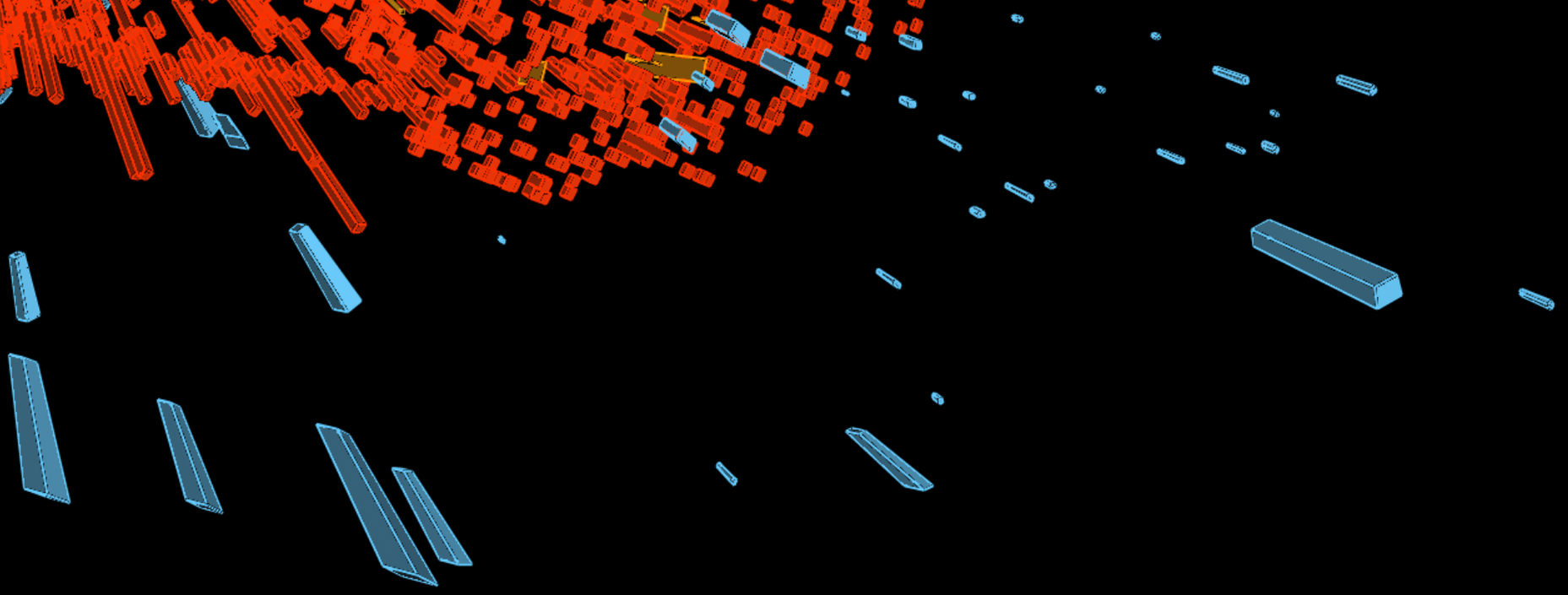
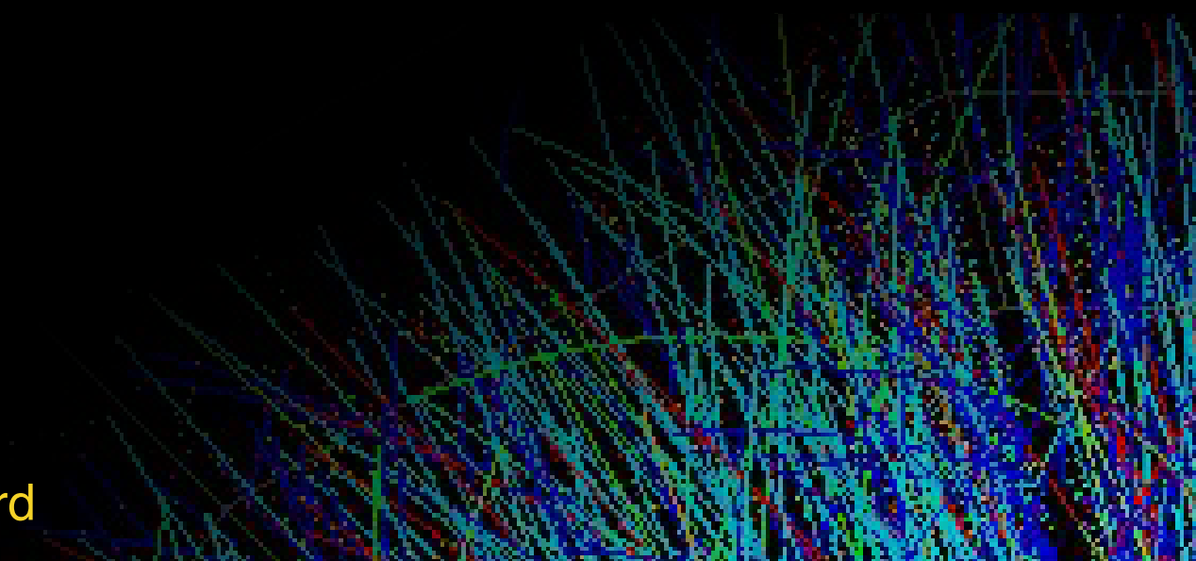


TRENTo: a new model for initial entropy deposition in ultra-relativistic nuclear collisions

