantum coherence measurements using 2-, 3-, and 4-pion Bose-Einstein correlations

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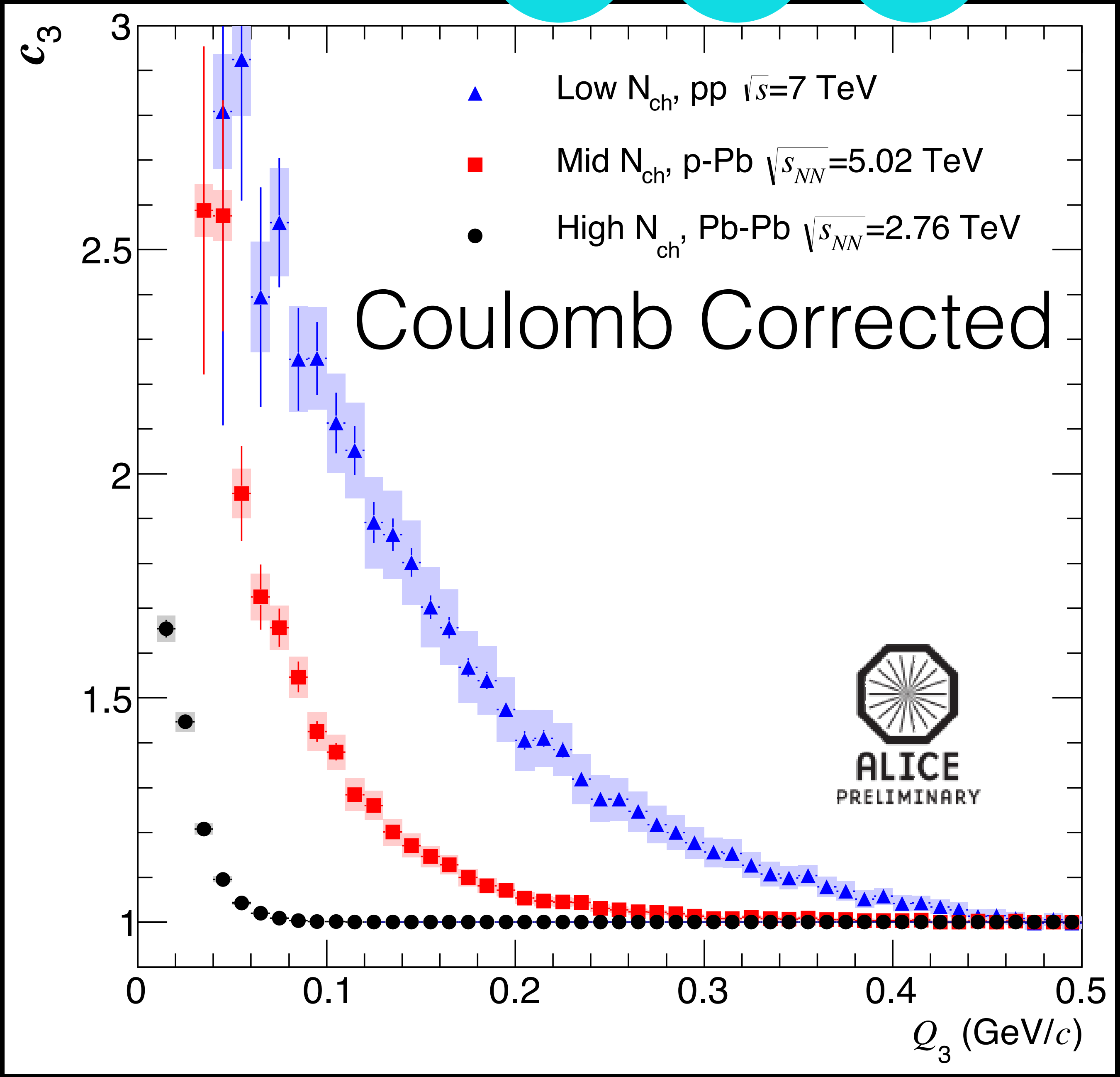
on behalf of the ALICE collaboration
IS 2014, Dec. 5th 2014

Femtoscopy (10^{-15})

The study of particle correlations at low relative momentum

3-pion Cumulant: π^+ π^+ π^+



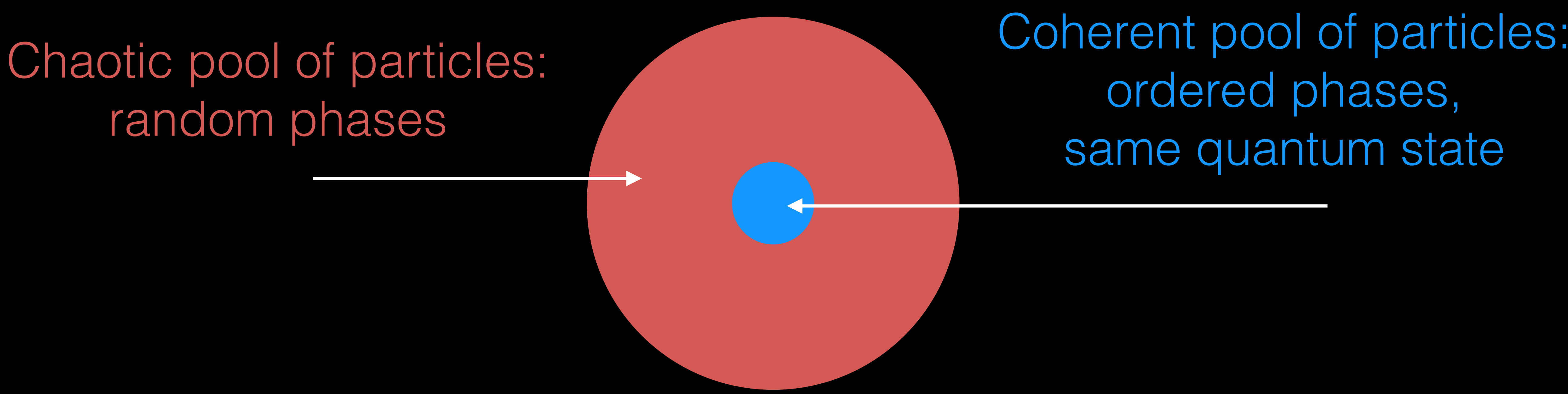
= Triplet relative momentum

Low Q region is dominated by Quantum Statistics (QS) and Coulomb correlations.

Clean region of study

3-pion cumulants are well defined.
ALICE data size allows
for 3- and 4-pion studies

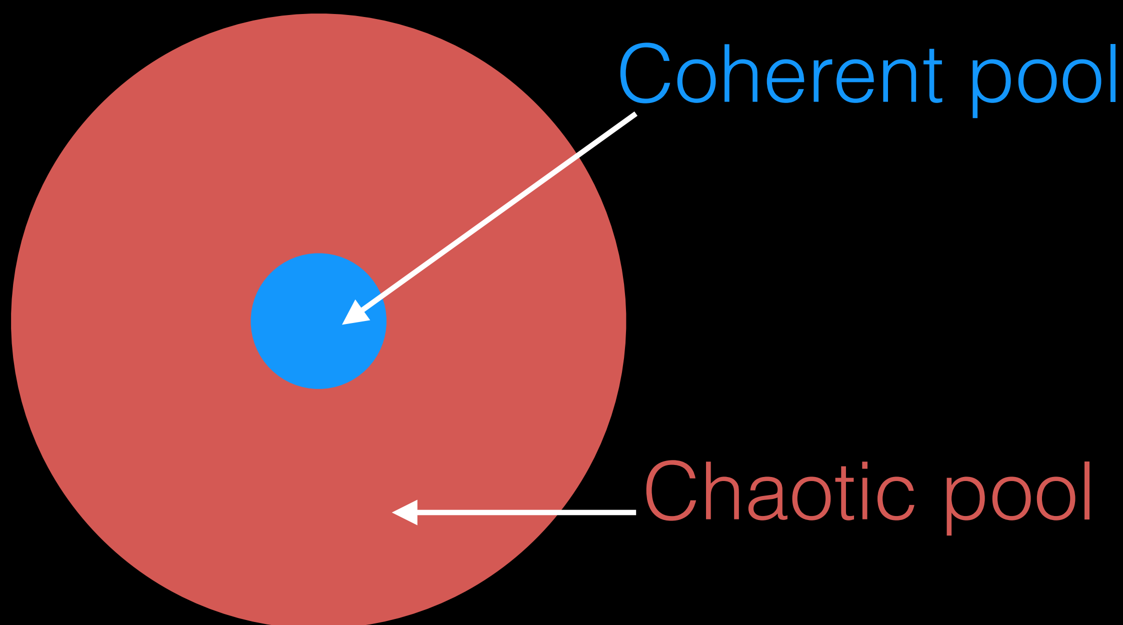
Measuring the Coherent Fraction of Pions



Pion condensation, Disoriented Chiral Condensates, +…..
may create a coherent pool of pions.

For coherence to survive in the final state,
the chaotic pool must not interact with the coherent pool.
Existence of such coherence would imply 2 disjunct sources!

2-pion Bose-Einstein Contributions



<u>2-pions</u>

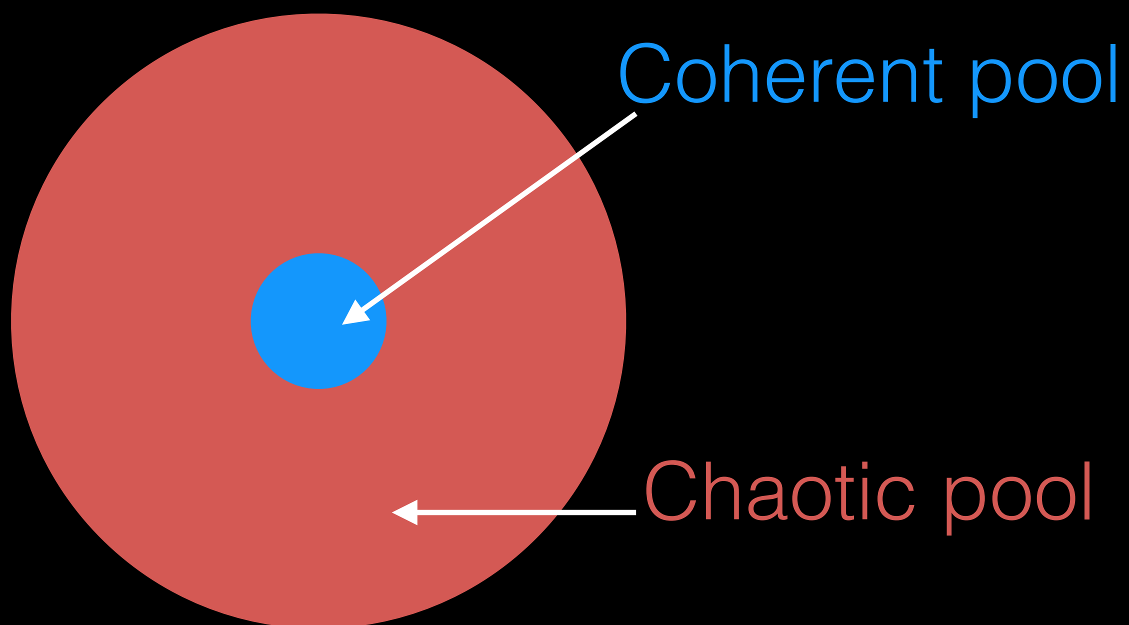
$\pi\ \pi$

$\pi\ \pi$

$\cancel{\pi\ \pi}$

1 suppressed combination

3-pion Bose-Einstein Contributions



3-pions

$\pi\ \pi\ \pi$

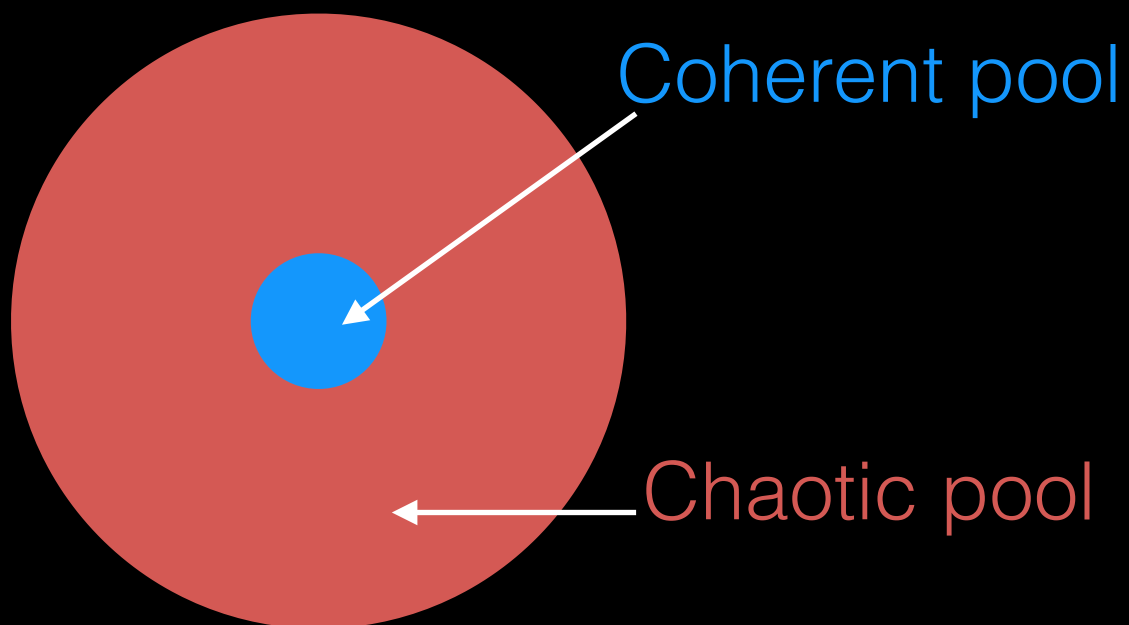
$\pi\ \pi\ \pi$

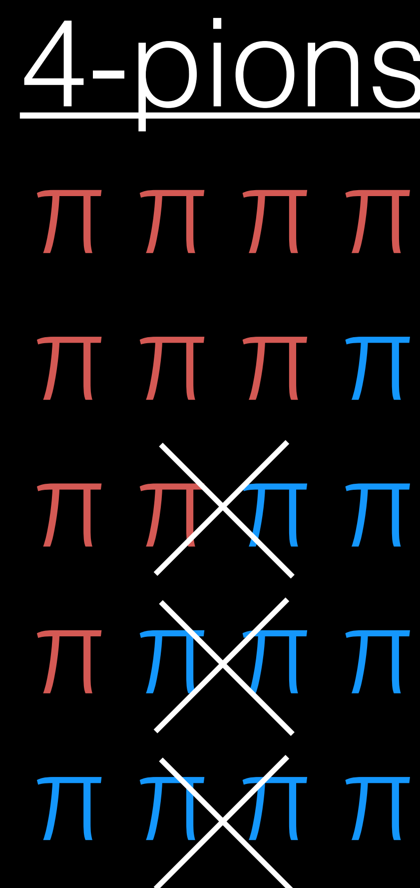
$\pi\ \cancel{\pi}\ \pi$

$\pi\ \cancel{\pi}\ \pi$

2 suppressed combinations

4-pion Bose-Einstein Contributions





3 suppressed combinations

Resolution of coherence increases with the number of pions used.

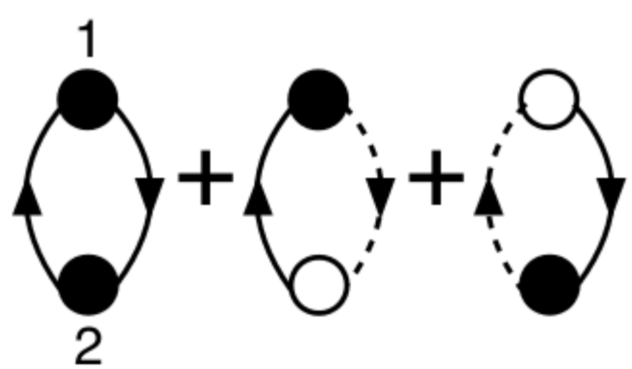
Pair Exchange Amplitude
— Building Blocks of Bose-Einstein Correlations

● = 1-G (chaotic pool)

○ = G (coherent pool)

$i \longrightarrow j \;=\; \underline{T_{ij} e^{i\Phi_{ij}}}$

$i \dashrightarrow j \;=\; \underline{t_{ij} e^{i\phi_{ij}}}$

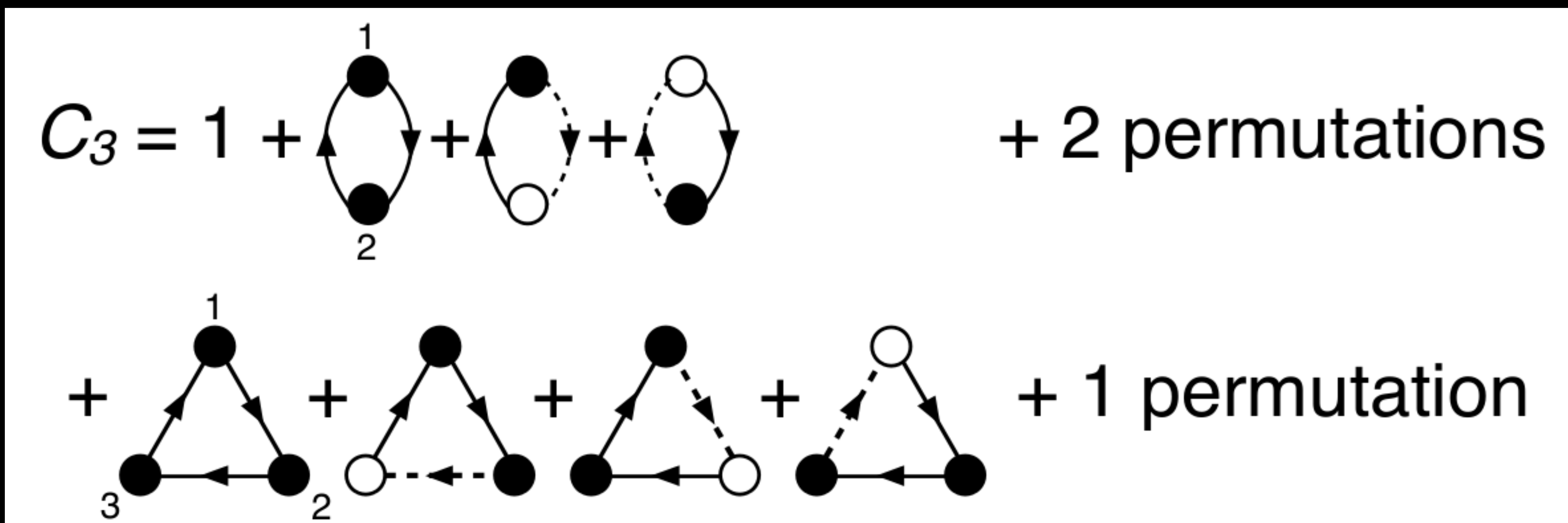
$C_2 = 1 +$ 

G = coherent fraction of pions

Pair exchange amplitudes:
Fourier Transform of source space-time distribution.
It is the building-block of all orders of Bose-Einstein correlations.

