Anisotropic hydrodynamics for conformal Gubser flow

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Motivation

- Dissipative hydrodynamics is commonly used to describe ultra-relativistic heavy-ion collisions

  - ✓ Ideal hydrodynamics
  - ✓ 2^{nd}-order hydrodynamics (multiple approaches)
  - ✓ Anisotropic hydrodynamics

- We derive the equations of (1+1)d aHydro for a system subject to Gubser flow by taking moments of the Boltzmann equation in relaxation-time approximation

- The obtained solution has both longitudinal and transversal expansion

- To accomplish this, we use a clever method introduced by Gubser that uses symmetries to construct a static flow in Weyl-rescaled de Sitter space

- Once the solution is determined in de Sitter space, we then map it back to Minkowski space which gives the full spatiotemporal evolution

- We then compare the predictions of aHydro against a recent obtained exact solution to the Boltzmann equation in relaxation-time approximation

Weyl rescaling & Gubser flow

- Conformality ⟷ Weyl rescaling invariance[1]

- Gubser flow[2]

$$\tilde{u}^\tau = \cosh\left[\tanh^{-1}\left(\frac{2q^2\tau r}{1+q^2\tau^2+q^2r^2}\right)\right]$$

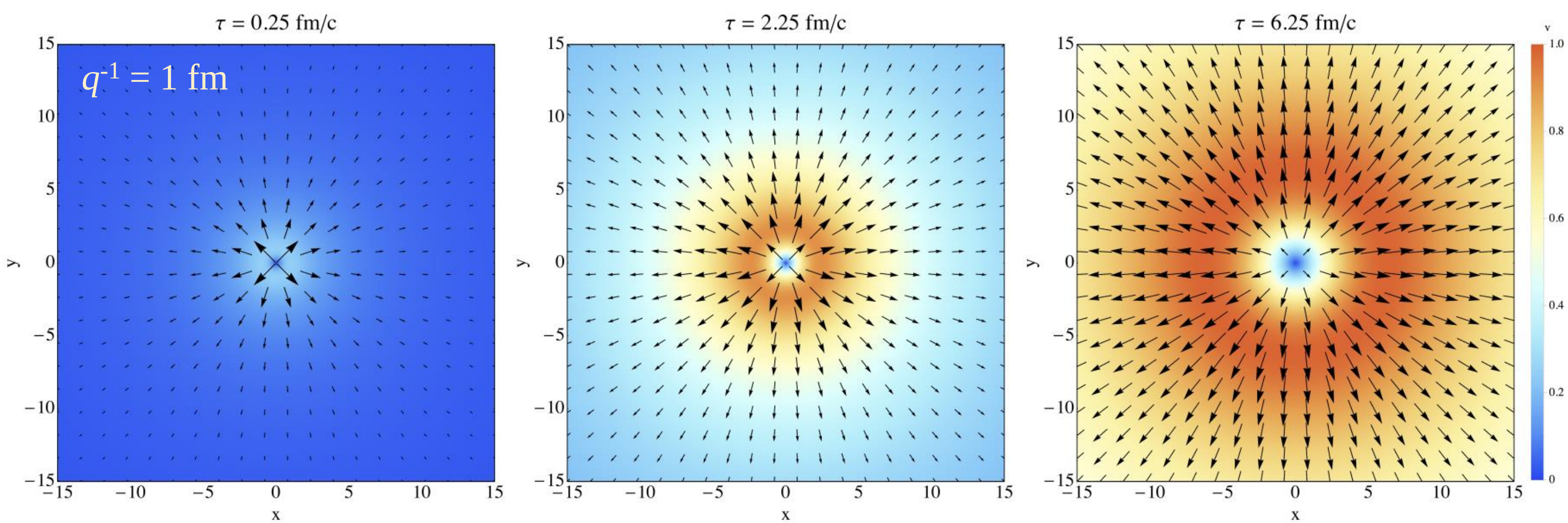
$$\tilde{u}^r = \sinh\left[\tanh^{-1}\left(\frac{2q^2\tau r}{1+q^2\tau^2+q^2r^2}\right)\right]$$

$$\tilde{u}^\phi = \tilde{u}^\varsigma = 0$$

$$r = \sqrt{x^2+y^2}$$

$$\varphi = tan^{-1}\left(\frac{y}{x}\right)$$

$$\varsigma = tanh^{-1}\left(\frac{z}{t}\right)$$



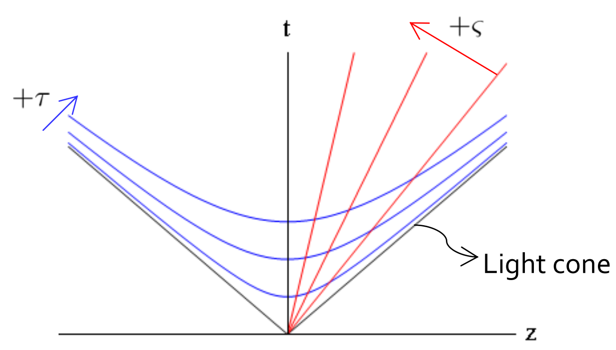
[1] S. S. Gubser, A. Yarom, arXiv: 1012.1314
[2] S. S. Gubser, arXiv: 1006.0006

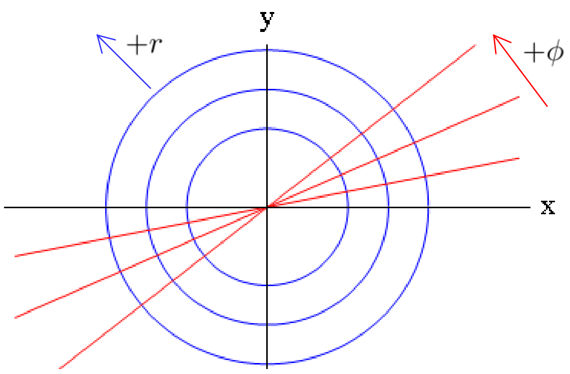
Basis vectors

- We defined the most general basis (LRF) and its parametrization for boost-invariant and cylindrically symmetric flow $x^\mu = (t, x, y, z)$

$$
\begin{aligned}
u^\mu_{LRF} &\equiv (1,0,0,0) \\
\mathcal{X}^\mu_{LRF} &\equiv (0,1,0,0) \\
\mathcal{Y}^\mu_{LRF} &\equiv (0,0,1,0) \\
\mathcal{Z}^\mu_{LRF} &\equiv (0,0,0,1)
\end{aligned}
\quad \xrightarrow{\text{Parametrization}} \quad
\begin{aligned}
u^\mu &= (\cosh\theta_\perp \cosh\varsigma, \sinh\theta_\perp \cos\phi, \sinh\theta_\perp \sin\phi, \cosh\theta_\perp \sinh\varsigma) \\
\mathcal{X}^\mu &= (\sinh\theta_\perp \cosh\varsigma, \cosh\theta_\perp \cos\phi, \cosh\theta_\perp \sin\phi, \sinh\theta_\perp \sinh\varsigma) \\
\mathcal{Y}^\mu &= (0, -\sin\phi, \cos\phi, 0) \\
\mathcal{Z}^\mu &= (\sinh\varsigma, 0, 0, \cosh\varsigma)
\end{aligned}
$$

