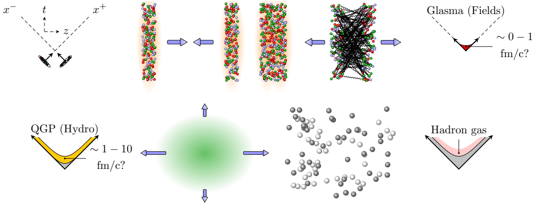

Thermalization: Results, Issues, Perspectives

IS 2014, Napa, 3rd December 2014

Thomas EPELBAUM
McGill

What is Hydrodynamics?

I) Macroscopic theory
II) Few field variables: P_L, P_T, ϵ, $\vec{u}$
III) Conservation law: $\partial_\mu T^{\mu\nu} = 0$
IV) Need input:

1) Equation of state $f(P) = \epsilon$
2) Small anisotropy
3) Short isotropization time
4) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
5) Viscous coefficients: shear viscosity η,...

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5) Viscous coefficients: shear viscosity η,...

None of this is easy to get from QCD

The Color Glass Condensate [McLerran, Venugopalan (1993)]

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + J_{\mu}\mathcal{A}^{\mu} \qquad \text{(no quarks)}$$

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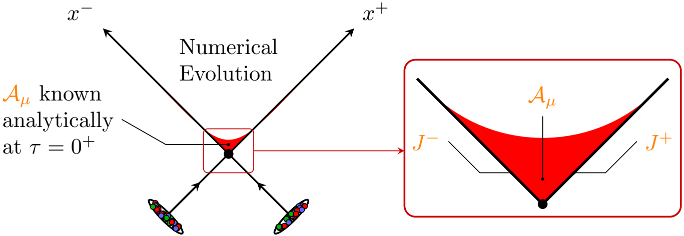

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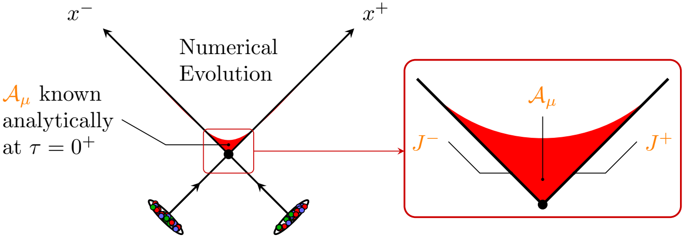


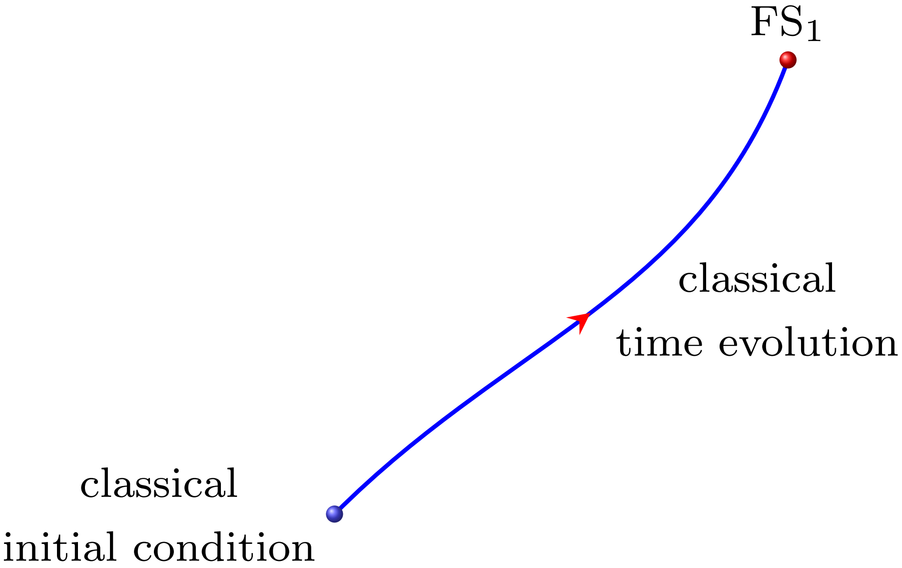
LO:

$$\underbrace{[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}] = J^\nu \sim \frac{Q_s^3}{g}}_{\text{Color sources on the light cone}}$$

