Thermalization process in ultra-relativistic heavy-ion collisions

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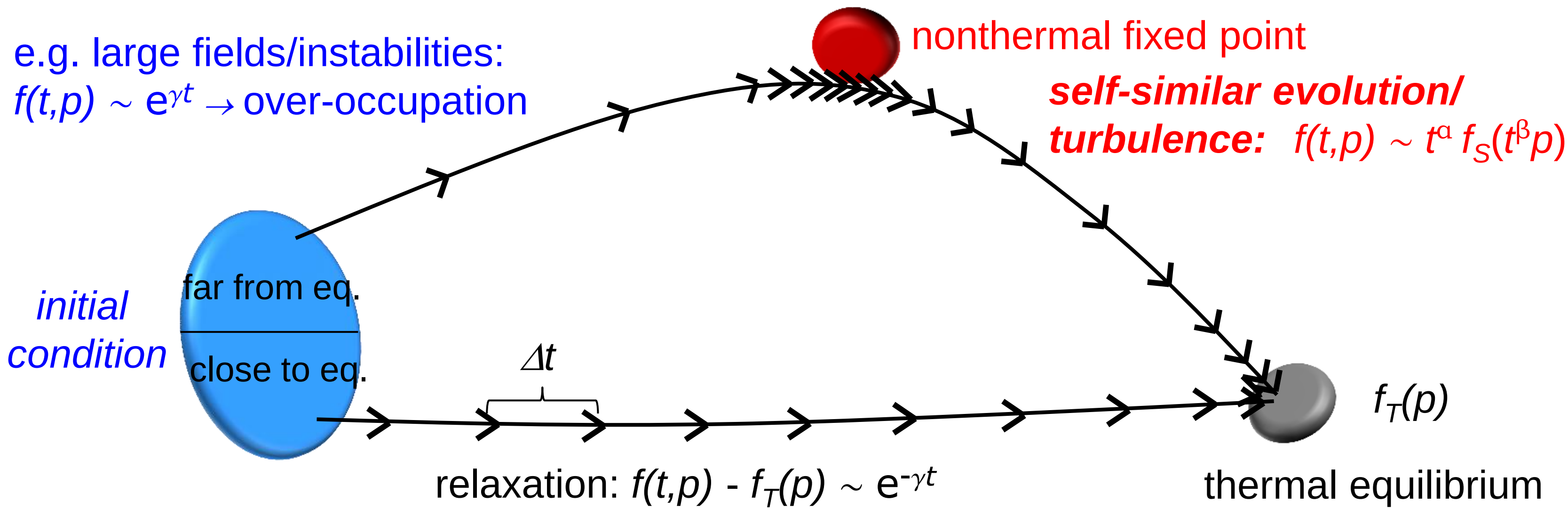
arXiv:1408.1670 & PRD 89 (2014) 074011 & 114007

Far-from-equilibrium vs. close-to-equilibrium dynamics

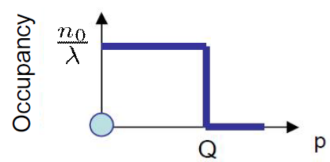


Far-from-equilibrium: ***Rapid loss of initial condition details / universality***

1) Large initial fields:

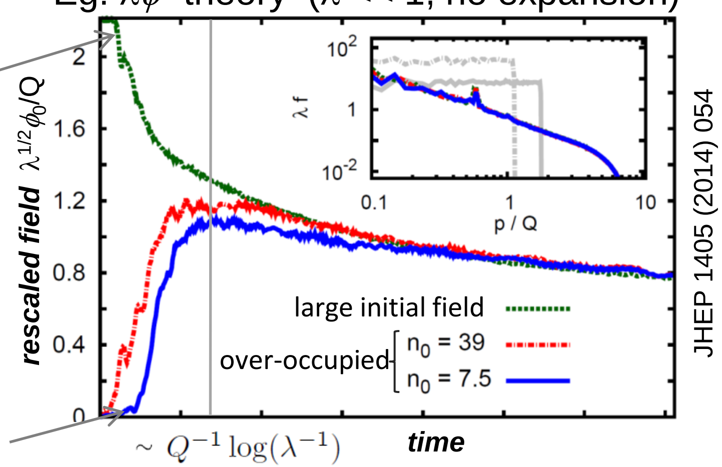
$\phi_0 \sim Q/\sqrt{\lambda}$ $\quad f = 0$

2) Initial over-occupation:



$\phi_0 = 0$ $\quad f(p \leq Q) = n_0/\lambda$

Eg: $\lambda\phi^4$ theory (λ << 1, no expansion)