- Polar Milne space $\tilde{x}^\mu = (\tau, r, \phi, \varsigma)$



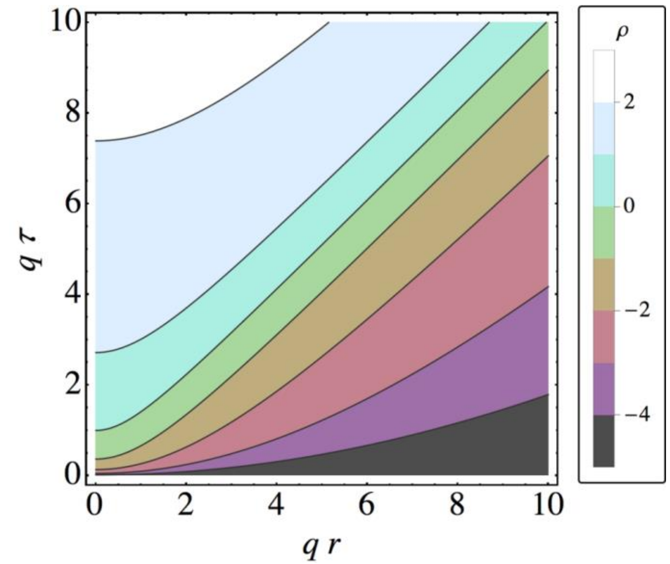


- De Sitter space $\hat{x}^\mu = (\rho, \theta, \phi, \varsigma)$

De Sitter-space basis vectors

- De Sitter $\hat{x}^\mu = (\rho, \theta, \phi, \varsigma)$ vs. Milne $\tilde{x}^\mu = (\tau, r, \phi, \varsigma)$ coordinates[1] (Figure from Ref. [2])

$$\sinh\rho = -\frac{1 - q^2\tau^2 + q^2 r^2}{2q\tau}$$
$$\tan\theta = \frac{2qr}{1 + q^2\tau^2 - q^2 r^2}$$



$$\lim_{\rho\to-\infty} \tau \to 0^+$$

- Metric (curved space)

$$\hat{g}_{\mu\nu} = \mathrm{diag}(-1,\ \cosh^2\rho,\ \cosh^2\rho\sin^2\theta,\ 1)$$

- Basis

$$u^\mu \to \hat{u}^\mu \qquad \hat{u}^\mu = (1,\ 0,\ 0,\ 0)$$
$$\mathcal{X}^\mu \to \hat{\Theta}^\mu \qquad \hat{\Theta}^\mu = (0,\ (\cosh\rho)^{-1},\ 0,\ 0)$$
$$\mathcal{Y}^\mu \to \hat{\Phi}^\mu \qquad \hat{\Phi}^\mu = (0,\ 0,\ (\cosh\rho\sin\theta)^{-1},\ 0)$$
$$\mathcal{Z}^\mu \to \hat{\varsigma}^\mu \qquad \hat{\varsigma}^\mu = (0,\ 0,\ 0,\ 1)$$

[1] S. S. Gubser, arXiv: 1006.0006
[2] G. Denicol, U. Heinz, M. Martinez, J. Noronha, M. Strickland, arXiv: 1408.7048

Boltzmann Eq. (BE) and distribution function

- The de Sitter-space Boltzmann equation in relaxation-time approximation is:

$$\hat{p}\cdot Df = \frac{\hat{p}\cdot\hat{u}}{\hat{\tau}_{\rm eq}}(f - f_{\rm iso}) \qquad \hat{\tau}_{eq} = \frac{5}{\hat{T}}\frac{\eta}{s}$$

- Leading-order aHydro ⟹ Ellipsoidal distribution function

$$f(\hat{x},\hat{p}) = f_{\rm iso}\left(\frac{1}{\hat{\lambda}}\sqrt{\hat{p}_\mu \hat{\Xi}^{\mu\nu}\hat{p}_\nu}\right) \qquad \begin{aligned}\hat{\Xi}^{\mu\nu} &= \hat{u}^\mu\hat{u}^\nu + \hat{\xi}^{\mu\nu} \\ \hat{\xi}^{\mu\nu} &= \hat{\xi}_\theta\hat{\Theta}^\mu\hat{\Theta}^\nu + \hat{\xi}_\phi\hat{\Phi}^\mu\hat{\Phi}^\nu + \hat{\xi}_\varsigma\hat{\varsigma}^\mu\hat{\varsigma}^\nu\end{aligned}$$

- Notation $\hat{\alpha}_i \equiv (1+\hat{\xi}_i)^{-1/2}$

- SO(3) symmetry (rotational symmetry in de Sitter space) requires $\hat{\alpha}_\theta = \hat{\alpha}_\phi$

- $n^{\rm th}$ moment of the distribution function ($\hat{g} \equiv \det \hat{g}_{\mu\nu}$)

$$\hat{\mathcal{I}}^{\mu_1\ldots\mu_n} \equiv \frac{1}{(2\pi)^3}\int \frac{d^3\hat{p}}{\sqrt{-\hat{g}}\,\hat{p}^0}\,\hat{p}^{\mu_1}\ldots\hat{p}^{\mu_n} f(\hat{x},\hat{p})$$

