Factorization in Saturation Physics

Bo-Wen Xiao

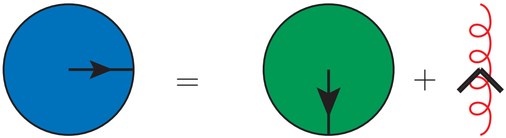
Institute of Particle Physics, Central China Normal University

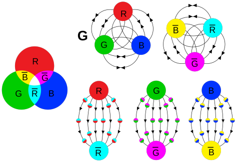
Initial Stages 2014

QCD and Gauge Symmetry

$$\text{QCD Lagrangian :} \qquad \mathcal{L} = i\bar{\psi}\gamma^{\mu}\mathcal{D}_{\mu}\psi - m\bar{\psi}\psi - \frac{1}{4}\left[\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\right]$$

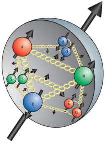
with $\mathcal{D}_{\mu} \equiv \partial_{\mu} + igA^{a}_{\mu}t^{a}$ and $F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - gf_{abc}A^{b}_{\mu}A^{c}_{\nu}$.

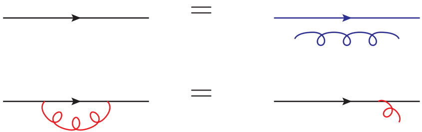




- Gauge symmetry is w.r.t. the local phase of charge particles.
- The same physics (phase) can be described using different orientations of the arrows (phases of the quark wavefunction) with a compensating gauge field.
- Non-Abelian gauge field theory. Invariant under SU(3) gauge transformation.
- High energy QCD, neglect light quark mass.
- Quarks are always accompanied by gluons.

Kinoshita-Lee-Nauenberg Theorem





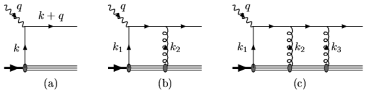
KLN theorem: In a theory with massless fields, transition rates are free of the infrared divergence (soft and collinear) if the summation over initial and final degenerate states is carried out.

- Long-distance (Infrared) physics, Non-perturbative.
- Infrared safe observables. e.g, Jet observables and e^+e^- total cross section.
- The KLN theorem: infrared divergences appear because some of states are physically "degenerate", but we treat them as different.
- A state with a quark accompanied by a collinear gluon is degenerate with a state with a single quark.
- A state with a soft gluon is almost degenerate with a state with no gluon (virtual).

The gauge invariant definition of parton distributions

The integrated quark distribution

$$f_q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P \left| \bar{\psi}(0)\gamma^+ \mathcal{L}(\xi^-)\psi(0,\xi^-) \right| P\rangle$$



- The gauge links come from the sum over all degenerate quark states.

$$|\psi_q(k)\rangle_{GI} = |\psi_q(k)\rangle + |\psi_q(k_1)g(k-k_1)\rangle + |\psi_q(k_1)g(k_2)g(k-k_1-k_2)\rangle + \cdots .$$

- Gauge invariant definition with $\mathcal{L}(\xi^-) \equiv \mathrm{P}\exp\left[-ig\int_0^{\xi^-} d\xi^{-\prime} A^+(\xi^{-\prime})\right]$.
- Light-cone gauge together with proper B.C. $\Rightarrow$ parton density interpretation.

The unintegrated (Transverse Momentum Dependent (TMD)) quark distribution

$$f_q(x,k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{4\pi(2\pi)^2} e^{ixP^+\xi^- + i\xi_\perp\cdot k_\perp} \langle P \left| \bar{\psi}(0)\mathcal{L}^\dagger(0)\gamma^+\mathcal{L}(\xi^-,\xi_\perp)\psi(\xi_\perp,\xi^-) \right| P\rangle$$

