Numerical solution to the NLO BK equation

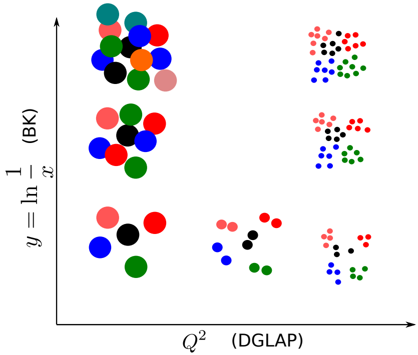
IS 2014

Heikki Mäntysaari
In collaboration with T. Lappi

University of Jyväskylä
Department of Physics

Dec 5, 2014

Introduction



- Study QCD at high energies
- Evolution in x (energy): BK equation
- Saturation phenomena described by CGC
- Saturation scale Q_s = characteristic momentum scale

CGC phenomenology

Non-perturbative input

Fit initial condition for the dipole amplitude to DIS data

Evolve dipole amplitude

Balitsky-Kovchegov equation LO and NLO (no solution so far)

- Current state of the art: LO + running coupling corrections

Calculate observables

- DIS (LO and NLO)
- Single inclusive spectra (LO and NLO)
- Two-particle correlations (LO)
- ...

BK and CGC phenomenology

BK: evolution equation for the correlator of two Wilson lines ("dipole")

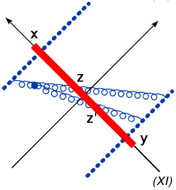
$$S(x-y) = \frac{1}{N_\mathrm{c}} \langle \, \mathrm{Tr} \, U_x U_y^\dagger \rangle$$

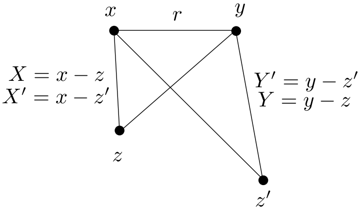
Running coupling corrections computed by Kovchegov, Weigert 2006; Balitsky, Chirilli, 2007

- Good fits with $\chi^2/N \sim 1$ to the HERA data
- Need an additional fit parameter in α_s slow the evolution
  $\Rightarrow$ NLO corrections to BK generally expected to slow evolution

NLO BK equation

Many diagrams, e.g.





$\otimes = \int_{zz'}$

Balitsky, Chirilli, arXiv:0710.4330 [hep-ph], here only large-N_c:

$$\partial_y S(r) = \frac{\alpha_s}{2\pi^2} K_1 \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 N_c^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$
$$+ \frac{\alpha_s^2 N_f N_c}{8\pi^4} K_f \otimes S(Y)[S(X') - S(X)]$$

NLO BK equation

Balitsky: scale of α_s is parametrically set by the smallest dipole.

- include terms $\sim \beta = \frac{11}{3}N_c - \frac{2}{3}N_f$ in the running coupling

$$\frac{\alpha_s}{2\pi^2}K_1 = \frac{\alpha_s}{2\pi^2}\frac{r^2}{X^2Y^2}\left\{1 + \frac{\alpha_s}{4\pi}\left[\beta \ln r^2\mu^2 - \beta\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \dots\right]\right\}$$

$$= K_{\text{Balitsky}} + \frac{\alpha_s^2}{8\pi^3}[\dots]$$

$$\to \frac{\alpha_s(r_{\min})}{2\pi^2}\frac{r^2}{X^2Y^2}\left\{1 + \frac{\alpha_s(r_{\min})}{4\pi}[\dots]\right\}$$

$r_{\min}$: smallest dipole

- Smallest dipole prescription: easy to generalize also to α_s^2 case with more coordinates
- Evolution speed depends strongly on the running coupling prescription

Conformal dipole

- Wilson lines are conformally invariant
- NLO BK equation is not due to double log term

$$K_1 = \cdots + \ln \frac{X^2}{r^2} \ln \frac{Y^2}{r^2}$$

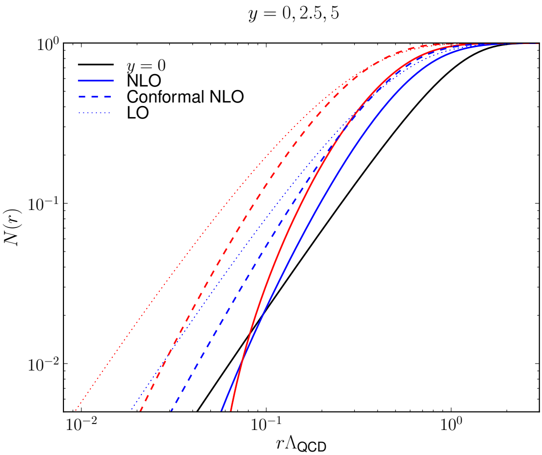
- Proposal by Balitsky and Chirilli (0903.5326), define

Conformal dipole

$$S(r)^{\mathrm{conf}} = S(r) - \frac{\alpha_{\mathrm{s}} N_{\mathrm{c}}}{4\pi^2} \int \mathrm{d}^2 z \frac{r^2}{X^2 Y^2} \ln \frac{a r^2}{X^2 Y^2} [S(X)S(Y) - S(r)]$$