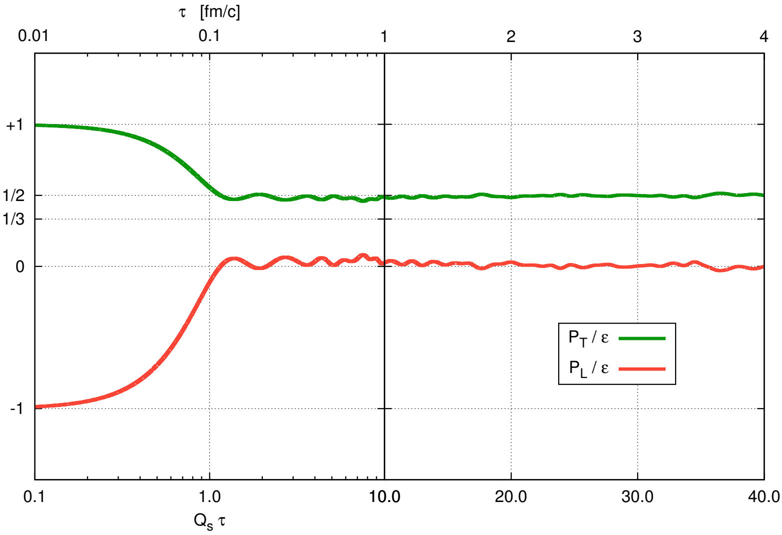
$$\epsilon = \underbrace{\frac{1}{2}\left(\vec{\mathcal{E}}^2 + \vec{\mathcal{B}}^2\right) \sim \frac{Q_s^4}{g^2}}_{\textbf{Classical}\text{ color fields}}$$

[Krasnitz, Venugopalan (1998)]

The Color Glass Condensate at its LO



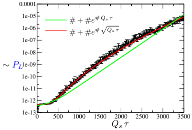
The Color Glass Condensate at its LO
Strong anisotropy at early time



[Lappi, McLerran (2006), Fukushima, Gelis (2012)...]

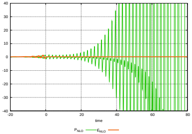
- Because of instabilities, the NLO correction eventually becomes as large as the LO ⇒ Important effect, should be included

Instabilities [MROWCZYNSKI (1988)...]



- NLO alone grows forever ⇒ unphysical effect, should be taken care of

Secular divergences



- Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?

- At the initial time $\tau = \tau_0$, take:

$$A_0{}^{\mu}(\tau_0, \boldsymbol{x}) = \mathcal{A}_0{}^{\mu}(\tau_0, \boldsymbol{x}_\perp) + \int_{\boldsymbol{k}} c_{\boldsymbol{k}} a^{\mu}_{\boldsymbol{k}}(\tau_0, \boldsymbol{x})$$

  where $c_{\boldsymbol{k}}$ are random coefficients: $\langle c_{\boldsymbol{k}} c_{\boldsymbol{k}'} \rangle \sim \delta_{\boldsymbol{k}\boldsymbol{k}'}$

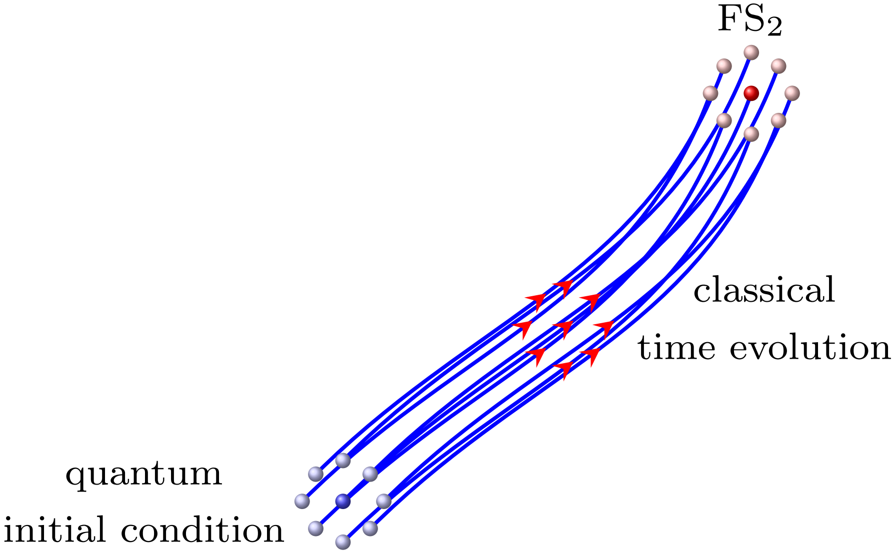
- Solve the **classical** Equation Of Motion (EOM) $[D_\mu, F^{\mu\nu}] = J^\nu$

- Compute $\left\langle \vec{E}^2(\tau, \vec{x}) \right\rangle$, where $\langle\rangle$ is the average on the $c_{\boldsymbol{k}}$ (Monte-Carlo)

- One can show that this resums all the fastest growing terms at each order, leading to a result that remain bounded when $\tau \to \infty$ [GELIS, LAPPI, VENUGOPALAN (2008)]

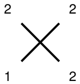
Provided the $a^{\mu}_{\boldsymbol{k}}$ are chosen correctly, gives

$\Rightarrow$ LO+NLO+Subset of higher orders


FS_2
classical
time evolution
quantum
initial condition

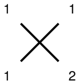
The classical Lagrangean reads

$$\mathcal{L}_{\text{clas}}[\phi_1, \phi_2] = \underbrace{\frac{1}{2}\left(\partial_\mu \phi_1\right)\left(\partial^\mu \phi_2\right)} - \underbrace{\frac{g^2}{3!}\phi_1\,\phi_2^3}$$