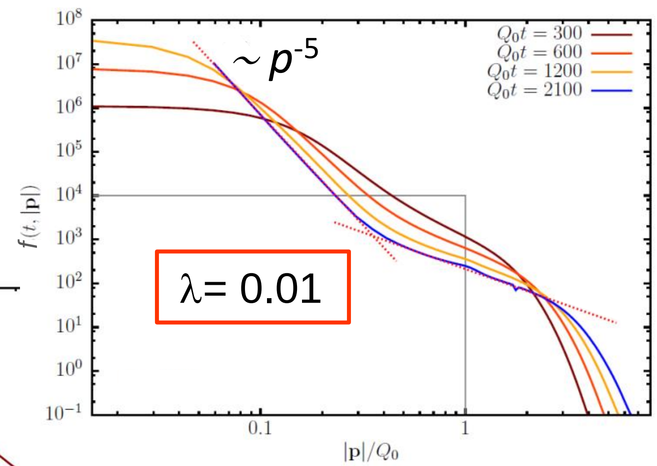
Universal scaling dynamics

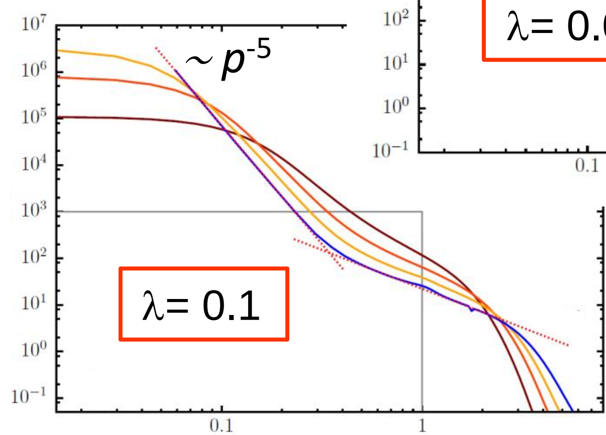
E.g. *N*-component $\lambda\phi^4$ theory (no expansion):

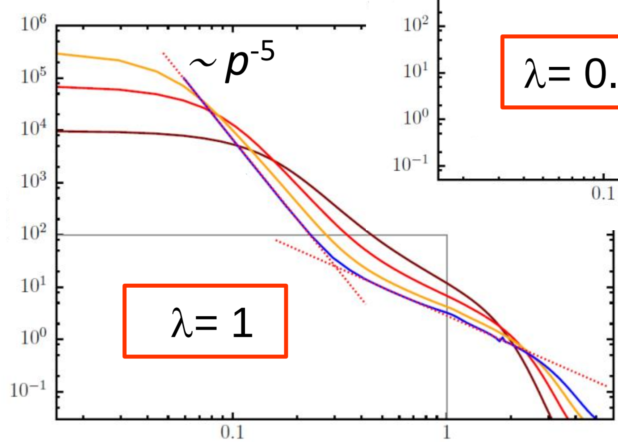
Son; Khlebnikov, Tkachev; Prokopec, Roos; Kofman, Felder; Garcia Bellido; Micha; JB, Rothkopf, Schmidt, Sexty; Gasenzer, Novak …

$f(t,p) \sim t^{\alpha} f_S(t^{\beta} p)$

α, β, f_S universal







B. Wallisch

***QFT* approach:**
2PI 1/*N* to NLO

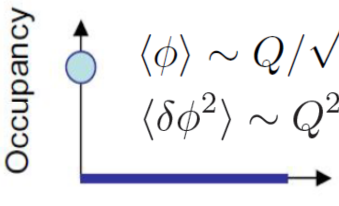
universal scaling behavior for wide range of couplings!

Suitable resummation techniques for scalar quantum theories exist beyond the weak coupling limit – but much more difficult for gauge theories!

Classical-statistical lattice simulations

Weak-coupling dynamics of large fields/high occupancies can be accurately mapped onto classical-statistical theory, which is simulated on a lattice

Two-step mapping: $\phi = \phi_0 + \delta\phi$ *Son (`96), Klebnikov, Tkachev (`96),…*



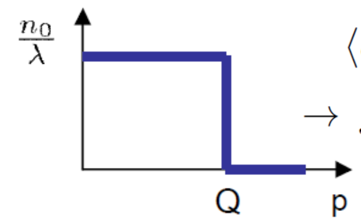
$\langle \phi \rangle \sim Q/\sqrt{\lambda}$

$\langle \delta\phi^2 \rangle \sim Q^2$

$$f(Q) \sim e^{\gamma_Q \Delta t}$$

$\Delta t \sim Q^{-1} \log \lambda^{-1}$

instability



$\langle \delta\phi^2 \rangle \sim Q^2/\lambda$

$\rightarrow f(Q) \sim 1/\lambda \gg 1$

1. Large field: ***linear regime*** *in* $\delta\phi$
   - solve linearized e.o.m.
     $\rightarrow$ well-defined continuum limit

2. High occupancy: ***non-linear regime***
   - finite if *f*(*p*) falls faster than p^{-1}
   - super-renormalizable if *f*(*p*) $\sim p^{-1}$

Aarts, Smit (`97)

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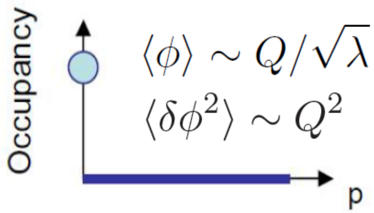