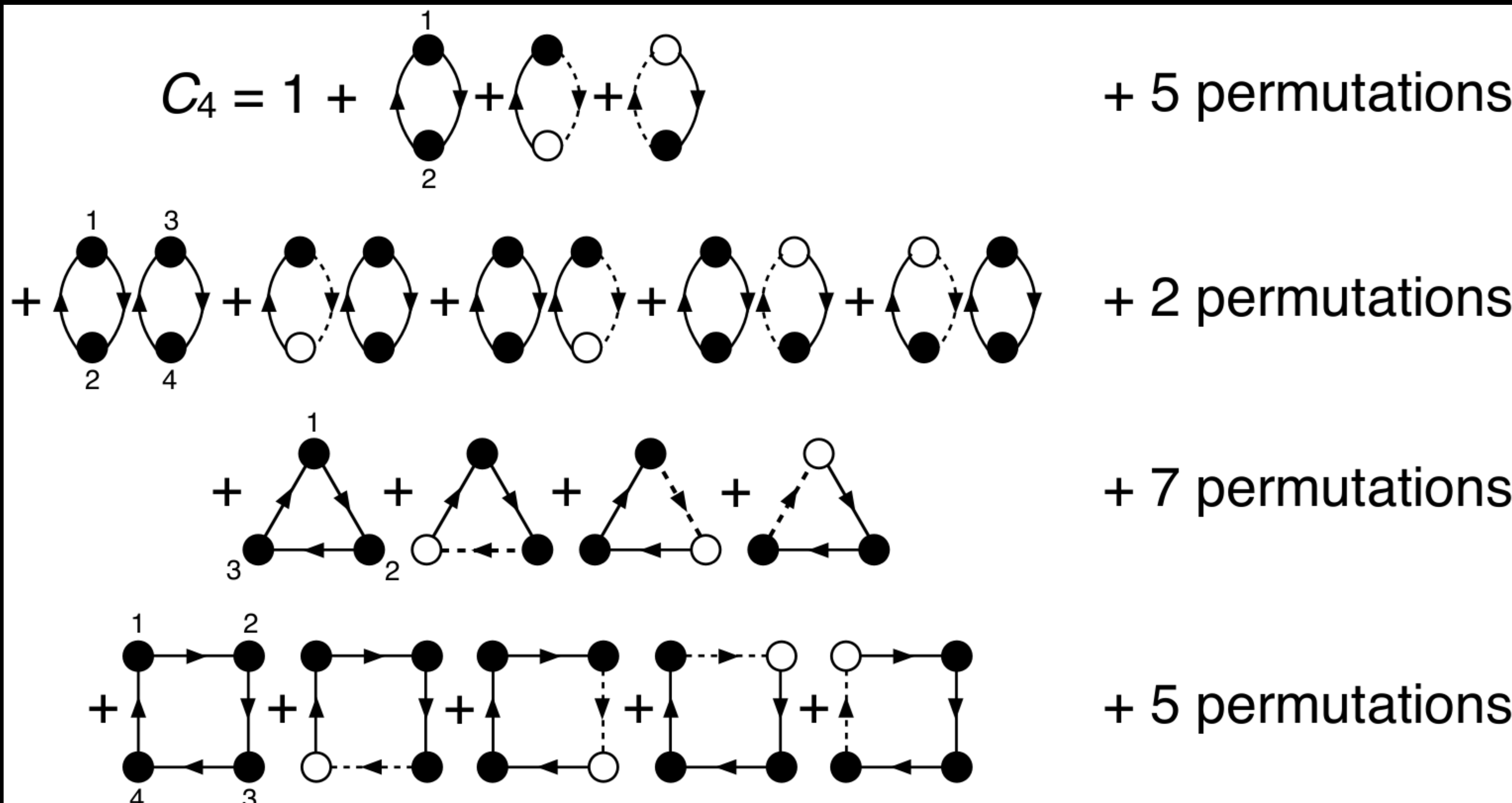
Diagrams derived from
T. Csorgo
*Heavy Ion Physics **15** 1-80*

3-boson Symmetrization



Diagrams derived from
T. Csorgo
*Heavy Ion Physics **15** 1-80*

4-boson Symmetrization



Diagrams derived from
T. Csorgo
Heavy Ion Physics ***15*** *1-80*

Standard Correlation Functions

$$C_n = \frac{N_n(\mathbf{p_1}, \mathbf{p_2}, ..., \mathbf{p_n})}{N_1(\mathbf{p_1})N_1(\mathbf{p_2})...N_1(\mathbf{p_n})}$$

p = momentum

<u>Projection Variables</u>

$$q_{ij} = \sqrt{-(p_i - p_j)_\mu (p_i - p_j)^\mu}$$

$$k_T = |\vec{p}_{\mathrm{T}1} + \vec{p}_{\mathrm{T}2}|/2$$

$$Q_3 = \sqrt{q_{12}^2 + q_{13}^2 + q_{23}^2}$$

$$K_{T,3} = |\vec{p}_{\mathrm{T}1} + \vec{p}_{\mathrm{T}2} + \vec{p}_{\mathrm{T}3}|/3$$

$$Q_4 = \sqrt{q_{12}^2 + q_{13}^2 + q_{14}^2 + q_{23}^2 + q_{24}^2 + q_{34}^2}$$

$$K_{T,4} = |\vec{p}_{\mathrm{T}1} + \vec{p}_{\mathrm{T}2} + \vec{p}_{\mathrm{T}3} + \vec{p}_{\mathrm{T}4}|/4$$

Coherence Measurements from 3-pion Cumulants

First measurement of coherent fraction and 3-pion phase in ALICE: r_3

A comparison of 3-pion to 2-pion Bose-Einstein correlation strengths

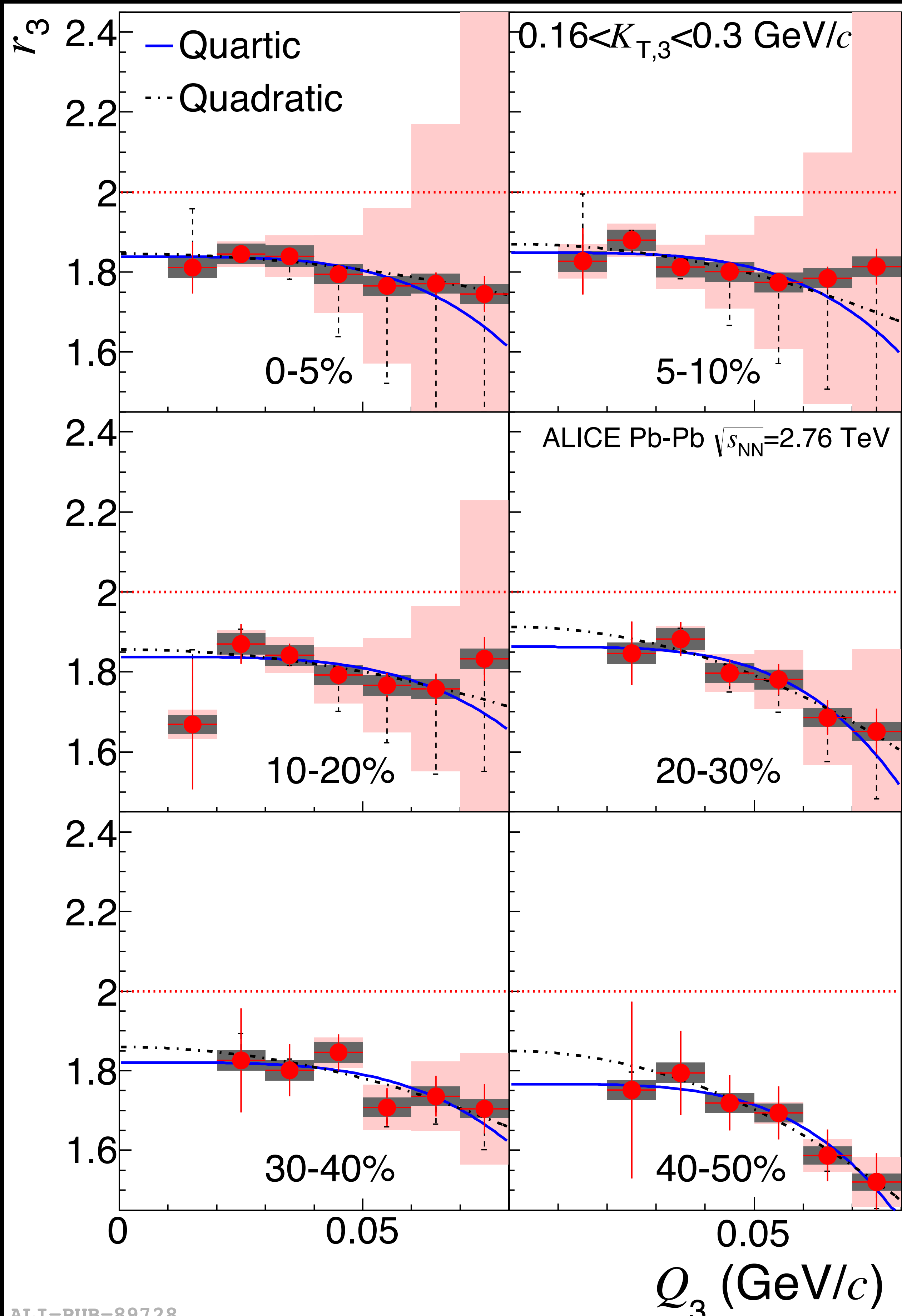
$$r_3(Q_3) = \frac{c_3(q_{12}, q_{23}, q_{31}) - 1}{\sqrt{(C_2(q_{12}) - 1)(C_2(q_{13}) - 1)(C_2(q_{23}) - 1)}}$$

$$= \underbrace{I(G)}_{\text{Intercept}} \underbrace{\cos(\Phi_{12} + \Phi_{23} + \Phi_{31})}_{\text{3-pion phase}}$$

$r_3(0) = 2.0$ for no coherence

$r_3(Q_3) = 2.0$ additionally for no 3-pion phase

Heinz & Zhang
*PRC **56** 426*



r_3 for 6 centrality bins in Pb-Pb

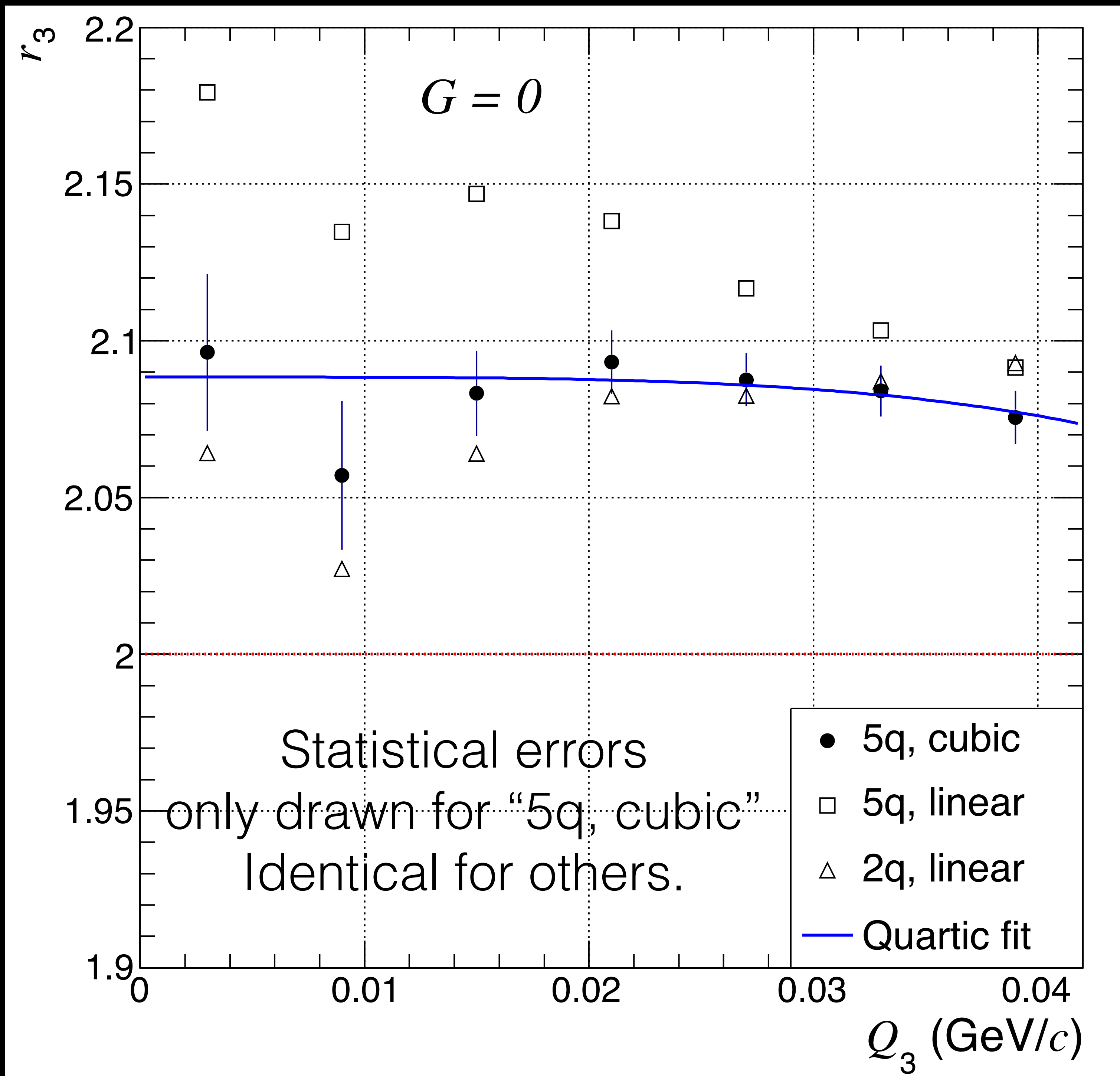
$$r_3(Q_3) = \frac{c_3(q_{12}, q_{23}, q_{31}) - 1}{\sqrt{(C_2(q_{12}) - 1)(C_2(q_{13}) - 1)(C_2(q_{23}) - 1)}}$$

All correlations are first Coulomb corrected.

r_3 is suppressed below 2.0. Intercept corresponds to **23% ± 8% coherence at low p_T.**

ALICE
PRC 89 024911 (2014)

r_3 Calculation in Therminator



Therminator 2 model:
Kisiel et al.,
Comput. Phys. Commun. 174, 669 (2006)

Thermintor model calculation without coherence (G=0).

No Q_3 dependence in this model.
= No effect of the 3-pion phase.

A upward bias is observed for each interpolation type.

5q = 5 MeV wide q bins with cubic or linear interpolation.

r_3 is ~flat in ALICE and in Therminator.
We ignore 3- and 4-pion phases
for the rest of the talk

Extracting the building block from 2-pion correlations

$$C_2 - 1 = (1-G)^2 T_{ij}^2 + 2G(1-G)T_{ij}t_{ij}$$

The coherent component can be parametrized with various assumptions.
t_{ij} can be assumed to be Gaussian with a certain radius (1 fm, 2 fm…)
G is the assumed coherent fraction (0, 2%,…, 50%).

T_{ij} can then be extracted and used to build higher order correlation functions.

Extracting the building block from 3-pion cumulants

$$
\begin{aligned}
\underline{T_{ij}} &= s e^{-R^2 q_{ij}^2/2} E_{\mathrm{w}}(R q_{ij}) \\
c_3(q_{12}, q_{23}, q_{31}) &= 1 + 2 T_{12} T_{23} T_{31} \\
&= 1 + 2 s^3 e^{-R^2 (q_{12}^2 + q_{23}^2 + q_{31}^2)/2} E_{\mathrm{w}}(R q_{12}) E_{\mathrm{w}}(R q_{23}) E_{\mathrm{w}}(R q_{31})
\end{aligned}
$$

- The Exchange amplitude can be parametrized with an Edgeworth expansion.
- One fits c_3 with the above parametrization.
- The extracted fit coefficients can then be used to build higher order correlation functions.