- Evolution equation for $S(r)^{\mathrm{conf}}$ is conformally invariant except running α_{s}

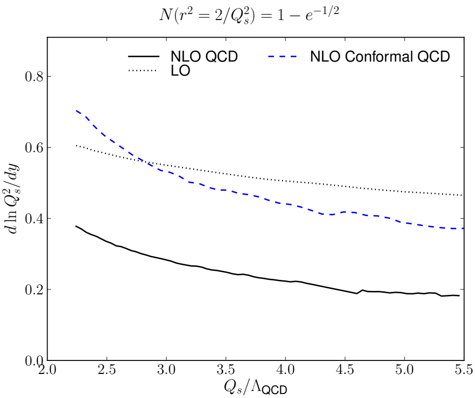
Solution to the NLO BK equation



Initial condition: MV model

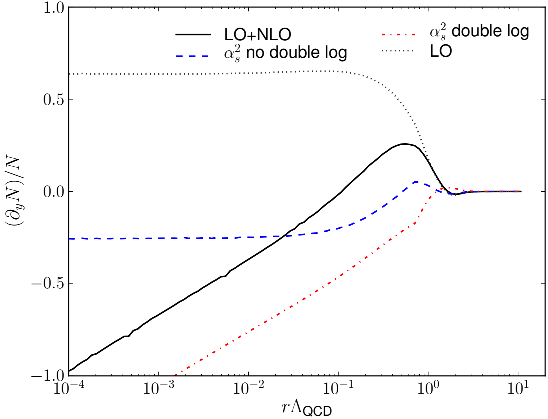
- Slower evolution speed compared to LO BK
- Amplitudes become negative at small dipoles

Evolution speed



- NLO corrections slow down the evolution
- Detailed evolution speed depends on
  - **RC prescription**
  - Initial condition
  - Definition of the saturation scale

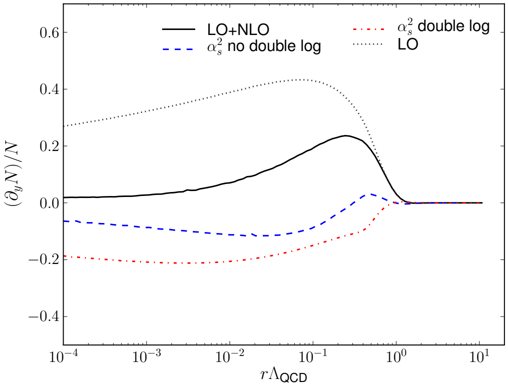
Dipole amplitude evolution speed $\partial_y N/N$



- Double log term $\sim \ln X^2/r^2 \ln Y^2/r^2$ drives the evolution speed negative

Dependence on the initial condition

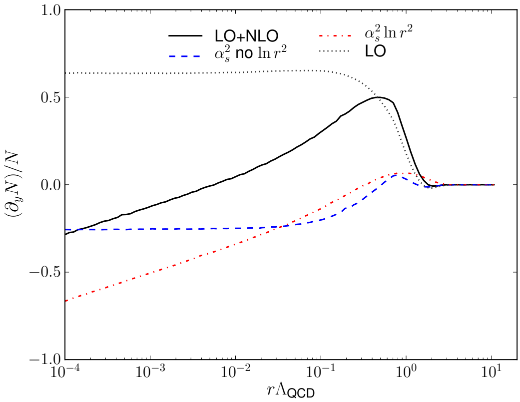
Modify MV model: introduce anomalous dimension $\gamma < 1$
$\Rightarrow$ positive evolution speed



- But HERA data (at least at LO) prefers $\gamma > 1$
- Evolution speed strongly depends on the running coupling prescription

Conformal dipole amplitude evolution speed $\partial_y N/N$

Conformal dipole, MV model initial condition



- Conformal dipole ⇒ no double log term
- Equation for the conformal dipole has an extra $\sim \ln r^2$ term ⇒ negative evolution speed

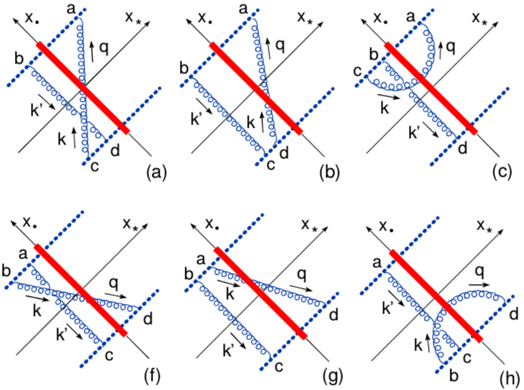
Conclusions

- First numerical solution to the NLO BK equation
- Also solved evolution equation for the conformal dipole
- NLO corrections
  - Decrease evolution speed
  - Depending on the details of the initial condition and running coupling prescription, can make the dipole amplitude negative at small dipoles
- Better understanding of the NLO equation is needed before NLO BK can be applied to phenomenology

Backups

Origin of the negative dipole

Problematic double logarithmic term comes from $1 \to 2$ gluon splitting diagrams where one gluon interacts with a shockwave



Balitsky, Chirilli, 0710.4330