Two Different Gauge Invariant Operator Definitions

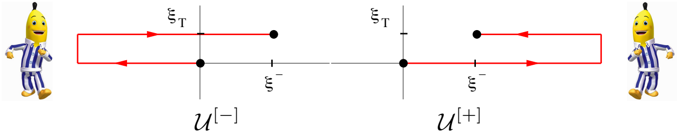
Known from TMD factorization *[F. Dominguez, BX and F. Yuan, Phys.Rev.Lett. 11]*

I. Weizsäcker Williams gluon distribution:

$$xG^{(1)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

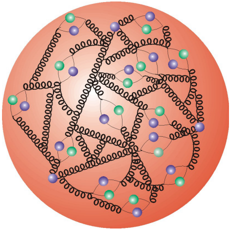
II. Color Dipole gluon distributions:

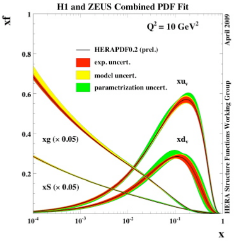
$$xG^{(2)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

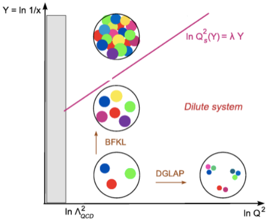


- The WW gluon distribution is the conventional gluon distributions.
- The dipole gluon distribution has no such interpretation.
- *[Balitsky, Tarasov, 14]* Starting from the same operator definition, $xG^{(1)}$: TMD (moderate $x \sim \frac{Q^2}{s}$) and W.W. (small-x, high energy with fixed Q^2).
- Sudakov resummation in small-x physics. *[Mueller, BX and Yuan, 13]*

Deep into small-x region





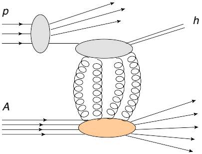


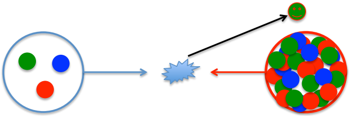
- Partons in the low-x region is dominated by gluons. See HERA data.
- BFKL equation $\Rightarrow$ Resummation of the $\alpha_s \ln \frac{1}{x}$.
- When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine $\Rightarrow$ Non-linear dynamics $\Rightarrow$ BK (JIMWLK) equation
- Use $Q_s(x)$ to separate the saturated dense regime from the dilute regime.
- Core ingredients: Multiple interactions + Small-x (high energy) evolution

Forward rapidity single hadron productions in pA collisions

Dilute-Dense factorizations *[Dumitru, Jalilian-Marian, 02; Hayashigaki, 06]*

$$p + A \to h(y, p_\perp) + X$$





projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$ gluon

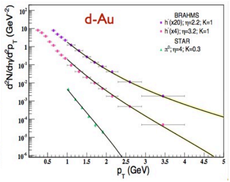
- Proton PDFs are under control at large-x, use collinear PDFs and FFs.
- Dense gluons at low-x in the nucleus target is described by saturation or CGC.
- Simpler and More interesting than middle rapidity.

Forward hadron production in pA collisions

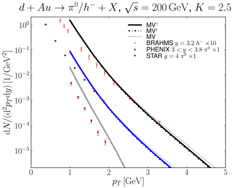
[Dumitru, Jalilian-Marian, 02] Inclusive forward hadron production in pA collisions

$$\frac{d\sigma_{\mathrm{LO}}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[x_1 q_f(x_1,\mu)\mathcal{F}_{x_2}(k_\perp)D_{h/q}(z,\mu) + x_1 g(x_1,\mu)\tilde{\mathcal{F}}_{x_2}(k_\perp)D_{h/g}(z,\mu)\right].$$