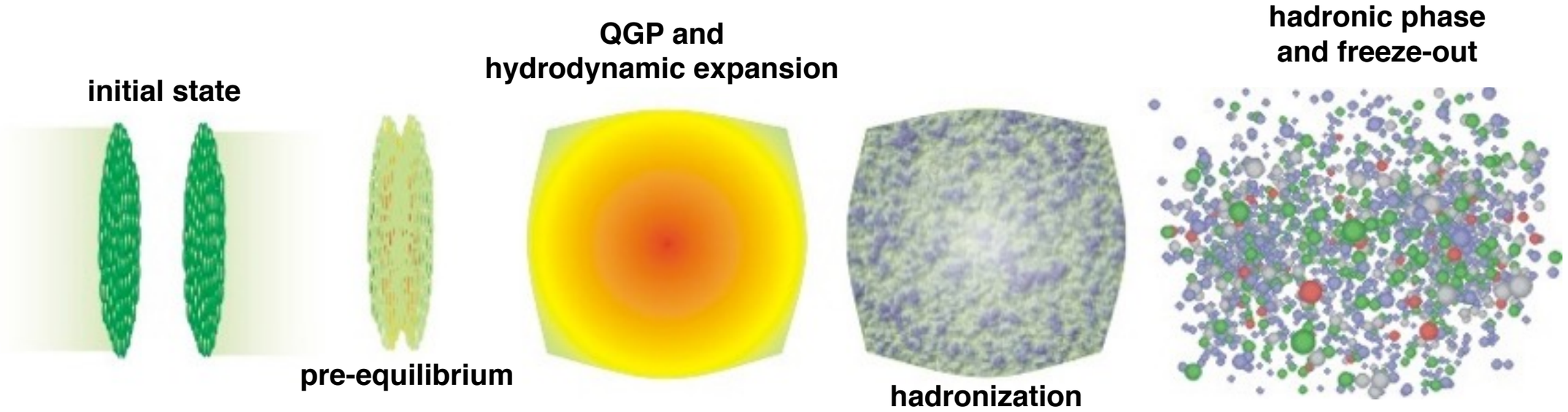
Steffen A. Bass
Duke University

in collaboration with J. Scott Moreland and Jonah E. Bernhard



Motivation

Time Evolution of a Heavy-Ion Collision



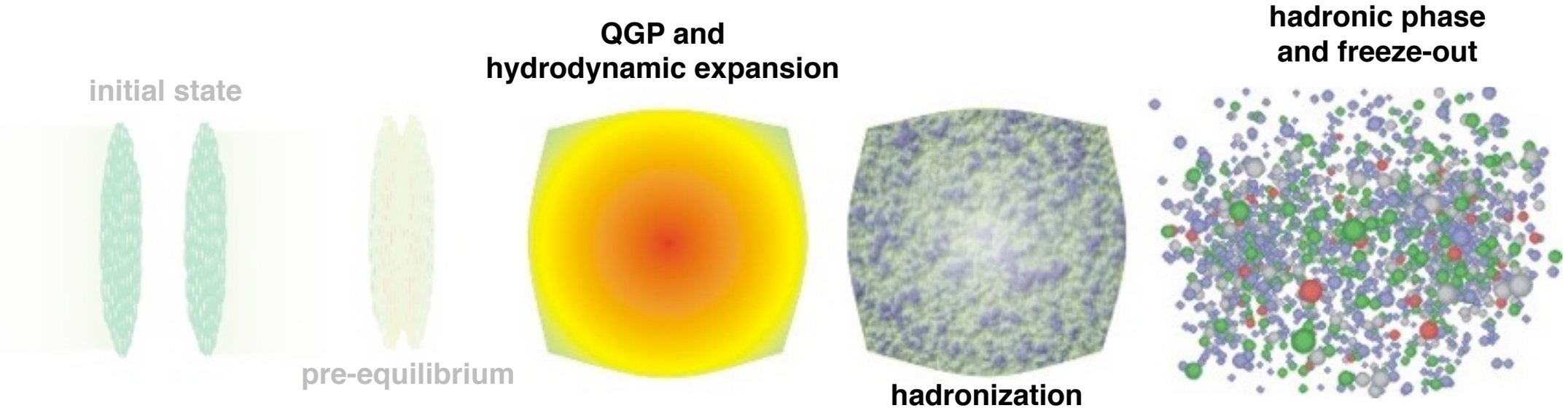
- **Initial State:**
  - fluctuates event-by-event
  - classical color-field dynamics

- **QGP and hydrodynamic expansion:**
  - proceeds via 3D viscous RFD
  - EoS from Lattice QCD

- **Pre-equilibrium:**
  - rapid change-over from glue-field dominated initial state to thermalized QGP
  - time scale: 0.15 to 2 fm/c in duration
  - build-up of transverse velocity fields?

- **hadronic phase & freeze-out**
  - interacting hadron gas
  - separation of chemical and kinetic freeze-out

Constraining the IS of Heavy-Ion Collisions



- treatment of QGP evolution and hadronic freeze-out is well established and largely understood
- major success: first extraction of QGP properties such as η/s
- major challenges:
  - quantify uncertainties in extracted QGP properties
  - temperature dependence of transport coefficients

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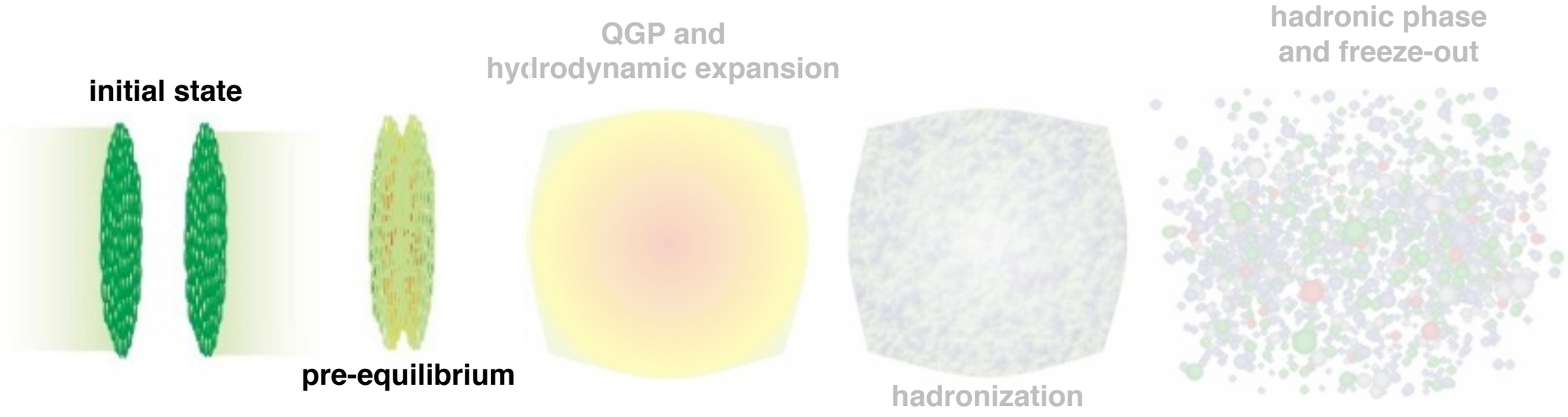

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- physics of initial state and pre-equilibrium dynamics are still conceptually challenging with many open questions
- a major source of uncertainty for the extraction of QGP properties

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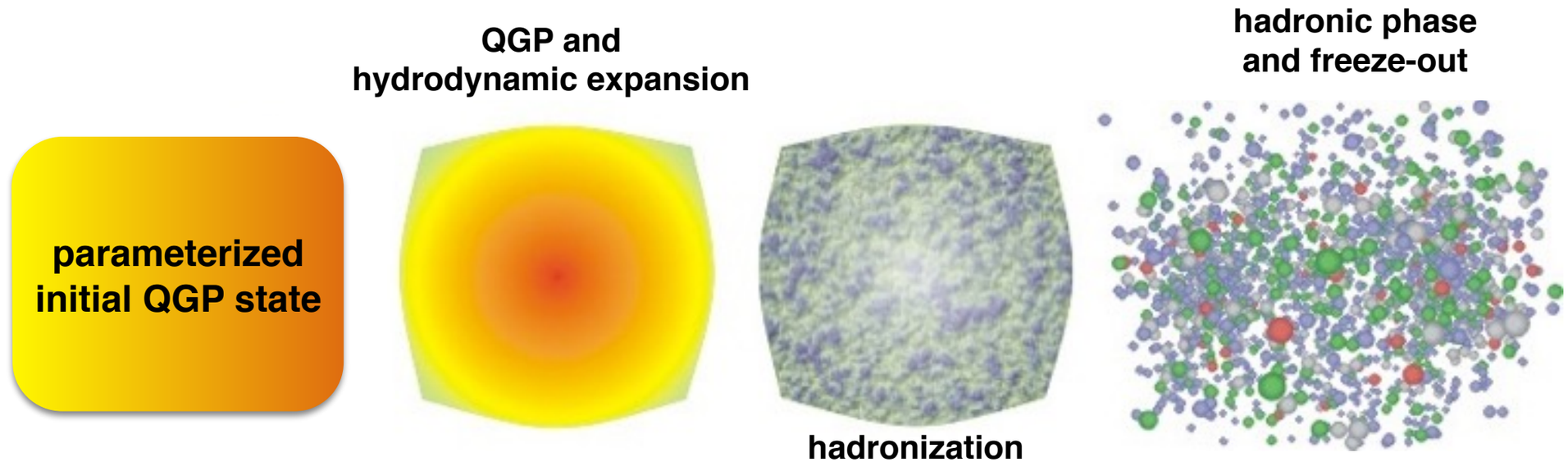

parameterized initial QGP state:

- based on simple phenomenological ideas for entropy deposition
- constrained by global model to data fit
- provides guidance to ab-initio IS models on features needed to describe the data

- **QGP and hydrodynamic expansion:**
  - proceeds via 3D viscous RFD
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Model Design

Basic Specifications

construction in similar fashion as a two-component Glauber model:

- a pair of projectiles A, B collide along beam axis z
- each projectile is represented by beam-integrated density of its nuclear matter, called thickness:

$$T_{A,B}(x,y) = \int dz\, \rho_{A,B}(x,y,z)$$

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model should meet following criteria:

