# Azimuthal correlations in pA collisions from CGC

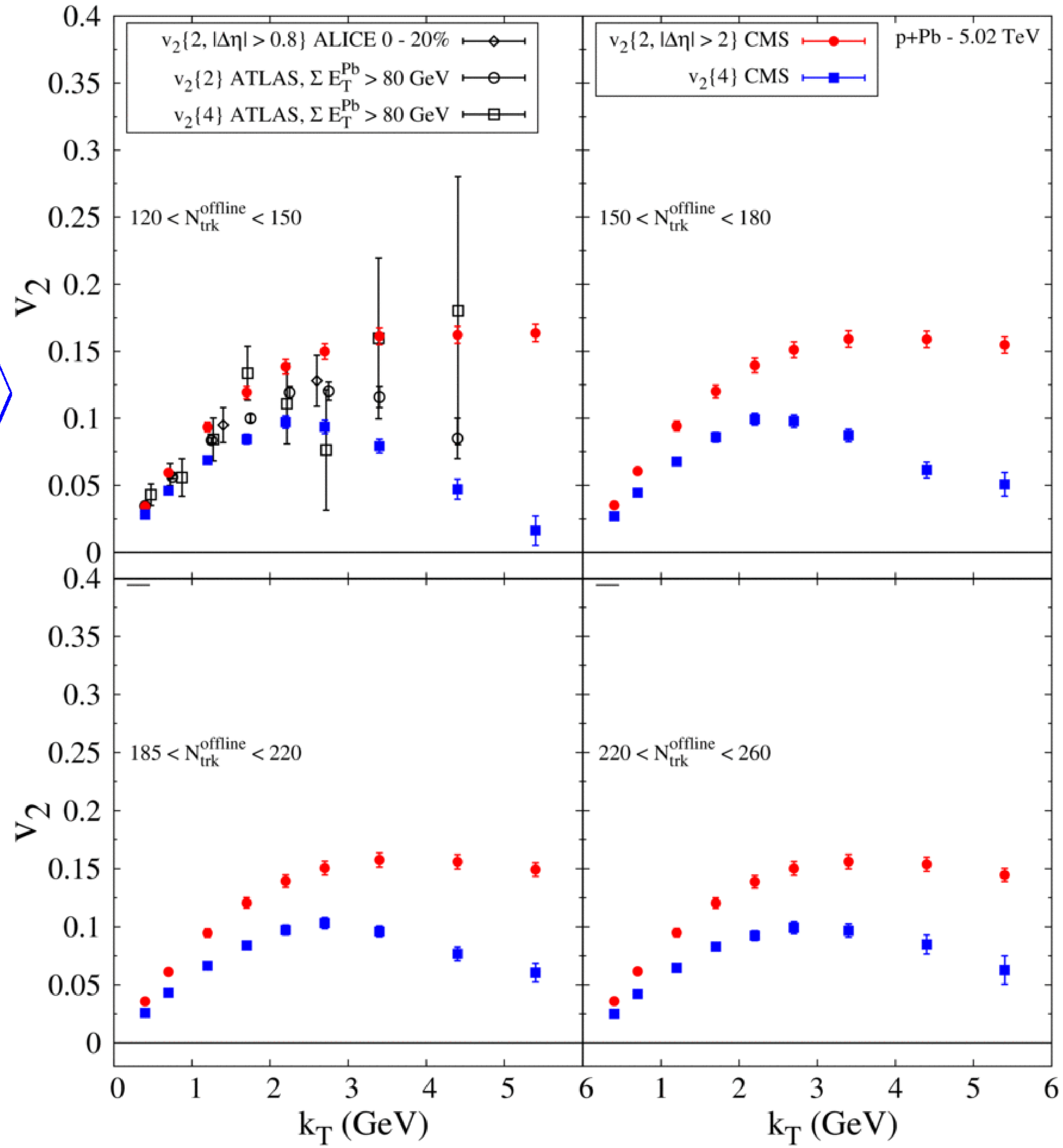
Adrian Dumitru
Baruch College, CUNY

Initial State 2014
Dec. 3 – 7, 2014
Napa, CA

# ATLAS, ALICE & CMS data for $v_2(p_T)$ in high mult. p+Pb @ 5TeV

$$v_n\{2\}^2 = \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$$

- perhaps hydro-like final-state effects at low $p_T$ ?
- $p_T$=2 GeV → $\Delta x \sim 0.1$ fm short-distance physics ?



# Goal here:

- are there initial-state effects that could generate such asymmetries ?

- can we understand / reproduce features at $p_T$ > 1-2 GeV from short distance physics (QCD) ?

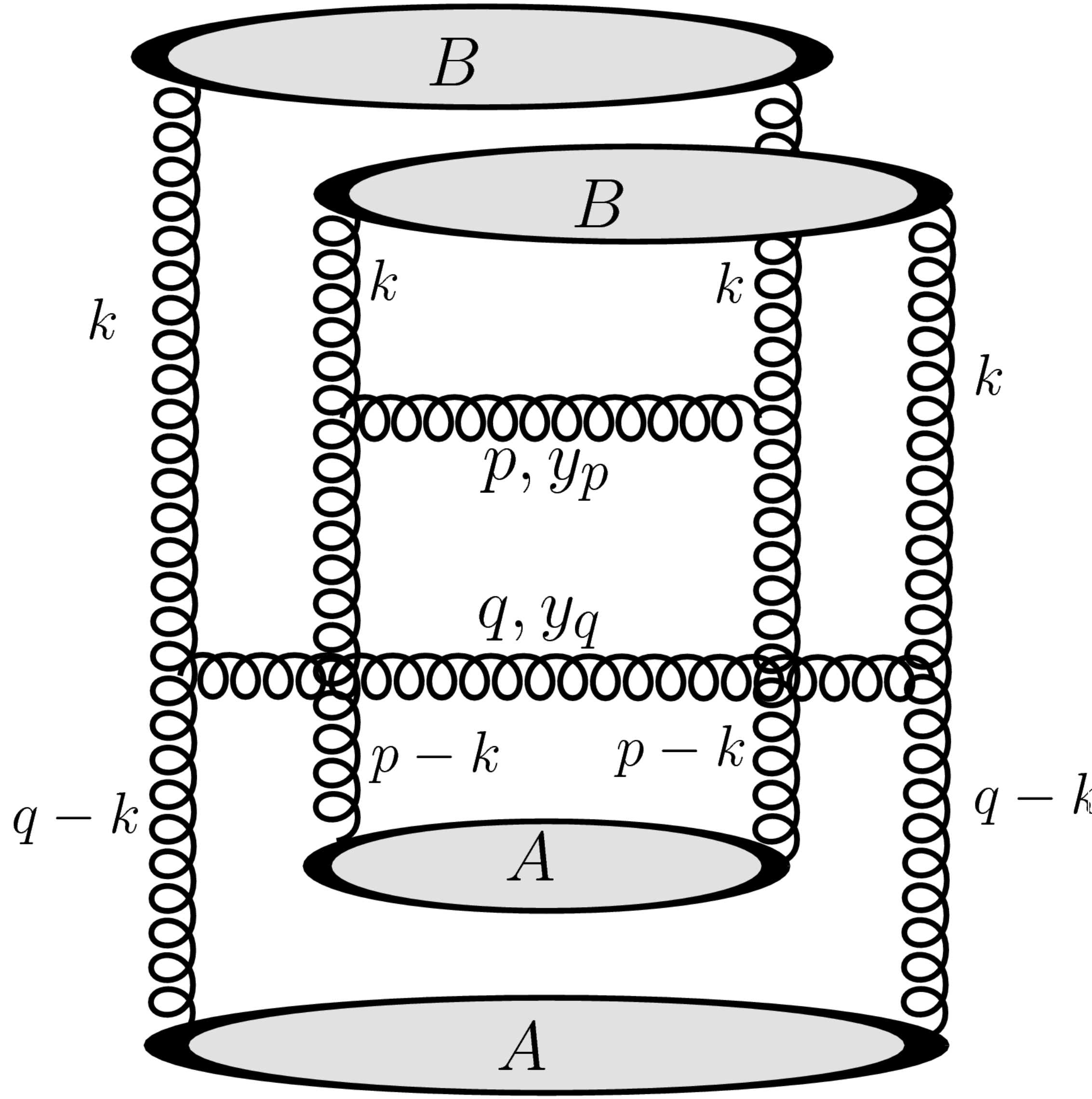
- going to focus on azimuthal asymmetries but very important feature is that correlations are *long range in rapidity* !
  (as opposed to transverse directions)