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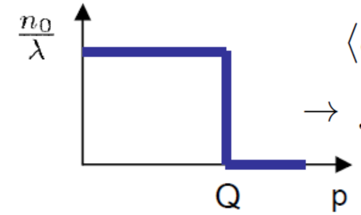

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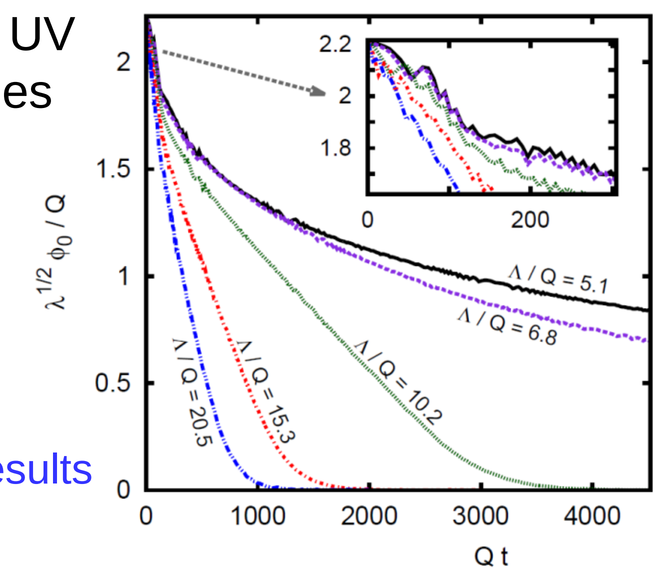
If replaced by non-linear theory from t=0 for **finite** UV cutoff $\Lambda >> Q$, then only accurate for small λ & times

$\to$ explains conflicting scalar results from Epelbaum et al. for λ = 1 (*NPA 872 (2011) 210,…*):

JB, Boguslavski, Schlichting, Venugopalan, JHEP 1405 (2014) 054; Epelbaum, Gelis, Wu, PRD90 (2014) 065029

$\to$ talk by T. Epelbaum for role of cutoff in their gauge results



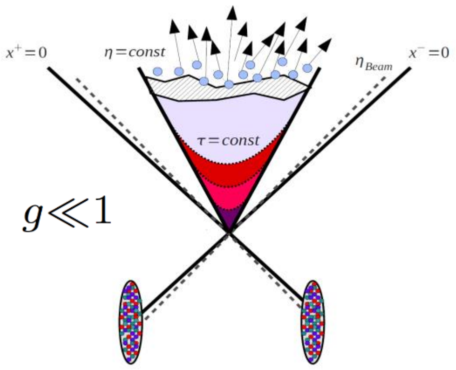
Heavy-ion collisions in the high-energy limit

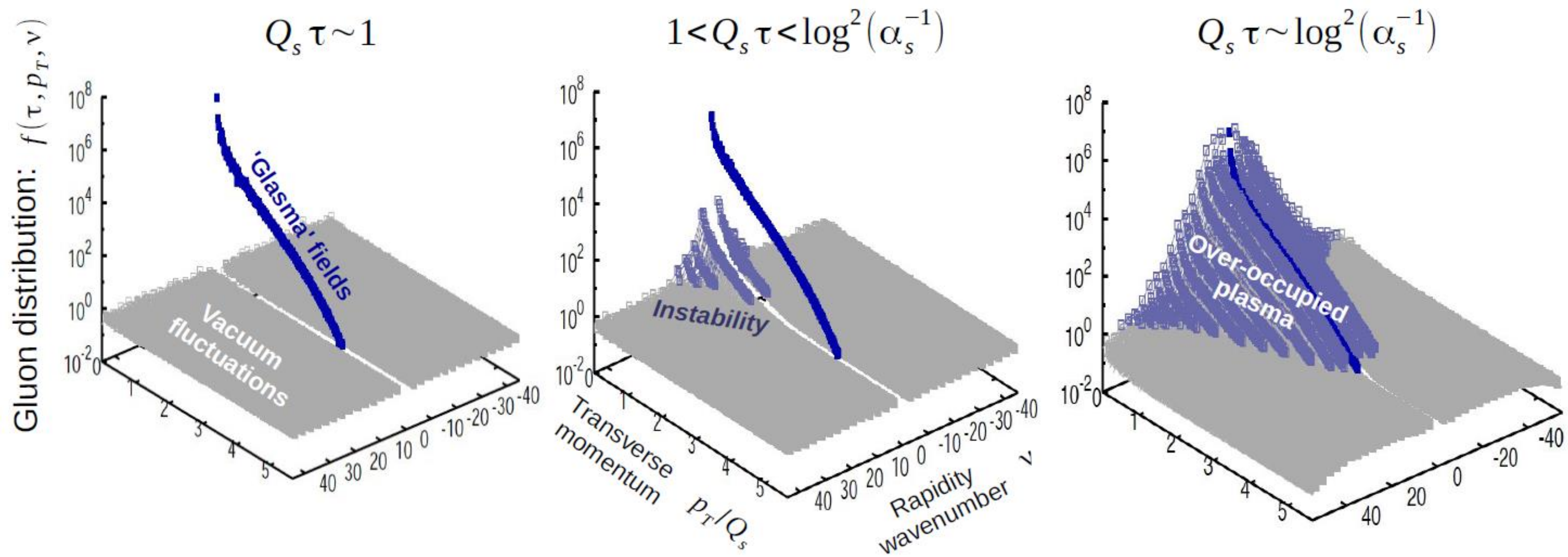
Large initial gauge fields: $\langle A \rangle \sim Q_s/g$