Albaete-Marquet, 2010



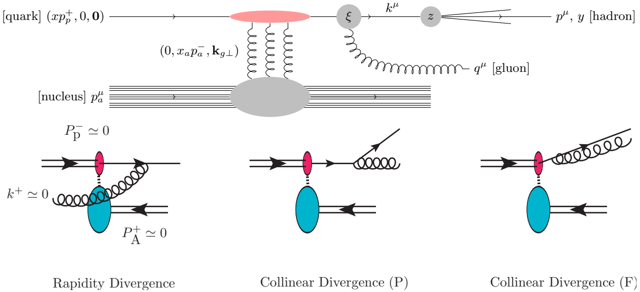
Lappi & Mantysaari 13



- $\mathcal{F}(k_\perp)$ is related to the dipole gluon distribution.
- Caveats: arbitrary choice of the renormalization scale μ and K factor.
- Need NLO correction! IR cutoff: *[Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14]* DR: *[Chirilli, BX and Yuan, 12]*

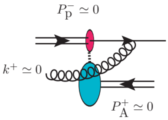
Factorization for single inclusive hadron productions

Factorization for the $p + A \to H + X$ process *[Chirilli, BX and Yuan, 12]*

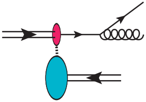


- Need to include all real and virtual graphs in all four channel $q \to q$, $q \to g$, $g \to q(\bar{q})$ and $g \to g$.
- Gluons in different kinematical region give different divergences.
- 1. soft, collinear to the target nucleus; $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$.
- 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs
- 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs.

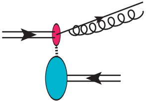
Divergences and Techniques at NLO



Rapidity Divergence



Collinear Divergence (P)



Collinear Divergence (F)

- When gluon is collinear to the initial state quark, they become degenerate with a single quark. ⇒ IR divergence appears according to KLN.
- According to gauge invariant def of PDFs, collinear gluons are included in the gauge links as "degenerate" states.
- Absorb IR divergence into the quark PDF $xq(x)$ via renormalization. ⇒ DGLAP evolution
- Must use dimensional regularization. Preserves gauge, Lorentz, etc invariances.
- IR cutoff may be more intuitive, but it breaks gauge invariance in QCD, prevent one from using most PDFs in $\overline{\rm MS}$ scheme.
- NLO is vital in establishing the QCD factorization in saturation physics.

Factorization and NLO Calculation

- Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (Non perturbative).

$$\sigma \sim xf(x) \otimes \mathcal{H} \otimes D_h(z) \otimes \mathcal{F}(k_\perp)$$

- NLO (1-loop) calculation always contains various kinds of divergences.
  - Some divergences can be absorbed into the corresponding evolution equations.
  - The rest of divergences should be cancelled.
- Hard factor

$$\mathcal{H} = \mathcal{H}_{\mathrm{LO}}^{(0)} + \frac{\alpha_s}{2\pi}\mathcal{H}_{\mathrm{NLO}}^{(1)} + \cdots$$

  should always be finite and free of divergence of any kind.
- NLO vs NLL Naive α_s expansion sometimes is not sufficient!

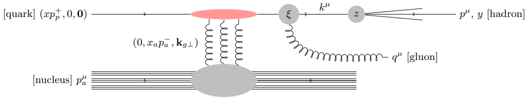
	LO	NLO	NNLO	$\cdots$
LL	1	$\alpha_s L$	$(\alpha_s L)^2$	$\cdots$
NLL		α_s	$\alpha_s\,(\alpha_s L)$	$\cdots$
$\cdots$			$\cdots$	$\cdots$

- Evolution $\rightarrow$ Resummation of large logs.
  LO evolution resums LL; NLO $\Rightarrow$ NLL.

Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

$$\begin{aligned} d\sigma &= \int xf_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} \\ &+ \frac{\alpha_s}{2\pi} \int xf_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}. \end{aligned}$$

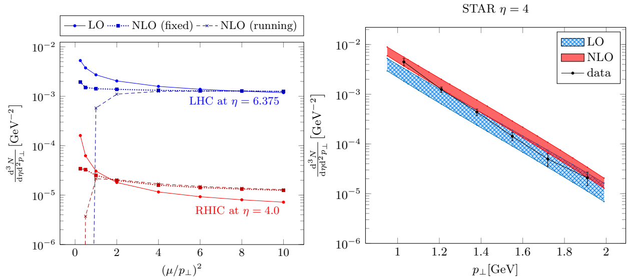


Consistent implementation should include all the NLO α_s corrections.

- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. Partially by *[Albacete, Dumitru, Fujii, Nara, 12]*
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution *[Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]*. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). *[Stasto, Xiao, Zaslavsky, 13]*

Numerical implementation of the NLO result

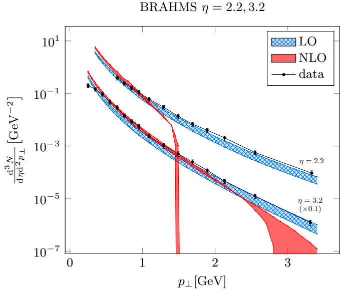
Saturation physics at One Loop Order (SOLO). *[Stasto, Xiao, Zaslavsky, 13]*

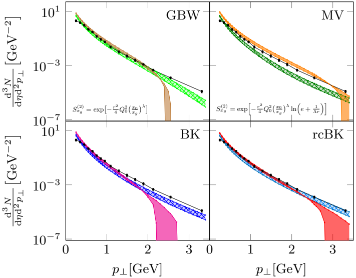


- Agree with data for $p_\perp < Q_s(y)$, and reduced scale dependence, no K factor.
- For more forward rapidity, the agreement gets better and better.

Numerical implementation of the NLO result

Saturation physics at One Loop Order (SOLO). *[Stasto, Xiao, Zaslavsky, 13]*

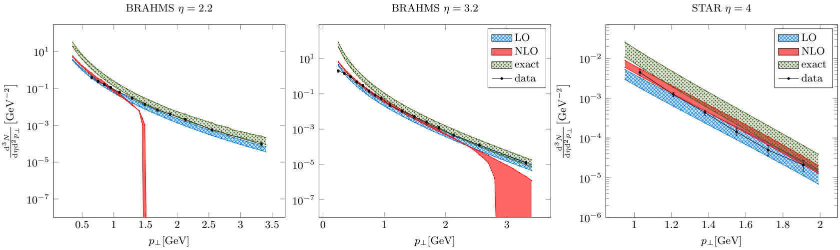




- The abrupt drop at NLO when $p_\perp > Q_s$ was surprising and puzzling.
- Fixed order calculation in field theories is not guaranteed to be positive.
- Failure of positivity is also seen in TMD factorization, where Y-term is devised to match collinear factorization.*[Collins, Foundations of perturbative QCD, 11]*
- Similar to TMD, saturation only applies at low-$k_\perp$ and x region in $s \to \infty$.

Matching to the collinear factorisation at large $k_\perp$

[Stasto, Xiao, Yuan, Zaslavsky, 14] $\sqrt{s} \to \infty$ vs $\sqrt{s} = 200\,\mathrm{GeV}$



- Adopt exact kinematics and match with collinear factorization at high $p_\perp$.
- See also *[G. Beuf, 14]* and *[Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14]*
- Systematic matching with collinear factorization for all channels at high $p_\perp$.
- The matching point increases with $Q_s \sim 1,\ 1.5,\ 2\,\mathrm{GeV}$ when $y \to 2.2,\ 3.2,\ 4$.
- Implement the exact kinematical constraint in saturation formalism. Extend the applicability of saturation formalism to larger $k_\perp$ window. *[Watanabe, Xiao, Yuan, Zaslavsky, work in progress]* Please stay tuned.

Conclusion

- Factorization for single and dihadron productions in pA collisions in the small-x saturation formalism at one-loop order. (More interesting).
- Towards the quantitative test of saturation physics beyond LL. (More precise).
- One-loop calculation for hard processes in pA collisions, Sudakov factor. (More complete understanding of TMD or UGD).
- Extension to larger $k_\perp$ region and matching to collinear factorization. Low-$k_\perp$ $\Leftrightarrow$ saturation; High-$k_\perp$ $\Leftrightarrow$ collinear dilute physics.
- Gluon saturation could be the next interesting discovery at the LHC and future EIC.