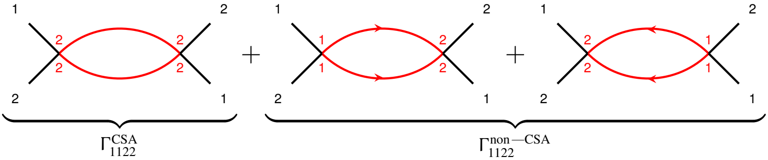


Differs from the full Lagrangean

$$\mathcal{L}_{\text{quant}}[\phi_1, \phi_2] = \mathcal{L}_{\text{clas}}[\phi_1, \phi_2] - \underbrace{\frac{g^2}{4!}\phi_1^3\,\phi_2}$$



$\Gamma_{1122} =$



Divergences? (ingoing momentum $P = (p^0, p)$, $P^2 = (p^0)^2 - |\boldsymbol{p}|^2$)

$$\Gamma^{\mathrm{CSA}}_{1122} - \frac{g^4\theta(-P^2)}{16\pi|\boldsymbol{p}|}\Lambda g^4 + \text{finite}$$

Linearly divergent, non analytic in P! $\Rightarrow$ non-renormalizable.

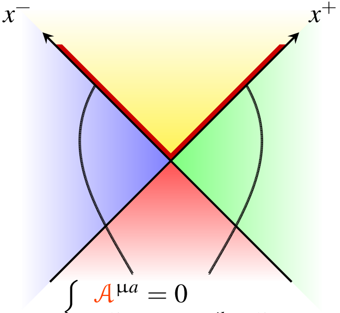
In the full theory: $\Gamma^{\mathrm{non-CSA}}_{1122} = \dfrac{g^4\theta(-P^2)}{16\pi|\boldsymbol{p}|}\Lambda g^4 + \text{finite}$

So in the full theory Γ_{1122} is finite.

No big Λ effect if $\Lambda \ll \dfrac{16\pi}{g^4} Q_{\text{phys}}$

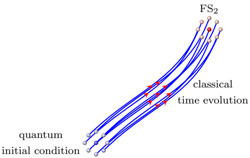
Effects at late times no matter what

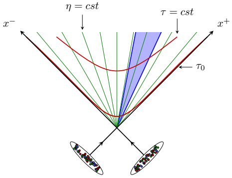
- Need to know $a^{\mu}_{\boldsymbol{k}}(\tau_0, \boldsymbol{x})$ at the time τ_0 we start the numerical simulation
- For practical reasons, we must start in the forward light cone $(\tau_0 > 0)$



$$\begin{cases} \mathcal{A}^{\mu a} = 0 \\ a^{\mu a}_{\boldsymbol{k}\lambda c} = \delta^a_c \, e^{ikx} \, \varepsilon^{\mu a}_{\boldsymbol{k}\lambda} \end{cases}$$

This can be done analytically
[TE,GELIS (2013)]





Initial condition

$$A_0{}^{\mu a}(\tau_0, \boldsymbol{x}_\perp, \eta) = \mathcal{A}_0{}^{\mu a}(\tau_0, \boldsymbol{x}_\perp) + \sum_{\lambda c} \int_{\nu \boldsymbol{k}_\perp} c^{c\lambda}_{\nu \boldsymbol{k}_\perp} \, a^{\mu a}_{\boldsymbol{k}_\perp \nu \lambda c}(\tau_0, \boldsymbol{x}_\perp, \eta)$$

Time evolution ($I = x, y, \eta$) for each configuration

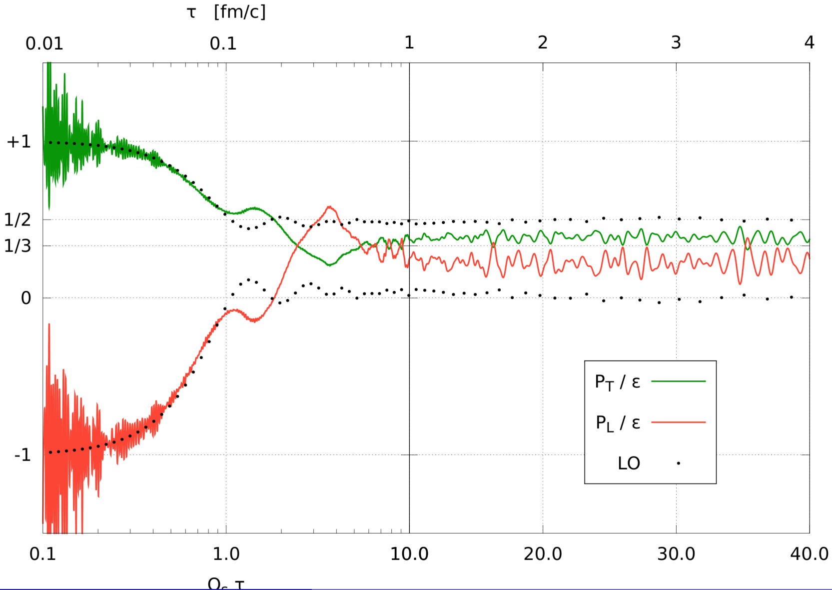
$$D_\mu F^{\mu I} = 0 \qquad \Rightarrow \qquad T^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} - F^{\mu\rho} F^{\nu}{}_{\rho}$$