1. eikonal approximation is valid; entropy is produced if T_A & T_B eikonally overlap
2. there exists a scalar field $f(T_A, T_B)$ which converts projectile thickness into entropy deposition
3. the function f is proportional to the entropy created at mid-rapidity and at the hydrodynamic thermalization time:

$$f \propto dS/dy\,|_{\tau=\tau_0}$$

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- model should provide an effective description of entropy production prior to thermalization, yet need not arise from first principles
- ▸ introduce **reduced thickness*** as ansatz for f:

$$f = T_R(p; T_A, T_B) \equiv \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

* for p=-1 T_R has same structure as reduced mass

Properties of the Reduced Thickness

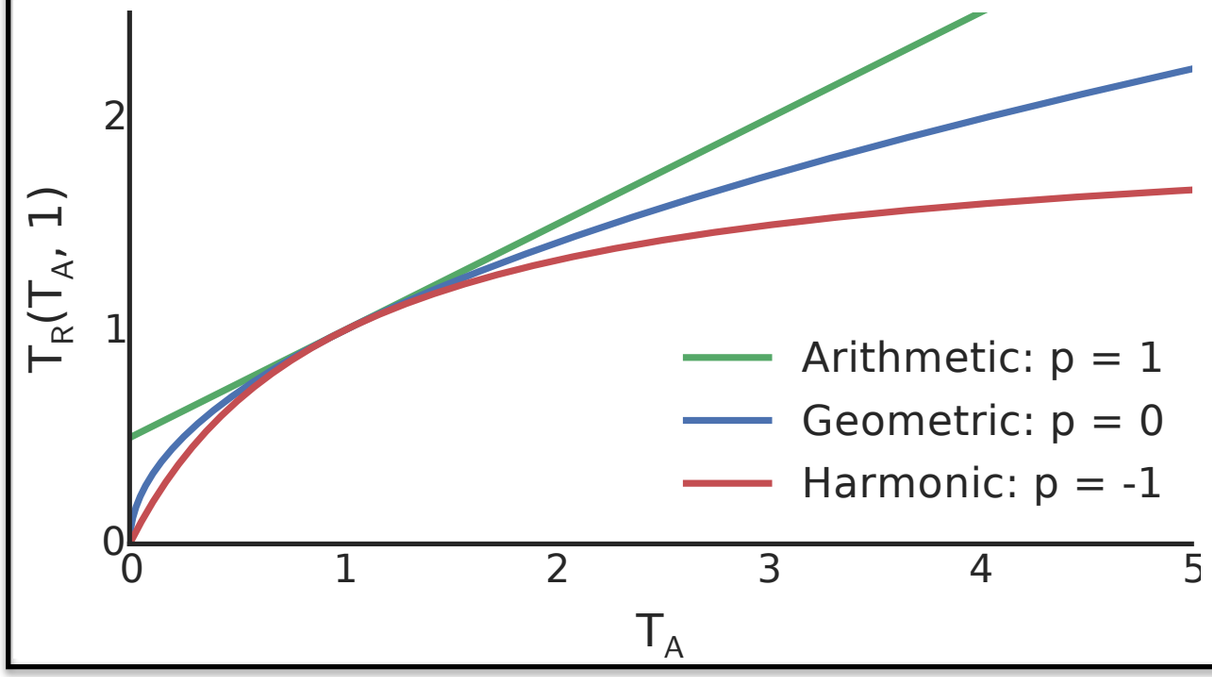
reduced thickness T_R:

$$T_R(p; T_A, T_B) \equiv \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

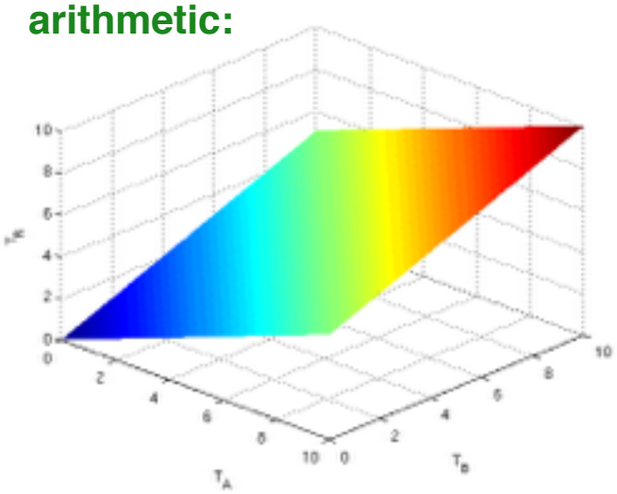
with $p \in (-\infty, \infty)$

T_R has formal structure of **generalized mean**:

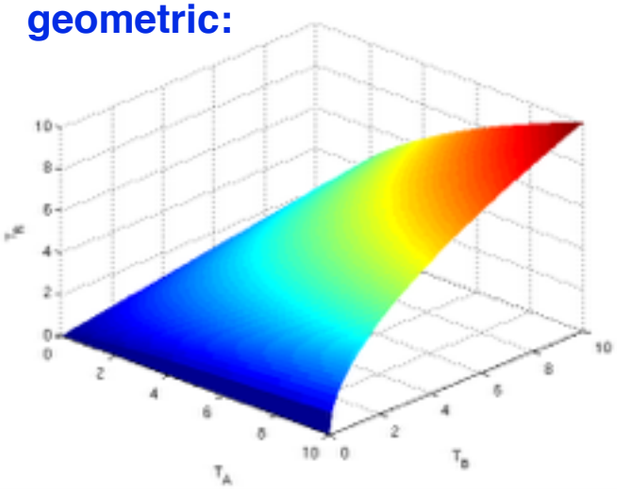
$$T_R(p; T_A, T_B) = \begin{cases} \dfrac{T_A + T_B}{2} & p = 1 \text{ (arithmetic)}, \\ \sqrt{T_A T_B} & p = 0 \text{ (geometric)}, \\ \dfrac{2 T_A T_B}{T_A + T_B} & p = -1 \text{ (harmonic)}. \end{cases}$$



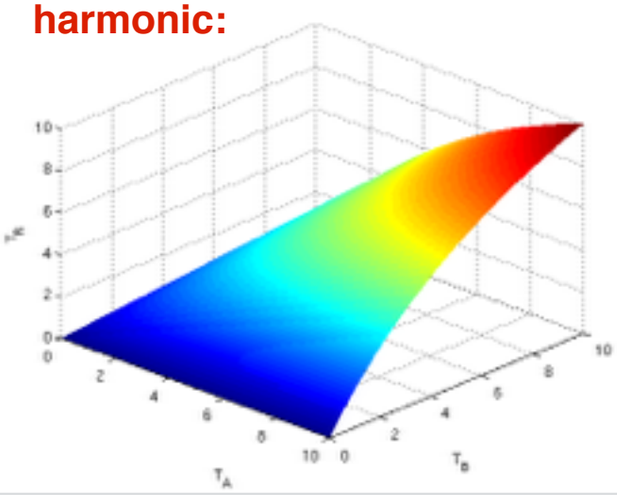
arithmetic:



geometric:



harmonic:



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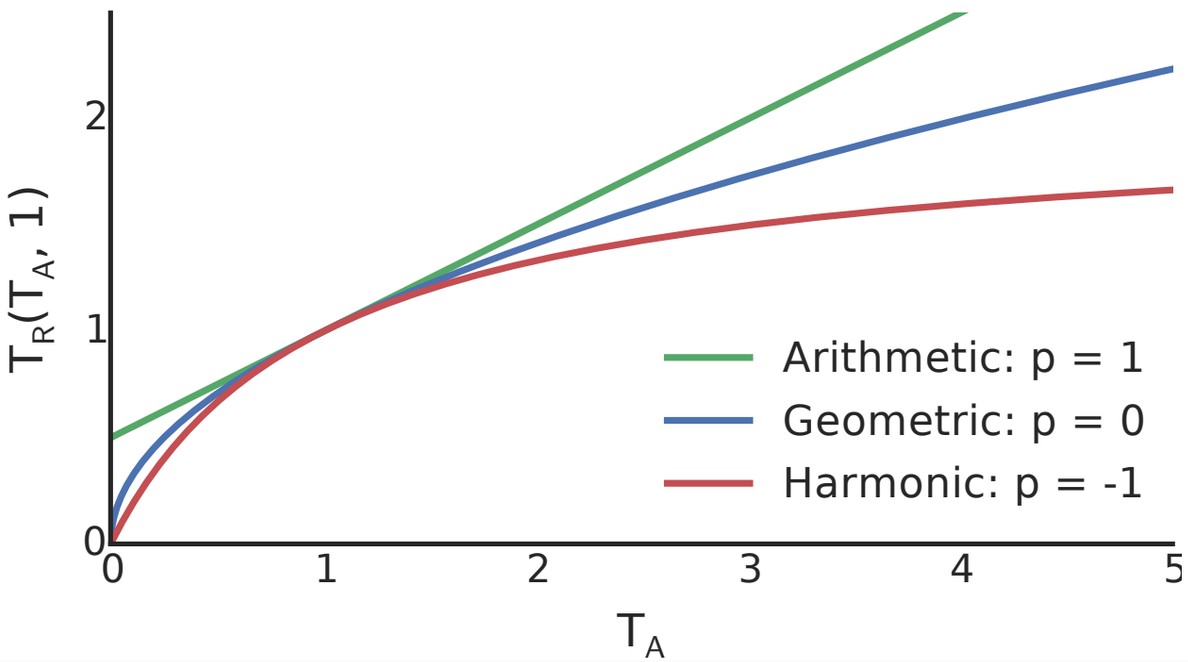
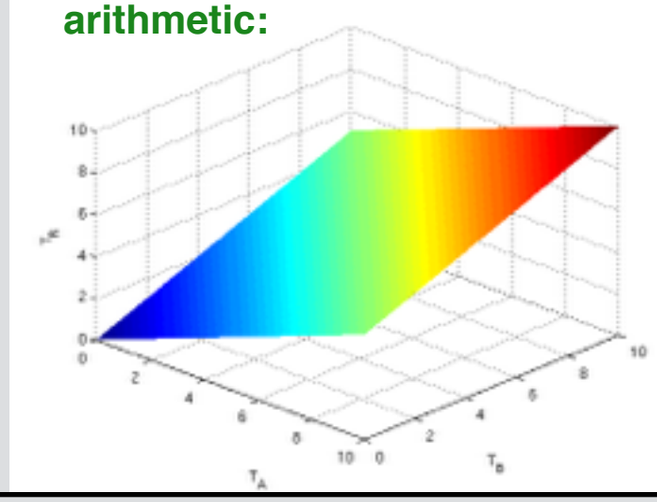
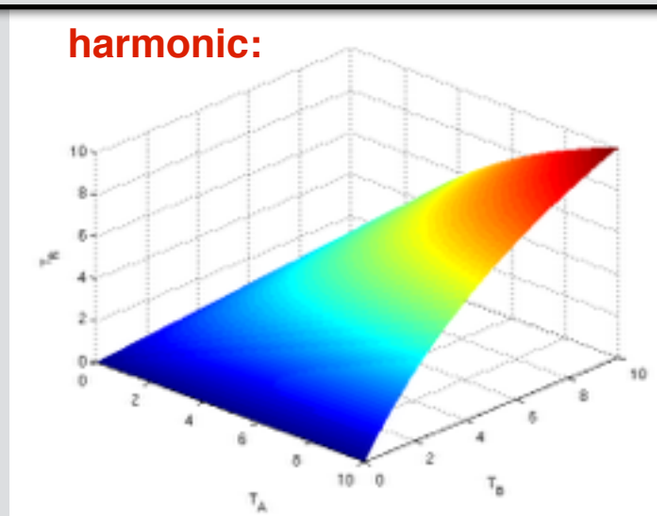




Features:

p quantifies attenuation of entropy production in off-diagonal regions:

- decreasing p, increasing attenuation
- **p=1**: wounded nucleon model
- **-1 < p < 0**: exhibits features seen in saturation models



Construction of the Nuclear Thickness

- determine participant nucleons in A, B
  by sampling for each nucleon pair: $P_{\rm coll} = 1 - \exp\left[-\sigma_{gg} \int dx\, dy \int dz\, \rho_A \int dz\, \rho_B\right]$

  Nuclear Thickness*: $T_A = \sum_i \gamma_i \int dz\, \rho_{\rm nucleon}(x - x_i, y - y_i, z - z_i)$

- sum is over participant nucleons with positions sampled from an uncorrelated Woods-Saxon distribution or correlated nuclear configurations when available
- introduce fluctuations via γ_i, sampled from a gamma distribution with unit mean: $P(\gamma; k) \propto \gamma^{k-1} e^{-\gamma/k}$
- nucleon density $\rho_{\rm nucleon}$ modeled as Gaussian in transverse plane $\int dz\, \rho_{\rm proton} = \frac{1}{2\pi B} \exp\left(-\frac{x^2 + y^2}{2B}\right)$

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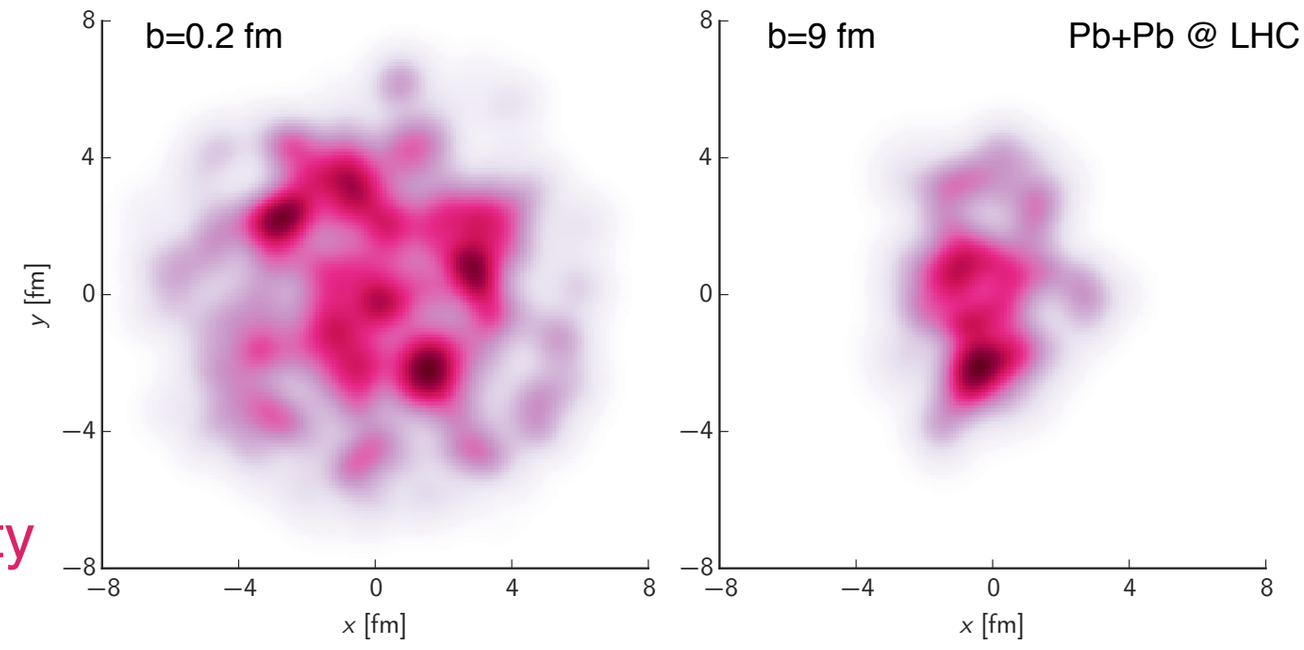
model parameters:

- attenuation parameter: p
- fluctuation parameter: k
- Gaussian width of nucleon: B
- overall normalization: $C_{\rm norm}$

model output:

- event by event spatial entropy density distribution at mid-rapidity at thermalization time τ_0



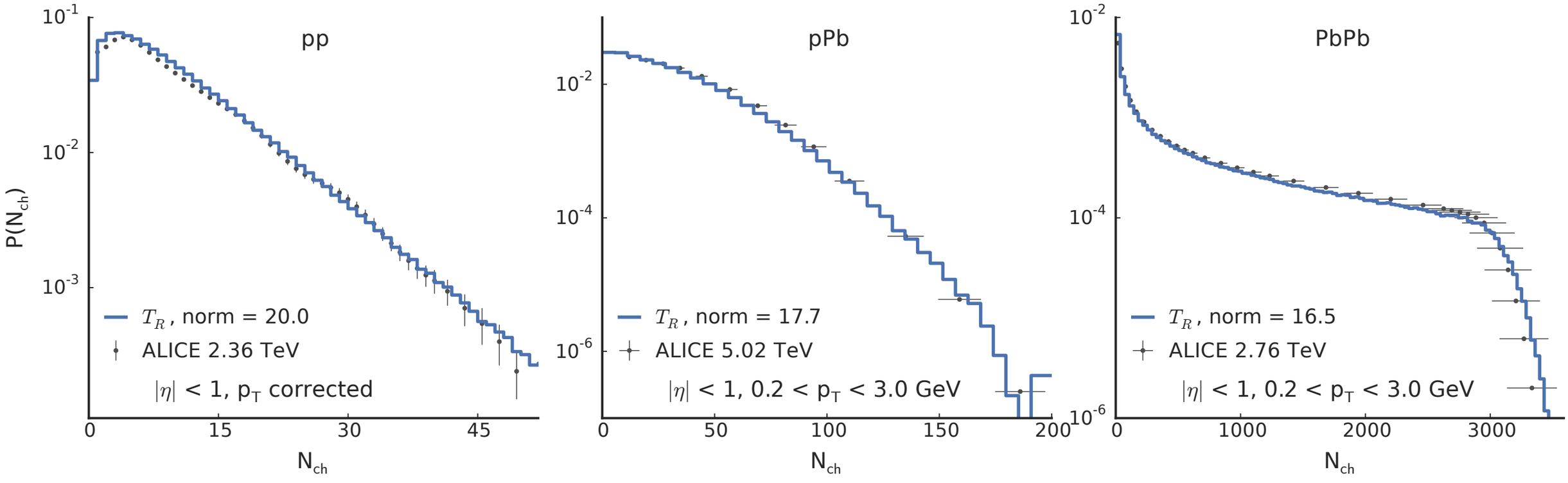
First Results

Charged Particle Multiplicity Distributions

- experimentally observed charged-particle multiplicity N_{ch} is to a very good approximation proportional to the total initial entropy
- proportional to the integrated reduced thickness:

$$N_{ch} = C_{\,\mathrm{norm}} \int dx\, dy\, T_R$$

- fix parameters of model: p=0 (geometric mean); k=0.8; B=(0.6 fm)^2
- normalization constant C_{norm} will depend on collision energy and kinematic cuts

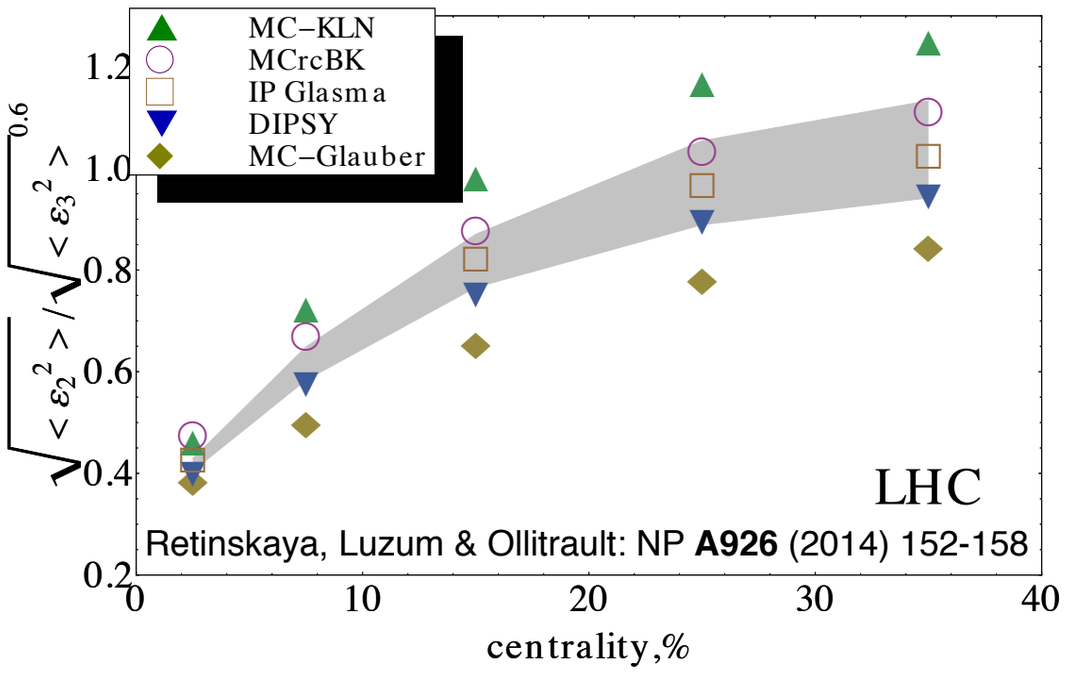


Eccentricity Harmonics

Analysis by Retinskaya, Luzum & Ollitrault:

- vRFD analysis of v_2 & v_3 at RHIC & LHC
- linear response ansatz: $v_n=C_n\varepsilon_n$ relates final state v_n to initial ε_n
- variation of vRFD parameters such as η/s provides bands for ε_n compatible to data
- findings can be summarized in centrality dependence of ratio

$$R = \sqrt{\langle \varepsilon_2^2 \rangle} / \sqrt{\langle \varepsilon_3^2 \rangle}^{0.6}$$