- this is an automatic prediction by small-x QCD in weak (running-) coupling limit

# So far, we've concentrated on connected diagrams

A.D., Dusling, Gelis, Jalilian-Marian, Venugopalan: PLB (2011)

Dusling and Venugopalan: PRL (2012), [PRD (2013)]$^3$

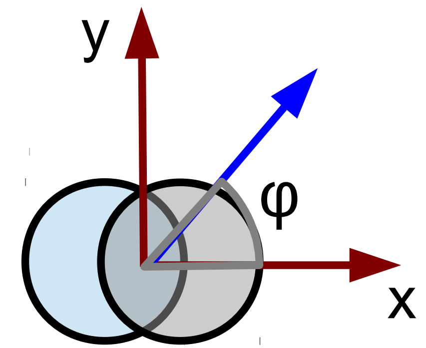



but this can not be the full story,
these diagrams do not give $c_2\{4\}<0$

# Angular asymmetries $v_n$

$$v_n = \langle \cos\, n\phi \rangle$$

avg on 1-particle distribution   (with Φ → -Φ symmetry)




classical impact parameter picture

**2D rotational symmetry spontaneously broken**

- for even n = 2m: $\langle \cos\, n\phi \rangle = +\langle \cos\, n(\phi + \pi) \rangle$   **P = +**
- for odd n = 2m+1: $\langle \cos\, n\phi \rangle = -\langle \cos\, n(\phi + \pi) \rangle$   **P = –**

☆ $-c_2\{4\} = (v_2\{4\})^4 > 0$ requires rot. sym. breaking of single-particle distribution (see below)

# Spontaneous breaking of rotational symmetry :

Kovner & Lublinsky:
PRD 84 (2011)

- $\vec{E}$ field “domains”

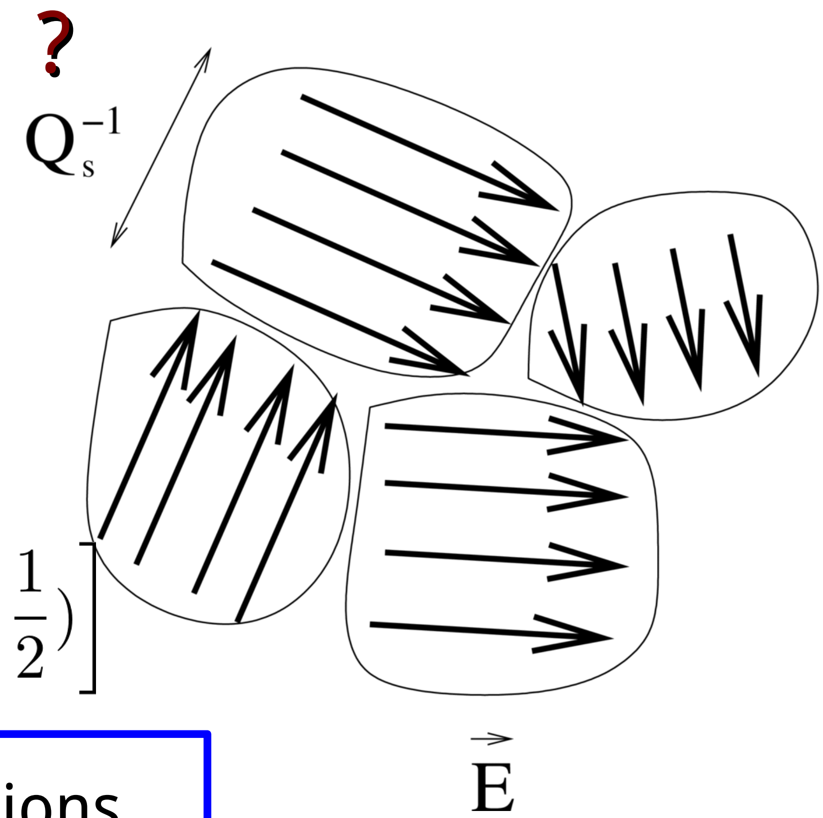
$$D(\vec{r}) = \left\langle e^{-\frac{1}{2}(g\,\vec{r}\cdot\vec{E})^2} \right\rangle$$

usually: avg over ALL configurations:

$$g^2\, r^i r^j \left\langle \mathrm{tr}\, E^i E^j \right\rangle \sim r^2\, Q_s^2$$

here: avg at fixed orientation of $\vec{E}$ :

$$g^2\, r^i r^j \left\langle \mathrm{tr}\, E^i E^j \right\rangle \sim r^2\, Q_s^2 \left[1 + 2\mathcal{A}(\cos^2\phi_r - \frac{1}{2})\right]$$




- $\mathcal{A} > 0$ arises for *individual* configurations due to *fluctuations* of distribution of sources

- $\longrightarrow$ Angular dependence of *single-particle* distribution

# *Single-inclusive* distribution in q+A elastic scattering:

amplitude: $V_x = \mathcal{P}\, e^{-ig\int dx^- A^+}$

$$\frac{dN}{d^2k} = \int d^2r\, e^{-i\vec{k}\cdot\vec{r}}\, S(\vec{r})$$

dipole S-matrix

$$= \int d^2r\, e^{-i\vec{k}\cdot\vec{r}}\, [D(\vec{r}) + iO(\vec{r})]$$

split into C-even and odd parts:

$v_2$, $v_4$

$$S(\vec{r}) = \frac{1}{N_c}\left\langle \mathrm{tr}\ V_x V_y^\dagger \right\rangle$$

$$= \frac{1}{2N_c}\mathrm{tr}\left\langle V_x V_y^\dagger + V_y V_x^\dagger + V_x V_y^\dagger - V_y V_x^\dagger \right\rangle$$