Edgeworth expansion:
Csorgo & Hegyi,
Phys. Lett. B 489, 15 (2000)

$$
E_{\mathrm{w}}(R_{\mathrm{inv}}\, q) = 1 + \sum_{n=3}^{\infty} \frac{\kappa_n}{n! (\sqrt{2})^n} H_n(R_{\mathrm{inv}}\, q)
$$

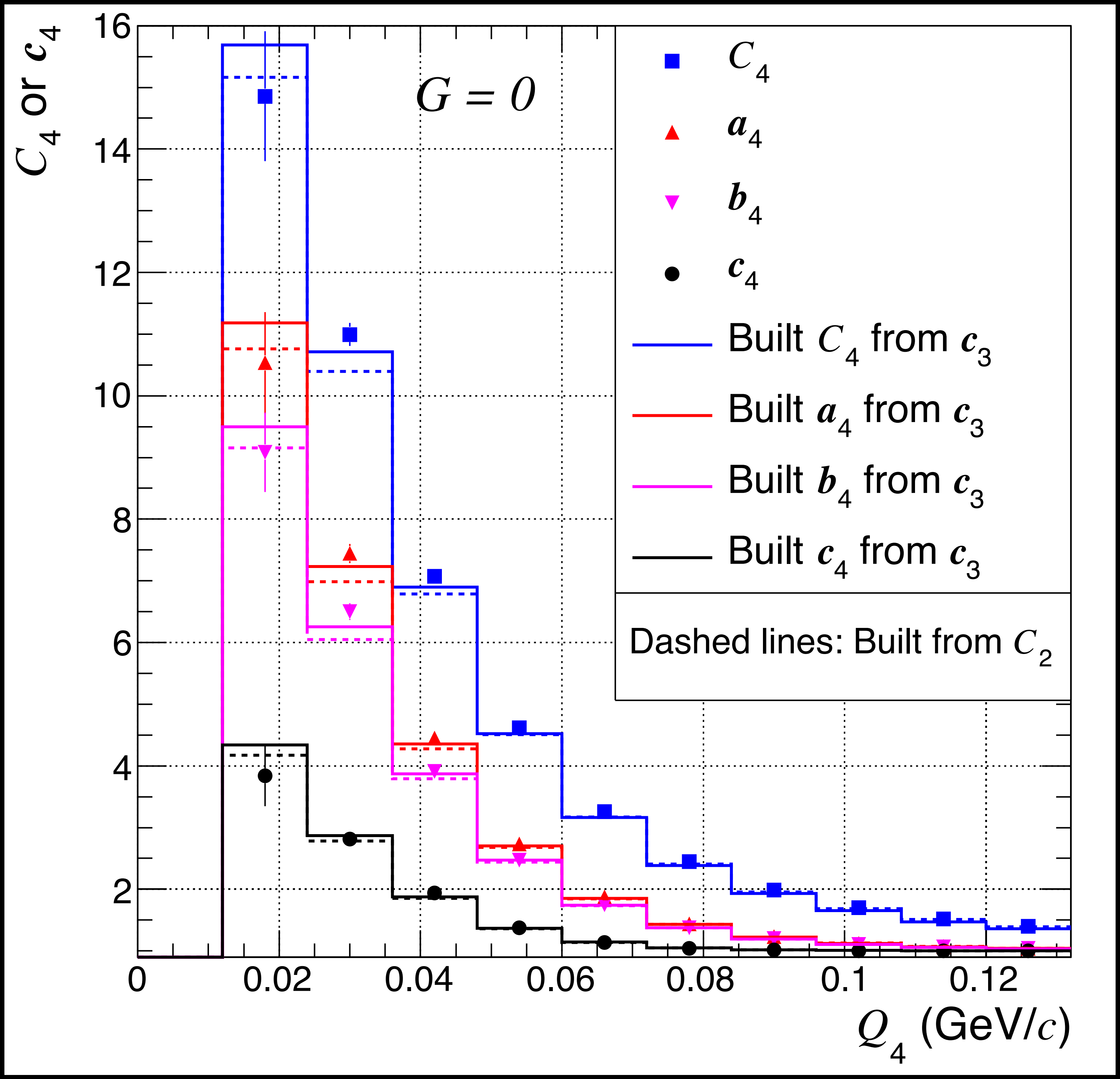
Equations to Build QS correlations with coherence

With the assumption of identical chaotic and coherent emission functions: $T_{ij} = t_{ij}$

$$
\begin{aligned}
C_2^{QS} - 1 &= (1-G^2)T_{12}^2 && (52) \\
C_3^{QS} - 1 &= (1-G)^2(T_{12}^2 + T_{13}^2 + T_{23}^2) && (53) \\
&+ (6G(1-G)^2 + 2(1-G)^3)T_{12}T_{13}T_{23} && (54) \\
C_4^{QS} - 1 &= (1-G^2)(T_{12}^2 + T_{13}^2 + T_{14}^2 + T_{23}^2 + T_{24}^2 + T_{34}^2) && (55) \\
&+ (4G(1-G)^3 + (1-G)^4(T_{12}^2 T_{34}^2 + T_{13}^2 T_{24}^2 + T_{14}^2 T_{23}^2) && (56) \\
&+ (6G(1-G)^2 + 2(1-G)^3)(T_{12}T_{13}T_{23} + T_{12}T_{14}T_{24} + T_{13}T_{14}T_{34} + T_{23}T_{24}T_{34}) && (57) \\
&+ (8G(1-G)^3 + 2(1-G)^4)(T_{12}T_{13}T_{24}T_{34} + T_{12}T_{14}T_{23}T_{34} + T_{13}T_{14}T_{23}T_{24}) && (58)
\end{aligned}
$$

Proof of Principle

Therminator, 0-5% Pb-Pb



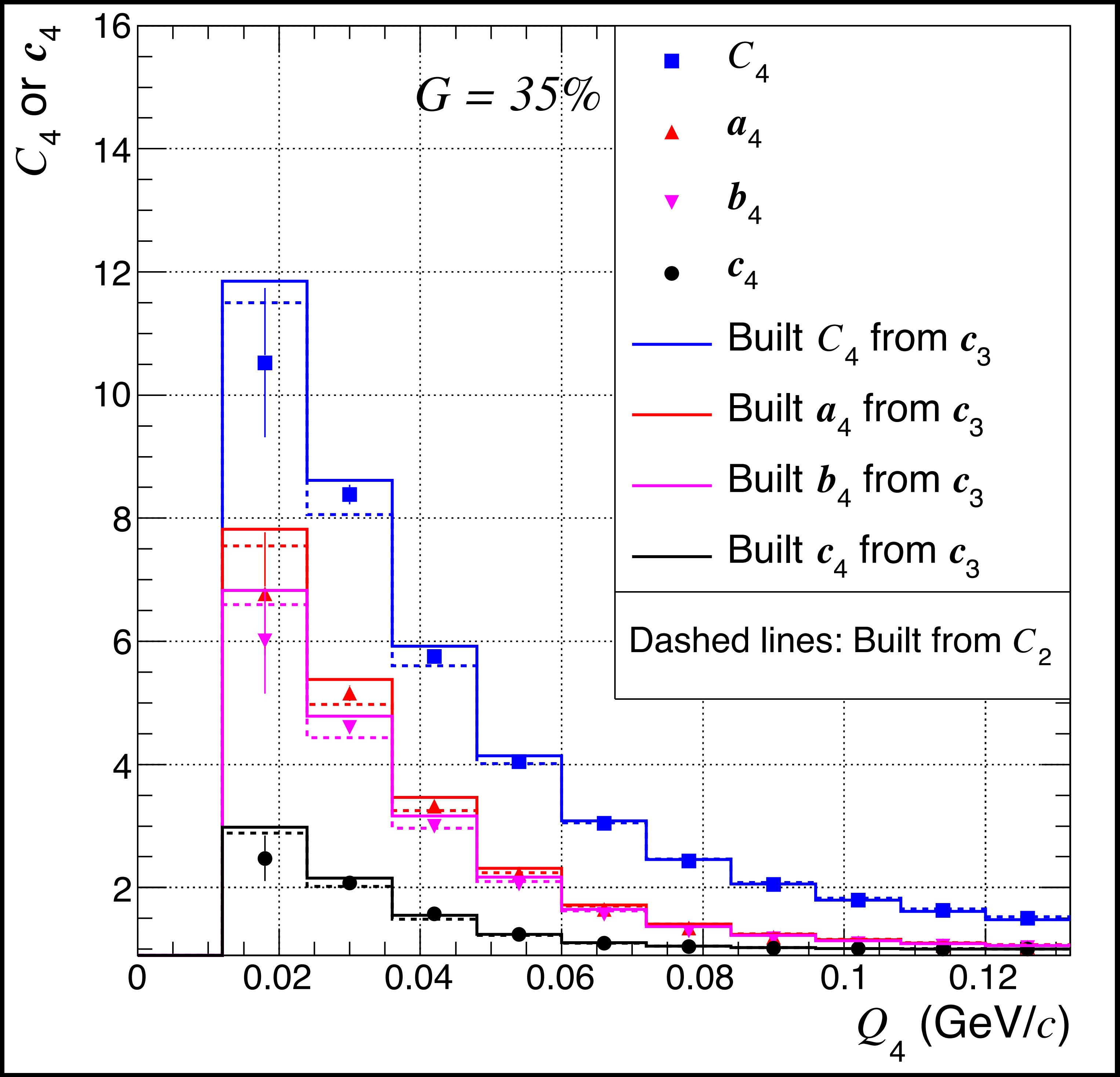
Therminator model calculation without coherence.

$a_4 = C_4$ - 2-pion symmetrizations.
$b_4 = C_4$ - 2-pion - 2-pair symmetrizations.
$c_4 =$ only 4-pion symmetrizations (cumulant)

Built correlations are within 3% of measured ones.

Proof of Principle

Therminator, 0-5% Pb-Pb

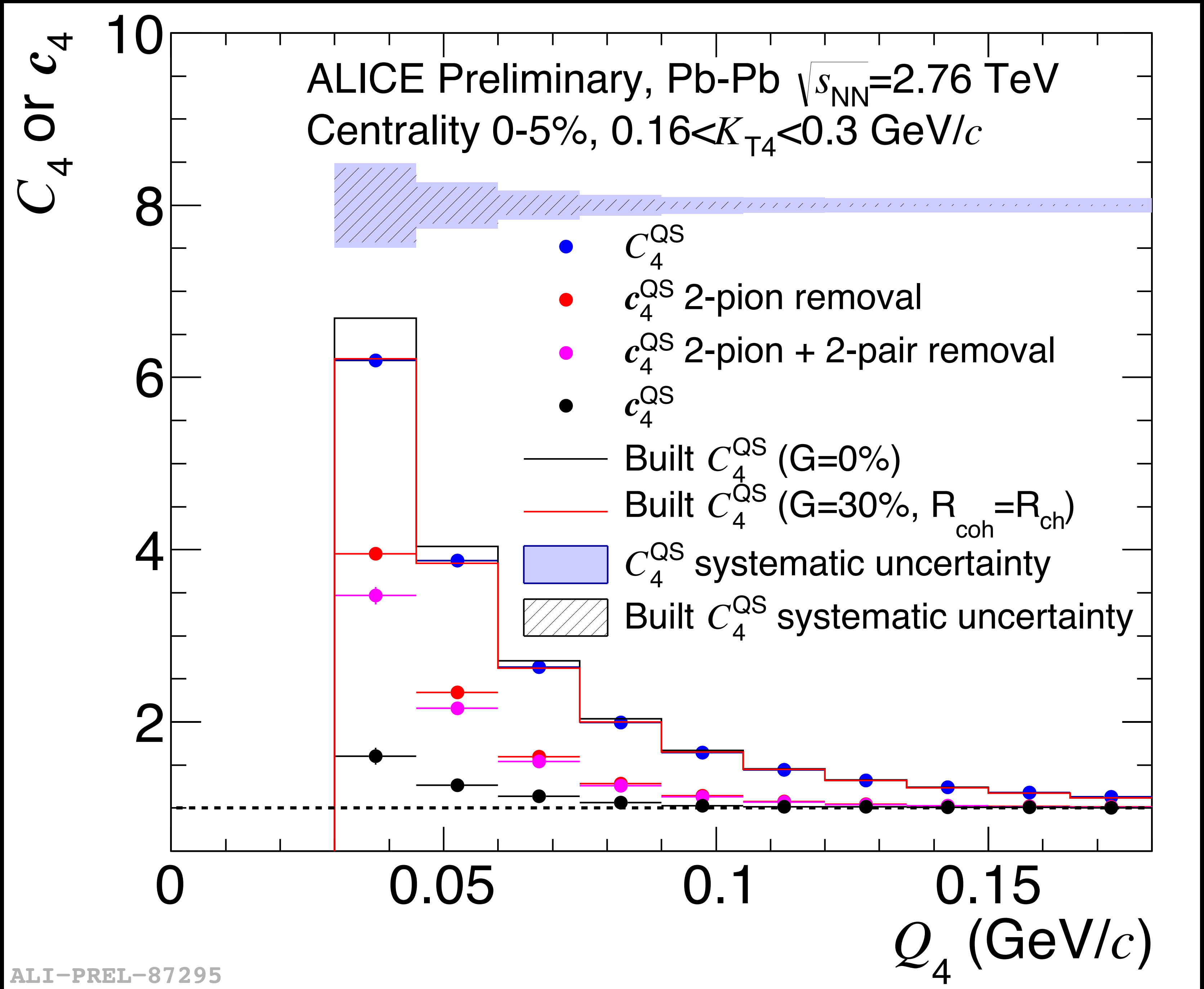


Therminator model calculation with G = 35%, 1 fm Gaussian coherent component.

$a_4 = C_4$ - 2-pion symmetrizations.
$b_4 = C_4$ - 2-pion - 2-pair symmetrizations.
c_4 = only 4-pion symmetrizations (cumulant)

Built correlations are within 5% of measured ones.

ALICE Measurement: 4-pion Bose-Einstein Correlation



Extracted coherent fractions are again non-zero at low p_T:
~30% for $R_{coh} = R_{ch}$
~15% for $R_{coh} = 0$

Signal is significantly smaller at higher p_T

Summary

Quantum Coherence at Freeze-out:

- Our results indicate that 15-30% of charged pions may be coherent at freeze-out.
- First seen with the 3-to-2 comparison (r_3).
- Confirmed with the 4-to-2 comparison.
- Ongoing work to check 4-to-3 comparison.
- Ongoing work to extract coherent fractions in pp and p-Pb.
- Ongoing estimate of multi-Boson distortions which may provide an alternative explanation to the observed suppressions.

Survival of partial coherence would imply:
→2 disjunct particle-emitting sources!
→Non-equilibrium phenomenon

Supporting Slides

Supporting ALICE Publications

"Two- and three-pion quantum statistics correlations in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV at the CERN Large Hadron Collider"
Phys. Rev. C **89** 024911 (2014)

"Freeze-out radii extracted from three-pion cumulants in pp, p-Pb and Pb-Pb collisions at the LHC"
Phys. Lett. B. **739** 139 (2014)

Data and Track Selection

Collision type

Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV

Track Selection

• **Pions** selected based on their specific energy loss in the Time Projection Chamber. Time of Flight also used for p > 0.6 GeV/c.

