Factorization in Saturation Physics

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**QCD and Gauge Symmetry**

QCD Lagrangian: \[ \mathcal{L} = i \bar{\psi} \gamma^\mu D_\mu \psi - m \bar{\psi} \psi - \frac{1}{4} [F^{\mu \nu} F_{\mu \nu}] \]

with \( D_\mu \equiv \partial_\mu + igA_\mu^a t^a \) and \( F_{\mu \nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf_{abc} A_\mu^b A_\nu^c \).

- **Gauge symmetry** is w.r.t. the local phase of charge particles.
- The same physics (phase) can be described using different orientations of the arrows (phases of the quark wavefunction) with a compensating gauge field.
- Non-Abelian gauge field theory. Invariant under SU(3) gauge transformation.
- High energy QCD, neglect light quark mass.
- Quarks are always accompanied by gluons.
**Kinoshita-Lee-Nauenberg Theorem**

KLN theorem: In a theory with massless fields, transition rates are free of the infrared divergence (soft and collinear) if the summation over initial and final degenerate states is carried out.

- Long-distance (Infrared) physics, Non-perturbative.
- Infrared safe observables. e.g, Jet observables and $e^+ e^-$ total cross section.
- The KLN theorem: infrared divergences appear because some of states are physically “degenerate”, but we treat them as different.
- A state with a quark accompanied by a collinear gluon is degenerate with a state with a single quark.
- A state with a soft gluon is almost degenerate with a state with no gluon (virtual).
The gauge invariant definition of parton distributions

The integrated quark distribution

\[ f_q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P \left| \bar{\psi}(0)\gamma^+ \mathcal{L}(\xi^-)\psi(0, \xi^-) \right| P \rangle \]

- The gauge links come from the sum over all degenerate quark states.

\[ |\psi_q(k)\rangle_{GI} = |\psi_q(k)\rangle + |\psi_q(k_1)g(k - k_1)\rangle + |\psi_q(k_1)g(k_2)g(k - k_1 - k_2)\rangle + \cdots. \]

- Gauge invariant definition with \( \mathcal{L}(\xi^-) \equiv P \exp \left[ -ig \int_0^{\xi^-} d\xi'^- A^+(\xi'^-) \right] \).

- Light-cone gauge together with proper B.C. ⇒ parton density interpretation.

The unintegrated (Transverse Momentum Dependent (TMD)) quark distribution

\[ f_q(x, k_{\perp}) = \int \frac{d\xi^- d^2\xi_{\perp}}{4\pi(2\pi)^2} e^{ixP^+\xi^- + i\xi_{\perp} \cdot k_{\perp}} \langle P \left| \bar{\psi}(0)\mathcal{L}^\dagger(0)\gamma^+ \mathcal{L}(\xi^-, \xi_{\perp})\psi(\xi_{\perp}, \xi^-) \right| P \rangle \]
Two Different Gauge Invariant Operator Definitions

Known from TMD factorization [F. Dominguez, BX and F. Yuan, Phys.Rev.Lett. 11]

I. Weizsäcker Williams gluon distribution:

\[ xG^{(1)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{i x P^+ \xi^- - i k_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^+ i (\xi^- , \xi_\perp) U^{[+]} \dagger F^+ i (0) U^{[+]} | P \rangle. \]

II. Color Dipole gluon distributions:

\[ xG^{(2)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{i x P^+ \xi^- - i k_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^+ i (\xi^- , \xi_\perp) U^{[-]} \dagger F^+ i (0) U^{[+]} | P \rangle. \]

- The WW gluon distribution is the conventional gluon distributions.
- The dipole gluon distribution has no such interpretation.
- [Balitsky, Tarasov, 14] Starting from the same operator definition, \( xG^{(1)} \): TMD (moderate \( x \sim \frac{Q^2}{s} \)) and W.W. (small-\( x \), high energy with fixed \( Q^2 \)).
- Sudakov resummation in small-\( x \) physics. [Mueller, BX and Yuan, 13]
Deep into small-x region

- Partons in the low-x region is dominated by gluons. See HERA data.
- BFKL equation $\Rightarrow$ Resummation of the $\alpha_s \ln \frac{1}{x}$.
- When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine $\Rightarrow$ Non-linear dynamics $\Rightarrow$ BK (JIMWLK) equation
- Use $Q_s(x)$ to separate the saturated dense regime from the dilute regime.
- Core ingredients: Multiple interactions $+$ Small-x (high energy) evolution
**Forward rapidity single hadron productions in pA collisions**

Dilute-Dense factorizations [*Dumitru, Jalilian-Marian, 02; Hayashigaki, 06*]

\[ p + A \rightarrow h(y, p_{\perp}) + X \]

- Proton PDFs are under control at large-\(x\), use collinear PDFs and FFs.
- Dense gluons at low-\(x\) in the nucleus target is described by saturation or CGC.
- Simpler and More interesting than middle rapidity.
**Forward hadron production in pA collisions**

[Dumitru, Jalilian-Marian, 02] Inclusive forward hadron production in pA collisions

\[
\frac{d\sigma_{LO}^{pA\rightarrow hX}}{d^2p_\perp dy_h} = \int_{\tau}^1 \frac{dz}{z^2} \left[ x_1 q_f(x_1, \mu) \mathcal{F}_{x_2}(k_\perp) D_{h/q}(z, \mu) + x_1 g(x_1, \mu) \tilde{\mathcal{F}}_{x_2}(k_\perp) D_{h/g}(z, \mu) \right].
\]