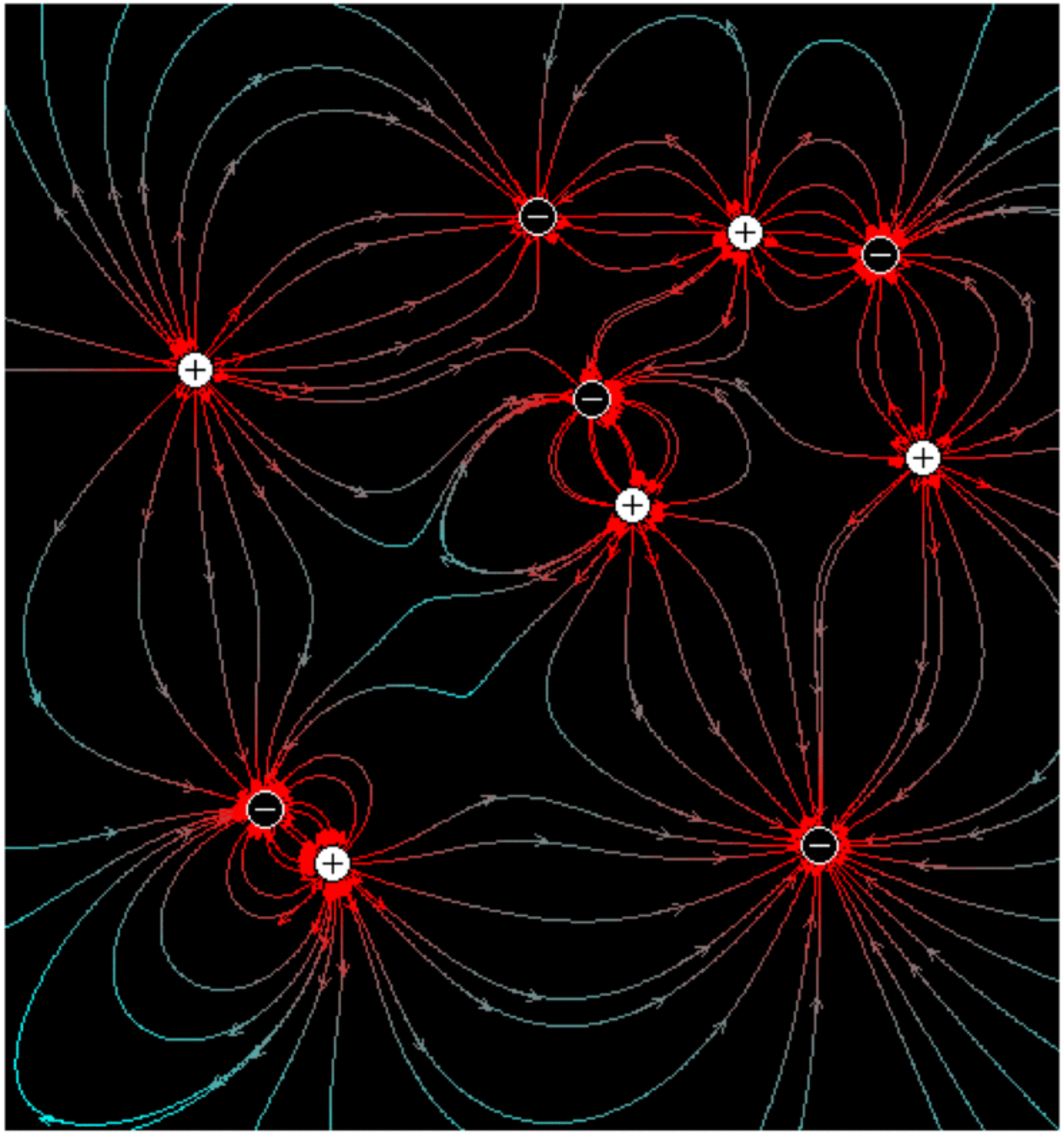
$$\equiv D(\vec{r}) + iO(\vec{r})$$

Parity: $D(\vec{r}) = D(-\vec{r})$ , $O(\vec{r}) = -O(-\vec{r})$

[ Note: $S_{\mathrm{adj}}(\vec{r}) \sim |S(\vec{r})|^2$ is real: only $v_{2n} \neq 0$ ]

# E-field of Abelian charges

- isotropic monopole fields
- dipole fields
- and some more “exotic” field line configurations, too

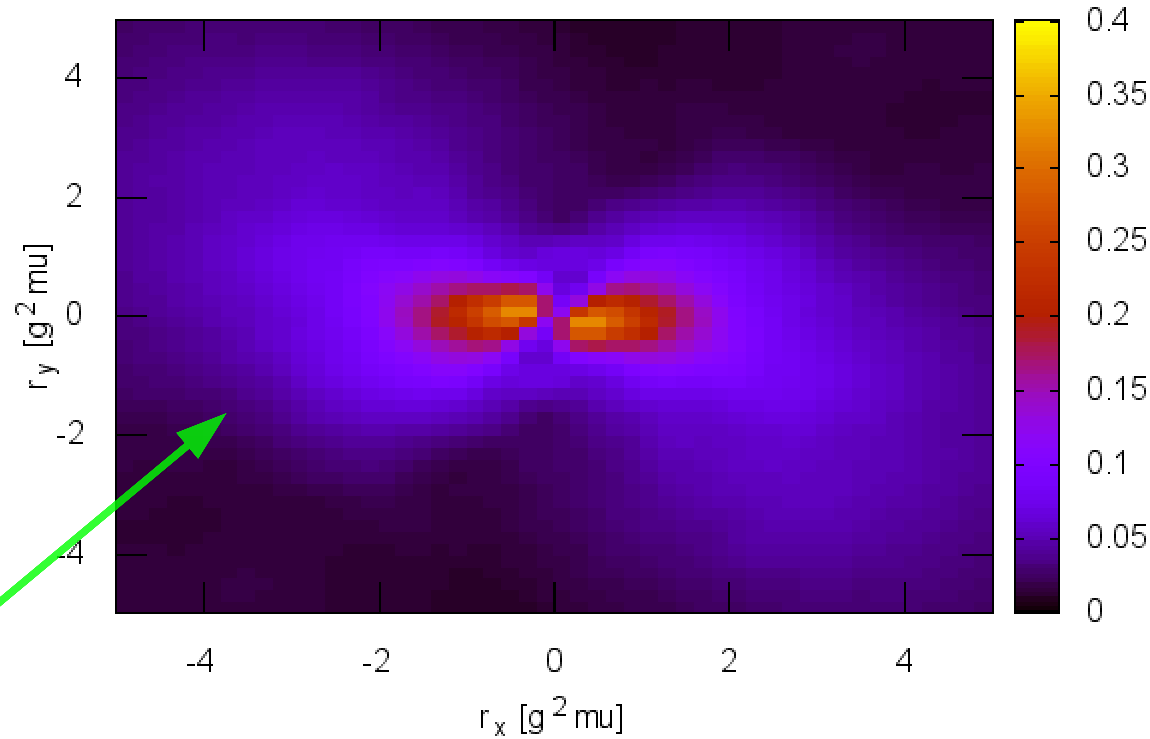


Java applet by M. J. McGuffin, Univ. of Toronto
(uses ~1/r Coulomb potential for d= 3)

# E-field in MV model (at some fixed b), *single configuration*

V. Skokov: talk at this meeting

A.D. & V. Skokov, arXiv:1411.6630

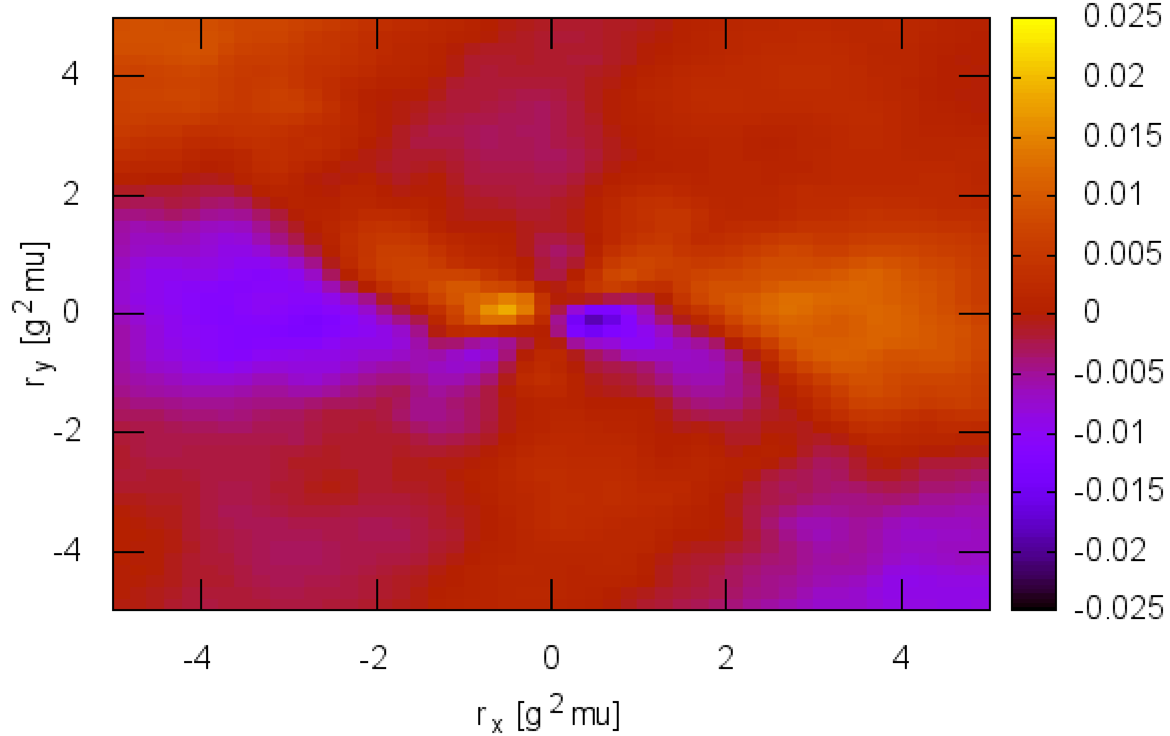



- clear cos(2φ)
  + other components

$$\boxed{\frac{1}{r^2}\left[1-\frac{1}{N}\mathrm{Re}\,\mathrm{tr}\,V_{\vec{x}}V^{\dagger}_{\vec{y}}\right] \sim g^2\,\mathrm{tr}\left(\hat{r}\cdot\vec{E}(\vec{b})\right)^2 + \mathcal{O}\left((igr)^4\right)}$$