CGC: Lappi, McLerran, Dusling, Gelis, Venugopalan, Epelbaum…

Small initial (vacuum) fluctuations: $\langle \delta A^2 \rangle \sim Q_s^2$

$\rightarrow$ ***plasma instabilities***

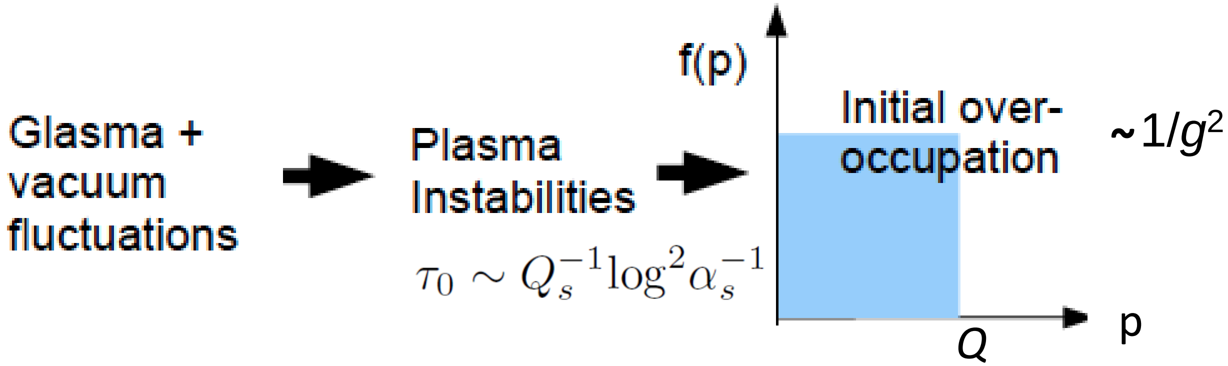




JB, Schenke, Schlichting, Venugopalan, arXiv:1409.1638 for initial spectrum from *Epelbaum, Gelis, PRD88 (2013) 085015*. ***Plasma instabilities from wide range of initial conditions:***

Mrowczynski; Rebhan, Romatschke, Strickland; Arnold, Moore, Yaffe; Bödecker; Attems, … Romatschke, Venugopalan; Berges, Scheffler, Schlichting, Sexty; Fukushima, Gelis …

Initial conditions in the over-occupied QGP



- To see attractor: Initial over-occupation described by family of distributions at τ_0 (Coulomb gauge)