Energy-momentum tensor

- 1st-moment of BE needs: $\hat{T}^{\mu\nu} \equiv \frac{1}{(2\pi)^3}\int \frac{d^3\hat{p}}{\sqrt{-\hat{g}}\,\hat{p}^0}\hat{p}^\mu \hat{p}^\nu f(\hat{x},\hat{p})$
- Energy-Momentum Tensor

$$\hat{T}^{\mu\nu} = \hat{\varepsilon}\hat{u}^\mu\hat{u}^\nu + \hat{P}_\theta\hat{\Theta}^\mu\hat{\Theta}^\nu + \hat{P}_\phi\hat{\Phi}^\mu\hat{\Phi}^\nu + \hat{P}_\varsigma\hat{\varsigma}^\mu\hat{\varsigma}^\nu$$

- Taking different projections

$$\hat{\varepsilon} = \frac{6\hat{\alpha}_\theta\hat{\alpha}_\phi}{(2\pi)^3}\hat{\lambda}^4\int_0^{2\pi} d\phi\,\hat{\alpha}_\perp^2 H_2(y)$$

$$\hat{P}_\theta = \frac{6\hat{\alpha}_\theta^3\hat{\alpha}_\phi}{(2\pi)^3}\hat{\lambda}^4\int_0^{2\pi} d\phi\cos^2\phi H_{2T}(y)$$

$$\hat{P}_\phi = \frac{6\hat{\alpha}_\theta\hat{\alpha}_\phi^3}{(2\pi)^3}\hat{\lambda}^4\int_0^{2\pi} d\phi\sin^2\phi H_{2T}(y)$$

$$\hat{P}_\varsigma = \frac{6\hat{\alpha}_\theta\hat{\alpha}_\phi}{(2\pi)^3}\hat{\lambda}^4\int_0^{2\pi} d\phi\,\hat{\alpha}_\perp^2 H_{2L}(y)$$

SO(3)-symmetry ⟹ $\hat{\alpha}_\theta = \hat{\alpha}_\phi$

$$\hat{\varepsilon} = \frac{3\,\hat{\alpha}_\theta^4\hat{\lambda}^4}{2\pi^2}H_2(\bar{y})$$

$$\hat{P}_\theta = \hat{P}_\phi = \frac{3\,\hat{\alpha}_\theta^4\hat{\lambda}^4}{4\pi^2}H_{2T}(\bar{y})$$

$$\hat{P}_\varsigma = \frac{3\,\hat{\alpha}_\theta^4\hat{\lambda}^4}{2\pi^2}H_{2L}(\bar{y})$$

$$H_{2L}(y) \equiv \frac{y^3}{(y^2-1)^{3/2}}\left(y\sqrt{y^2-1} - \tanh^{-1}\frac{\sqrt{y^2-1}}{y}\right)$$

$$H_{2T}(y) \equiv \frac{y}{(y^2-1)^{3/2}}\left((2y^2-1)\tanh^{-1}\frac{\sqrt{y^2-1}}{y} - y\sqrt{y^2-1}\right)$$

$$H_2(y) \equiv H_{2T}(y) + H_{2L}(y)$$

$$\hat{\alpha}_\perp \equiv \sqrt{\hat{\alpha}_\theta^2\cos^2\phi + \hat{\alpha}_\phi^2\sin^2\phi}$$

$$y \equiv \hat{\alpha}_\varsigma/\hat{\alpha}_\perp$$

$$\bar{y} \equiv \hat{\alpha}_\varsigma/\hat{\alpha}_\theta$$

Energy-momentum tensor

- 1^{st}-moment of BE

$$D_\mu \hat{T}^{\mu\nu} = 0 \qquad \hat{T}^{\mu\nu} = \hat{\varepsilon}\hat{u}^\mu\hat{u}^\nu + \hat{P}_\theta\hat{\Theta}^\mu\hat{\Theta}^\nu + \hat{P}_\phi\hat{\Phi}^\mu\hat{\Phi}^\nu + \hat{P}_\varsigma\hat{\varsigma}^\mu\hat{\varsigma}^\nu$$

- Dynamical Landau-matching condition can be used to fix the "effective temperature"

$$\Rightarrow \quad \hat{T} = \frac{\hat{\alpha}_\varsigma}{\bar{y}}\left(\frac{H_2(\bar{y})}{2}\right)^{1/4}\hat{\lambda}$$

$$\begin{gathered}\partial_\rho\hat{\varepsilon} + \tanh\rho\,(2\hat{\varepsilon} + \hat{P}_\theta + \hat{P}_\phi) = 0\\ \partial_\theta\hat{P}_\theta + (\hat{P}_\theta - \hat{P}_\phi)\cot\theta = 0\\ \partial_\phi\hat{P}_\phi = 0\\ \partial_\varsigma\hat{P}_\varsigma = 0\end{gathered}$$

SO(3)-symmetry $\Rightarrow$ $\hat{\alpha}_\theta = \hat{\alpha}_\phi$

$$\begin{gathered}\partial_\rho\hat{\varepsilon} + 2\tanh\rho\,(\hat{\varepsilon} + \hat{P}_\theta) = 0\\ \partial_\theta\hat{P}_\theta = \partial_\phi\hat{P}_\phi = \partial_\varsigma\hat{P}_\varsigma = 0\end{gathered}$$

$$4\frac{d\log\hat{\lambda}}{d\rho} + \frac{3\hat{\alpha}_\varsigma^2\left(\frac{H_{2L}(\bar{y})}{H_2(\bar{y})}+1\right)-4}{3\hat{\alpha}_\varsigma^2-1}\frac{d\log\hat{\alpha}_\varsigma}{d\rho} + \tanh\rho\left(\frac{H_{2T}(\bar{y})}{H_2(\bar{y})}+2\right) = 0$$

2nd-moment of the BE

- 2nd moment of BE needs:

$$\hat{\mathcal{I}}^{\lambda\mu\nu} = \int \frac{d^3\hat{p}}{\sqrt{-\hat{g}}\,\hat{p}^0}\,\hat{p}^\lambda\hat{p}^\mu\hat{p}^\nu f(\hat{x},\hat{p})$$