• p_T > 0.16 GeV/c

• p < 1.0 GeV/c

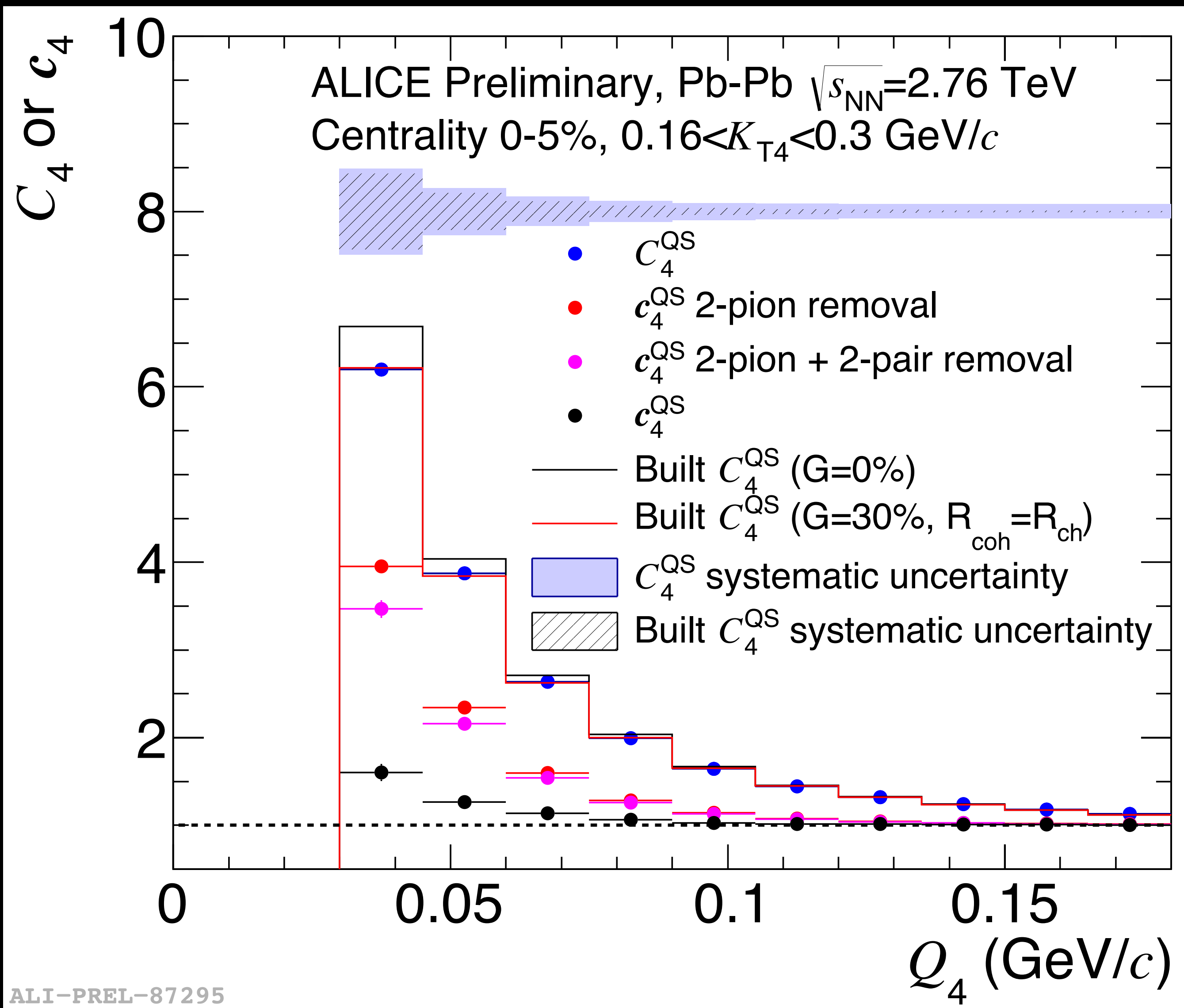
• $|\eta| < 0.8$

Pair Cuts

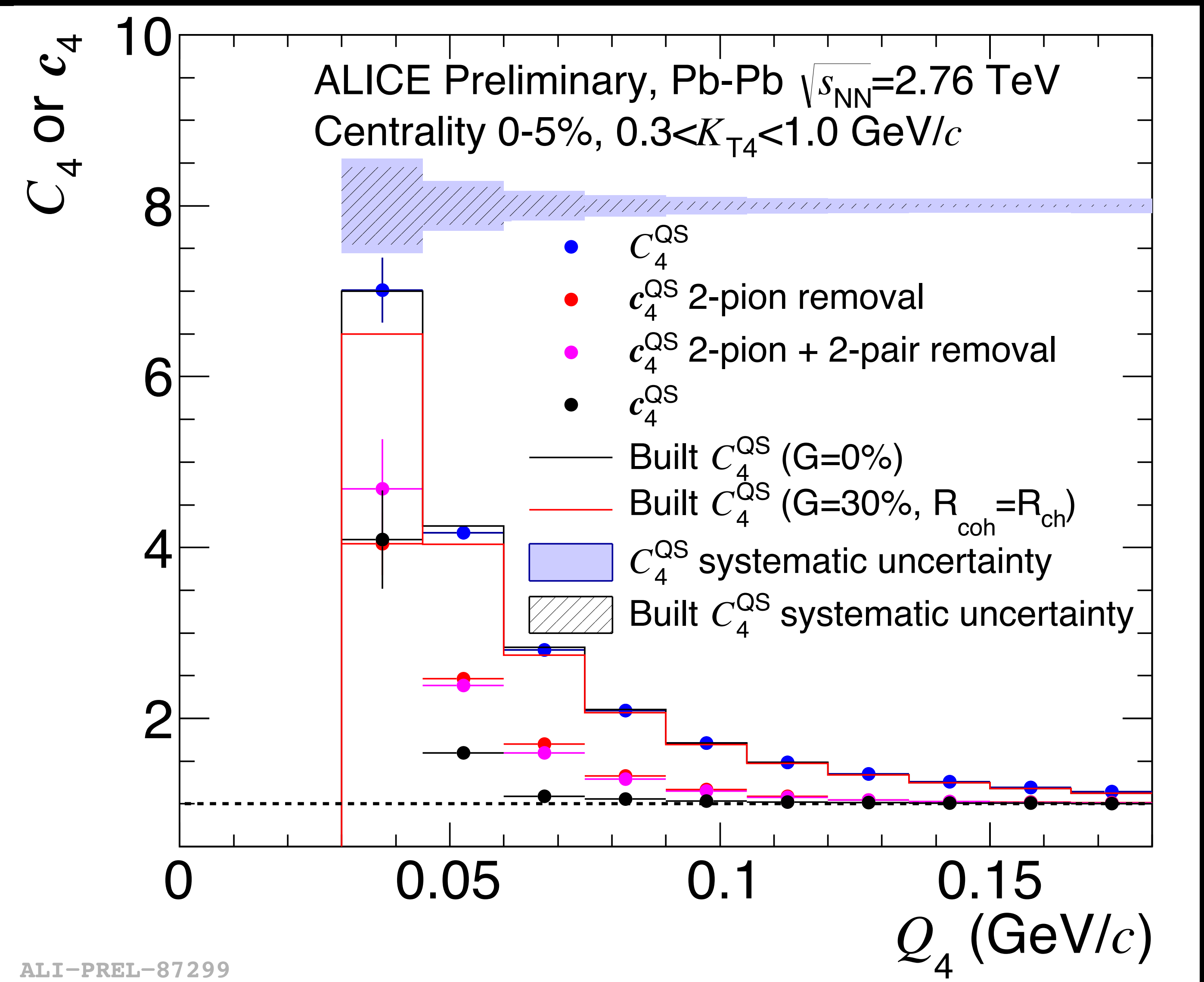
• Track merging and splitting: pair angular separation

• For 3 (4) pions, pair cuts applied to all 3 (6) pairs in the triplet (quadruplet).

Low K_{T4} (low p_T)

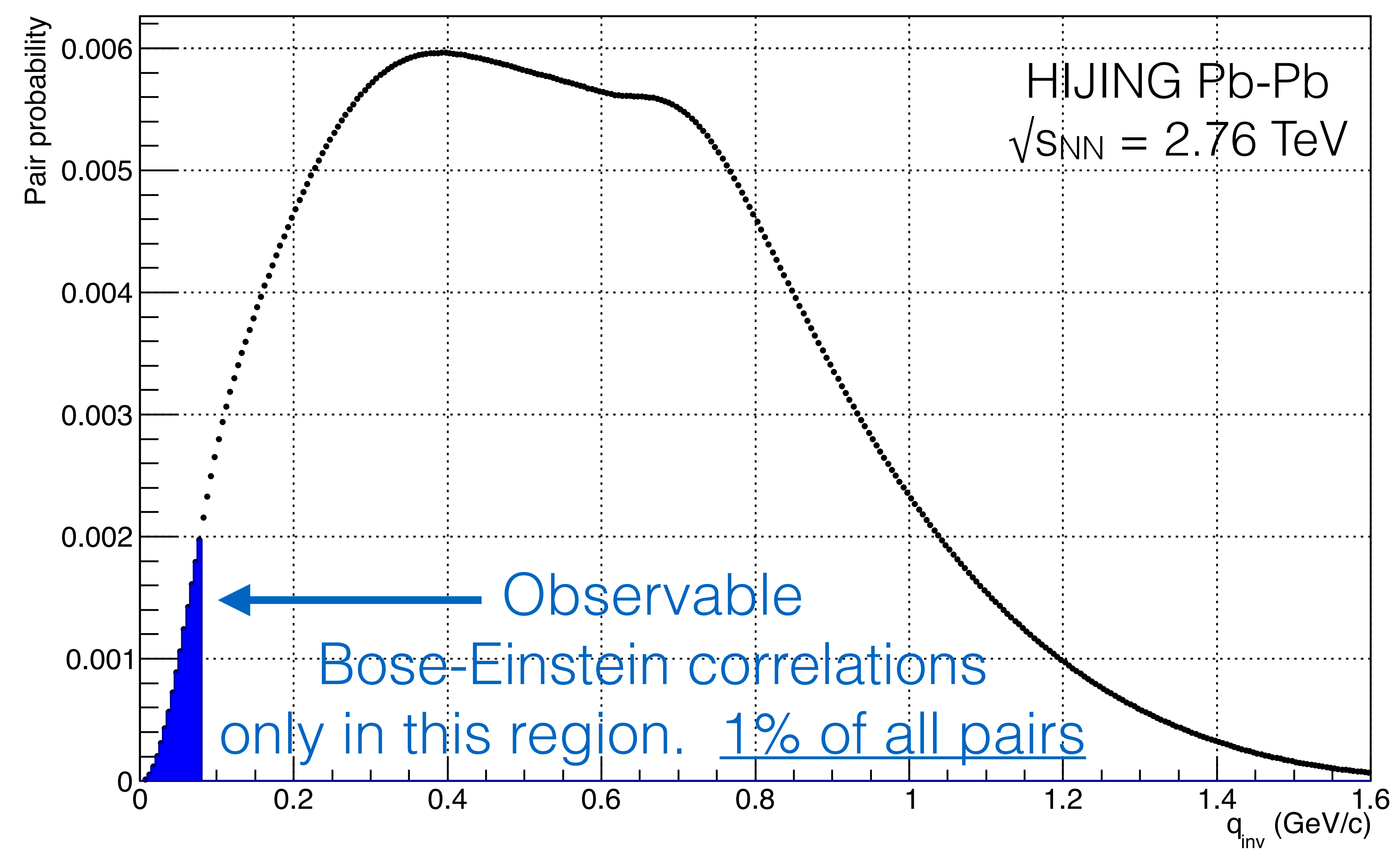


High K_{T4} (high p_T)



Bose-Einstein correlations are in a very narrow region of phase-space

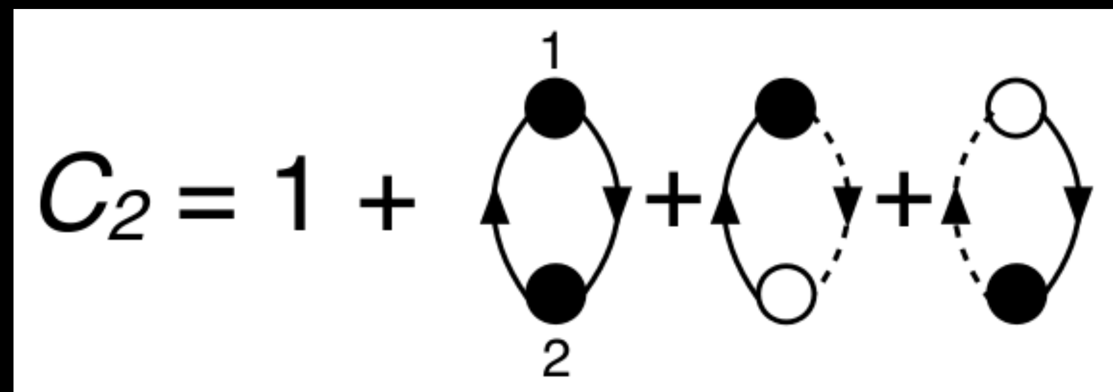
$$\Delta x \Delta p \leq 2\pi\hbar$$



$$q_{\text{inv}} = \sqrt{(\vec{p}_1 - \vec{p}_2)^2 - (E_1 - E_2)^2}$$

HIJING
Wang & Gyulassy
PRD **44** 3501

2-boson Symmetrization



Diagrams derived from
T. Csorgo
Heavy Ion Physics ***15*** *1-80*

$$
\begin{aligned}
C_2(1,2) \quad &= \quad 1 + (1-G)^2 (T_{12}^{\mathrm{ch}})^2 \\
&+ \quad 2G(1-G) T_{12}^{\mathrm{ch}} T_{12}^{\mathrm{coh}} \cos(\phi_{12}^{\mathrm{ch-coh}})
\end{aligned}
$$

phase of chaotic-coherent interference.

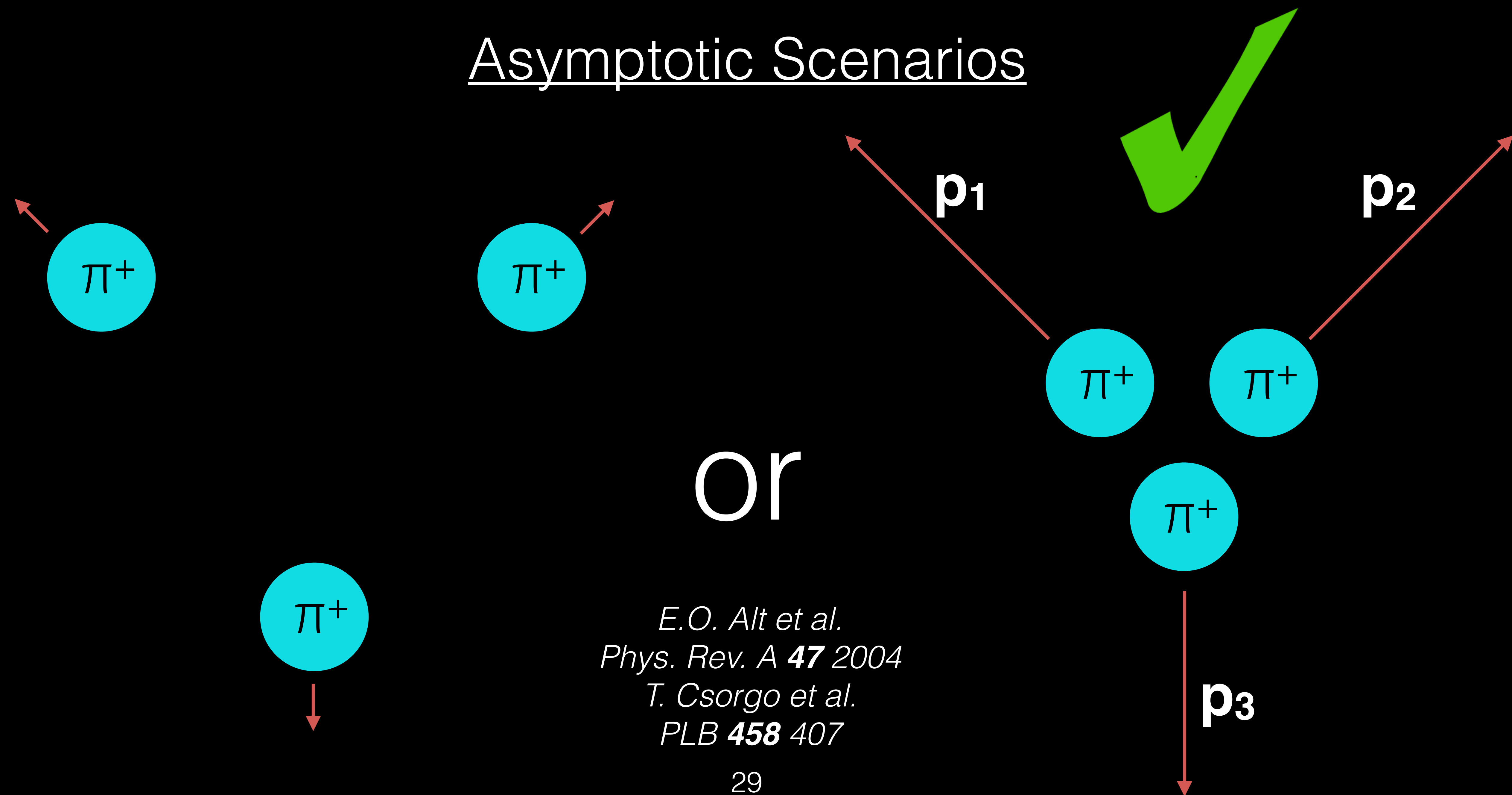
coherent fraction of pions

Equations derived from
I. Andreev et al.
Int. J. Mod. Phys. A ***8*** *4577*

Multi-Pion Coulomb Interaction

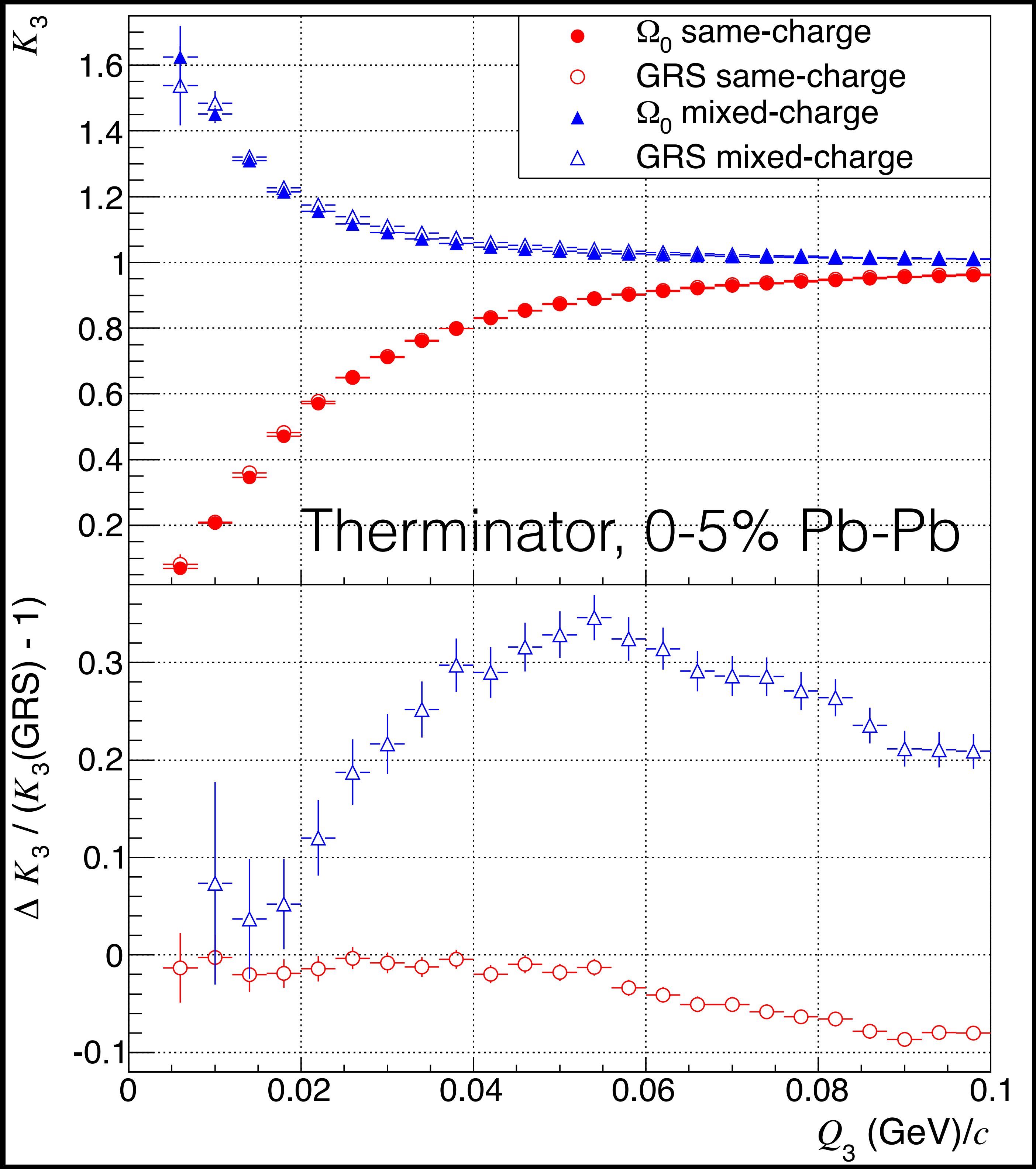
Multi-body Coulomb wave-functions are not known exactly. However, asymptotic solutions exist which are applicable to high-energy collisions.