Cross checks: Gauss's law and Bjorken's law

$$D_\mu E^\mu = 0 \qquad \tau \partial_\tau \epsilon = -\epsilon - P_L$$

NUMERICAL RESULTS [TE,GELIS (2013)]

$\alpha_s = 2\,10^{-2}$ $(g = 0.5)$



Assuming simple first order viscous hydrodynamics

$$\epsilon \approx \underbrace{\epsilon_0\tau^{-\frac{4}{3}}}_{\text{ideal hydro}} - \underbrace{2\eta_0\tau^{-2}}_{\text{first order correction}}$$

we can compute the dimensionless ratio ($\eta = \eta_0\tau^{-1}$)

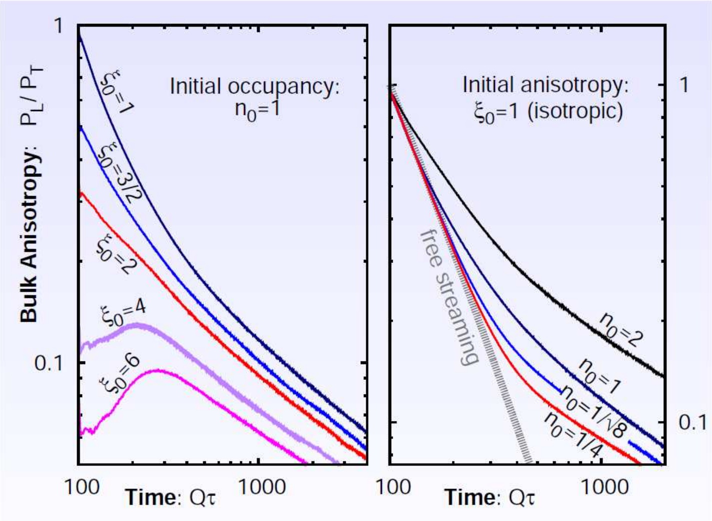
$$\eta\epsilon^{-\frac{3}{4}} \lesssim 1$$

In contrast, perturbation theory at LO gives $\eta\epsilon^{-\frac{3}{4}} \sim 300$.

If the system is nearly thermal

$$\epsilon^{\frac{3}{4}} \sim s \Longrightarrow \frac{\eta}{s} \text{ close to } \frac{1}{4\pi} \text{ (ideal fluid)}$$

TG VERSUS BBSV



BBSV Scenario

- Start at $Q\tau \sim 100$ with $\mathcal{A} = 0$ and $a \sim \frac{1}{g} \to g$ scales out
- at $Q\tau \gtrsim 300$: $f(p_\perp, p_z) = (Q_s\tau)^\alpha f_0((Q_s\tau)^\beta p_\perp, (Q_s\tau)^\gamma p_z)$
- $\alpha, \beta, \gamma = (-\frac{2}{3}, 0, \frac{1}{3})$ "universals". deduced from

$$\epsilon = \text{cst} \times \tau^{-1} \qquad n = \text{cst} \times \tau^{-1} \qquad \partial_\tau f = \hat{q}\partial_z^2 f$$

Other possible explanation? [TE, MOORE (Work in progress)]

BBSV Scenario

- Start at $Q\tau \sim 100$ with $\mathcal{A} = 0$ and $a \sim \frac{1}{g} \to g$ scales out
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$$\epsilon = \mathrm{cst} \times \tau^{-1} \qquad n = \mathrm{cst} \times \tau^{-1} \qquad \partial_\tau f = \hat{q}\partial_z^2 f$$

TG: LO+NLO fully; $Q_s\tau_{\text{init}} \ll 1$; $g \lesssim 0.5$

upside: close to real situation

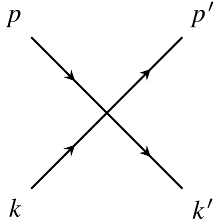
downside: Λ effects ?
$Q_s\tau_{\text{init}} \gg 1$ not accesible

BBSV: not clear LO, not NLO; $Q_s\tau_{\text{init}} \gg 1$; $g \lesssim 10^{-6}$

upside: Almost no Λ effects

downside: Phenomenological relevance?
Fixed point IC dependent?