occupancy parameter

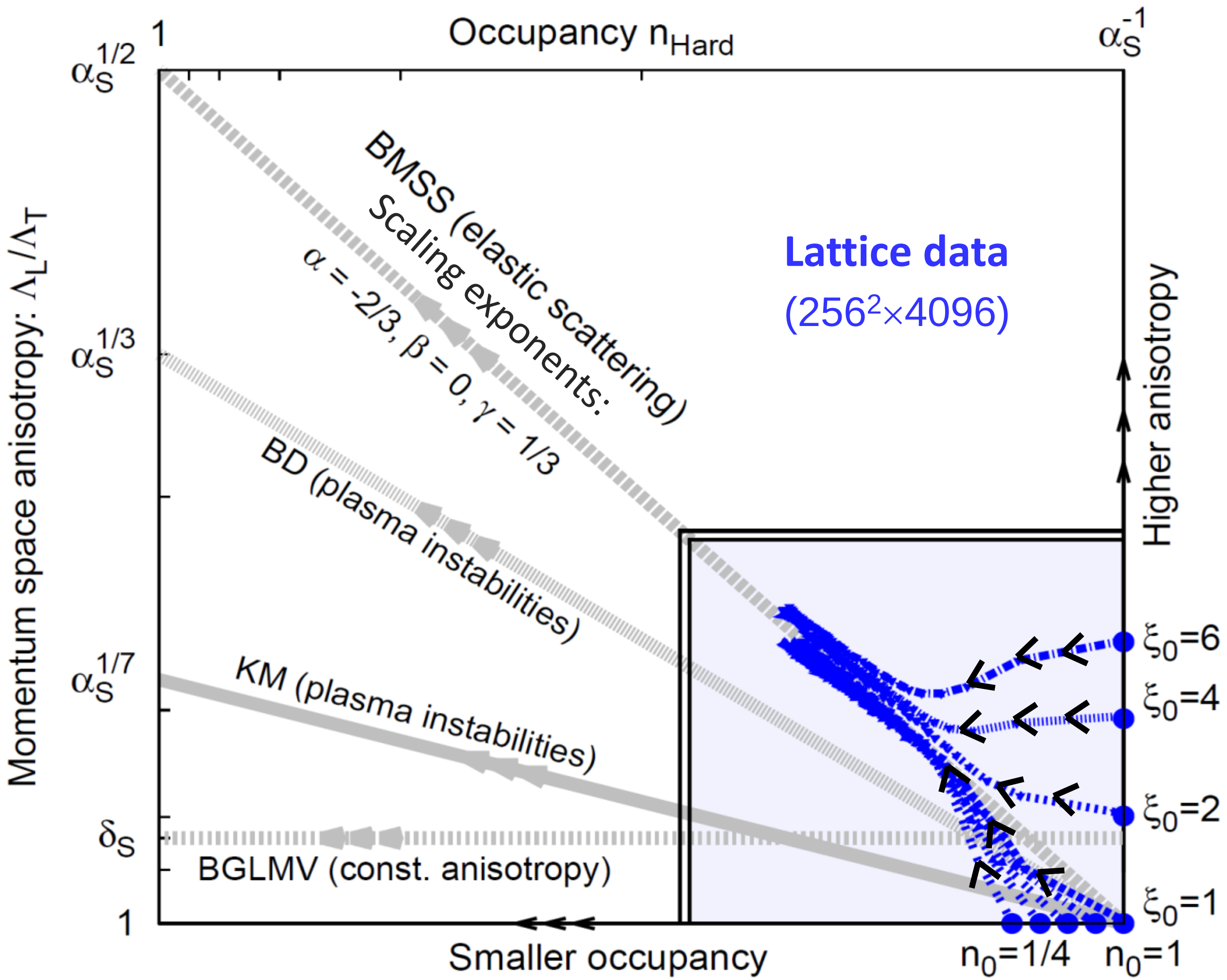
$$f(\mathrm{p_T}, \mathrm{p_z}, \tau_0) = \frac{n_0}{2g^2}\,\Theta\left(Q_s - \sqrt{\mathrm{p_T^2} + (\xi_0 \mathrm{p_z})^2}\right)$$

anisotropy parameter
(controls "prolateness" or "oblateness" of initial momentum distribution)

- Computations performed at very weak coupling, as $\alpha_S = 10^{-5}$ for accurate description at all times in simulation corresponds to $Q\tau_0 \approx \log^2(1/\alpha_S) \approx 100$