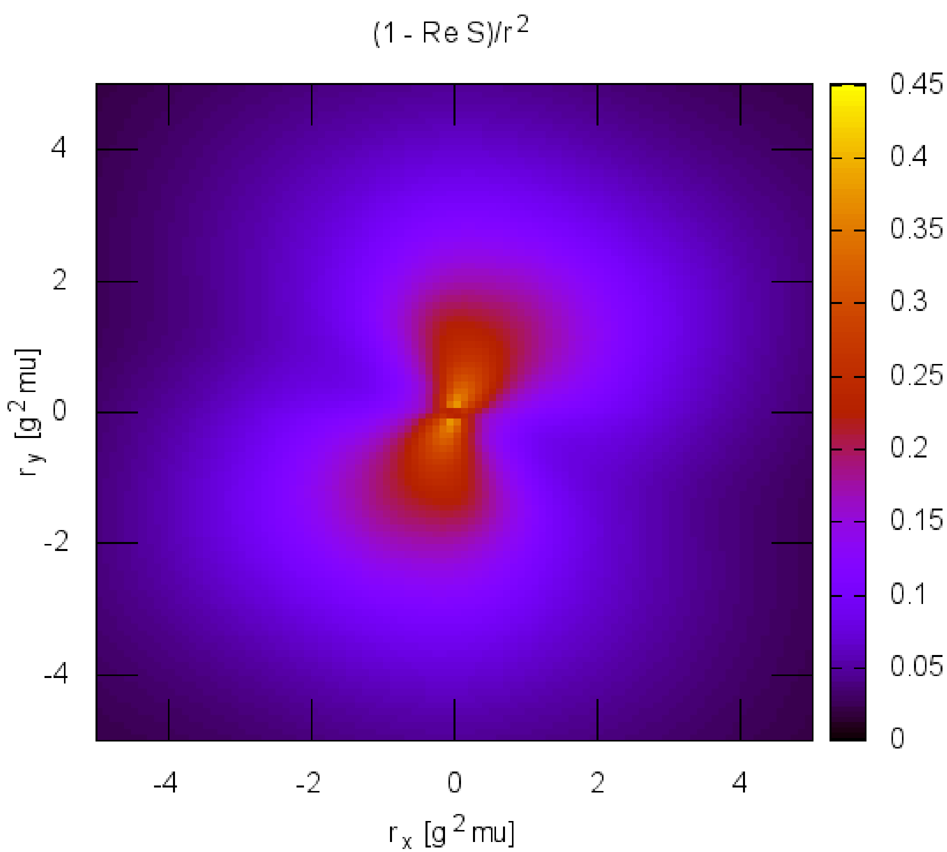
- clear cos(φ) + higher components

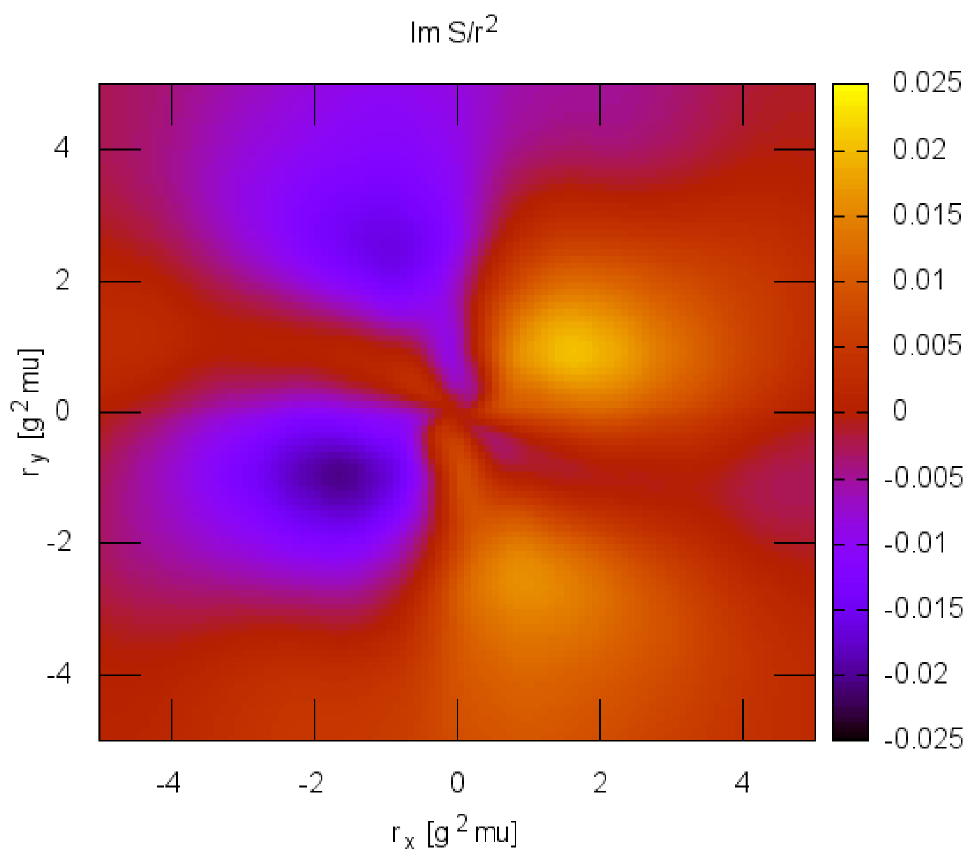
$$\frac{1}{r^2}\frac{1}{N}\mathrm{Im}\,\mathrm{tr}\,V_{\vec{x}}V_{\vec{y}}^{\dagger}$$



# another nice configuration:

V. Skokov (2014)




cos 2φ

cos 3φ

# MV model dipole:

polarization amplitude

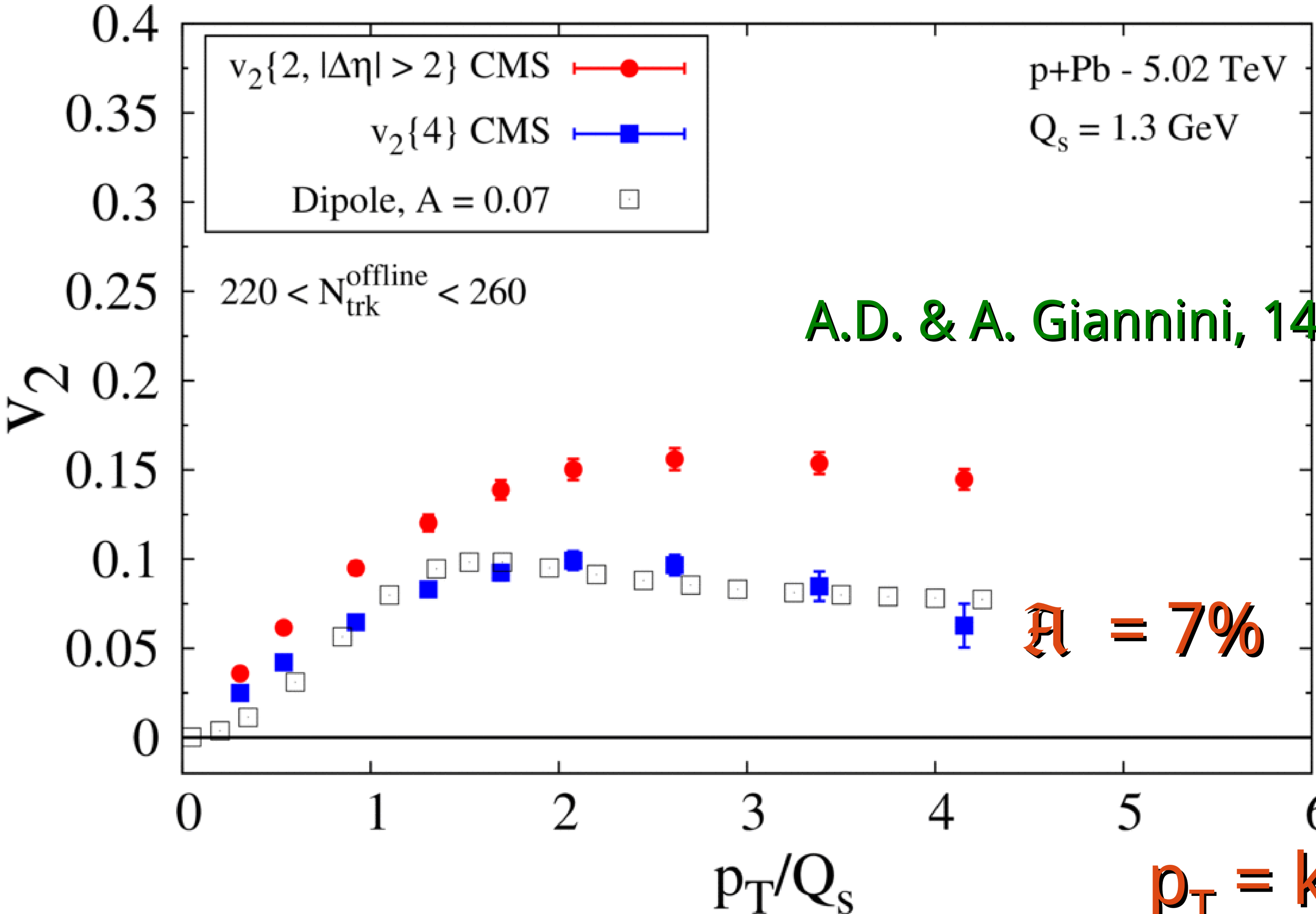
$$D(\vec{r}) = \exp\left[-\frac{1}{4}r^2 Q_s^2 (1 - \mathcal{A} + 2\mathcal{A}\cos^2\phi_r)\log\frac{1}{r\Lambda}\right]$$