- Expanding over non-zero components gives:

$$\begin{aligned}\hat{\mathcal{I}} \equiv \hat{\mathcal{I}}_\rho &\Big[\hat{u}\otimes\hat{u}\otimes\hat{u}\Big]\\ +\hat{\mathcal{I}}_\theta &\Big[\hat{u}\otimes\hat{\Theta}\otimes\hat{\Theta} + \hat{\Theta}\otimes\hat{u}\otimes\hat{\Theta} + \hat{\Theta}\otimes\hat{\Theta}\otimes\hat{u}\Big]\\ +\hat{\mathcal{I}}_\phi &\Big[\hat{u}\otimes\hat{\Phi}\otimes\hat{\Phi} + \hat{\Phi}\otimes\hat{u}\otimes\hat{\Phi} + \hat{\Phi}\otimes\hat{\Phi}\otimes\hat{u}\Big]\\ +\hat{\mathcal{I}}_\varsigma &\Big[\hat{u}\otimes\hat{\varsigma}\otimes\hat{\varsigma} + \hat{\varsigma}\otimes\hat{u}\otimes\hat{\varsigma} + \hat{\varsigma}\otimes\hat{\varsigma}\otimes\hat{u}\Big]\end{aligned}$$

- Taking projections gives:

$$\hat{\mathcal{I}}_i = \hat{\alpha}\hat{\alpha}_i^2\hat{\mathcal{I}}_{\rm iso} \quad (i \in \theta,\phi,\varsigma)$$

$$\hat{\mathcal{I}}_\rho = \hat{\alpha}\left[\sum_{i=\theta,\phi,\varsigma}\hat{\alpha}_i^2\right]\hat{\mathcal{I}}_{\rm iso}$$

$$\hat{\alpha} \equiv \hat{\alpha}_\theta\hat{\alpha}_\phi\hat{\alpha}_\varsigma$$

$$\hat{\mathcal{I}}_{\rm iso} \equiv 4\hat{\lambda}^5/\pi^2$$

$$\hat{\mathcal{I}}_\rho = \sum_{i=\theta,\phi,\varsigma}\hat{\mathcal{I}}_i$$

2^{nd}-moment of the BE

- Taking the 2^{nd}-moment of BE

$$D_\lambda \hat{\mathcal{I}}^{\lambda\mu\nu} = -\frac{1}{\hat{\tau}_{\rm eq}}\left(\hat{u}_\lambda \hat{\mathcal{I}}^{\lambda\mu\nu}_{\rm iso} - \hat{u}_\lambda \hat{\mathcal{I}}^{\lambda\mu\nu}\right)$$

- Using the expanded form of $\hat{\mathcal{I}}^{\lambda\mu\nu}$

$$\begin{gathered}
\partial_\rho \hat{\mathcal{I}}_\theta + 4\tanh\rho\, \hat{\mathcal{I}}_\theta = \frac{1}{\hat{\tau}_{\rm eq}}\left[\hat{\mathcal{I}}_{\theta,{\rm iso}} - \hat{\mathcal{I}}_\theta\right] \\
\partial_\rho \hat{\mathcal{I}}_\phi + 4\tanh\rho\, \hat{\mathcal{I}}_\phi = \frac{1}{\hat{\tau}_{\rm eq}}\left[\hat{\mathcal{I}}_{\phi,{\rm iso}} - \hat{\mathcal{I}}_\phi\right] \\
\partial_\rho \hat{\mathcal{I}}_\varsigma + 2\tanh\rho\, \hat{\mathcal{I}}_\varsigma = \frac{1}{\hat{\tau}_{\rm eq}}\left[\hat{\mathcal{I}}_{\varsigma,{\rm iso}} - \hat{\mathcal{I}}_\varsigma\right] \\
\partial_\theta \hat{\mathcal{I}}_\theta + \cot\theta(\hat{\mathcal{I}}_\theta - \hat{\mathcal{I}}_\phi) = 0 \\
\partial_\phi \hat{\mathcal{I}}_\phi = 0 \\
\partial_\varsigma \hat{\mathcal{I}}_\varsigma = 0
\end{gathered}$$

SO(3)-symmetry → $\hat{\alpha}_\theta = \hat{\alpha}_\phi$

$$\begin{gathered}
\partial_\rho \hat{\mathcal{I}}_\theta + 4\tanh\rho\, \hat{\mathcal{I}}_\theta = \frac{1}{\hat{\tau}_{\rm eq}}\left[\hat{\mathcal{I}}_{\theta,{\rm iso}} - \hat{\mathcal{I}}_\theta\right] \\
\partial_\rho \hat{\mathcal{I}}_\varsigma + 2\tanh\rho\, \hat{\mathcal{I}}_\varsigma = \frac{1}{\hat{\tau}_{\rm eq}}\left[\hat{\mathcal{I}}_{\varsigma,{\rm iso}} - \hat{\mathcal{I}}_\varsigma\right] \\
\partial_\theta \hat{\mathcal{I}}_\theta = \partial_\phi \hat{\mathcal{I}}_\phi = \partial_\varsigma \hat{\mathcal{I}}_\varsigma = 0
\end{gathered}$$

$$\frac{6\hat{\alpha}_\varsigma}{1-3\hat{\alpha}_\varsigma^2}\frac{d\hat{\alpha}_\varsigma}{d\rho} - \frac{3\left(3\hat{\alpha}_\varsigma^4 - 4\hat{\alpha}_\varsigma^2 + 1\right)}{4\hat{\tau}_{\rm eq}\hat{\alpha}_\varsigma^5}\left(\frac{\hat{T}}{\hat{\lambda}}\right)^5 + 2\tanh\rho = 0$$

Limiting cases

- It is possible to solve the aHydro equations analytically in two limiting cases
- Ideal limit ($\hat{\alpha}_\varsigma \to 1$, $\partial_\rho \hat{\alpha}_\varsigma \to 0$, $\hat{\tau}_{\rm eq} \to 0$)[1]

$$\hat{T}(\rho) = \hat{T}_0 \left(\frac{\cosh \rho_0}{\cosh \rho} \right)^{2/3}$$

- Free-streaming limit ($\hat{\tau}_{\rm eq} \to \infty$)[2,3]

$$\hat{\varepsilon}_{\rm FS} = \frac{3\hat{\lambda}_0^4 \hat{\alpha}_{\varsigma,0}^4}{\pi^2} \mathcal{H}_\varepsilon(\mathcal{C}_{\rho_0,\rho})$$

$$(\hat{\pi}_\varsigma^\varsigma)_{\rm FS} = \frac{\hat{\lambda}_0^4 \hat{\alpha}_{\varsigma,0}^4}{\pi^2} \mathcal{H}_\pi(\mathcal{C}_{\rho_0,\rho}^{-1})$$

$$\mathcal{H}_\varepsilon(x) \equiv \frac{x^2}{2} + \frac{x^4}{2} \frac{\tanh^{-1}\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$\mathcal{H}_\pi(x) \equiv \frac{x\sqrt{x^2-1}(1+2x^2) + (1-4x^2)\coth^{-1}(x/\sqrt{x^2-1})}{2x^3(x^2-1)^{3/2}}$$

$$\mathcal{C}_{\rho_0,\rho} \equiv \frac{\hat{\alpha}_{\theta,0}\cosh\rho_0}{\hat{\alpha}_{\varsigma,0}\cosh\rho}$$

- aHydro gives both the ideal <u>and</u> free streaming limits!