CSA non renormalizable: How to really see the Λ effects?[1]



Prerequisites

$g \ll 1$

$f \ll g^{-2}$ $(Qt \gg 1)$

f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|) = f_p$

[1] Gauge case: [ABRAAO, KURKELA, LU, MOORE (2014)]

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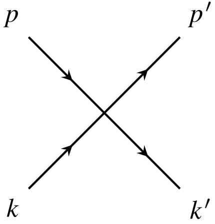

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f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|) = f_p$

$$\partial_t f_p = \frac{(2\pi)^4 g^4}{4E_p} \int_{p',k,k'} \delta^4(P + K - P' - K') \underbrace{\left[(1+f_p)(1+f_k) f_{p'} f_{k'} - f_p f_k (1+f_{p'})(1+f_{k'})\right]}_{F[f]}$$

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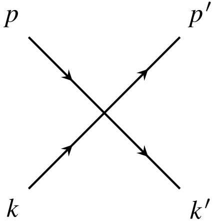

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$g \ll 1$
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f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|) = f_p$

Quantum theory, Q: keep everything

$$F_{\mathrm{Q}}[f] = \underbrace{(f_p + f_k)f_{p'}f_{k'} - f_p f_k(f_{p'} + f_{k'})}_{\sim f^3} + \underbrace{f_{p'}f_{k'} - f_p f_k}_{\sim f^2}$$

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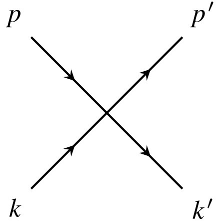
CSA non renormalizable: How to really see the Λ effects?[1]



Prerequisites
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$f \ll g^{-2}$ $(Qt \gg 1)$
f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|) = f_p$

Classical approximation, $\mathcal{C}^0$: $f \gg 1$, keep the dominant term in $F_{\mathcal{Q}}[f]$

$$F_{\mathcal{C}^0}[f] = \underbrace{(f_p + f_k) f_{p'} f_{k'} - f_p f_k (f_{p'} + f_{k'})}_{\sim f^3}$$

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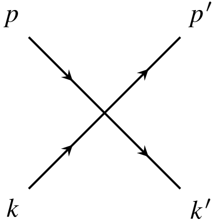
CSA non renormalizable: How to really see the Λ effects?[1]



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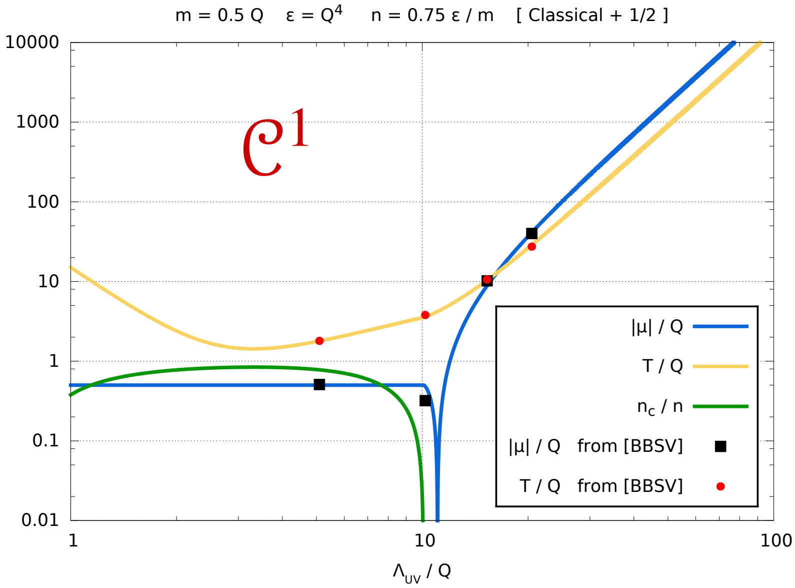
f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|) = f_p$

CSA, $\mathcal{C}^1$: $\mathcal{C}^0$ and then $f \to f + \frac{1}{2}$ [MUELLER, SON (2002)]

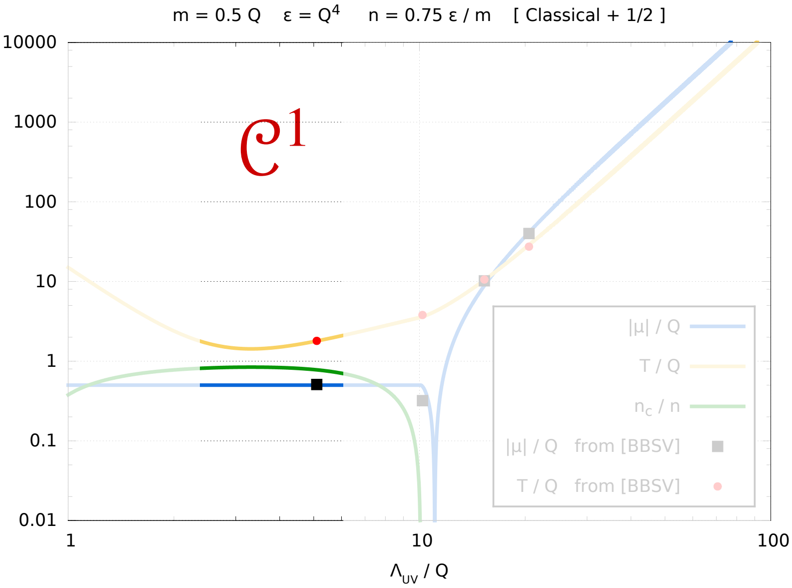
$$F_{\mathcal{C}^1}[f] = \underbrace{F_{\mathcal{Q}}[f]}_{\sim f^3+f^2} + \underbrace{\frac{1}{4}(f_{p'} + f_{k'} - f_p - f_k)}_{\sim f}$$