Fourier transform at $k_T \gg Q_s$ :

$$\frac{dN}{k_T dk_T\, d\phi_k} = \frac{1}{2\pi}\frac{Q_s^2}{k_T^4}\left[1 - 2\mathcal{A} + 4\mathcal{A}\cos^2\phi_k\right]$$

- $v_2 = \mathcal{A}$, flat in $k_T$ (!)
- $v_4 = 0$

# Numerical Fourier transform




A.D. & A. Giannini, 1406.5781

$\mathfrak{A} = 7\%$

$p_T = k_T/2$

- $\mathfrak{A}$ ~ 7% ?
- → try domain model (talk by Andre Giannini)

Genuine 2-particle correlation from MV model (high $p_T$):

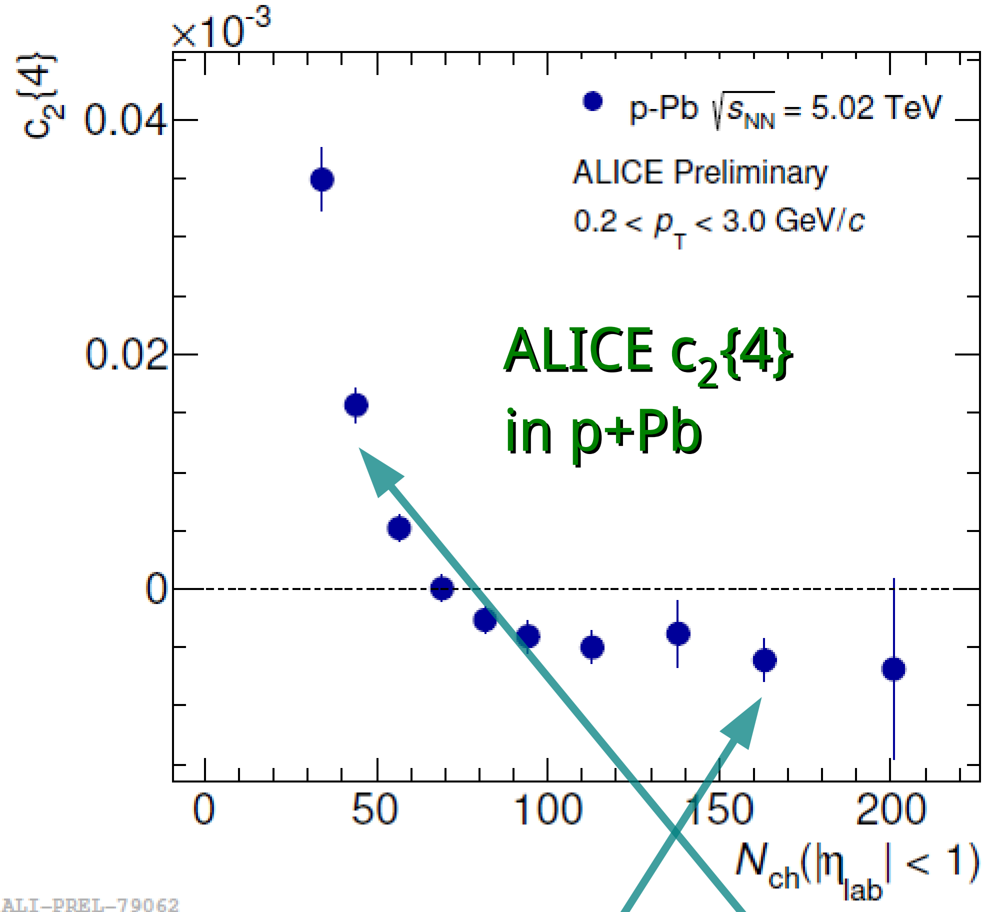
$$\frac{(ig)^4}{4N_c^2}\left\langle \mathrm{tr}\ \left(\vec{r}_1\cdot\vec{E}(\vec{b}_1)\right)^2\ \mathrm{tr}\ \left(\vec{r}_2\cdot\vec{E}(\vec{b}_2)\right)^2\right\rangle_{\mathrm{conn.}}$$

$$= \quad \frac{1}{4N_D}\frac{1}{N_c^2-1}+\mathcal{O}\left(N_c^{-4}\right)$$

In all:

$$\begin{aligned}(v_2\{2\})^2 \quad &\equiv \quad \left\langle e^{2i(\phi_1-\phi_2)}\right\rangle = \frac{1}{N_D}\left(\mathcal{A}^2+\frac{1}{4(N_c^2-1)}\right)\\ &= \quad \frac{1}{N_D}\left(v_2\{1\}^2+\frac{1}{4(N_c^2-1)}\right)\end{aligned}$$

- disconnected ~ connected when $\mathcal{A}\sim 1/N_c$
- can we tell which regime we're in ?
- *** aside: no initial state $v_2$ in AA: $N_D \approx \infty$




$$v_2^4\{4\} = -c_2\{4\}$$

- How to explain sign flip ?
- (hydro gives $c_2\{4\} < 0$)

$$c_n\{4\} = \left\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \right\rangle - 2\left\langle e^{in(\phi_1-\phi_2)} \right\rangle \left\langle e^{in(\phi_3-\phi_4)} \right\rangle$$

$$\simeq -v_n\{1\}^4 + \left\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \right\rangle_{\mathrm{conn.}}$$

$$\left\langle (\vec{r}_1 \cdot \vec{E}^a(\vec{b}_1))^2 \cdots (\vec{r}_4 \cdot \vec{E}^b(\vec{b}_4))^2 \right\rangle_c \to c_2\{4\} > 0 \ !$$

A.D., L. McLerran.
V. Skokov: 1410.4844

$$
\begin{aligned}
c_2\{4\} &= -\frac{1}{N_D^3}\left[\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3}\right] \\
&= -\frac{1}{N_D^3}\left[(v_2\{1\})^4 - \frac{1}{4(N_c^2-1)^3}\right]
\end{aligned}
$$

(around $c_2\{4\}$~0)

- disconnected ~ connected (resp. $c_2\{4\}$~0) when $\mathcal{A} \sim 1/N_c^{3/2}$
- 4-particle correlation dominated by disconnected part ***before*** 2-particle correlation ! (i.e. for smaller $\mathcal{A}$)
- analogous to BBGKY / Dyson-Schwinger hierarchy of n-particle correlations