<u>Asymptotic Scenarios</u>



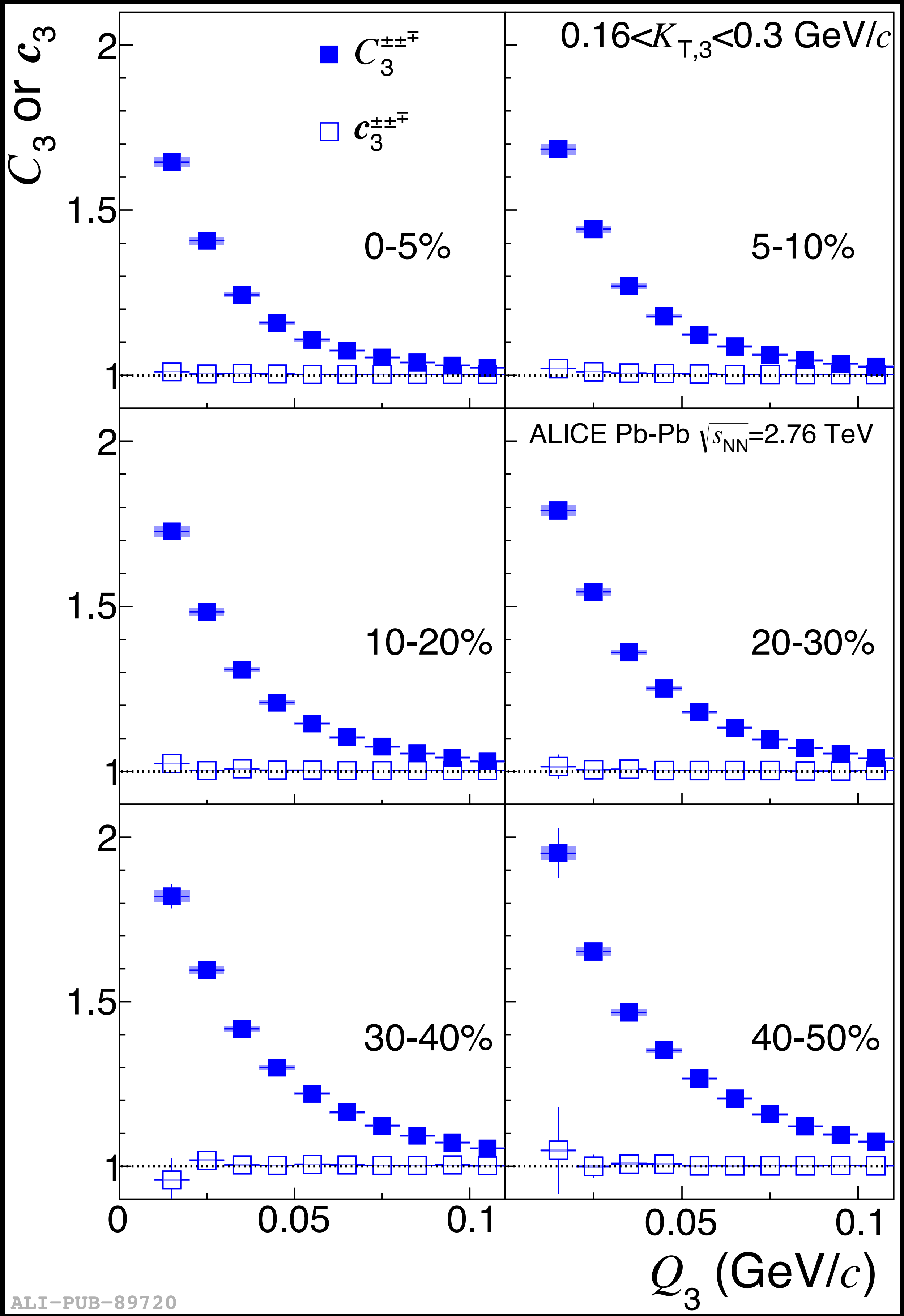
E.O. Alt et al.
Phys. Rev. A ***47*** *2004*
T. Csorgo et al.
PLB ***458*** *407*

Near Equivalence between 2 types of Coulomb Calculations

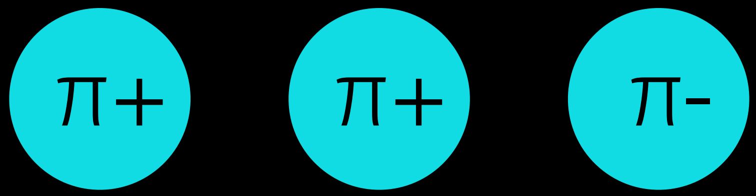


Ω_0 = Full Asymptotic wave-function calculation.

GRS = Generalized Riverside $= K_{12}K_{13}K_{23}$



Check that 3-body Coulomb corrections work

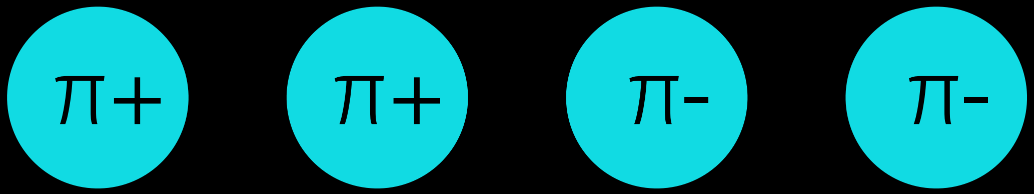


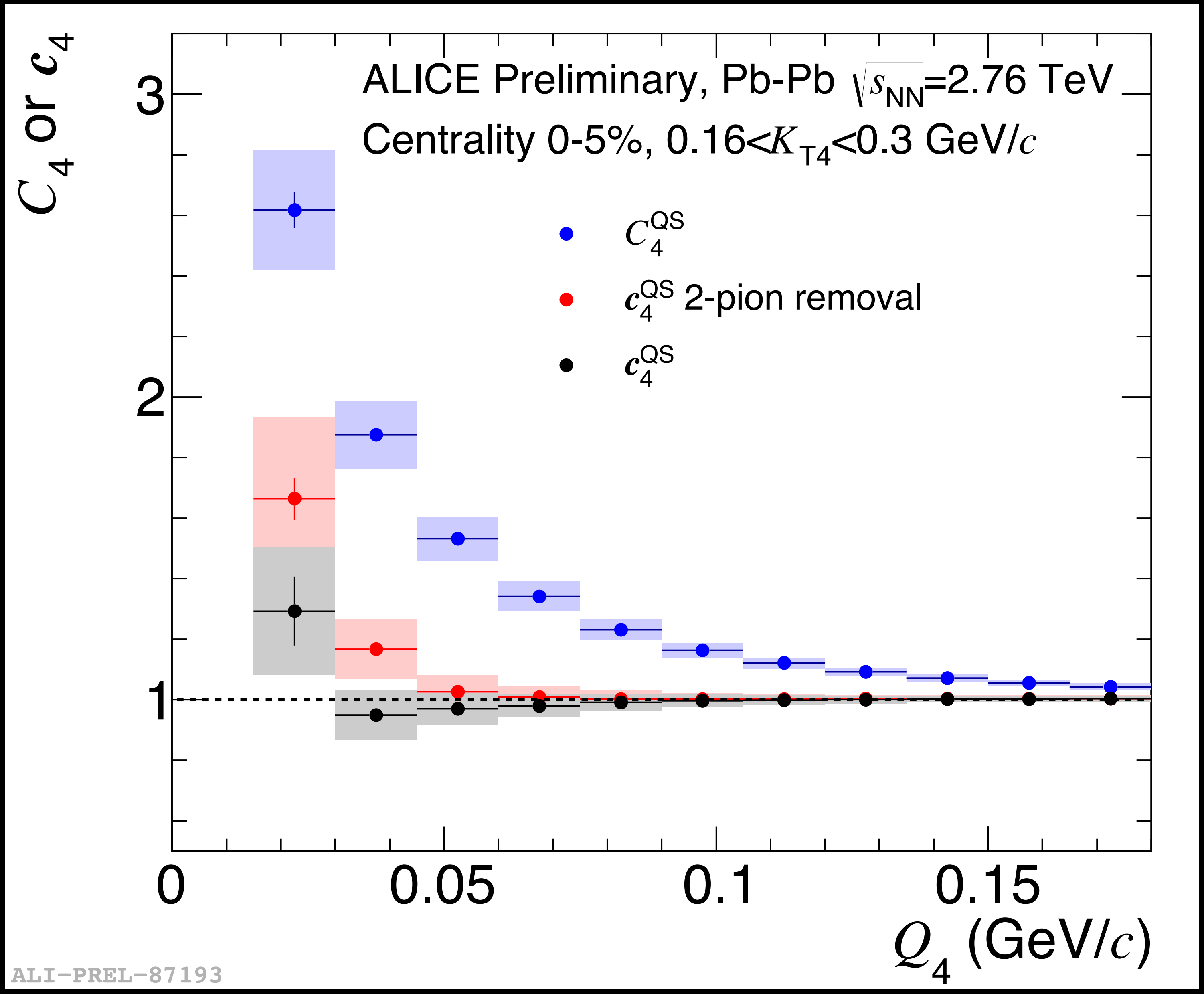
The cumulant (hollow points) are Coulomb corrected.

Consistency with unity demonstrates success of 3-body Coulomb ansatz

ALICE PRC 89 024911 (2014)

New: 4-pion Coulomb Check

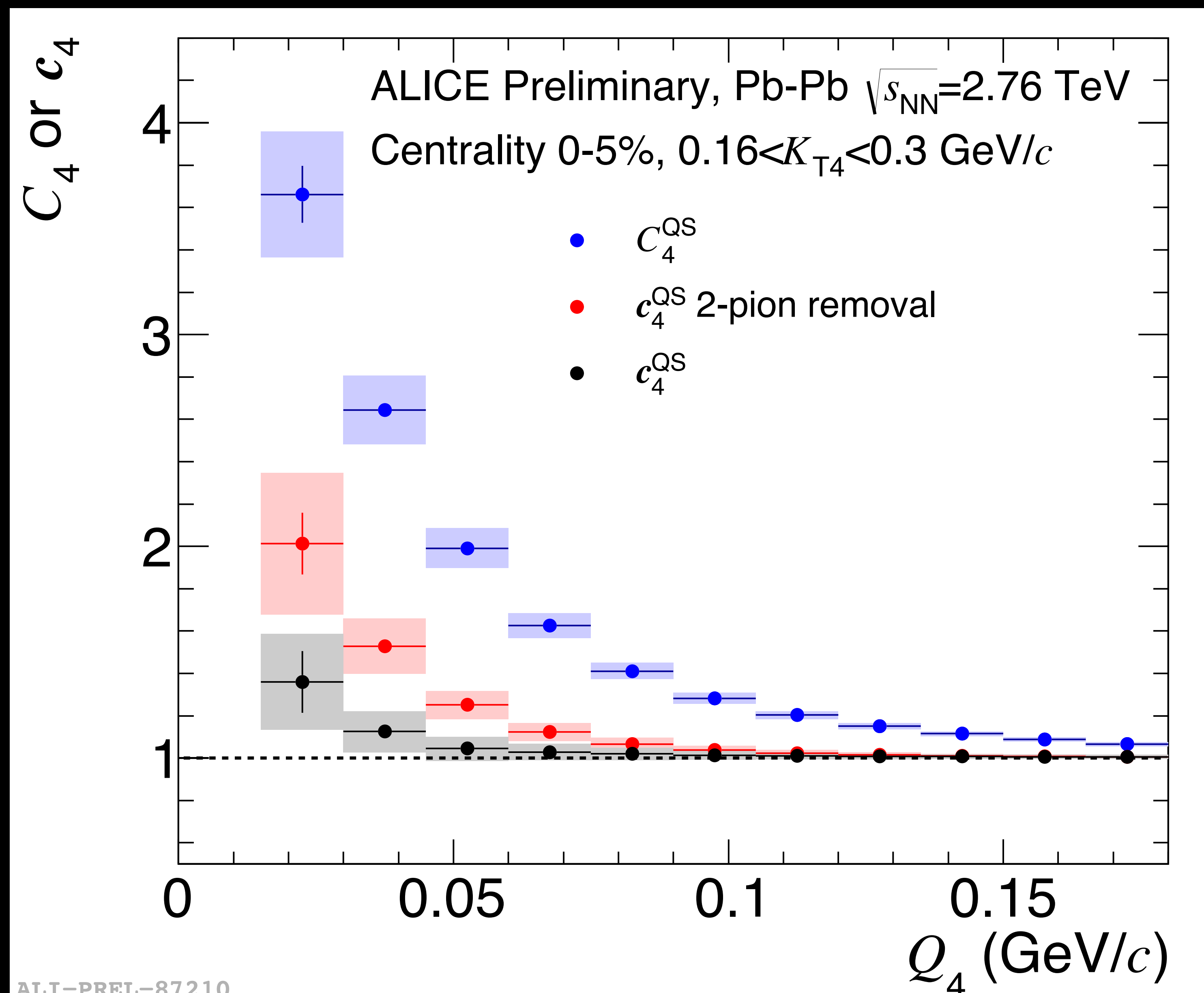




- - - + + correlation well understood. Cumulant (black) near unity.

New: 4-pion Coulomb Check

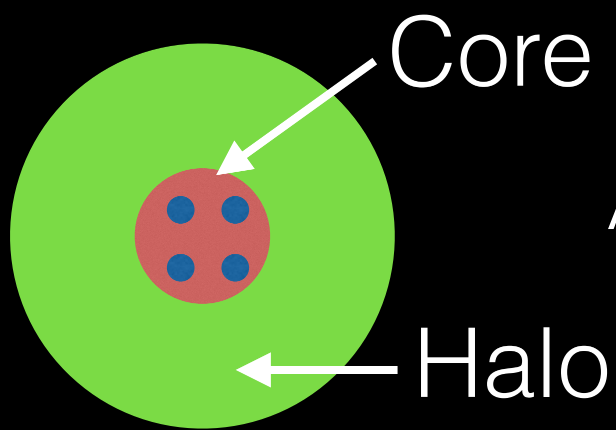
$\pi+$ $\pi+$ $\pi+$ $\pi-$



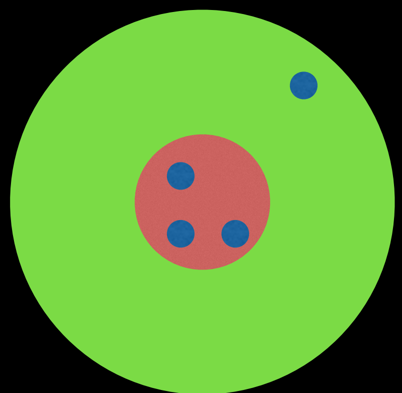
- - - - + correlation mostly understood. Cumulant (black) near unity.
- Ongoing studies in pp and p-Pb suggest that the reside is not Coulomb related.

4-pion possibilities in the Core/Halo picture

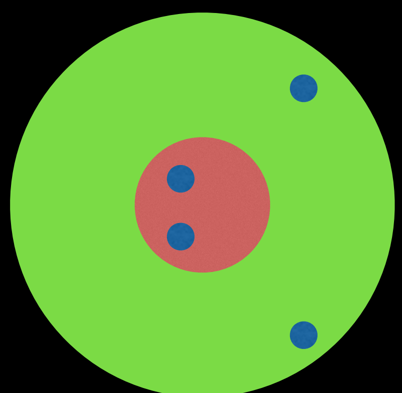
T. Csorgo et al.
Z. Phys. C ***71*** *491*



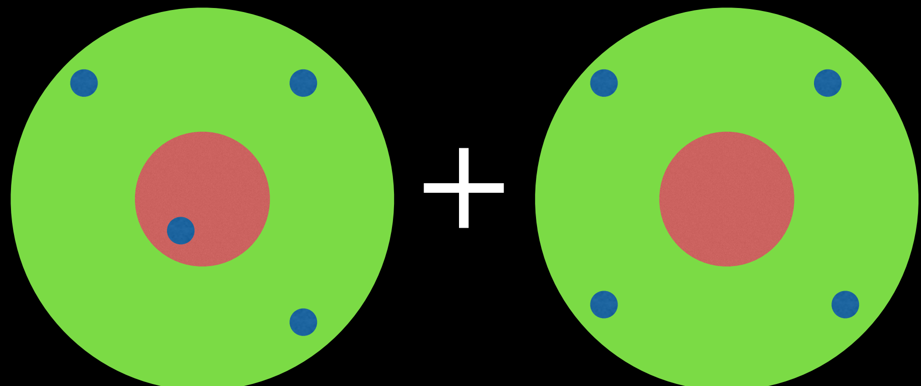
All 4 originating from the core (short-lived emitters)



3 originating from the core



2 originating from the core



1 or 0 originating from the core.
<u>No observable</u> Bose-Einstein correlations

Multi-pion Distributions

4-pion Distributions

$N_4(p_1, p_2, p_3, p_4)$ — 4 pions from same event

$N_3(p_1, p_2, p_3)\ N_1(p_4)$ — 3 pions from same event

$N_2(p_1, p_2)\ N_1(p_3)\ N_1(p_4)$ — 2 pions from same event

$N_2(p_1, p_2)\ N_2(p_3, p_4)$ — 2 pairs from same event

$N_1(p_1)\ N_1(p_2)\ N_1(p_3)\ N_1(p_4)$ — All from different events

$K_4 = K_2^{12}\ K_2^{13}\ K_2^{14}\ K_2^{23}\ K_2^{24}\ K_2^{34}$ — 4-body Final-State-Interaction