[1] Gauge case: [ABRAAO, KURKELA, LU, MOORE (2014)]

Kinetic treatment: Numerical results for isotropic f
[TE, Gelis, Tanji, Wu (2014)]



KINETIC TREATMENT: NUMERICAL RESULTS FOR ISOTROPIC f [TE, GELIS, TANJI, WU (2014)]



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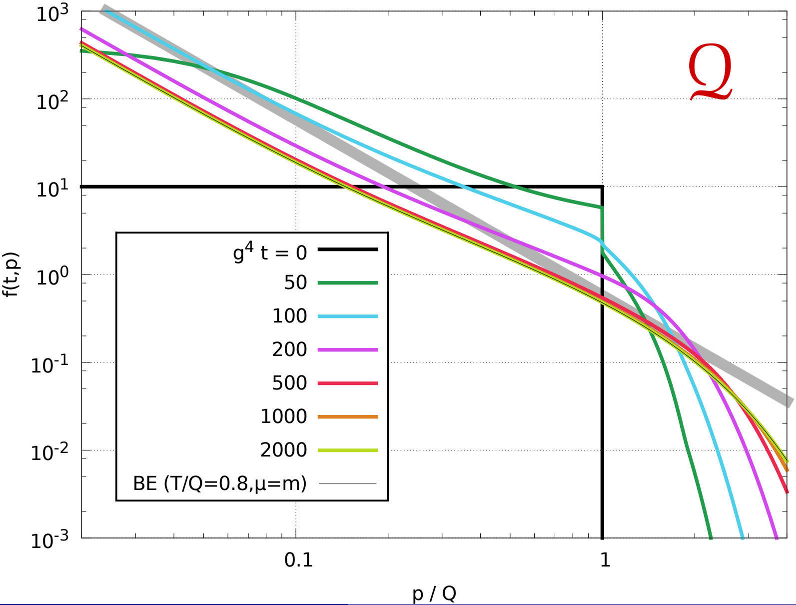

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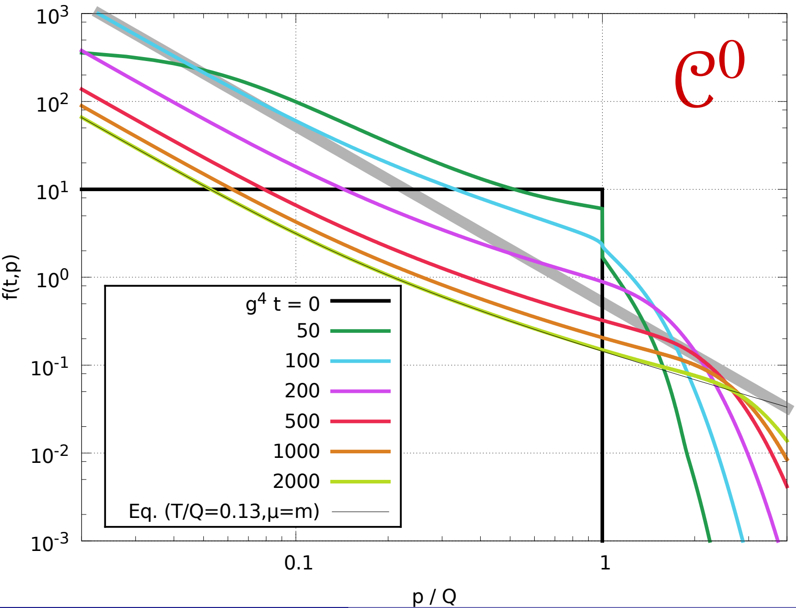

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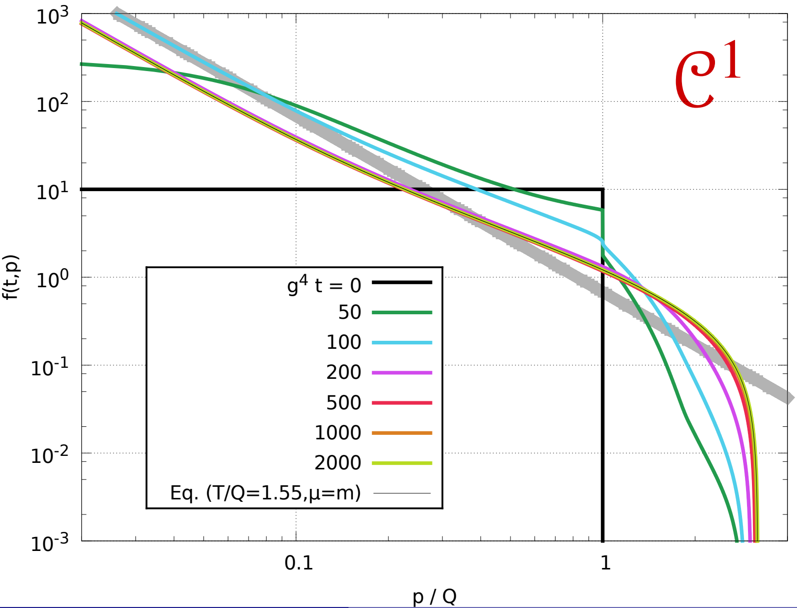
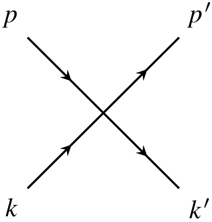