# How about 3-particle correlations ? (again: high pT)

with Giannini & Skokov

$$c_2\{3\} = \left\langle e^{2i(\phi_1+\phi_2-2\phi_3)} \right\rangle$$

requires v4-like contribution from particle 3 → expand S-matrix to order $r_1^2$, $r_2^2$, $r_3^4$

$$\frac{1}{2}\left(\frac{(ig)^2}{2N_c}\right)^4 \left\langle \mathrm{tr}\,(\vec{r}_1\cdot\vec{E}(\vec{b}_1))^2\;\mathrm{tr}\,(\vec{r}_2\cdot\vec{E}(\vec{b}_2))^2\;\left[\mathrm{tr}\,(\vec{r}_3\cdot\vec{E}(\vec{b}_3))^2\right]^2 \right\rangle$$

gives ($A \sim 1/N_c$):

$$c_2\{3\} \sim \frac{Q_s^2}{k_{\perp 3}^2}\frac{1}{N_D^2}\left[\frac{1}{2}\mathcal{A}^4 + \frac{1}{2}\mathcal{A}^2\frac{1}{N_c^2-1} + \frac{1}{16}\left(\frac{1}{N_c^2-1}\right)^2\right]$$

disconnected
(4 dipoles)

connected,
1-3 dipoles

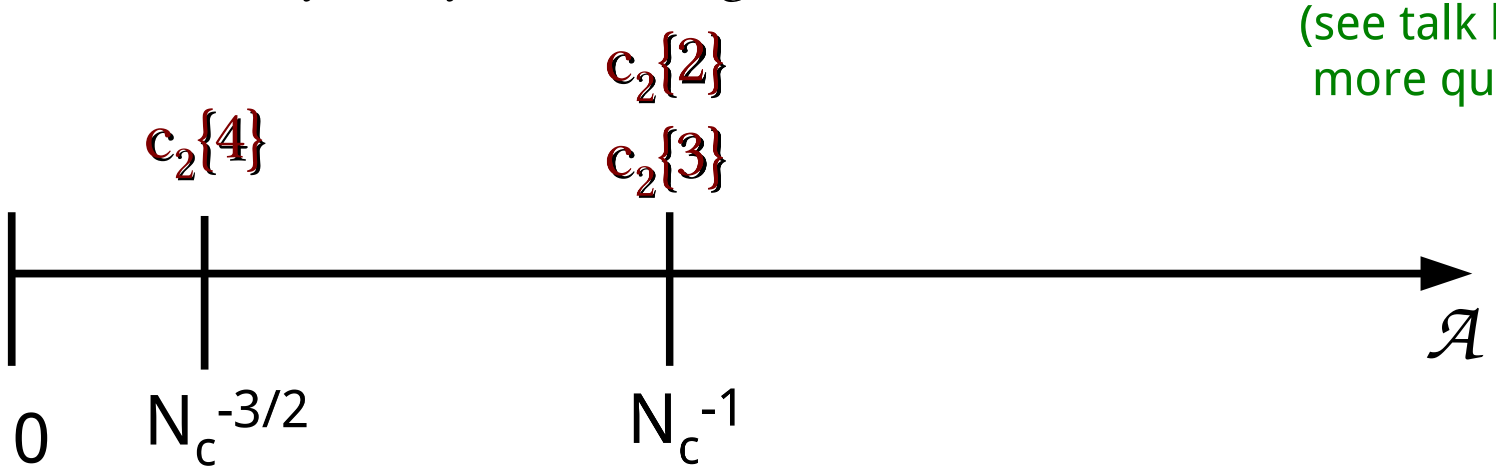
connected,
2-2 dipoles

## Discussion $c_2\{3\}$ :

- fully connected contribution small
  [ suppressed by an extra $1/(N_c^2-1)$ ]
- however, 2-particle connected contributions are large; fully disconnected / factorizable term dominates only beyond $\mathcal{A} \sim 1/N_c$
- same scale (parametrically) as $c_2\{2\}$ !
  (***) not like "naive" hierarchy

**dominated by fully disc. diagrams:**

(see talk by A. Giannini for more quantitative plots)



# Summary

- Pretty clear that there are initial state anisotropies in pp/pA
- Fluctuating valence color charges produce anisotropic **E**-fields
- simple S-matrix (real part) of the form exp(- #($\mathbf{r}^*\mathbf{E})^2$) already gives quite decent $p_T$ dependence of $v_2$
- rot. sym. breaking gives rise to disconnected $(dN_1)^m$ contribution to m-particle cumulants
- naive hierarchy of m-particle correlations not quite valid, transition to disconnected diagrams at

$c_2\{2\}$ : $\mathcal{A} \sim 1/N_c$

$c_2\{3\}$ : $\mathcal{A} \sim 1/N_c$ ($\mathcal{A}$ is **E**-field anisotropy amplitude)

$c_2\{4\}$ : $\mathcal{A} \sim 1/N_c^{3/2}$

# Backup Slides

# Aside: gluon TMDs

(A. Metz, talk at POETIC V, Yale U., New Haven, Sept. 2014)

## Gluon TMDs: Unpolarized Target

- Gluon-gluon correlator

$$\int \frac{d\xi^-}{2\pi} \frac{d^2\vec{\xi}_T}{(2\pi)^2} e^{ik\cdot\xi} \langle P| F^{+i}(0) \mathcal{W}^g_{TMD} F^{+j}(\xi) |P\rangle \Big|_{\xi^+=0}$$

$$\sim \delta_T^{ij} f_1^g(x, \vec{k}_T^2) + \left( \hat{k}_T^i \hat{k}_T^j - \frac{1}{2}\delta_T^{ij} \right) h_1^{\perp g}(x, \vec{k}_T^2)$$

 – two gluon TMDs for unpolarized target

 – $h_1^{\perp g}$ describes distribution of linearly polarized gluons (Mulders, Rodrigues, 2000) (notation from Meißner, AM, Goeke, 2007)

 – gauge link $\mathcal{W}^g_{TMD}$ ensures color gauge invariance

 – $h_1^{\perp g}$ is nonzero without $\mathcal{W}^g_{TMD}$ (T-even) $\rightarrow$ different from $h_1^{\perp q}$

 – gauge link $\mathcal{W}^g_{TMD}$ generally depends on process $\rightarrow$ universality of TMDs ?
 $\rightarrow$ talks by Mulders, Yuan
 (Bomhof, Mulders, Pijlman, 2006 ... / Dominguez, Marquet, Xiao, Yuan, 2010, 2011 / Buffing, Mukherjee, Mulders, 2013)

 – $h_1^{\perp g}$ most likely very large (at small $x$), but so far no experimental information

# Multi-particle correlations

$$
\begin{aligned}
v_n^2\{2\} &= \frac{1}{\mathcal{N}^2}\int \mathcal{D}\rho\, W[\rho] \\
&\quad \int \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi}\, e^{in(\phi_1-\phi_2)}\,\frac{1}{N_c}\mathrm{tr}\, V_{x_1}^\dagger V_{y_1}\,\frac{1}{N_c}\mathrm{tr}\, V_{x_2}^\dagger V_{y_2} \\
&\neq \frac{1}{\mathcal{N}^2}\int \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi}\, e^{in(\phi_1-\phi_2)} \\
&\quad \int \mathcal{D}\rho_1\, W[\rho_1]\frac{1}{N_c}\mathrm{tr}\, V_{x_1}^\dagger V_{y_1}\int \mathcal{D}\rho_2\, W[\rho_2]\frac{1}{N_c}\mathrm{tr}\, V_{x_2}^\dagger V_{y_2} \\
&= 0
\end{aligned}
$$