Isolation of 4-pion QS

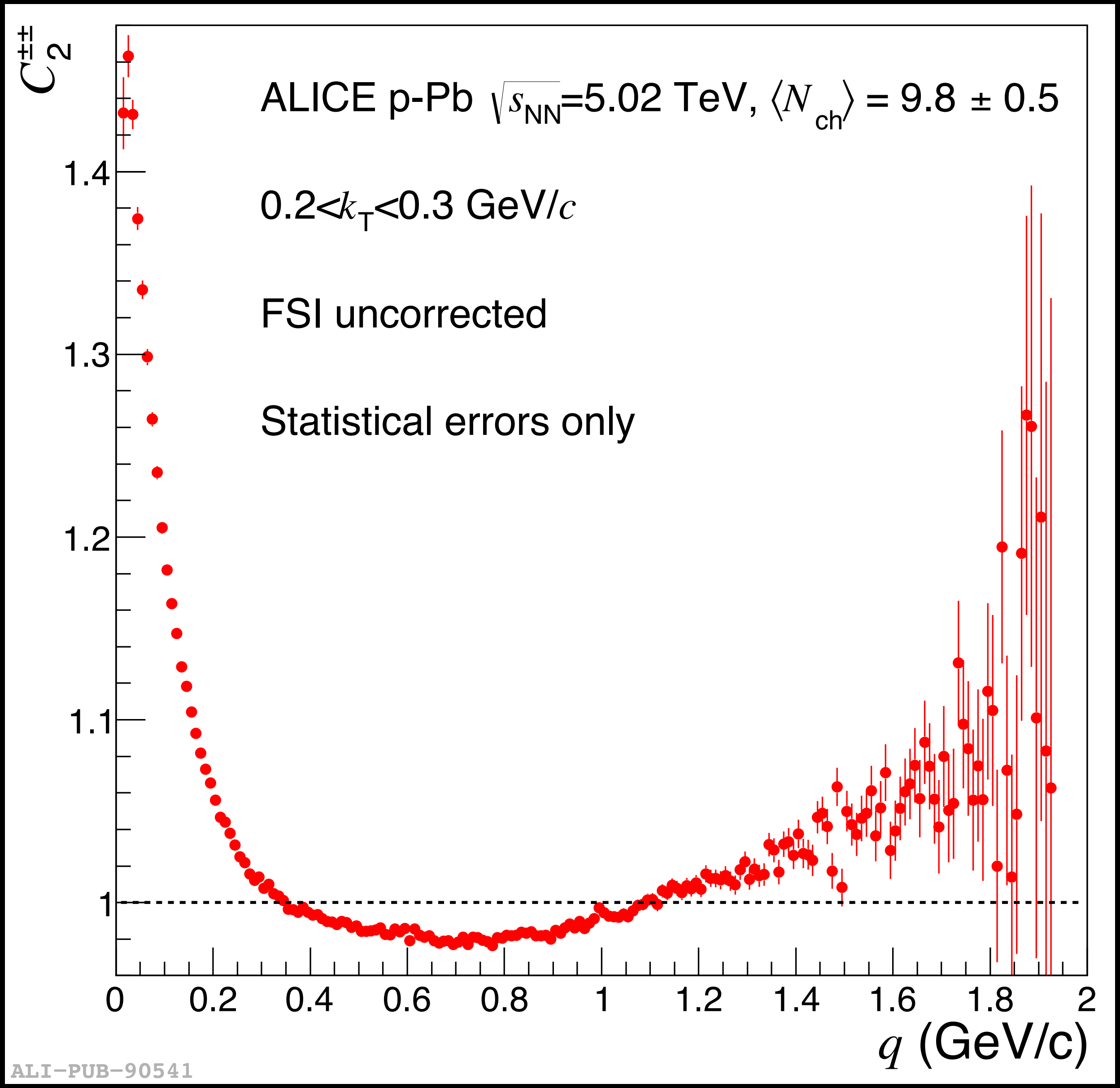
Quantity of Interest

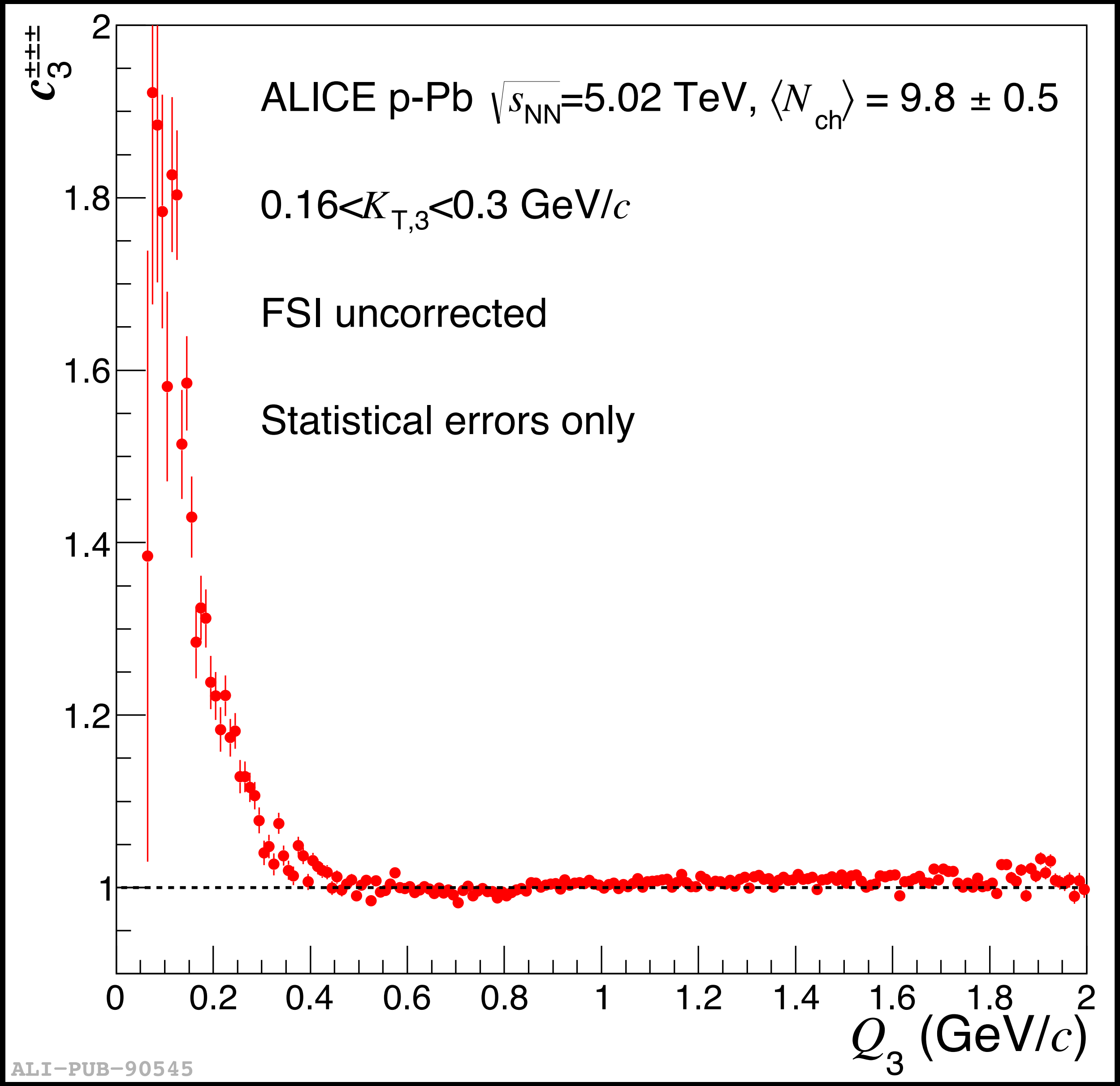
$$
\begin{aligned}
N_4(p_1,p_2,p_3,p_4) &= f_{41}N_1(p_1)N_1(p_2)N_1(p_3)N_1(p_4)\\
&+ f_{42}N_2(p_1,p_2)N_1(p_3)N_1(p_4)\\
&+ f_{43}N_3(p_1,p_2,p_3)N_1(p_4)\\
&+ f_{44}K_4(q_{12},q_{13},q_{14},q_{23},q_{24},q_{34})N_4^{QS}(p_1,p_2,p_3,p_4)
\end{aligned}
$$

$$
\begin{aligned}
f_{41}^{Core/Halo} &= -3(1-f_c)^4 - 8f_c(1-f_c)^3 + 6(1-f_c^2)(1-f_c)^2\\
f_{42}^{Core/Halo} &= -6(1-f_c)^2\\
f_{43}^{Core/Halo} &= 4(1-f_c)\\
f_{44}^{Core/Halo} &= f_c^4.
\end{aligned}
$$

f_c^2 = “lambda” = 0.7 +- 0.05 (fraction of correlated pairs)

C_2 & c_3 in an extended range





The baseline for 3-pion cumulants is more flat than for 2-pion correlations.

2-boson Symmetrization

We consider 2 extreme cases for the size of the coherent source radius

<u>Point source</u>

$R_{coh} = 0$

$$\begin{aligned} C_2(1,2) &= 1 + (1-G)^2 (T_{12}^{ch})^2 \\ &+ 2G(1-G) T_{12}^{ch} \end{aligned}$$

<u>Full size source</u>

$R_{coh} = R_{ch}$

$$\begin{aligned} C_2(1,2) &= 1 + (1-G)^2 (T_{12}^{ch})^2 \\ &+ 2G(1-G) (T_{12}^{ch})^2 \end{aligned}$$