Nonthermal fixed point

Evolution in the `anisotropy-occupancy plane´

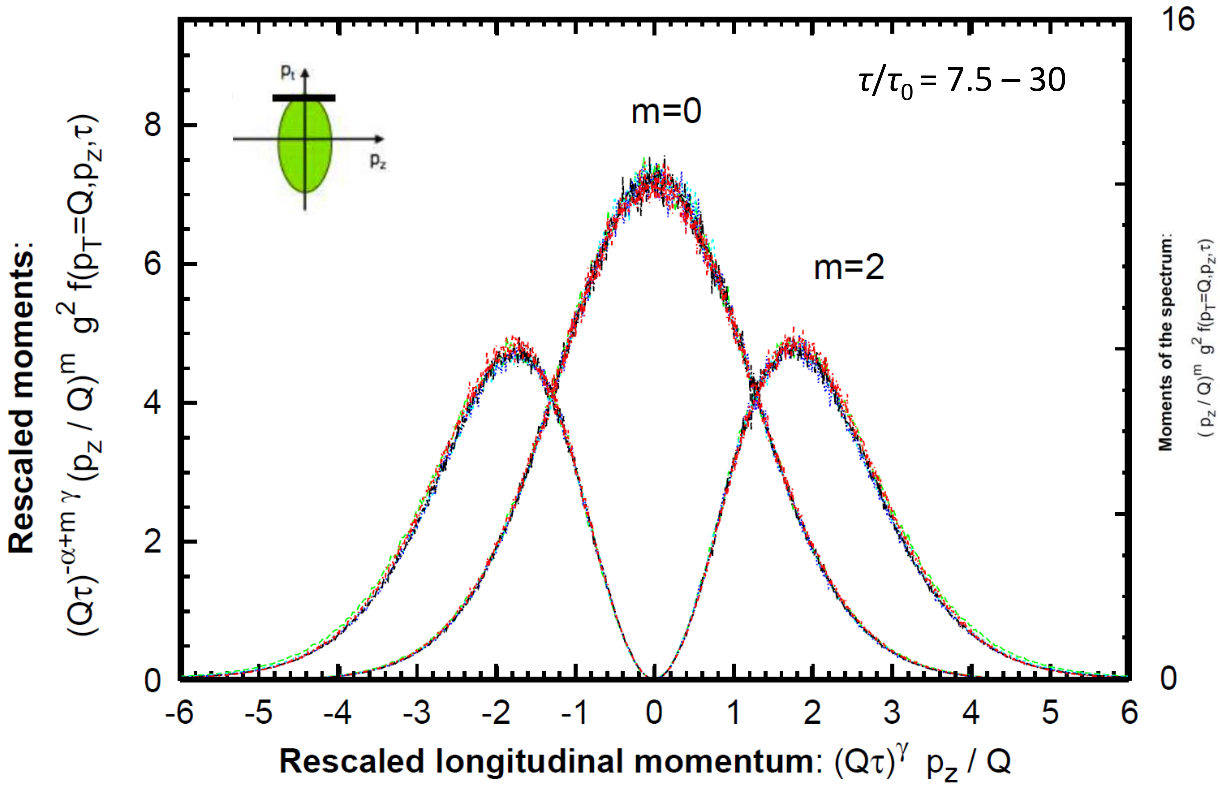


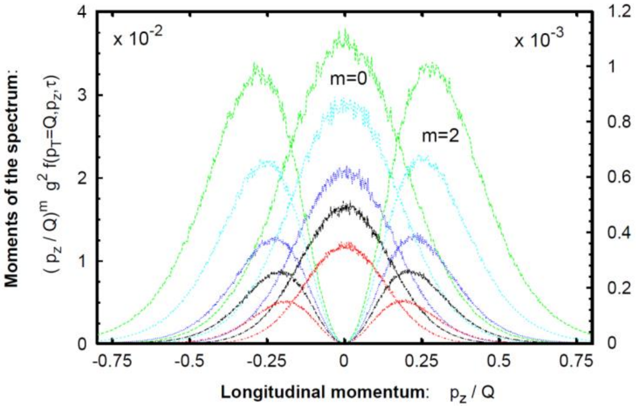
JB, Boguslavski, Schlichting, Venugopalan, PRD 89 (2014) 074011; ibid. 114007

`Bottom-up´* scaling emerges as a consequence of the fixed point!

**Baier et al, PLB 502 (2001) 51*

Self-similar evolution





Scaling exponents: $\alpha = -2/3\,, \qquad \beta = 0\,, \qquad \gamma = 1/3$

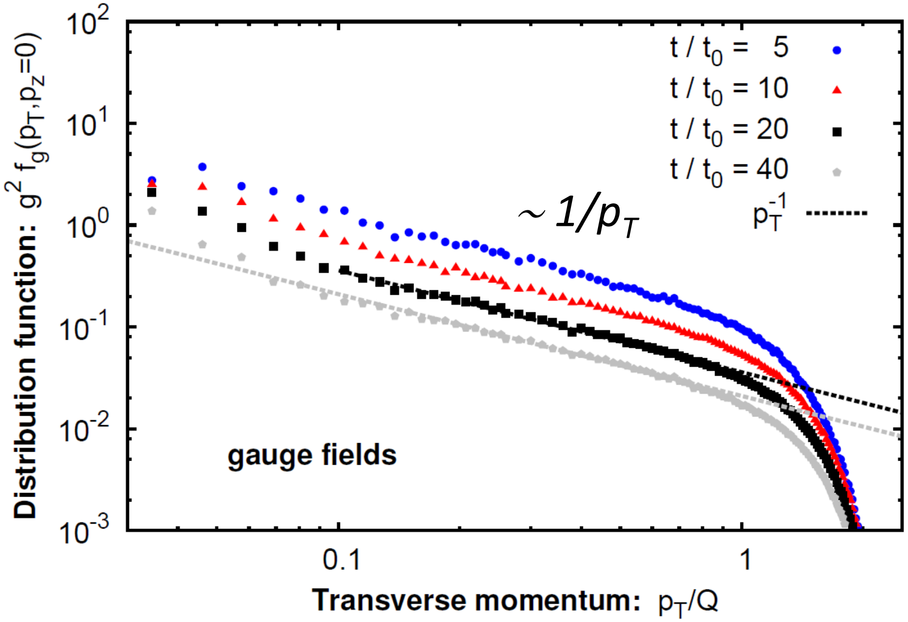
and scaling distribution function f_S:

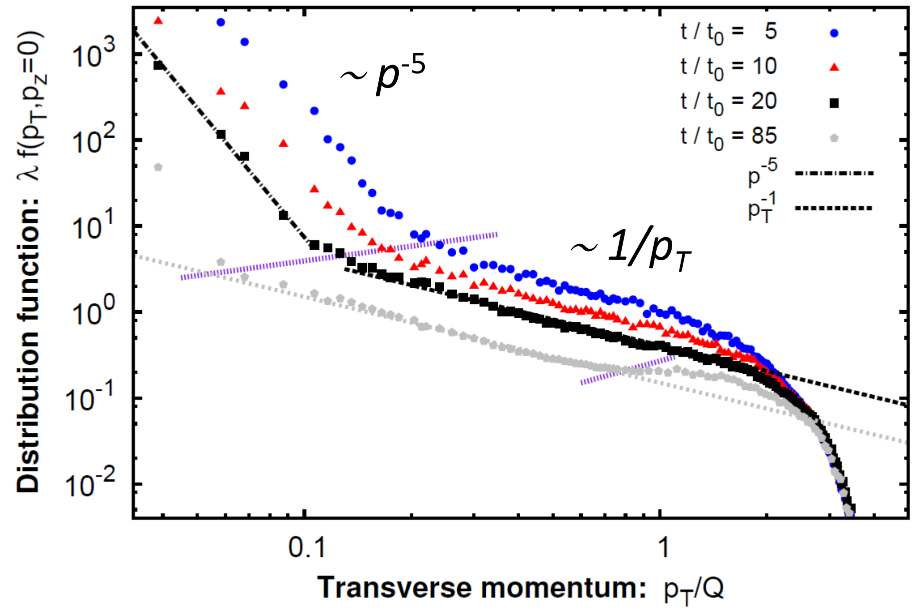
$$f(p_T, p_z, \tau) = (Q\tau)^{\alpha} f_S\Big((Q\tau)^{\beta} p_T, (Q\tau)^{\gamma} p_z\Big)$$

stationary fixed-point distribution

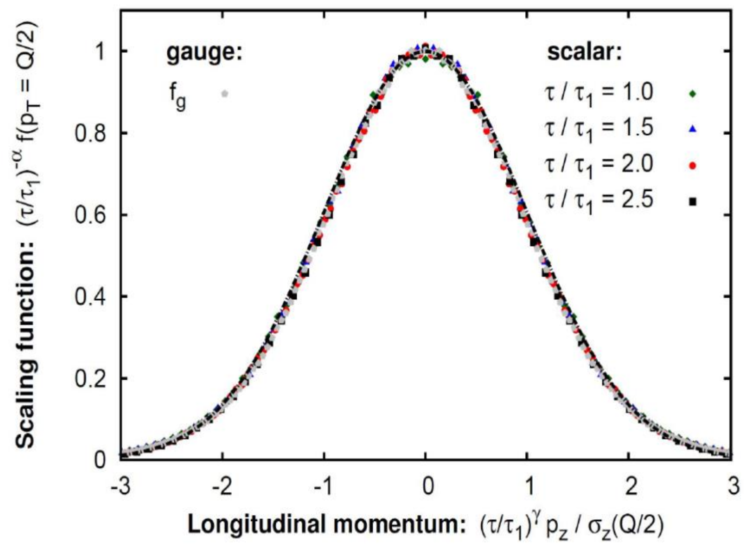
Comparing gauge and scalar field theories

(with longitudinal expansion)





Thermal-like transverse spectrum ~$1/p_T$ even as longitudinal distribution is being \`squeezed´



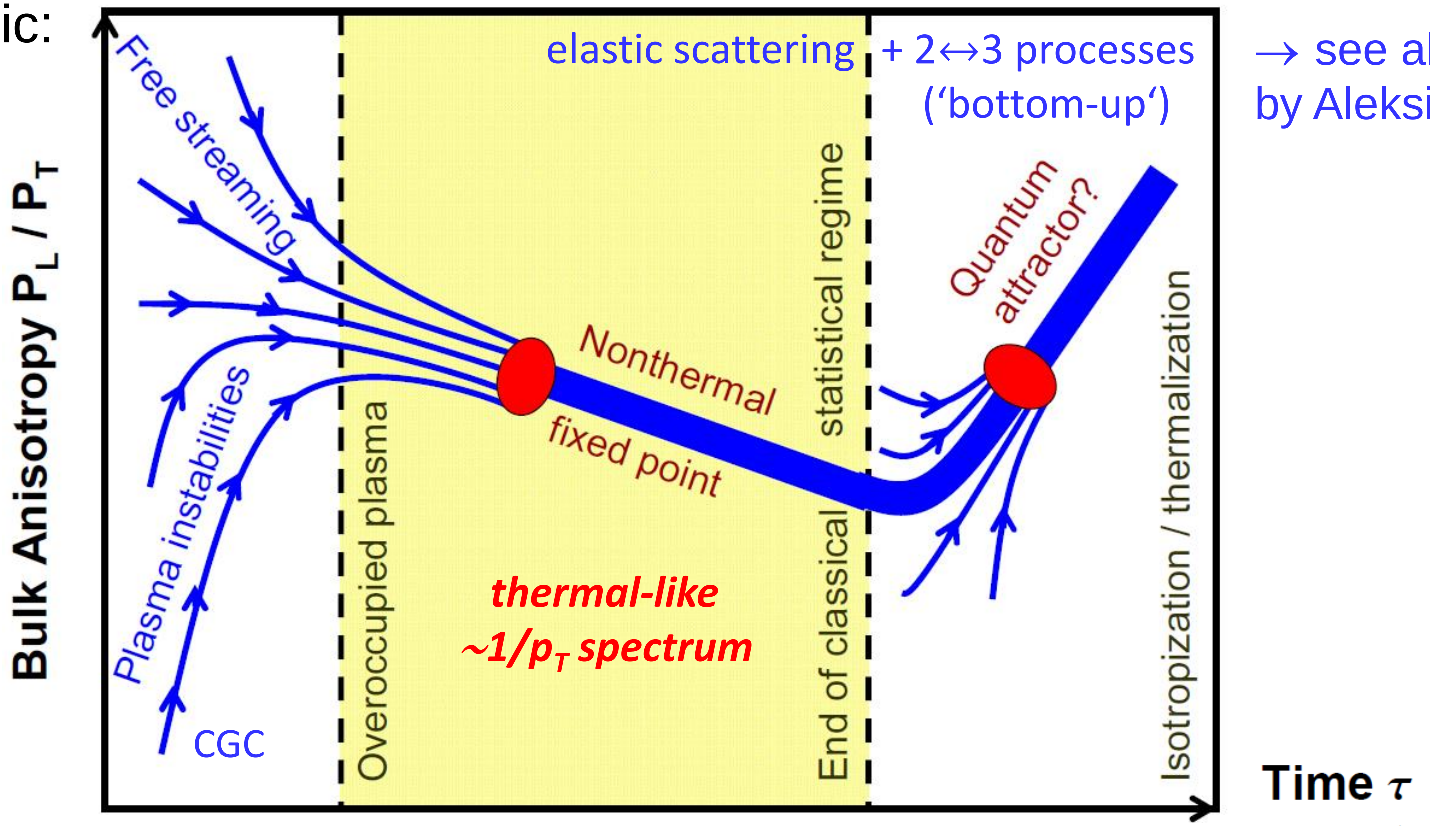
Agreement of α, β & scaling function f_S in inertial range $\sim 1/p_T$

