Continuous description of fluctuating eccentricities

Initial Stages in High Energy Nuclear Collisions
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Based on work done in collaboration with W. Broniowski and J.-Y. Ollitrault


What can we learn from initial state fluctuations

- Initial fluctuations affect final hydrodynamical flows

From B. Schenke @ QM 2012

- Which dynamical information can we extract from the measurements?
  Present descriptions seem to rely on too many ‘details’....
Various sources of fluctuations

Positions of individual nucleons (Glauber Monte Carlo)

(Note: nucleon-nucleon correlations are small corrections)

Subnucleonic fluctuations

usually added on top of position fluctuations

Popular modelling: sources of local energy density

\[ \rho(x) = \sum_{i=1}^{N} w_i \delta(x - x_i) \]

fluctuating quantities (=unknown): position, strength, shape, etc.

One needs a description where irrelevant details do not stand prominently

Consider the local energy density as a random field

(Note: related effort by S. Floerchinger and U. Wiedemann)
Note: focus here on central collisions (zero impact parameter)

Consider the energy density as a random field

\[ \rho(z) \]

(Note: analog to the shift from Lagrangian to Eulerian coordinates in fluid dynamics)

The probability distribution for this random field,

\[ P[\rho(z)] \]

encodes all sources of fluctuations, irrespective of their nature

What is \( P \)?

Essentially (not exactly) a Gaussian, with a short range 2-point function

(Effective theory for long wavelength fluctuations)
Relevant fluctuations are those that enter the calculation of eccentricities

\[ \varepsilon_n \equiv \frac{\int_z z^n \rho(z)}{\int_z |z|^n \rho(z)} \quad \text{(L. Yan and D. Teaney, 1010.1876)} \]

Only the 2-point function of \( P \) is needed to calculate the fluctuations of eccentricities

\[ \rho(z) = \langle \rho(z) \rangle + \delta \rho(z) \]

\[ S(z_1, z_2) \equiv \langle \delta \rho(z_1) \delta \rho(z_2) \rangle = \langle \rho(z_1) \rho(z_2) \rangle - \langle \rho(z_1) \rangle \langle \rho(z_2) \rangle \]

\[ \langle \Delta \varepsilon_n^2 \rangle = \langle \varepsilon_n \bar{\varepsilon}_n \rangle = \frac{\int_{z_1 z_2} z_1^n \bar{z}_2^n S(z_1, z_2)}{(\int_z |z|^n \langle \rho(z) \rangle)^2} \quad \text{(small fluctuations)} \]

- \( S \) summarizes the main information on local fluctuations and correlations
- Eccentricities and their variances are sensitive only to the « long wavelength » fluctuations
Simple ansatz for the 2-point function

\[ S(z_1, z_2) = A(z_1)\delta(z_1 - z_2) + Bf(z_1)f(z_2) \]

Fluctuations are correlated on short distances (subnucleonic) (B is determined by global constraints, e.g. \( \int_z \langle \delta \rho(z) \rangle = 0 \))

Short wavelength fluctuations simply renormalize \( A \)

They are not directly visible in the eccentricities

Typical result

with \( A(z) = A\langle \rho(z) \rangle \)

\[ \langle \varepsilon_n \bar{\varepsilon}_n \rangle = \frac{A}{\int_z \langle \rho(z) \rangle} \frac{\langle r^{2n} \rangle}{\langle r^n \rangle^2} \]

NB. This reproduces results obtained previously with independent sources (Bhalerao, Luzum, Ollitrault, 1107.5485)

NB. A is the only thing that one can extract from the data
The gaussian ansatz

\[ P[\delta \rho] \propto \exp \left\{ -\frac{1}{2} \int_{z_1, z_2} \delta \rho(z_1) K(z_1, z_2) \delta \rho(z_2) \right\} \]

\[ \int_z K(z_1, z) S(z, z_2) = \delta(z_1 - z_2) \]

K is a local operator \[ K(z_1, z_2) \sim \delta(z_1 - z_2) \]

- Why it is a good approximation
- Why it is not quite enough
Why a Gaussian is a good approximation

Useful to make contact with the model of independent sources

\[ \rho(z) = E_0 \sum_{i=1}^{N_s} \delta(z - z_i) \quad \int z \rho(z) = N_s E_0 \]

\[ S(z_1, z_2) = E_0 \langle \rho(z_1) \rangle \delta(z_1 - z_2) - \frac{1}{N_s} \langle \rho(z_1) \rangle \langle \rho(z_2) \rangle \]

Statistics of density fluctuations are those of an ideal gas in the plane

\[ \langle \delta \rho^2 \rangle = \frac{E_0}{\sigma} \langle \rho \rangle \left( 1 - \frac{\sigma}{\Sigma} \right) \quad \langle \rho \rangle = E_0 N_s / \Sigma \]

When probing large areas, one probes long wavelength fluctuations. Eventually for large areas, the distribution becomes gaussian, and the fluctuations have small amplitude.
Why P is not quite a Gaussian

• The distribution of eccentricity is not gaussian

  • it would be only if the relation between eccentricity and density fluctuation was linear, and it is not
  • besides, there is a constraint $\epsilon \leq 1$

$$P(\epsilon) = 2\alpha \epsilon (1 - \epsilon^2)^{\alpha-1}$$

Ollitrault 1992, Ollitrault and Yan, 2014

• Non trivial cumulants

  • do they signal non trivial dynamics ?
  • or reflect simply the fact that the probability distribution obeys simple constraints, e..g. $\rho(z) = \langle \rho(z) \rangle + \delta \rho(z) \geq 0$
Conclusions

• Field theoretical description of energy density fluctuations in the transverse plane

• One basic ingredient $P[\rho(z)]$

• $P[\rho(z)]$ is essentially gaussian with short range 2-point function

• Deviations from the gaussian have presumably a non-dynamical origin (conjecture: come from simple constraints, such as the positivity constraint)

• Short range fluctuations play no other role than renormalizing the (short range part of the) 2-point function

• Correlation length is sub nucleonic ($1/Q_s$ ?)
  Continuity of physics from pp to pA to AA ?