[1] S. S. Gubser and A. Yarom, arXiv:1012.1314.
[2] G. S. Denicol, U. W. Heinz, M. Martinez, J. Noronha, and M. Strickland, arXiv:1408.5646.
[3] G. S. Denicol, U. W. Heinz, M. Martinez, J. Noronha, and M. Strickland, arXiv:1408.7048.

Numerical results (setup)

- Solve two ODEs numerically subject to BCs at $\rho \to -\infty$ $(\tau \to 0^+)$

$$4\frac{d\log\hat{\lambda}}{d\rho} + \frac{3\hat{\alpha}_\varsigma^2\left(\frac{H_{2L}(\bar{y})}{H_2(\bar{y})}+1\right)-4}{3\hat{\alpha}_\varsigma^2-1}\frac{d\log\hat{\alpha}_\varsigma}{d\rho} + \tanh\rho\left(\frac{H_{2T}(\bar{y})}{H_2(\bar{y})}+2\right) = 0$$

$$\frac{6\hat{\alpha}_\varsigma}{1-3\hat{\alpha}_\varsigma^2}\frac{d\hat{\alpha}_\varsigma}{d\rho} - \frac{3\left(3\hat{\alpha}_\varsigma^4-4\hat{\alpha}_\varsigma^2+1\right)}{4\hat{\tau}_{\rm eq}\hat{\alpha}_\varsigma^5}\left(\frac{\hat{T}}{\hat{\lambda}}\right)^5 + 2\tanh\rho = 0 \qquad \hat{T} = \frac{\hat{\alpha}_\varsigma}{\bar{y}}\left(\frac{H_2(\bar{y})}{2}\right)^{1/4}\hat{\lambda}$$

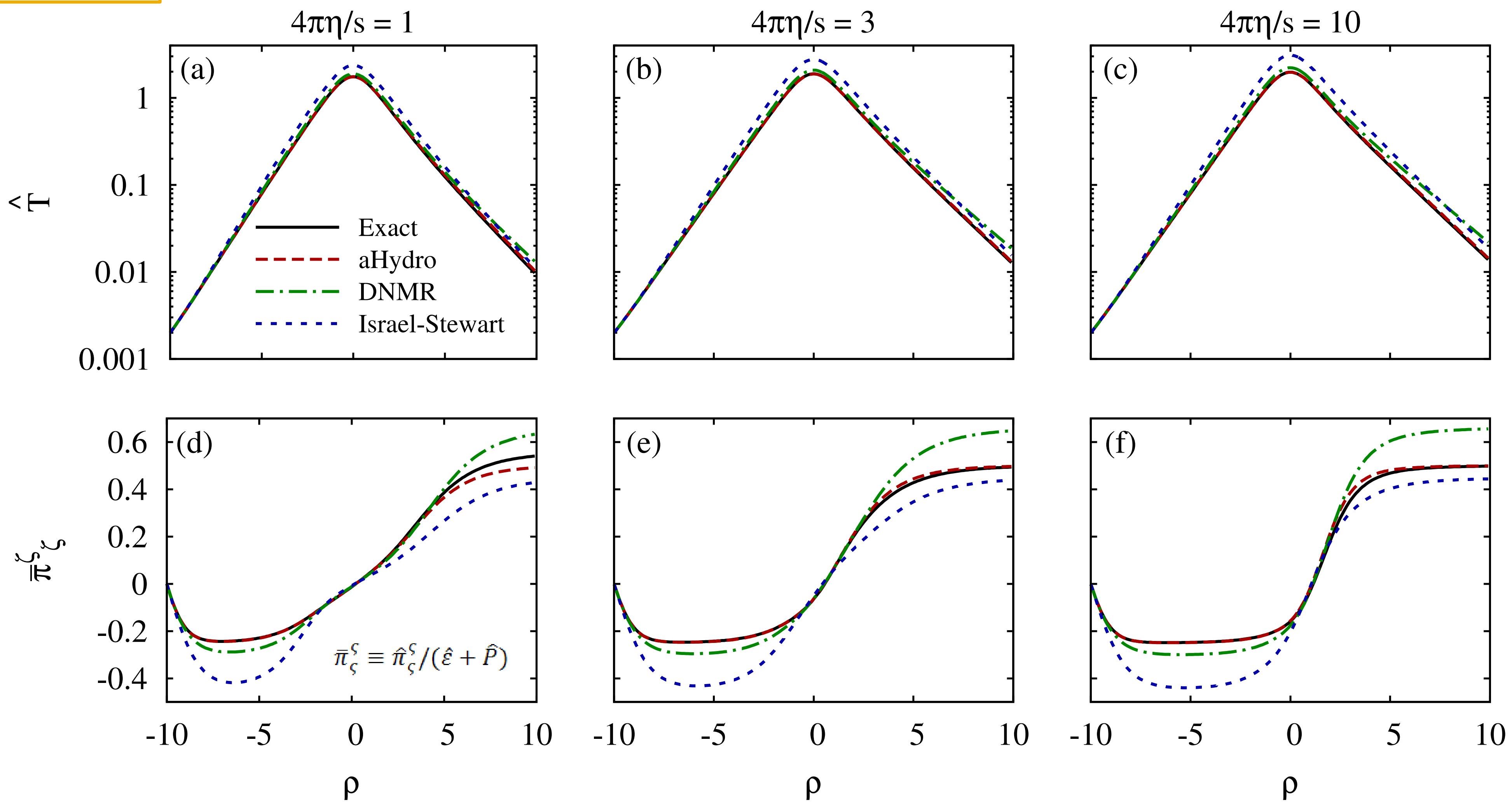
- Compare to exact solution obtained via iterative method[1,2]
- Compare to Israel-Stewart approximation[3]
- Compare to Denicol-Niemi-Molnar-Rischke (DNMR) approximation[1]