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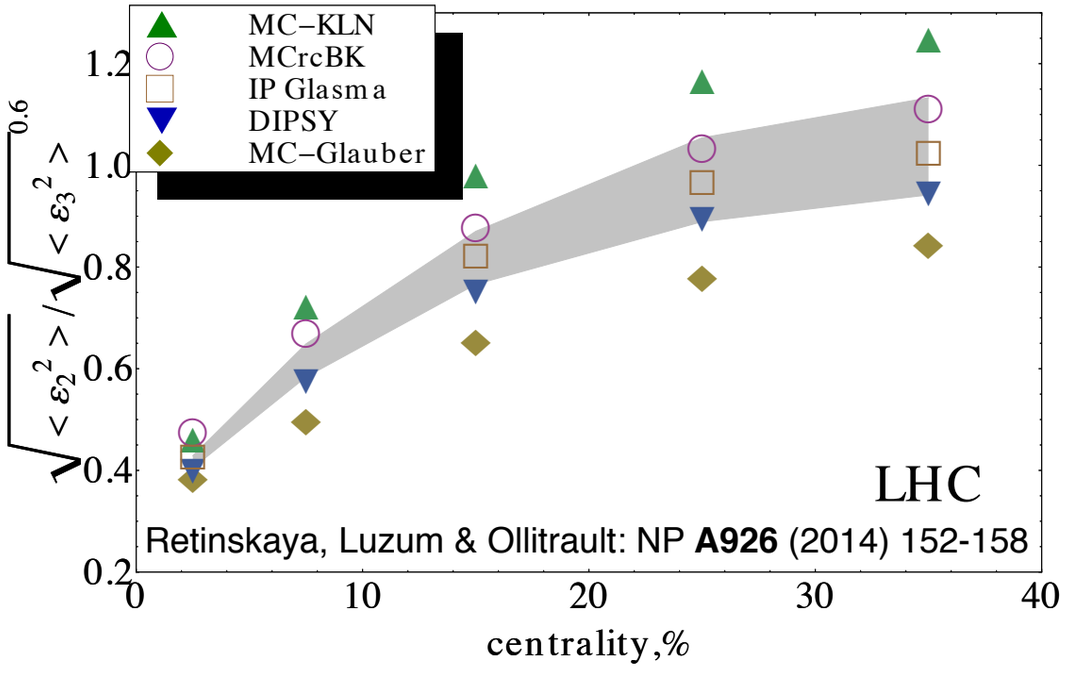
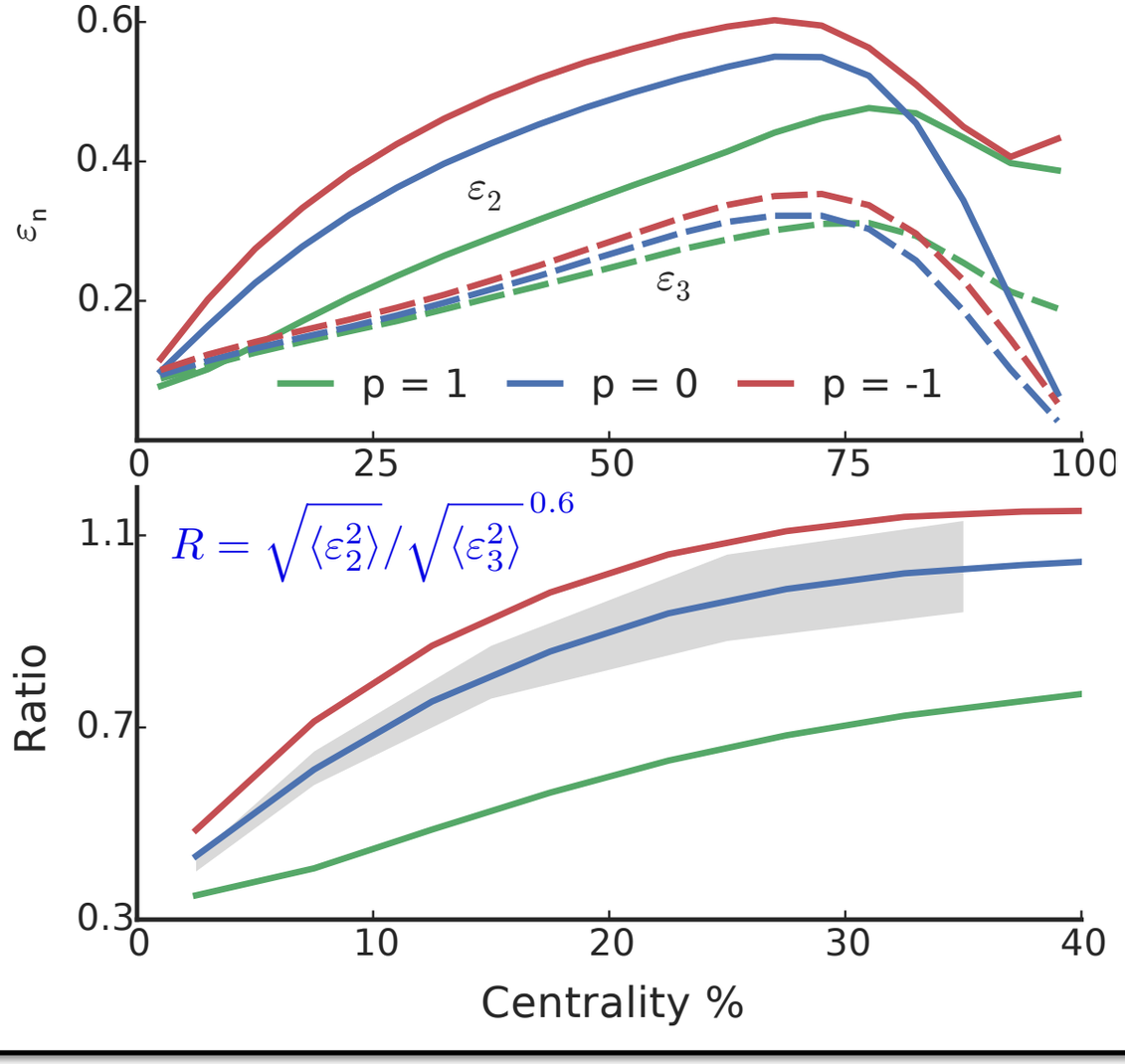


TRENTo:

- calculate eccentricities via:

$$\varepsilon_n e^{in\phi} = -\frac{\int dx\,dy\,r^n e^{in\phi}\,T_R}{\int dx\,dy\,r^n\,T_R}$$

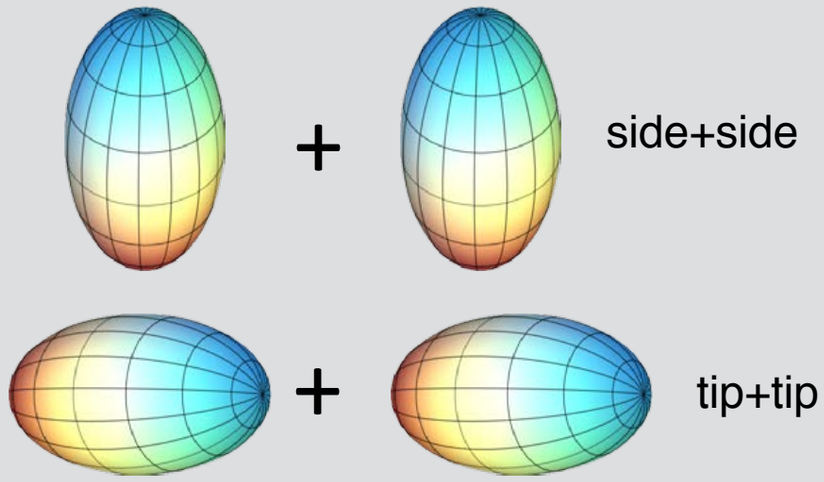
- attenuation p=0 tracks band for ε_2 and ε_3 compatible with LHC data



Eccentricities in ultracentral U+U collisions

exploit deformation of Uranium nuclei:

- larger maximal compression for tip+tip geometry
- improved control on path length dependence for jet energy-loss
- validation of models for particle production, e.g. disentangle N_{ch} dependence on N_{part} & N_{coll}



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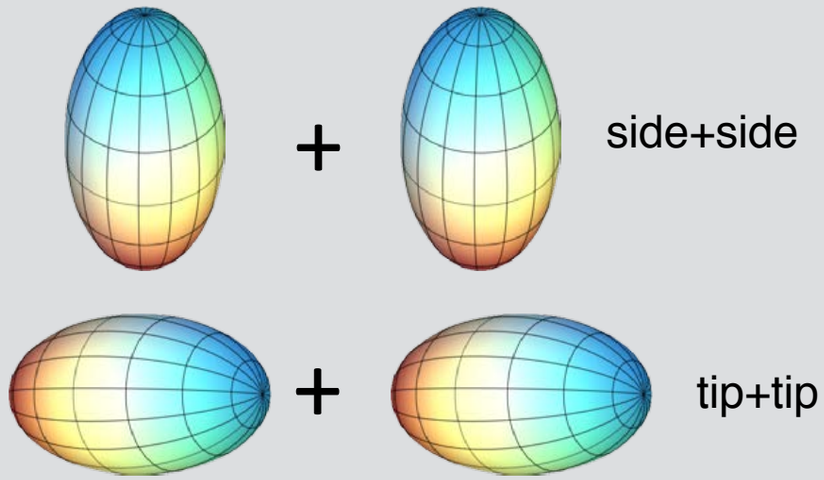
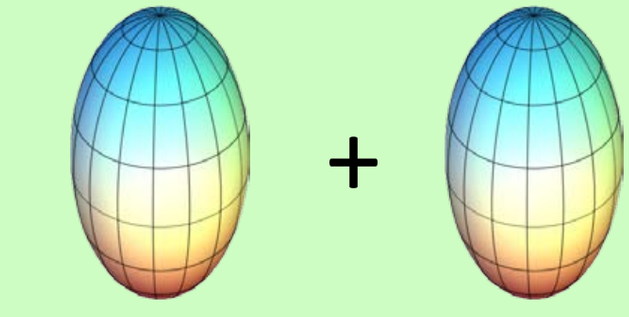
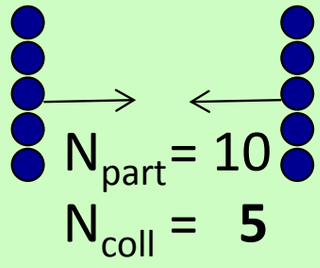