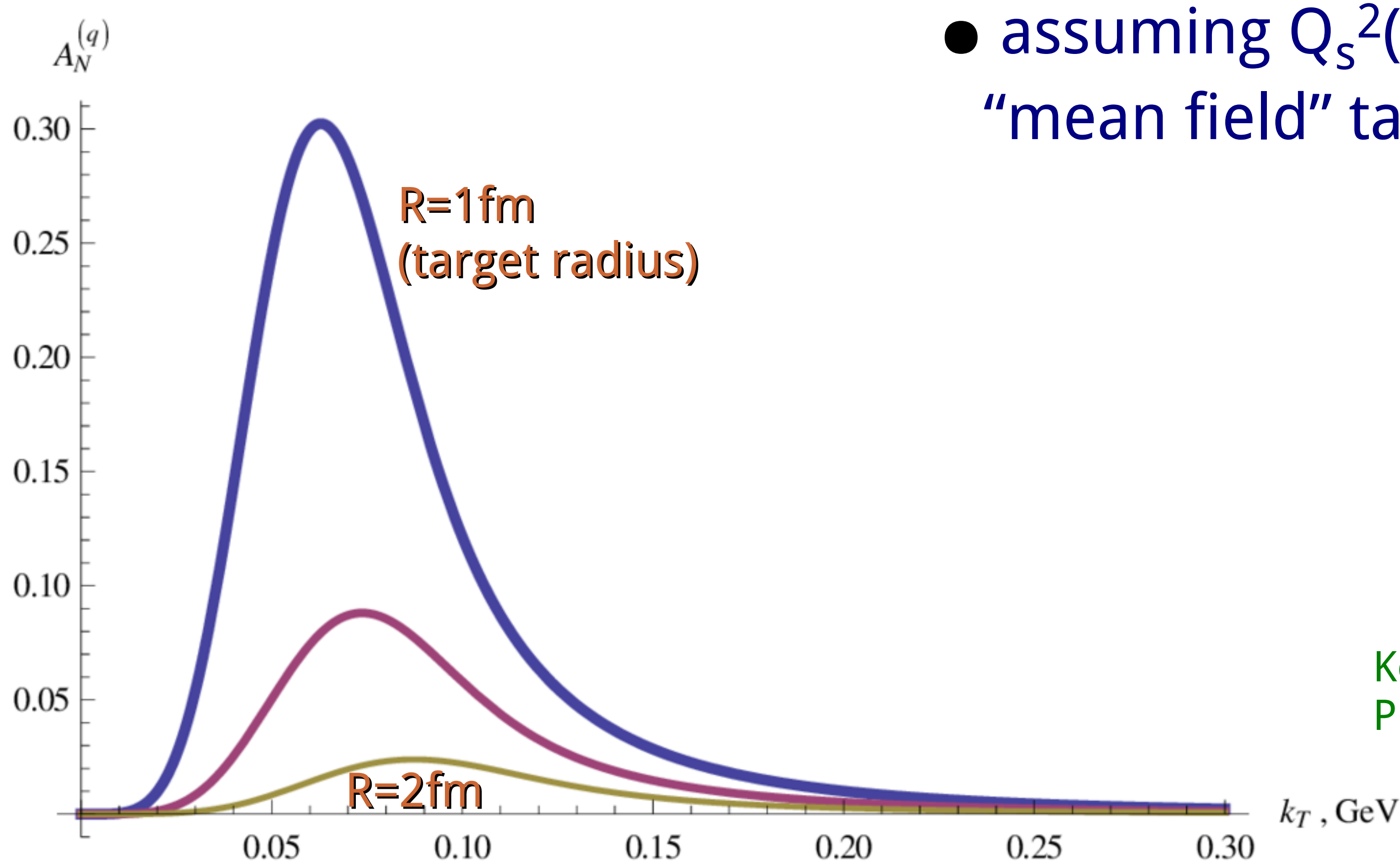
*W*[ρ] normalized: $\int \mathcal{D}\rho\, W[\rho] = 1$

# Single transverse spin asymmetry in pA

In quasi-classical approximation:

$$iO(\vec{r},\vec{b}) \sim i\alpha_s \vec{r}\cdot\vec{\nabla}_b D(\vec{r},\vec{b})$$

$$D(\vec{r},\vec{b}) = e^{-\frac{1}{4}r^2 Q_s^2(b)\log\frac{1}{r\Lambda}}$$




- assuming $Q_s^2(b) \sim T(b)$, "mean field" target thickness

Kovchegov & Sievert: PRD 86 (2012)

# semi-cl. odderon cont'd

Target fluctuations (single mode):

$$\frac{Q_s^2(\vec{s})}{Q_s^2} = 1 + \int \frac{d^2q}{(2\pi)^2}\, \delta f(\vec{q})\, e^{i\vec{q}\cdot\vec{s}}$$

$$\delta f(\vec{q}) = \frac{(2\pi)^2}{2} \mathcal{B}(q_0) \left[(1+i)\delta(\vec{q}-\vec{q}_0) + (1-i)\delta(\vec{q}+\vec{q}_0)\right]$$

$$\rightarrow$$

$$iO(\vec{r}) \sim \frac{i}{2}\, \alpha_s\, r^2\, Q_s^2\, \mathcal{B}(q_0) \sin\left(\frac{1}{2} r q_0 \cos\phi_r\right) \log\frac{1}{\Lambda r}\, e^{-\frac{1}{4}r^2 Q_s^2 \log\frac{1}{\Lambda r}}$$

high $k_T$ / small r :

$$\frac{dN}{d^2k_T} \sim \mathcal{B}(q_0) \frac{q_0}{k_T} \frac{Q_s^2}{k_T^4} \cos(\phi_k)$$

- $v_1 \neq 0$
- $v_3 = 0$

# semi-cl. odderon cont'd

for $v_3 \neq 0$ we need a "string" (analogous to AdS/CFT calculation):

$$iO(\vec{r}) \sim i\alpha_s \vec{r}\cdot\vec{\nabla}_b D(\vec{r},\vec{b}) \to i\alpha_s \int_{\vec{y}}^{\vec{x}} d\vec{s}\cdot\vec{\nabla}_s\, D(\vec{r},\vec{s})$$

$$r = x - y \quad , \quad \vec{b} = \frac{\vec{x}+\vec{y}}{2}$$

Target fluctuations ($dq_0/q_0$ spectrum with exp. cutoff):

$$\frac{Q_s^2(\vec{s})}{Q_s^2} = 1 + \int \frac{d^2q}{(2\pi)^2}\,\delta f(\vec{q})\, e^{i\vec{q}\cdot\vec{s}}$$

$$\delta f(\vec{q}) = \frac{(2\pi)^2}{2}\mathcal{B}\int \frac{dq_0^2}{q_0^2} e^{-q_0/Q_c}\left[(1+i)\delta(\vec{q}-\vec{q}_0) + (1-i)\delta(\vec{q}+\vec{q}_0)\right]$$

$$\boxed{iO(\vec{r}) \sim i\,\alpha_s\, r^2\, Q_s^2\, \mathcal{B}\arctan\left(\frac{1}{2} r Q_c \cos\phi_r\right) D(\vec{r})\, \log\frac{1}{r\Lambda}}$$

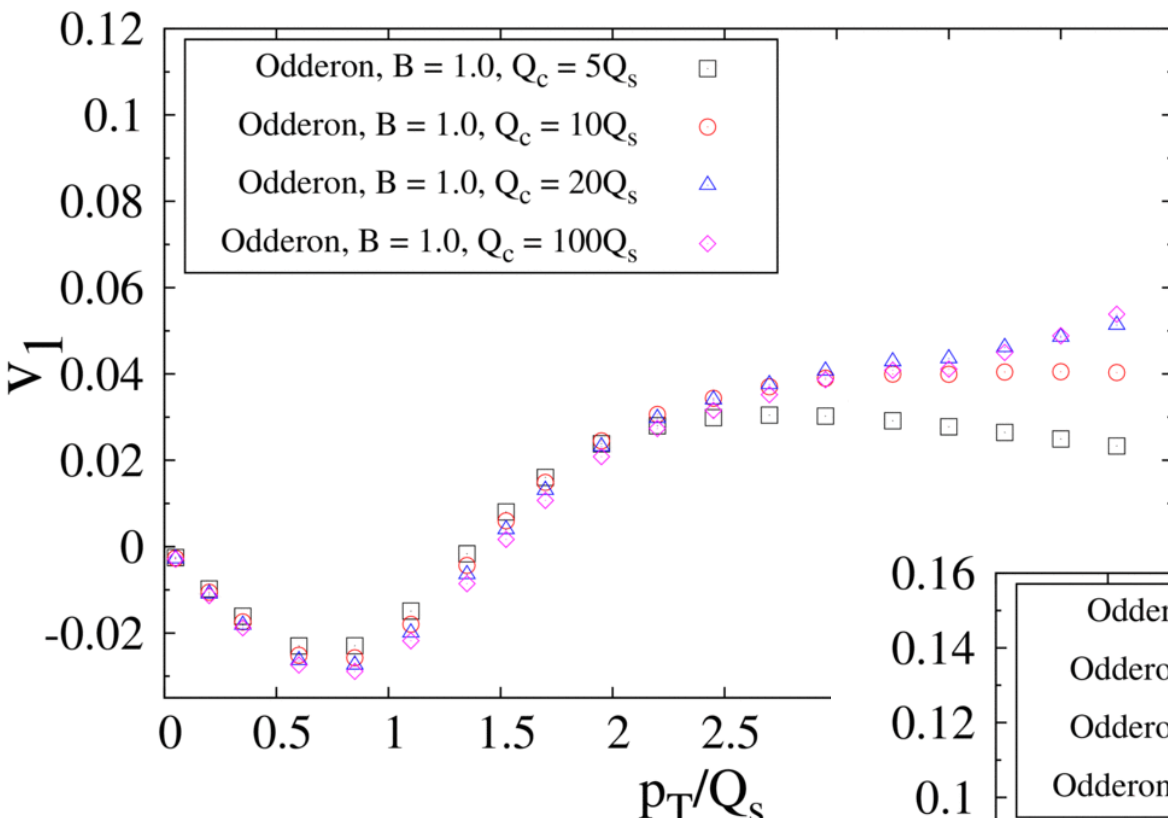
all $v_{2n+1}$, awesome...