[1] G. S. Denicol, U. W. Heinz , M. Martinez, M. Strickland, arXiv:1408.7048
[2] S. G. Denicol, U. W. Heinz, M. Martinez, J. Noronha, M. Strickland, arXiv:1408.5646
[3] H. Marrochio, J. Noronha, G. S. Denicol, M. Luzum, S. Jeon, C. Gale, arXiv:1307.6130

Numerical results (isotropic IC)

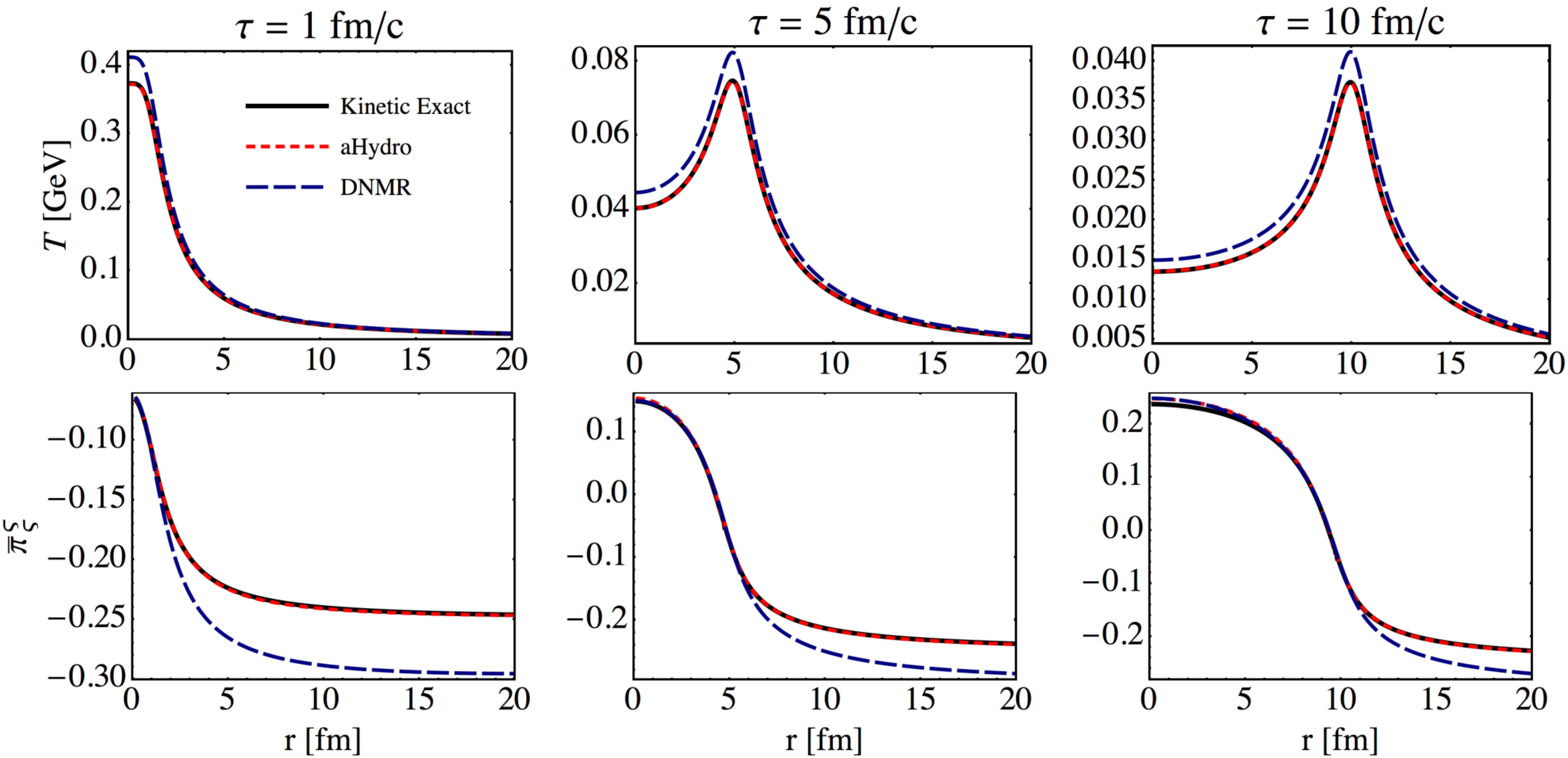
$\hat{\alpha}_{\varsigma,0} = 1$

$\hat{T}_0 = 0.002$



Mapping back to Milne coordinates

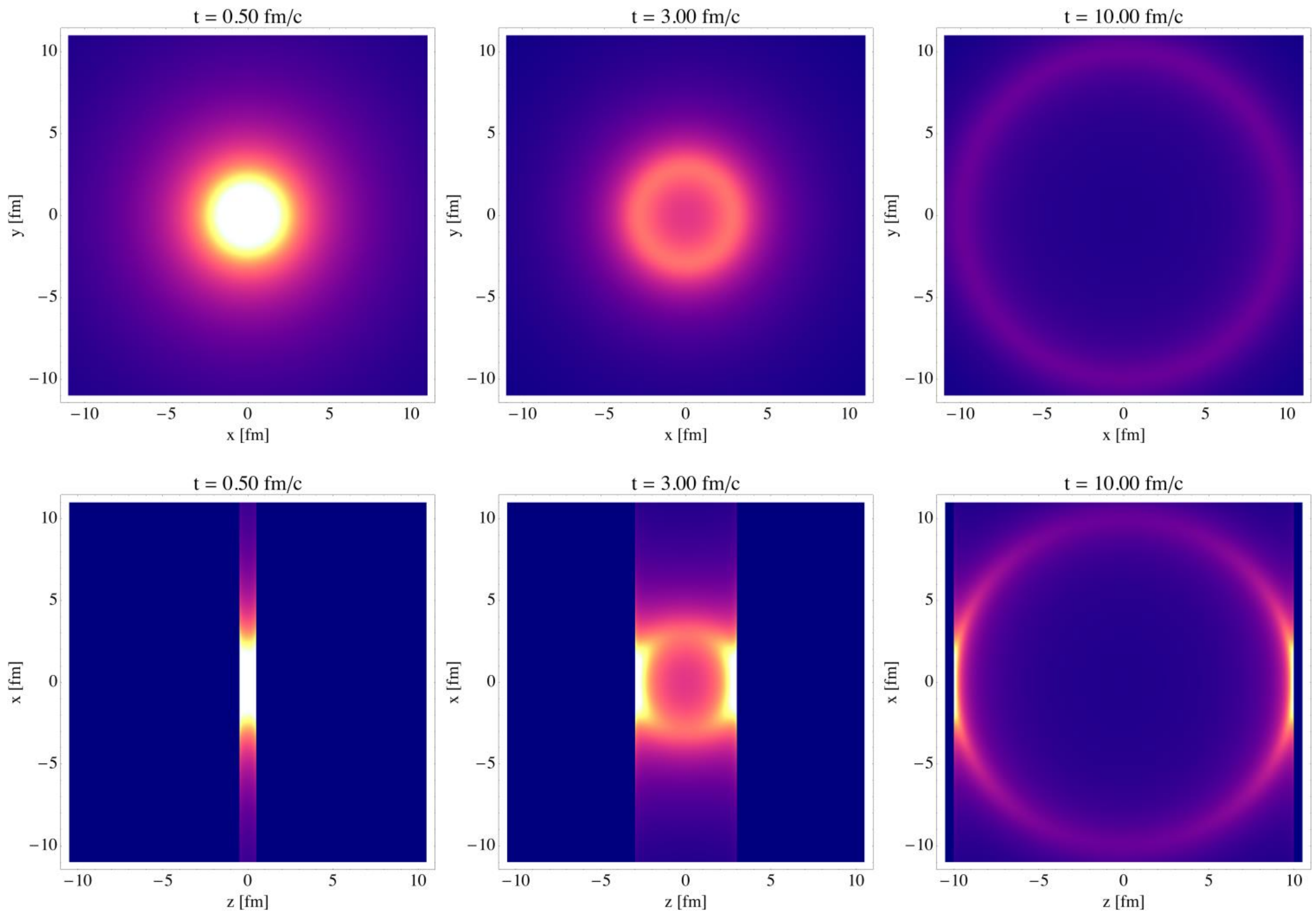
Taking $4\pi\eta/s = 3$ and $q = (1fm)^{-1}$



Mapping back to Minkowski Space

Slice through the x-y plane;
$4\pi\eta/s = 3$ and $q = (1fm)^{-1}$

Slice through the x-z plane;