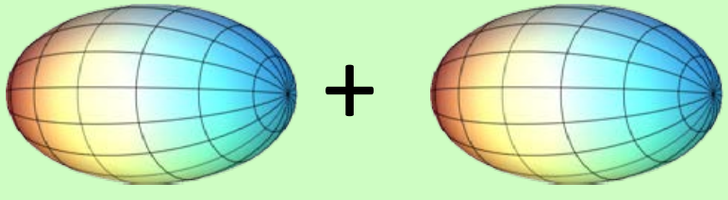
- two component model: $N_{ch} \propto \left((1 - x_{hard}) \frac{N_{part}}{2} + x_{hard} N_{coll} \right)$
- v_2 depends on initial eccentricity

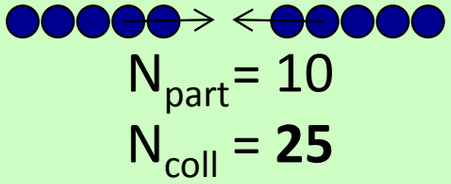
full overlap:





- large v_2
- small N_{ch}



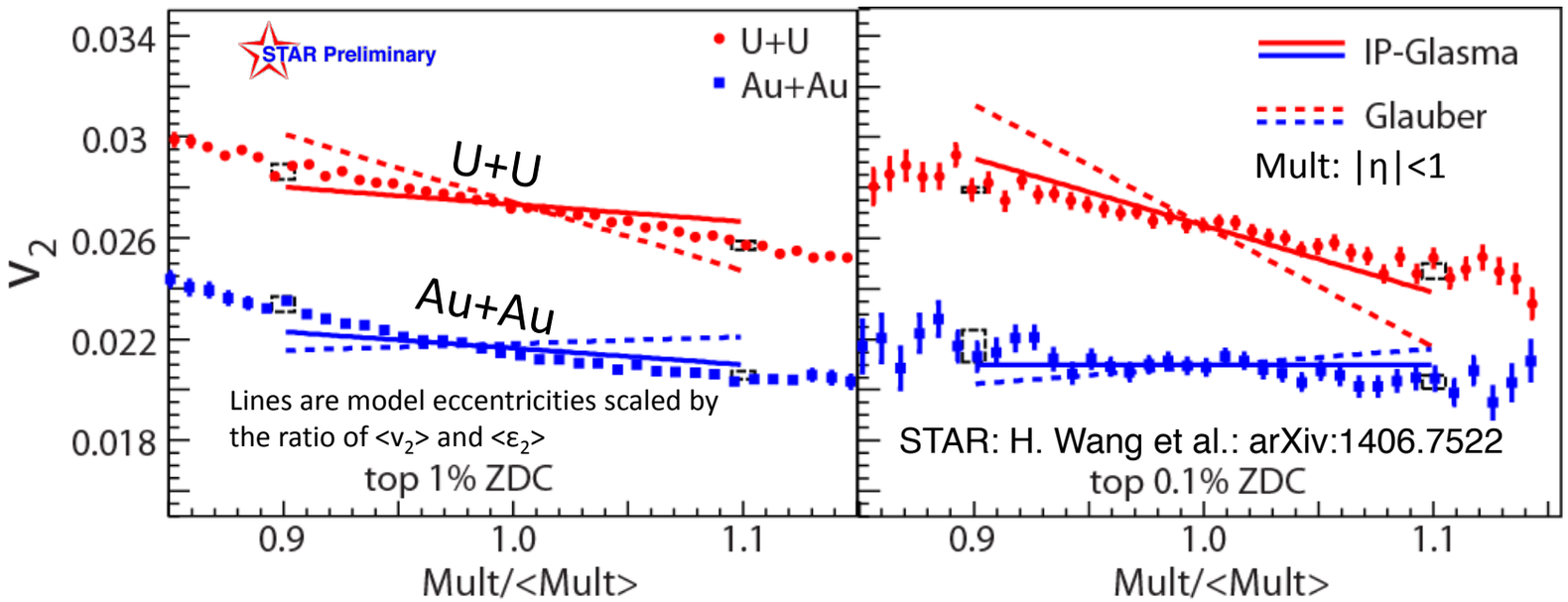


- small v_2
- large N_{ch}

- for highest multiplicity collisions, a large N_{ch} should correlate with a small v_2
- ▸ N_{ch} vs. v_2 can serve as discriminator between different initial state models

Central U+U: Data vs. Models

- STAR data shows expected N_{ch} vs. v_2 correlation in central U+U vs. Au+Au
- model validation:
  - Glauber exhibits slopes not compatible with data
  - IP Glasma shows good agreement to data