- \(\mathcal{F}(k_\perp)\) is related to the dipole gluon distribution.
- **Caveats:** arbitrary choice of the renormalization scale \(\mu\) and \(K\) factor.
- **Need NLO correction! IR cutoff:** [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14]
- **DR:** [Chirilli, BX and Yuan, 12]
Factorization for single inclusive hadron productions

Factorization for the $p + A \rightarrow H + X$ process \cite{Chirilli, BX and Yuan, 12}

- Need to include all real and virtual graphs in all four channel $q \rightarrow q, q \rightarrow g, g \rightarrow q(\bar{q})$ and $g \rightarrow g$.
- Gluons in different kinematical region give different divergences.
  - 1. soft, collinear to the target nucleus; $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$.
  - 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs
  - 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs.
Divergences and Techniques at NLO

- When gluon is collinear to the initial state quark, they become degenerate with a single quark. $\Rightarrow$ IR divergence appears according to KLN.
- According to gauge invariant def of PDFs, collinear gluons are included in the gauge links as “degenerate” states.
- Absorb IR divergence into the quark PDF $xq(x)$ via renormalization. $\Rightarrow$ DGLAP evolution
- Must use dimensional regularization. Preserves gauge, Lorentz, etc invariances.
- IR cutoff may be more intuitive, but it breaks gauge invariance in QCD, prevent one from using most PDFs in $\overline{MS}$ scheme.
- NLO is vital in establishing the QCD factorization in saturation physics.
Factorization and NLO Calculation

- Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (Non perturbative).

\[ \sigma \sim x f(x) \otimes H \otimes D_h(z) \otimes F(k_\perp) \]

- NLO (1-loop) calculation always contains various kinds of divergences.
  - Some divergences can be absorbed into the corresponding evolution equations.
  - The rest of divergences should be cancelled.

- Hard factor

\[ H = H^{(0)}_{\text{LO}} + \frac{\alpha_s}{2\pi} H^{(1)}_{\text{NLO}} + \cdots \]

should always be finite and free of divergence of any kind.

- NLO vs NLL  
  Naive \( \alpha_s \) expansion sometimes is not sufficient!

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<th>LO</th>
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<tr>
<td>LL</td>
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<td>( \alpha_s L )</td>
<td>( (\alpha_s L)^2 )</td>
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<tr>
<td>NLL</td>
<td>( \alpha_s )</td>
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- Evolution \( \rightarrow \) Resummation of large logs.
  LO evolution resums LL; NLO \( \Rightarrow \) NLL.
Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

\[
d\sigma = \int x f_a(x) \otimes D_a(z) \otimes F^x_g(k_\perp) \otimes \mathcal{H}^{(0)}
\]

\[
+ \frac{\alpha_s}{2\pi} \int x f_a(x) \otimes D_b(z) \otimes F^x_g(N)_{ab} \otimes \mathcal{H}^{(1)}_{ab}.
\]

Consistent implementation should include all the NLO $\alpha_s$ corrections.

- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. Partially by [Albacete, Dumitru, Fujii, Nara, 12]
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]
Numerical implementation of the NLO result

Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

- Agree with data for $p_\perp < Q_s(y)$, and reduced scale dependence, no $K$ factor.
- For more forward rapidity, the agreement gets better and better.
Numerical implementation of the NLO result

Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

- The abrupt drop at NLO when $p_\perp > Q_s$ was surprising and puzzling.
- Fixed order calculation in field theories is not guaranteed to be positive.
- Failure of positivity is also seen in TMD factorization, where $Y$-term is devised to match collinear factorization. [Collins, Foundations of perturbative QCD, 11]
- Similar to TMD, saturation only applies at low-$k_\perp$ and $x$ region in $s \to \infty$. 

**Summary**

**INTRODUCTION**

**FACTORIZATION AT ONE-LOOP ORDER**

**PHENOMENOLOGICAL APPLICATION**

**NUMERICAL IMPLEMENTATION**

**CONCLUSION**

**APPENDIX**
Matching to the collinear factorisation at large $k_{\perp}$

[Stasto, Xiao, Yuan, Zaslavsky, 14] \( \sqrt{s} \to \infty \) vs \( \sqrt{s} = 200 \text{ GeV} \)

- Adopt exact kinematics and match with collinear factorization at high $p_{\perp}$.
- See also [G. Beuf, 14] and [Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14]
- Systematic matching with collinear factorization for all channels at high $p_{\perp}$.
- The matching point increases with $Q_s \sim 1, 1.5, 2 \text{ GeV}$ when $y \to 2.2, 3.2, 4$.
- Implement the exact kinematical constraint in saturation formalism. Extend the applicability of saturation formalism to larger $k_{\perp}$ window.

[Watanabe, Xiao, Yuan, Zaslavsky, work in progress] Please stay tuned.
Conclusion

- **Factorization** for single and dihadron productions in $pA$ collisions in the small-$x$ saturation formalism at one-loop order. (More interesting).
- Towards the **quantitative** test of saturation physics beyond LL. (More precise).
- **One-loop** calculation for hard processes in $pA$ collisions, Sudakov factor. (More complete understanding of TMD or UGD).
- Extension to larger $k_\perp$ region and matching to collinear factorization. Low-$k_\perp \Leftrightarrow$ saturation; High-$k_\perp \Leftrightarrow$ collinear dilute physics.
- **Gluon saturation** could be the next interesting discovery at the LHC and future EIC.