$4\pi\eta/s = 3$ and $q = (1fm)^{-1}$



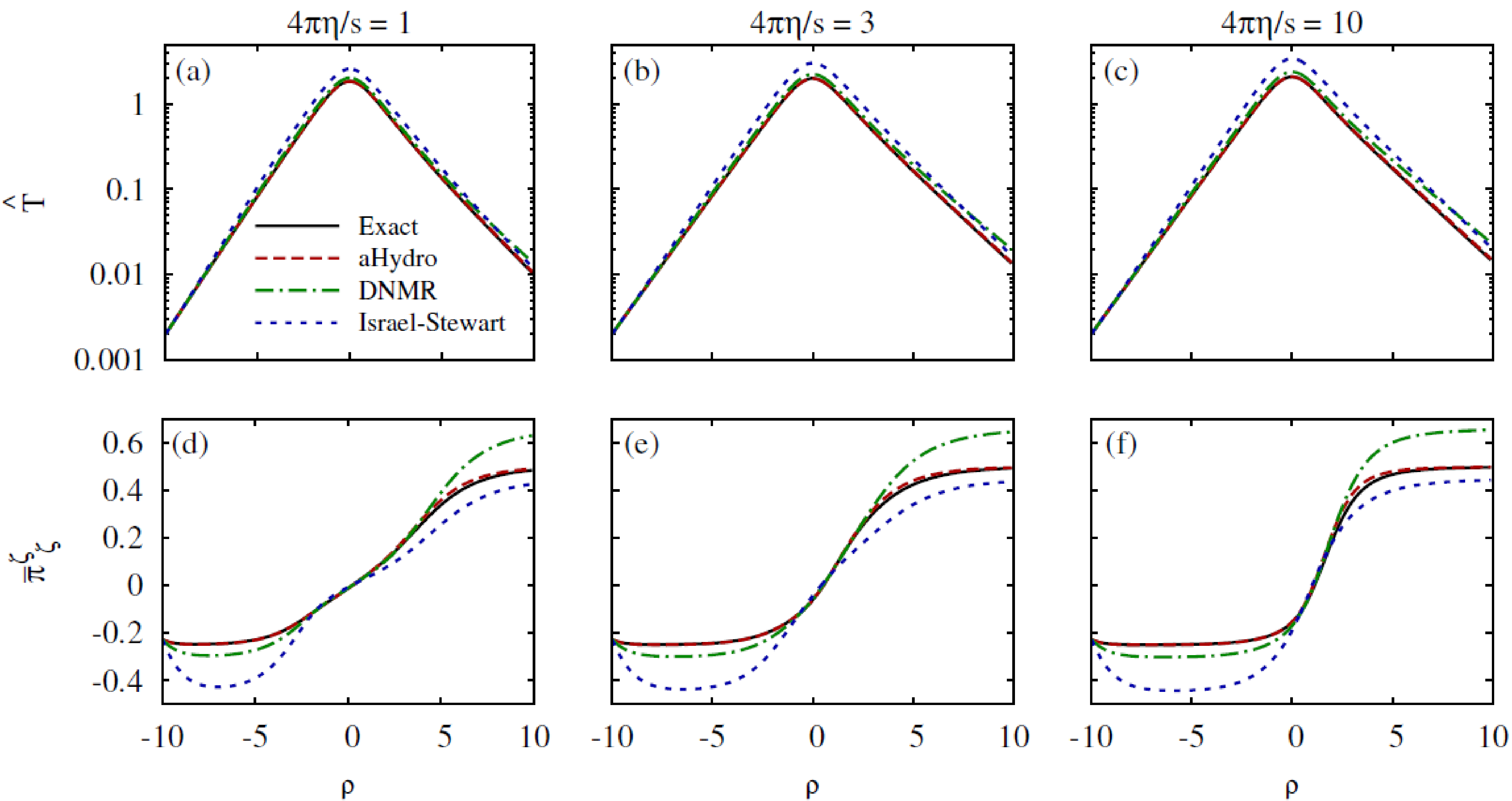
Conclusions and outlook

- Dynamical equations in the aHydro framework were derived
- Weyl rescaling + coordinate transformation makes the velocity profile static and simplifies the problem; 1+1d Minkowski flow ⟹ 0+1d de Sitter flow
- The exact solution of RTA Boltzmann equation was used to test different frameworks
- aHydro reproduces the exact solution better than both the DNMR and IS approximations
- The aHydro equations analytically reproduce the exact solutions in both the ideal ($\eta/s \to 0$) and free streaming ($\eta/s \to \infty$) limits.