→ universality far from equilibrium

JB, Boguslavski, Schlichting, Venugopalan, arXiv:1408.1670

Thermalization process

Schematic:



→ see also talk by Aleksi Kurkela

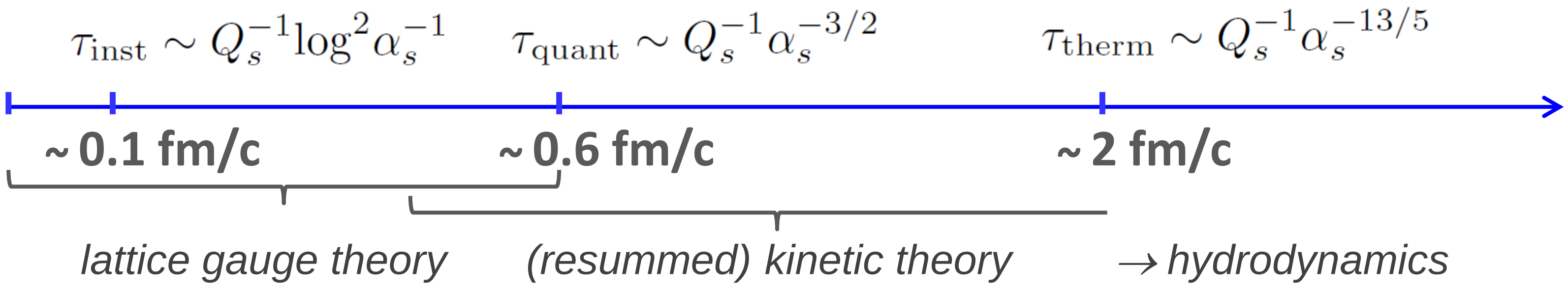
$\tau_{\rm inst} \sim Q_s^{-1} \log^2 \alpha_s^{-1}$ $\tau_{\rm quant} \sim Q_s^{-1} \alpha_s^{-3/2}$ $\tau_{\rm therm} \sim Q_s^{-1} \alpha_s^{-13/5}$

Extrapolation to realistic coupling $\alpha_s \sim 0.3$ for $Q_s \sim 2$ GeV:

$\tau_{inst} \sim$ **0.1 fm/c**	$\tau_{quant} \sim$ **0.6 fm/c**	$\tau_{therm} \sim$ **2 fm/c**
$P_L/P_T \sim$ **20-30%**	$P_L/P_T \sim$ **10-20%**	$P_L \sim P_T$

Conclusions

- Entire thermalization process can be computed from interplay of methods



- Lattice theory & (resummed) kinetic theory have overlapping range of validity

  → ***for the first time quantitative agreements on very large lattices***
  → ***self-similar attractor of longitudinally expanding plasma***

- ***Early thermal-like transverse spectrum ~1/p_T*** *even though the system is still far from equilibrium*

- ***Universality of gauge & scalar dynamics in inertial range ~1/p_T***

  → points to general principle (not small angle scattering? vertex corrections?)