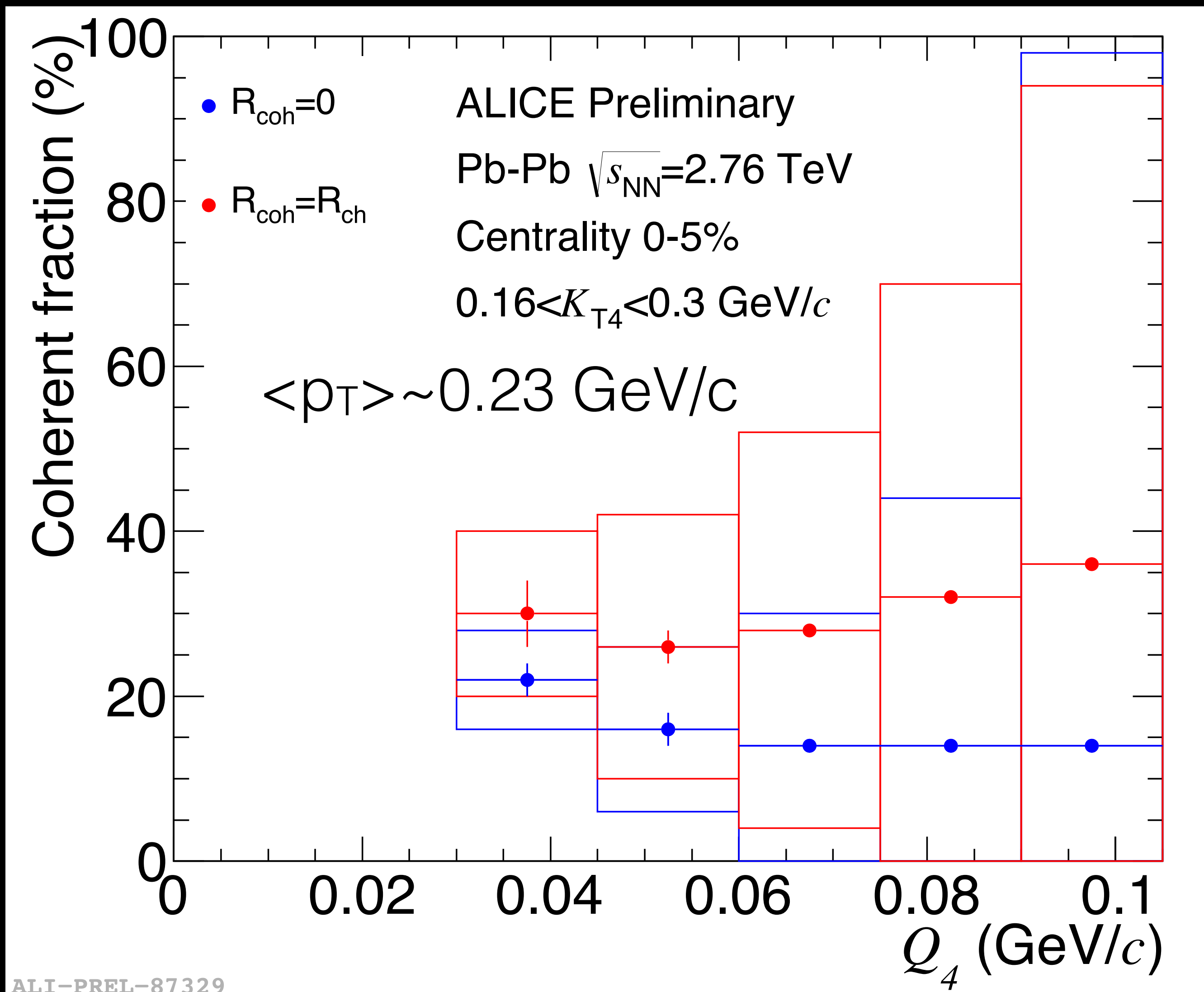
Measured

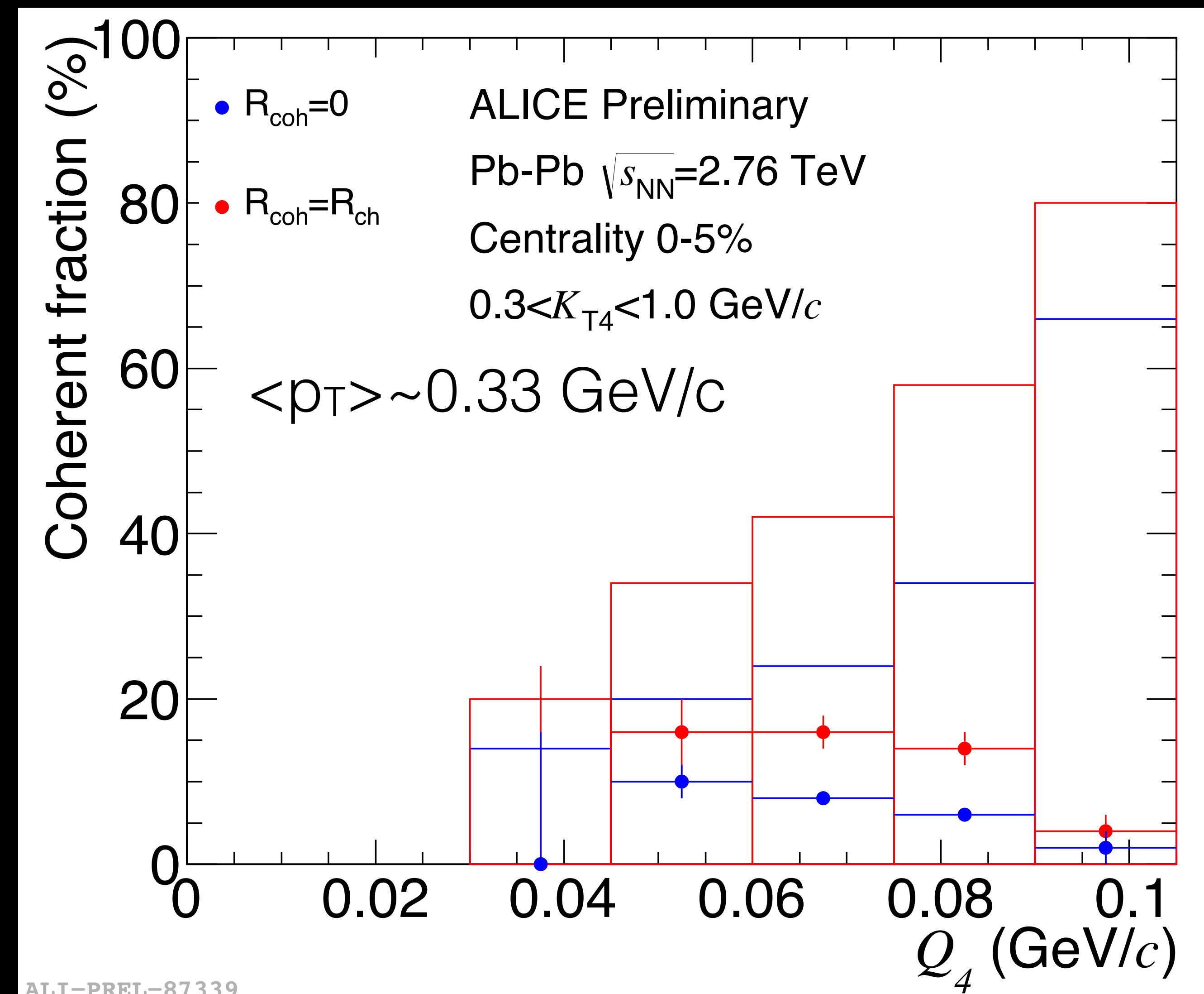
Assumed value

Extracted

Equations derived from
I. Andreev et al.
Int. J. Mod. Phys. A ***8*** *4577*

Coherent Fractions vs. Q_4

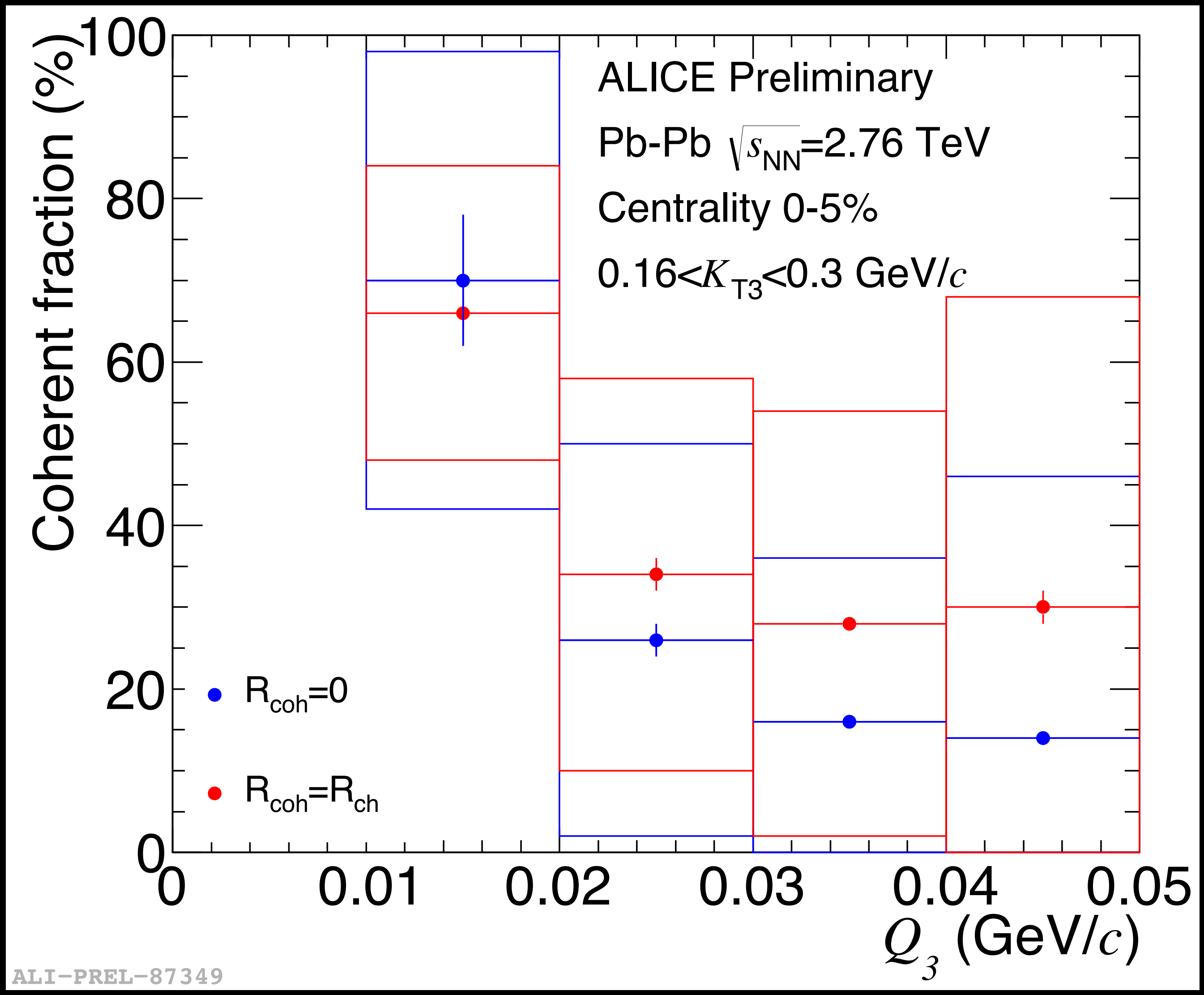


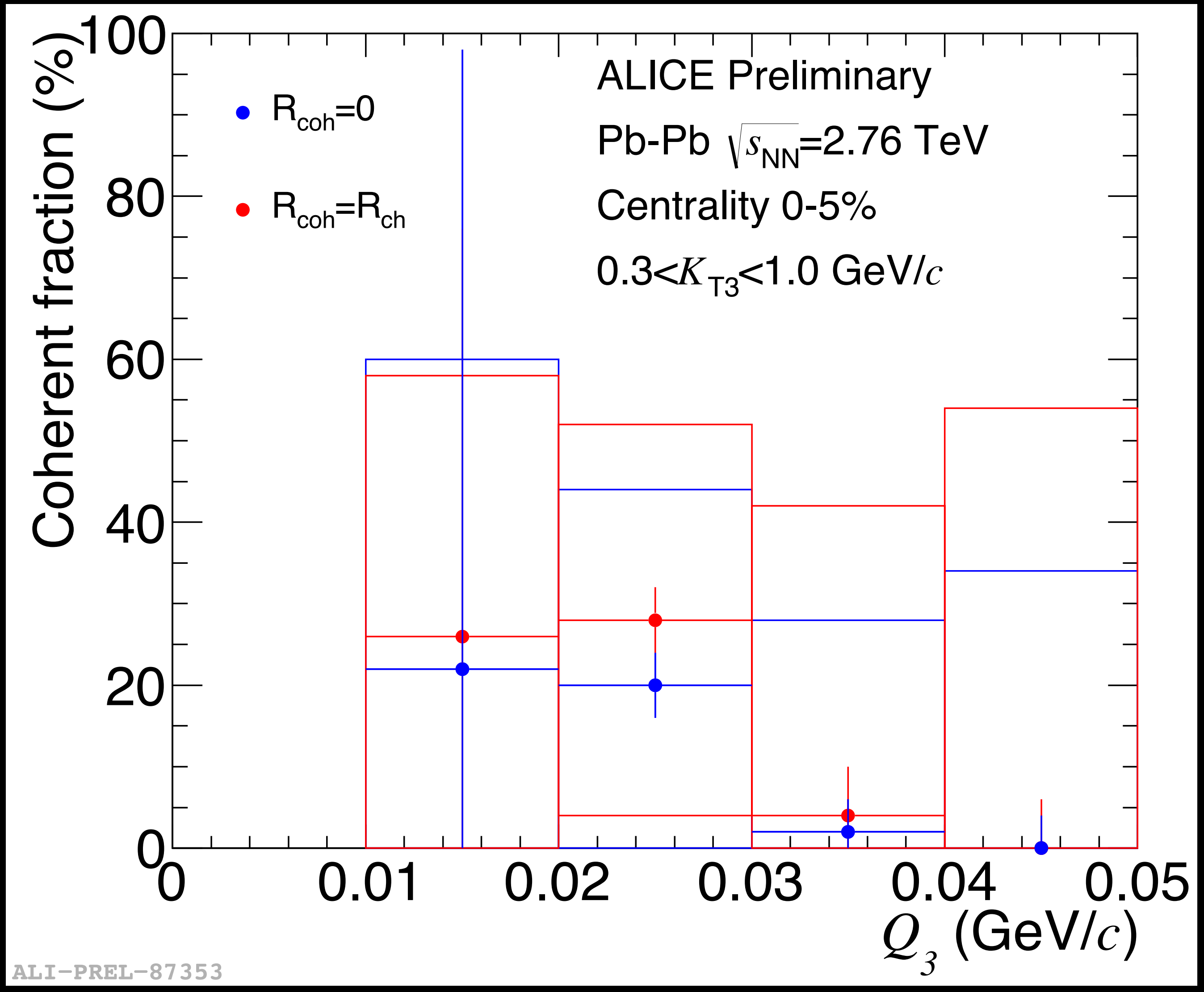


Low K_{T4} **High K_{T4}**

- Coherent fraction is fairly stable with Q_4.
- Systematics dominated by - - - + residual correlation.

3-pions (C_3^{QS}) : Minima vs. Q_3





<p_T> ~0.23 GeV/c

Low K_{T3}

<p_T> ~0.33 GeV/c

High K_{T3}

- Coherent fraction is fairly stable with Q_4.
- Systematics dominated by - - - + residual correlation.

Isolation of 2-pion Quantum Statistics (QS)

Quantity of Interest

$$N_2(p_1,p_2) = f_{21}N_1(p_1)N_1(p_2) + f_{22}K_2(q_{12})N_2^{QS}(p_1,p_2)$$

Sinyukov et al.,
Phys. Lett. B 432, 249 (1998)

f_{22} estimated to be 0.7 +- 0.05 for this analysis
f_{21} = 1 - f_{22}

f_{22} previously estimated in
ALICE 2014
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Isolation of 3-pion QS

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Quantity of Interest

$$
\begin{aligned}
N_3(p_1,p_2,p_3) &= f_{31}N_1(p_1)N_1(p_2)N_1(p_3) \\
&+ f_{32}N_2(p_1,p_2)N_1(p_3) \\
&+ f_{33}K_3(q_{12},q_{13},q_{23})N_3^{QS}(p_1,p_2,p_3)
\end{aligned}
$$

f coefficients derived in the core/halo picture as:

$$
\begin{aligned}
f_{31} &= (1-f_c)^3 + 3f_c(1-f_c)^2 - 3(1-f_c)(1-f_c^2) \\
f_{32} &= 3(1-f_c) \\
f_{33} &= f_c^3
\end{aligned}
$$

f_c^2 = "lambda" = 0.7 +- 0.05 (fraction of correlated pairs)

Systematics Checked

Those which pertain to both measured and built C_4^{QS}

Systematics are Q_4 dependent

- - vs. + pions — 0.1%.
- TPC B field orientation — negligible.
- Tracking efficiency — 0.4% at low Q_4.
- variation of f_c^2 (pair dilution). Default = 0.7, tried 0.65 and 0.75 — 6% at low Q_4
- Momentum resolution corrections — 1% at low Q_4
- Muon correction uncertainties — 2% at low Q_4.

High degree of correlation between measured and built C_4^{QS} for each of these variations.

Systematics Checked

Measured C_4^{QS} only

Systematics are Q_4 dependent

- variation of f_{41}, f_{42}, f_{43}, f_{44} from Therminator as compared to Core/Halo prescription — 0.4% at high Q_4
- Residue of mixed-charge (- - - +) cumulant — 5%
- K_4 FSI factor — 1% uncertainty at low Q_4.
  test of factorization: $K_4 = K_2^{12} K_2^{13} K_2^{14} K_2^{23} K_2^{24} K_2^{34}$

These systematics are the least understood sources of uncertainties. Future studies may reveal smaller values.

Systematics Checked

Built C_4^{QS} only

Systematics are Q_4 dependent

- Interpolator of 2-particle weights (C_2-1 = T_{ij}) — 0.7% at low Q_4.

Cubic interpolation used in between bins of q_{out}, q_{side}, q_{long} by default. Linear interpolation used as a variation.

- 2-particle weight problem at high q_{inv}

Statistical fluctuations at high q_{inv} can give a negative T_{ij} which is not allowed in theory (Bose-Einstein correlations are positive). In these cases Tij is set to zero.

— 0.3% at high Q_4, Low K_{T4}

— 4% at high Q_4, High K_{T4}

Equations to Build QS correlations with coherence, R_{coh}=0

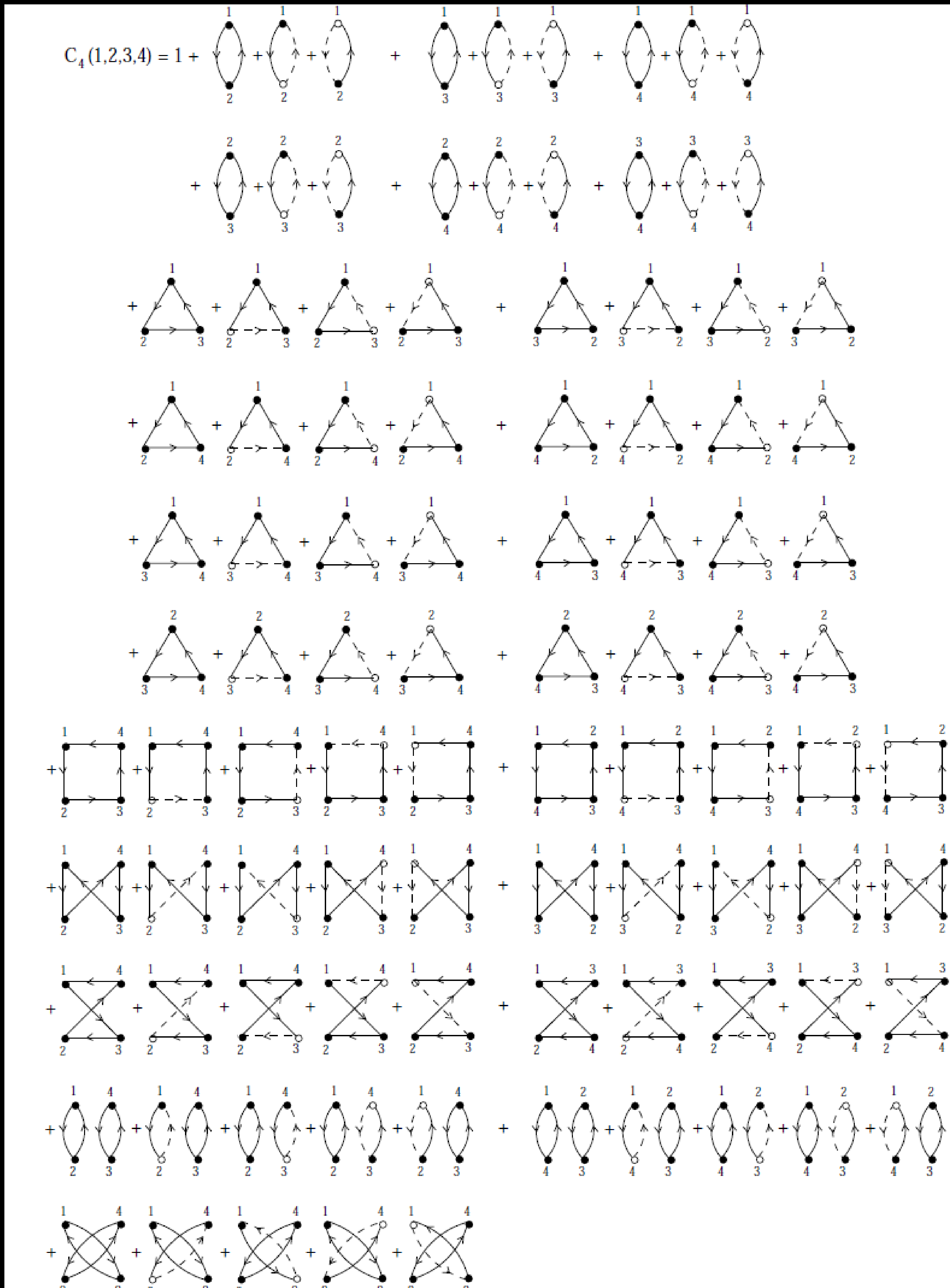
$$C_2^{QS} - 1 = 2G(1-G)T_{12} + (1-G)^2 T_{12}^2$$

$$
\begin{aligned}
C_3^{QS} - 1 &= 2G(1-G)(T_{12}+T_{13}+T_{23}) + (1-G)^2(T_{12}^2+T_{13}^2+T_{23}^2) \\
&+ 2G(1-G)^2(T_{12}T_{13}+T_{12}T_{23}+T_{13}T_{23}) + 2(1-G)^3(T_{12}T_{13}T_{23}) \\
C_4^{QS} - 1 &= 2G(1-G)(T_{12}+T_{13}+T_{14}+T_{23}+T_{24}+T_{34}) \\
&+ (1-G)^2(T_{12}^2+T_{13}^2+T_{14}^2+T_{23}^2+T_{24}^2+T_{34}^2) \\
&+ 2G(1-G)^3(T_{12}T_{34}^2+T_{12}^2T_{34}+T_{13}T_{24}^2+T_{13}^2T_{24}+T_{14}T_{23}^2+T_{14}^2T_{23}) \\
&+ (1-G)^4(T_{12}^2T_{34}^2+T_{13}^2T_{24}^2+T_{14}^2T_{23}^2) \\
&+ 2G(1-G)^2(T_{12}T_{13}+T_{12}T_{23}+T_{13}T_{23}+T_{12}T_{14}+T_{12}T_{24}+T_{14}T_{24}) \\
&+ 2G(1-G)^2(T_{13}T_{14}+T_{13}T_{34}+T_{14}T_{34}+T_{23}T_{24}+T_{23}T_{34}+T_{24}T_{34}) \\
&+ 2(1-G)^3(T_{12}T_{13}T_{23}+T_{12}T_{14}T_{24}+T_{13}T_{14}T_{34}+T_{23}T_{24}T_{34}) \\
&+ 2G(1-G)^3(T_{12}T_{14}T_{34}+T_{12}T_{14}T_{23}+T_{12}T_{23}T_{34}+T_{14}T_{23}T_{34}) \\
&+ 2G(1-G)^3(T_{12}T_{13}T_{34}+T_{12}T_{34}T_{24}+T_{12}T_{24}T_{13}+T_{13}T_{24}T_{34}) \\
&+ 2G(1-G)^3(T_{14}T_{13}T_{23}+T_{14}T_{13}T_{24}+T_{13}T_{23}T_{24}+T_{14}T_{24}T_{23}) \\
&+ 2(1-G)^4(T_{12}T_{13}T_{24}T_{34}+T_{12}T_{14}T_{23}T_{34}+T_{13}T_{14}T_{23}T_{24})
\end{aligned}
$$