- ❑ In the future, we plan to derive and test NLO aHydro for Gubser flow.
- ❑ In addition, we are working on phenomenological applications of aHydro for general 2+1d and 3+1d systems.

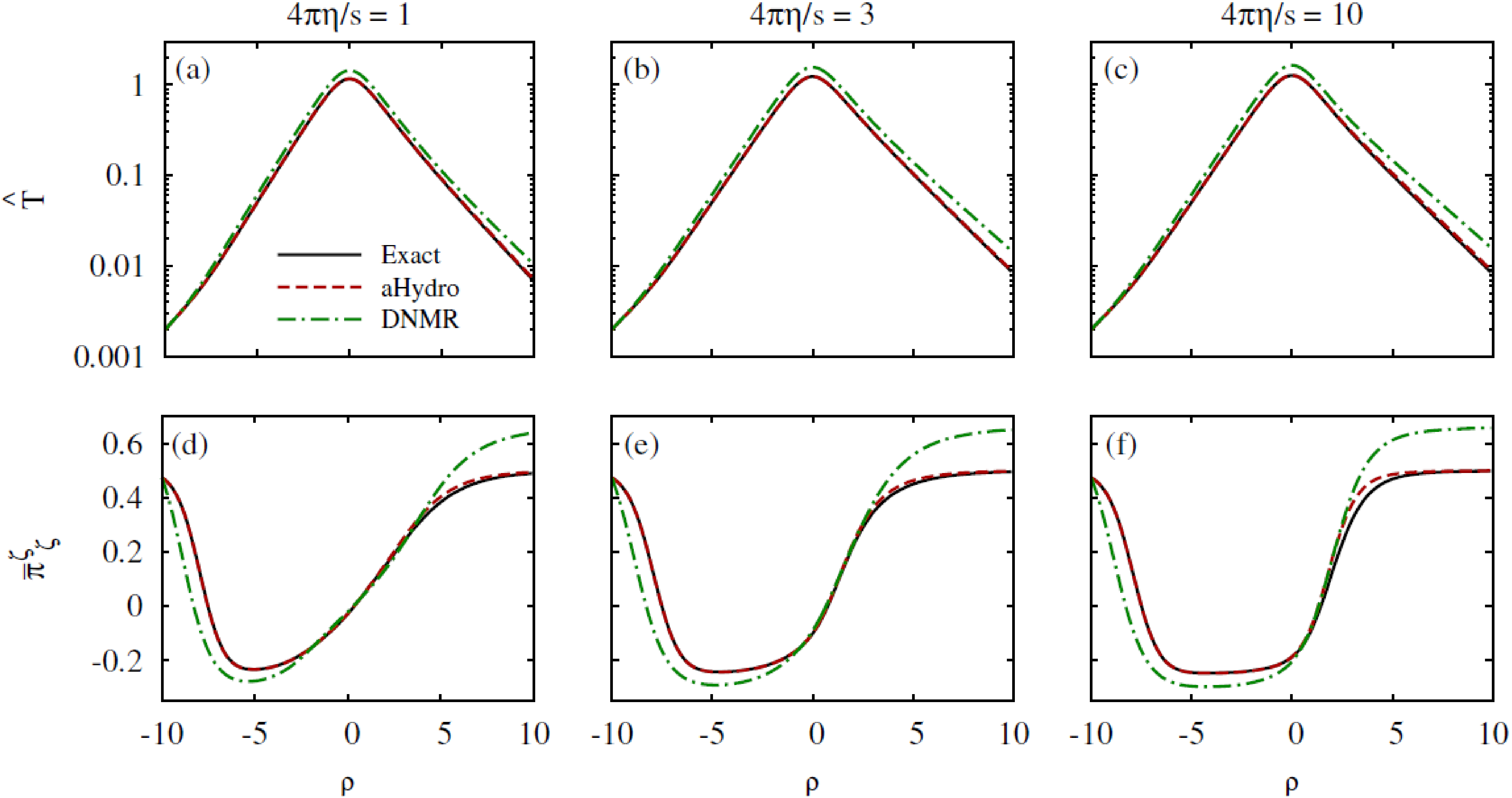
Numerical results (oblate IC)

$\hat{\alpha}_{\varsigma,0} = 0.6$
$\hat{T}_0 = 0.002$



Numerical results (prolate IC)

$\hat{\alpha}_{\varsigma,0} = 10$
$\hat{T}_0 = 0.002$



Connection to 2nd-order viscous hydro

$$\partial_\rho \hat{\varepsilon} + \tanh\rho\,(2\hat{\varepsilon} + \hat{P}_\theta + \hat{P}_\phi) = 0$$

$$\hat{\pi}_{\mu\nu} = \hat{\pi}^\theta_\theta \hat{\Theta}_\mu \hat{\Theta}_\nu + \hat{\pi}^\phi_\phi \hat{\Phi}_\mu \hat{\Phi}_\nu + \hat{\pi}^\varsigma_\varsigma \hat{\varsigma}_\mu \hat{\varsigma}_\nu$$

$$\hat{P}_i = \hat{P}_{\text{iso}} + \hat{\pi}^i_i$$

$$\hat{T}\hat{s} = 4\hat{\varepsilon}/3$$

$$\hat{P}_{\text{iso}} = \hat{\varepsilon}/3$$

- 2nd order viscous hydrodynamics[1]

$$\boxed{\frac{\partial_\rho \hat{T}}{\hat{T}} + \frac{2}{3}\tanh\rho = \frac{1}{3}\bar{\pi}^\varsigma_\varsigma \tanh\rho}$$

[1]H. Marrochio, J. Noronha, G. S. Denicol, M. Luzum, S. Jeon, et al., , arXiv:1307.6130.