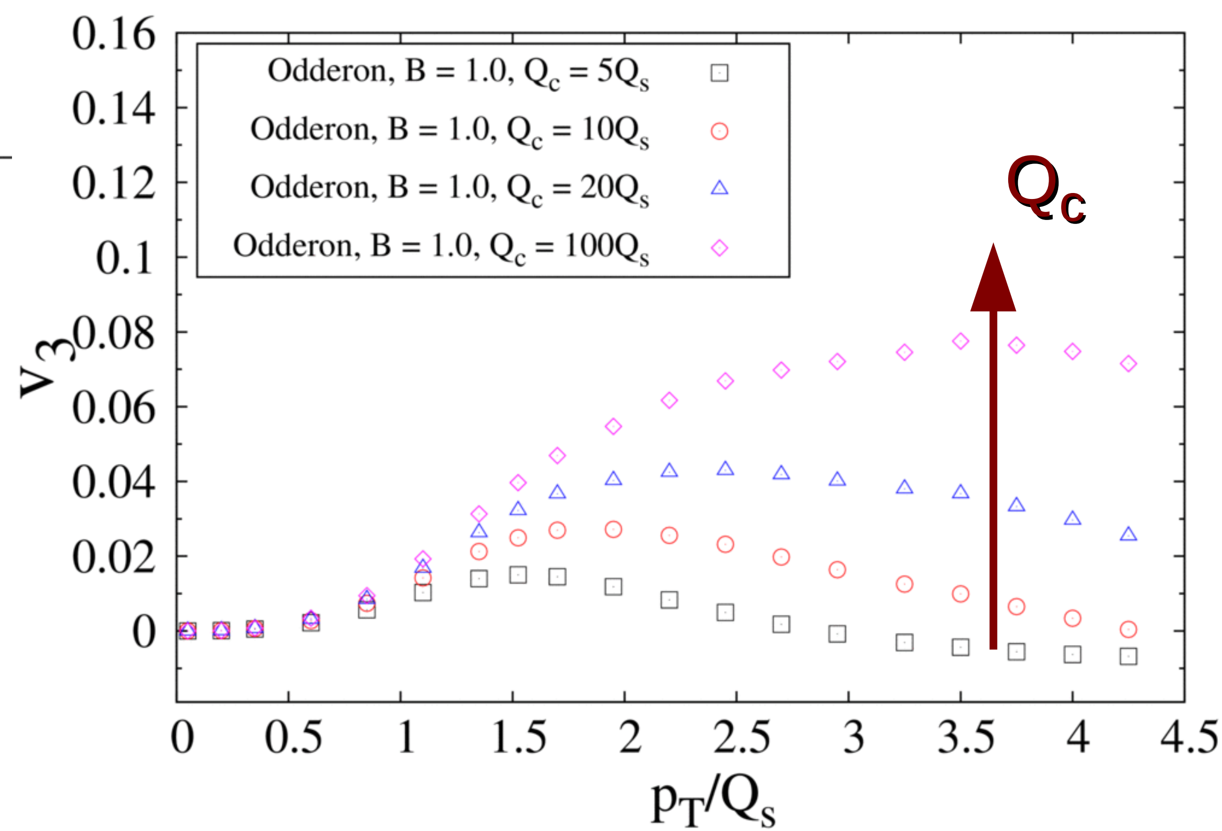
Expansion for 1/r » $Q_s$, $Q_c$ :

$$iO(\vec{r}) \sim r^3 Q_c \cos\phi_r \left[1 - \frac{r^2}{4}\left(Q_s^2 \log\frac{1}{r} + \frac{1}{3}Q_c^2 \cos^2\phi_r\right)\right]$$

- isotr.: $\sim r^2 \rightarrow 1/k_T^4$
- $v_1$: $\sim r^3 \rightarrow 1/k_T^5$
- $v_3$: $\sim r^5 \rightarrow 1/k_T^7$

# Numerical F.T. of O(r) : behavior of cutoff $Q_c$

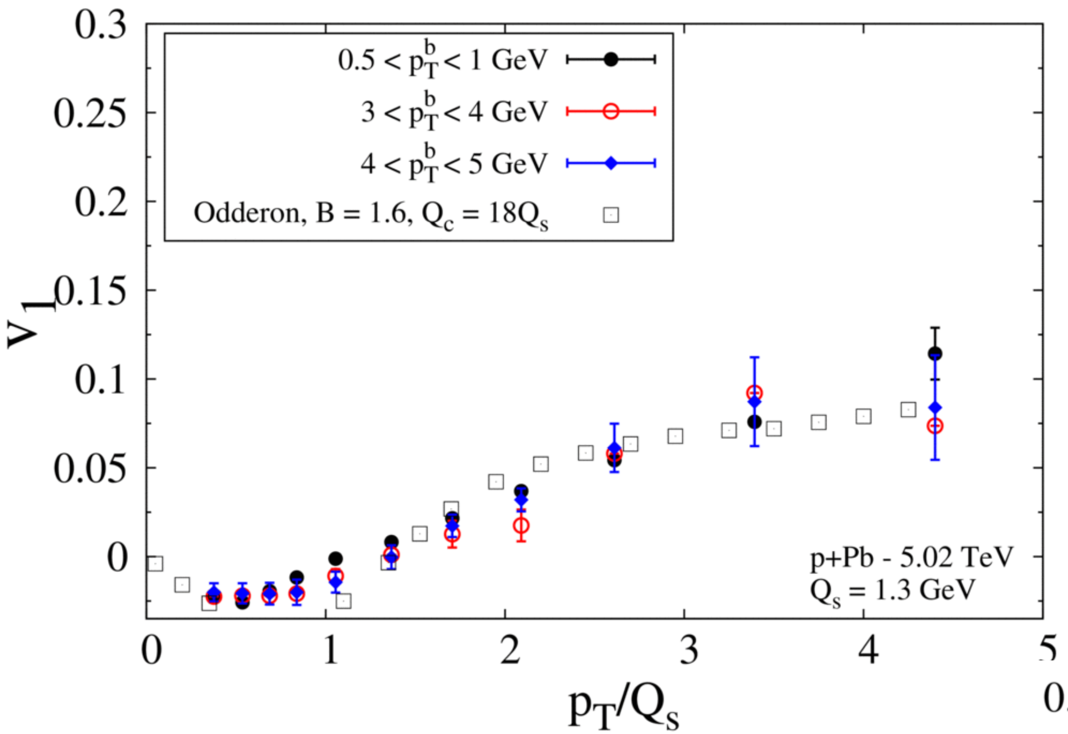



- $v_1$ affected only at high $p_T$
- low $Q_c$ kills $v_3$:

$$\int d\vec{s} \cdot \vec{\nabla}_s \, Q_s^2 \rightarrow \vec{r} \cdot \vec{\nabla}_b \, Q_s^2$$

# Numerical F.T. of O(r) : comparison to data; $v_1$ and $v_3$




- one fluctuation amplitude B=1.6 fits $v_1$ and $v_3$ simultaneously
- large $Q_c \sim 18\ Q_s$ gives decent $p_T$ dependence for both $v_1$ and $v_3$