G = coherent fraction of pions

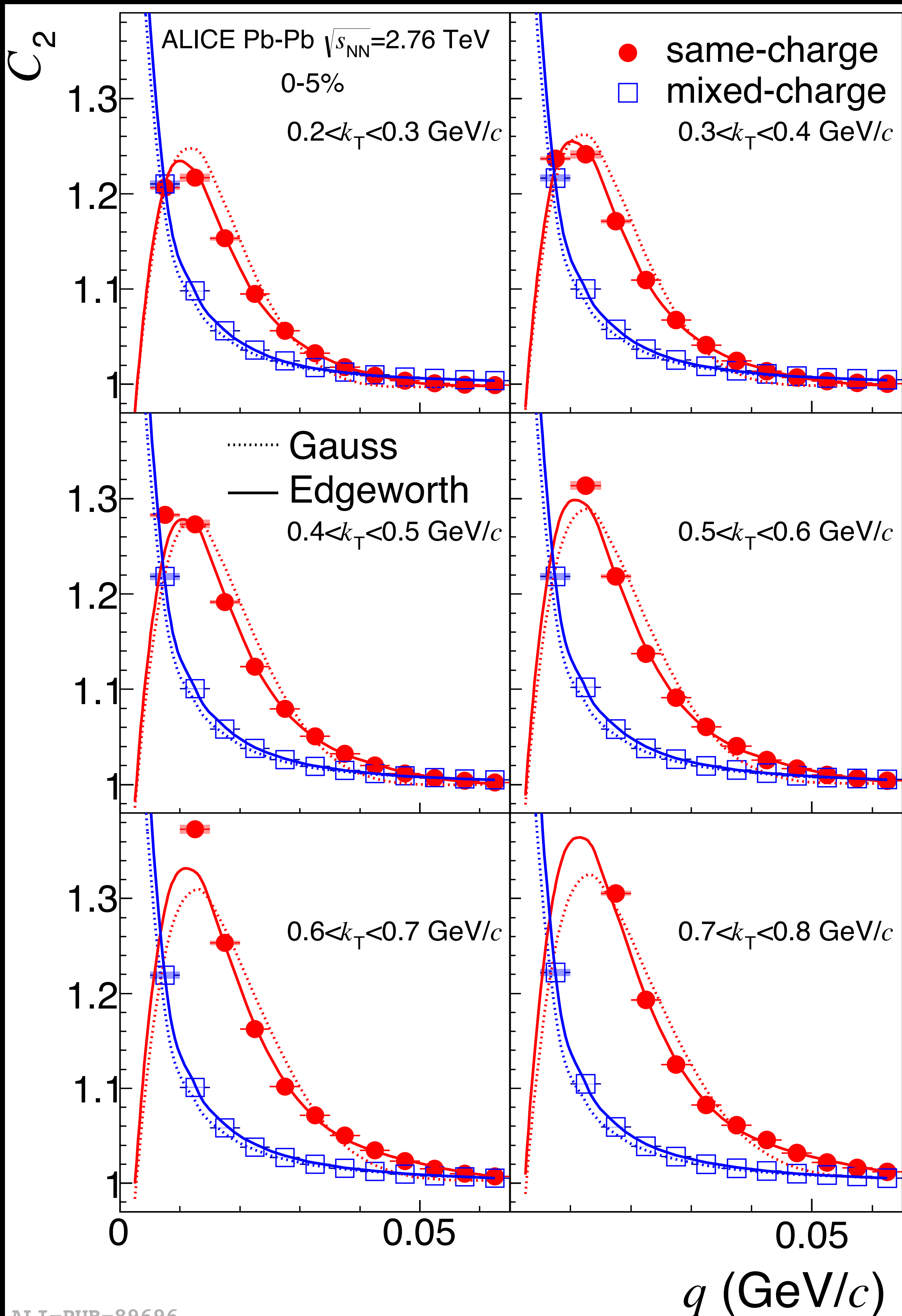
Weiner et al.
Int.J.Mod.Phys.A.
26 4577 (1993)

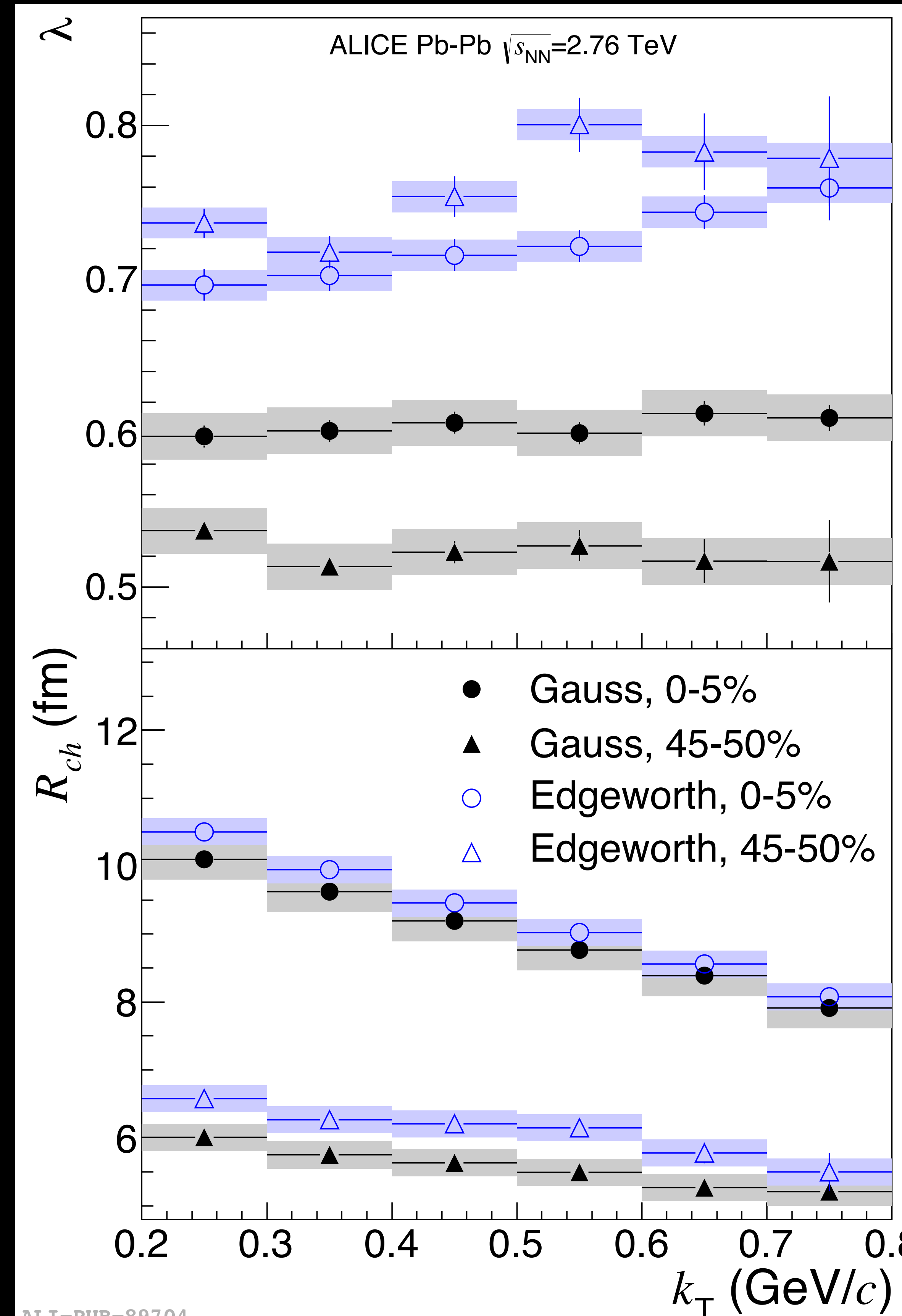
T. Csorgo.
Heavy Ion Phys.
15 1 (2002)



Full 4-pion Quantum Interference Diagrams

T. Csorgo
Heavy Ion Physics ***15*** *1-80*





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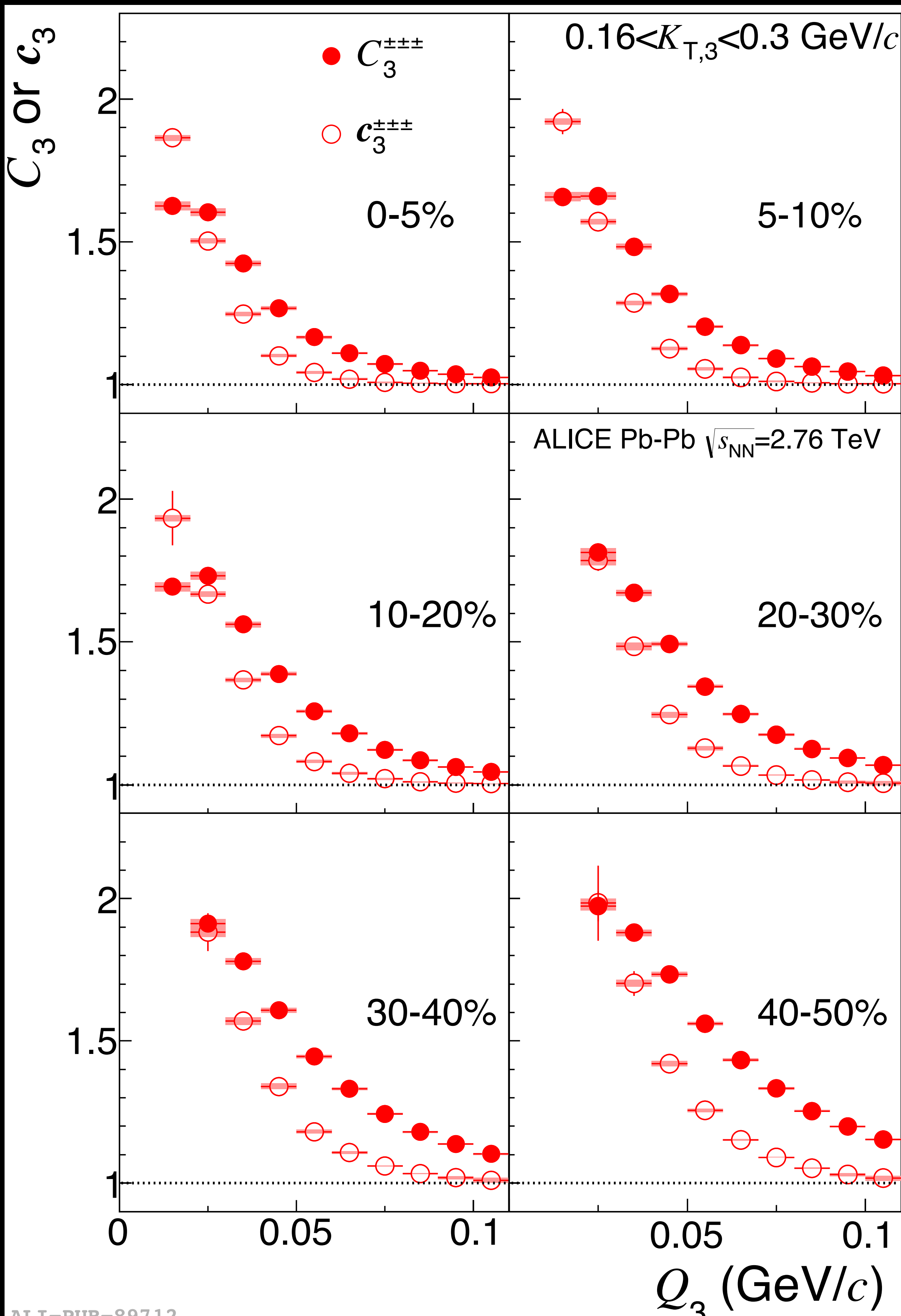
3-pion Correlation Functions in Pb-Pb



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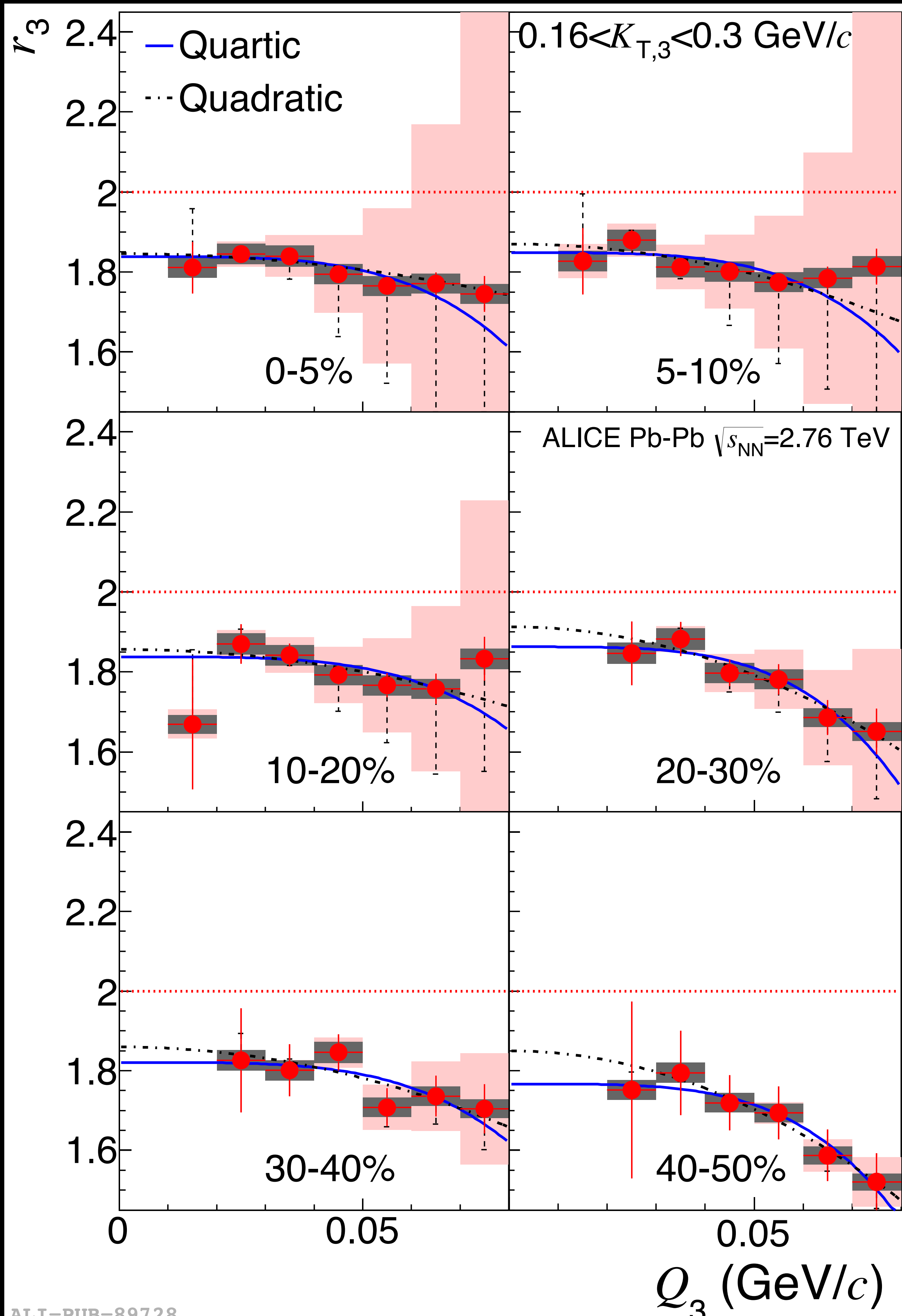

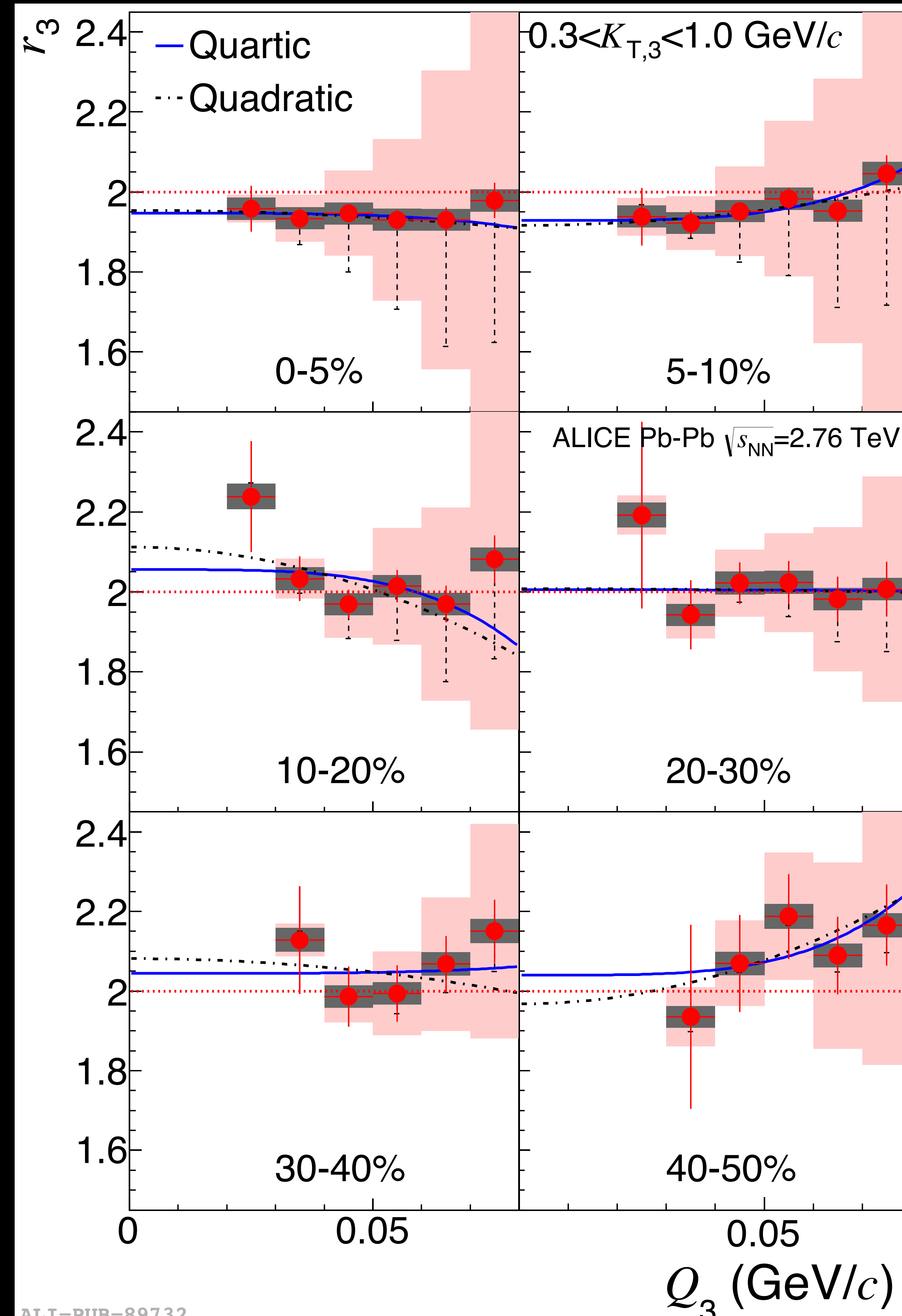

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PRC 89 024911 (2014)