Prerequisites

$g \ll 1$

$f \ll g^{-2}$ $(Qt \gg 1)$

f anisotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}_\perp|, p_z) = f_{p_\perp z}$

Boltzmann equation for $2 \leftrightarrow 2$ elastic scattering

$$\partial_t f_{p_\perp z} = \frac{(2\pi)^4 g^4}{4E_p} \int_{p',k,k'} \delta^4(P + K - P' - K') F[f]$$

Now suppose f very anisotropic initially

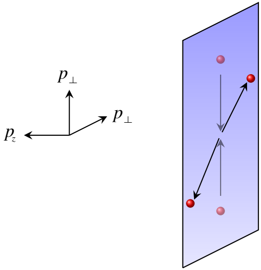
$$f_{p_{\perp z}} \sim \delta(p_z) f_0(p_\perp)$$

What can happen?

Now suppose f very anisotropic initially

$$f_{p_\perp z} \sim \delta(p_z) f_0(p_\perp)$$

What can happen?



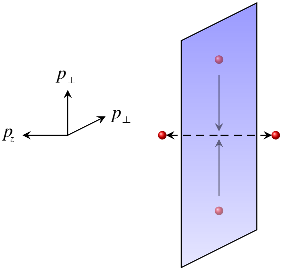
In plane collisions $\rightarrow$ no isotropization

KINETIC TREATMENT: WHAT TO EXPECT FOR ANISOTROPIC f?

Now suppose f very anisotropic initially

$$f_{p_{\perp z}} \sim \delta(p_z) f_0(p_\perp)$$

What can happen?

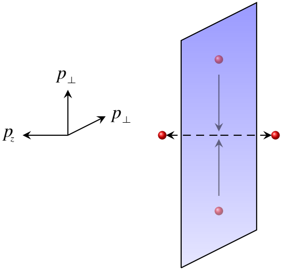


Out of plane collisions $\rightarrow$ isotropization

Now suppose f very anisotropic initially

$$f_{p_\perp z} \sim \delta(p_z) f_0(p_\perp)$$

What can happen?



Out of plane collisions $\rightarrow$ isotropization

Can these large angle collisions happen?

Remember

$$F_{\mathcal{C}^0}[f] \sim f^3$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_{p'}f_{k'} - f_p f_k$$

Now take initially $f_{p_{\perp z}} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

Remember

$$F_{\mathcal{C}^0}[f] \sim f^3$$

$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_{p'} f_{k'} - f_p f_k$$

Now take initially $f_{p_{\perp z}} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

$\mathcal{C}^0$

$$\partial_t f_{p_{\perp z}} \sim \delta(p_z) \int_{p'_\perp, k_\perp, k'_\perp} \delta^3(P_\perp + K_\perp - P'_\perp - K'_\perp) f^3$$

Remember

$$F_{\mathcal{C}^0}[f] \sim f^3$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_{p'}f_{k'} - f_p f_k$$

Now take initially $f_{p_{\perp z}} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

$\mathcal{Q}$

$$\partial_t f_{p_{\perp z}} \sim \delta(p_z) \int_{p'_\perp, k_\perp, k'_\perp} \delta^3(P_\perp + K_\perp - P'_\perp - K'_\perp) f^3$$
$$+ \int_{p'_\perp, k, k'_\perp} \delta^3(P_\perp + K_\perp - P'_\perp - K'_\perp)\delta(k_z + p_z) f_{p'_{\perp z}} f_{k'_{\perp z}}$$
$$- \delta(p_z) \int_{p', k_\perp, k'} \delta^3(P_\perp + K_\perp - P'_\perp - K'_\perp)\delta(k'_z + p'_z) f_{p_{\perp z}} f_{k_{\perp z}}$$

Remember

$$F_{\mathcal{C}^0}[f] \sim f^3$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_{p'}f_{k'} - f_p f_k$$

Now take initially $f_{p_{\perp z}} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

$\mathcal{Q}$

$$\partial_t f_{p_{\perp z}} = \delta(p_z) \int_{p_\perp', k_\perp, k_\perp'} \delta^3(P_\perp + K_\perp - P_\perp' - K_\perp') f^3$$
$$+ \int_{p_\perp', k, k_\perp'} \delta^3(P_\perp + K_\perp - P_\perp' - K_\perp') \delta(k_z + p_z) f_{p'_{\perp z}} f_{k'_{\perp z}}$$
$$-\delta(p_z) \int_{p', k_\perp, k'} \delta^3(P_\perp + K_\perp - P_\perp' - K_\perp') \delta(k_z' + p_z') f_{p_{\perp z}} f_{k_{\perp z}}$$

Remember

$$F_{\mathcal{C}^0}[f] \sim f^3$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_{p'}f_{k'} - f_p f_k$$

Now take initially $f_{p_{\perp z}} = 2\pi\delta(p_z)f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

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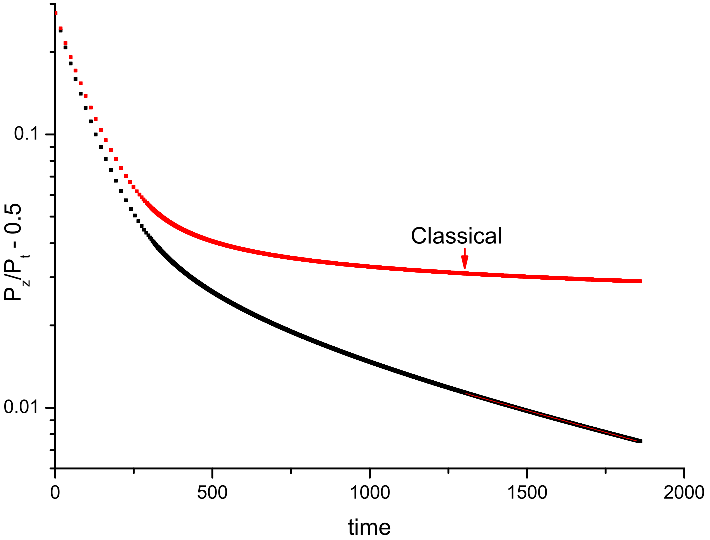
Could it be the reason why $\mathcal{C}^0$ never isotropizes?

ANISOTROPIC f: ILLUSTRATION OF THE PROBLEM WITH THE $\mathcal{C}^0$ SCHEME
[BLAIZOT, JIANG, LIAO] WORK IN PROGRESS



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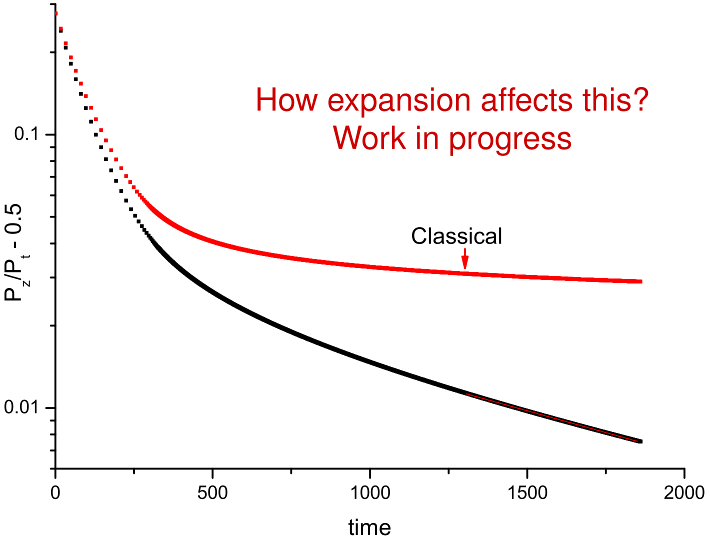


ARE THE POSTULATES OF HYDRODYNAMICS SATISFIED DURING THE EARLY STAGES OF A HEAVY-ION COLLISION?

Conclusion

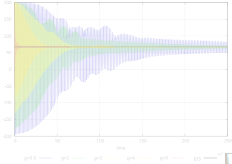
- Scalar models: strong hints of hydrodynamical behaviour ☑
- Yang-Mills: Evidences for an early hydrodynamical onset ☑
- Hydrodynamization already happens at weak coupling ☑
- CSA non-renormalizable $\Rightarrow$ Effective theory ☑

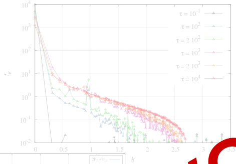
ARE THE POSTULATES OF HYDRODYNAMICS SATISFIED DURING THE EARLY STAGES OF A HEAVY-ION COLLISION?

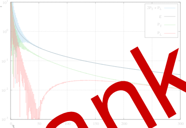
Perspectives

- Boltzmann treatment in the expanding case
- Understanding the BSSV exponants
- Going beyond the CSA $\Rightarrow$ Quantum evolution

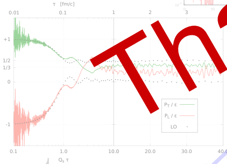
FS_2







$\frac{\eta}{s}$ close to $\frac{1}{4\pi}$



Thank you!

classical

time evolution

quantum

initial condition

KINETIC TREATMENT: NUMERICAL RESULTS FOR ISOTROPIC f

[TE, GELIS, TANJI, WU (2014)]