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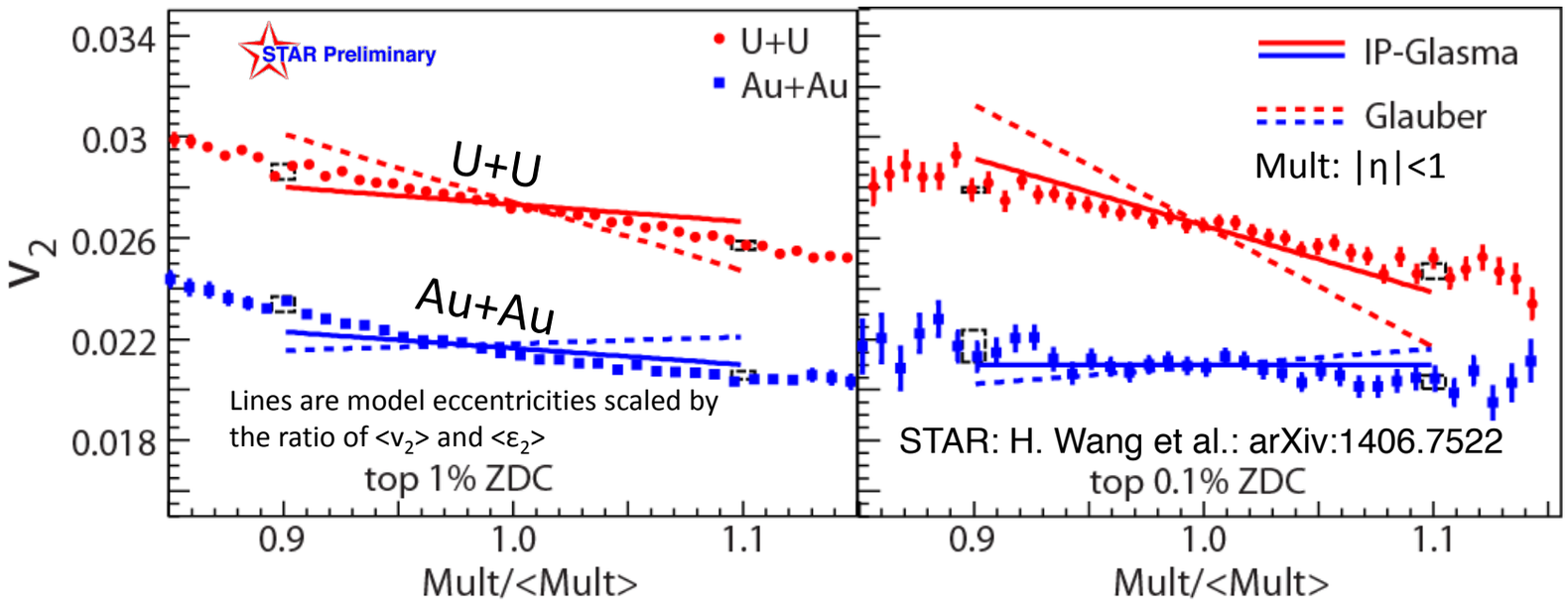
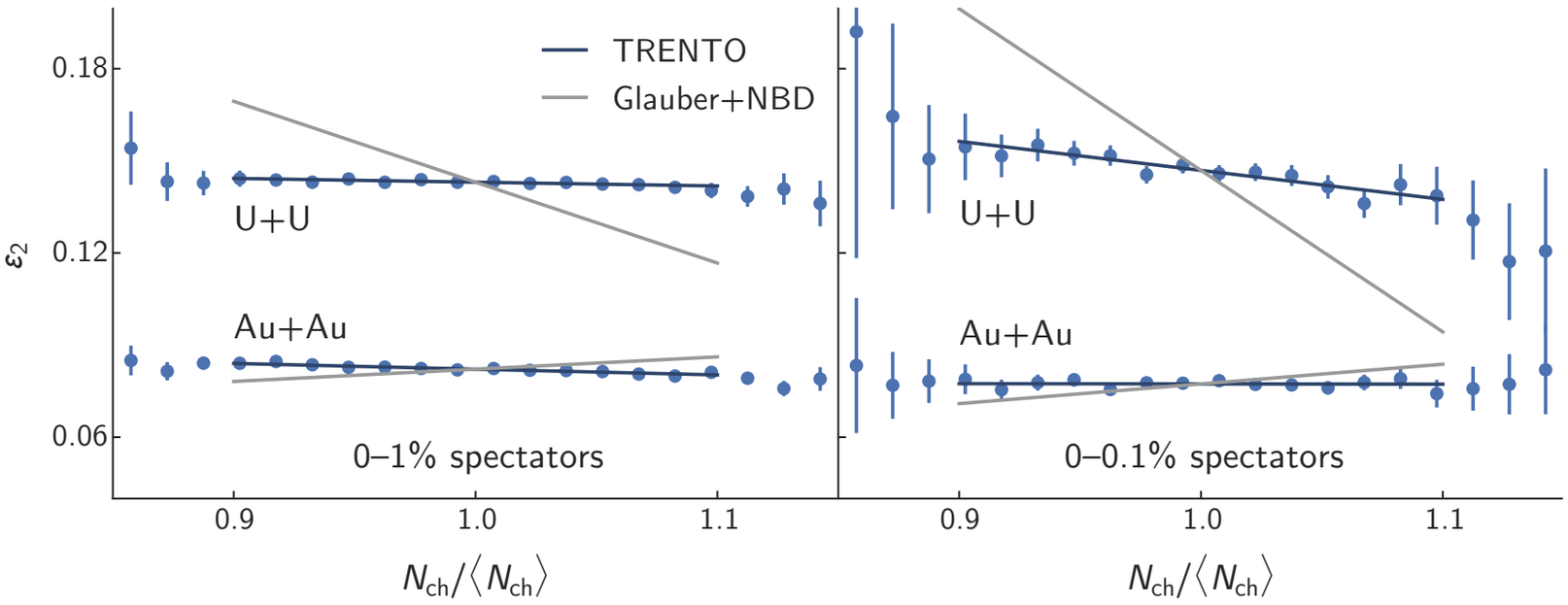


TRENTo:

- shows qualitative agreement with data
- scaling very different from Glauber approach
- features similar to those of the IP Glasma model



Summary & Next Steps

Technical Specifications & Usability

- programmed in C++; multithreaded
- output uses standard HDF5 binary format
- minimal memory requirements* (could run on your iPhone!)
- 300 - 400 Pb+Pb events per second per core

*HDF5 library buffers output: uses large amounts of memory but can optionally be disabled

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- available for download & use in early 2015 from the Github account of the Duke QCD group: https://github.com/Duke-QCD

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Summary & Outlook

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- a new model for the initial entropy deposition in ultra-relativistic nuclear collisions, applicable from p+p to p+A and A+A
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T_RENTo: **T**hickness-**R**educed **E**vent-by-event **N**uclear **To**pology

The End

Knowledge Extraction via Model-to-Data Analysis

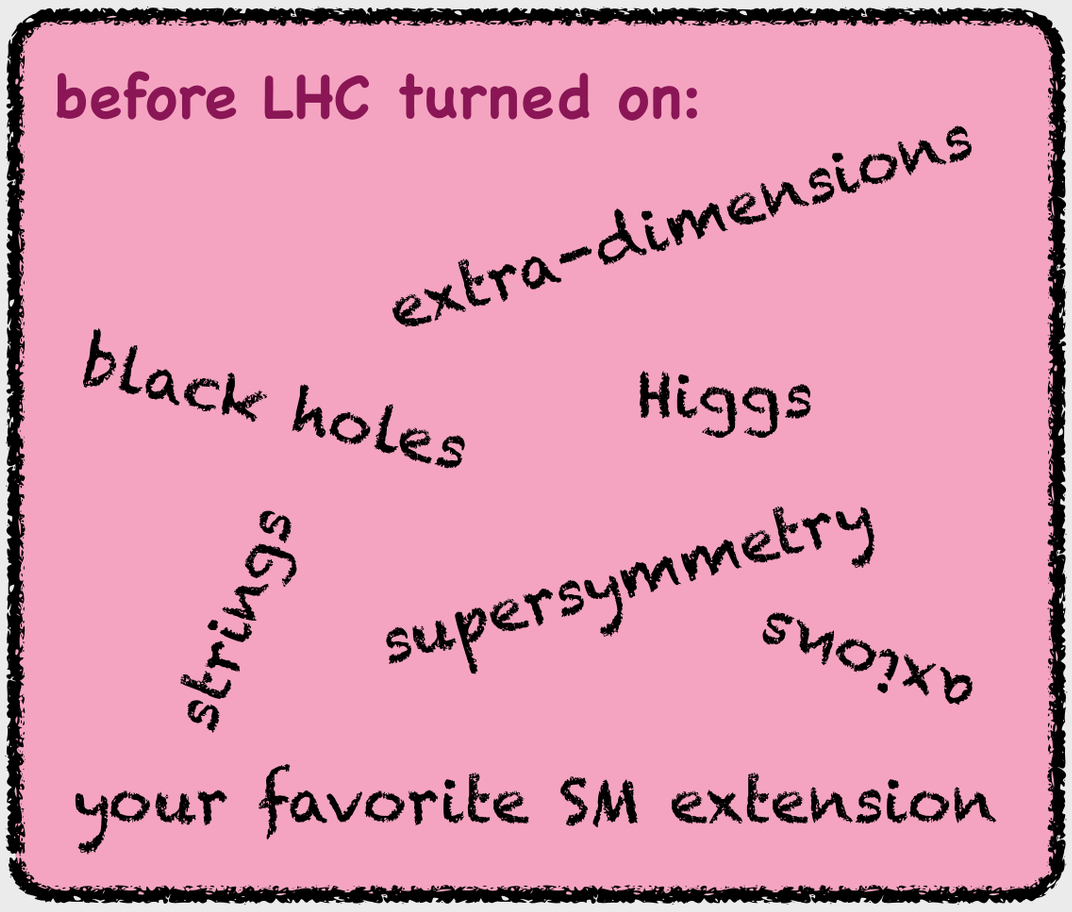
The Impact of Data on Theory...

The landscape of Particle Theory

The Impact of Data on Theory...

The landscape of Particle Theory

before LHC turned on:



The Impact of Data on Theory...

The landscape of Particle Theory

before LHC turned on:

extra-dimensions

black holes

Higgs

strings

supersymmetry

axions

your favorite SM extension

after LHC turned on:

Higgs

The Impact of Data on Theory...

The landscape of Particle Theory

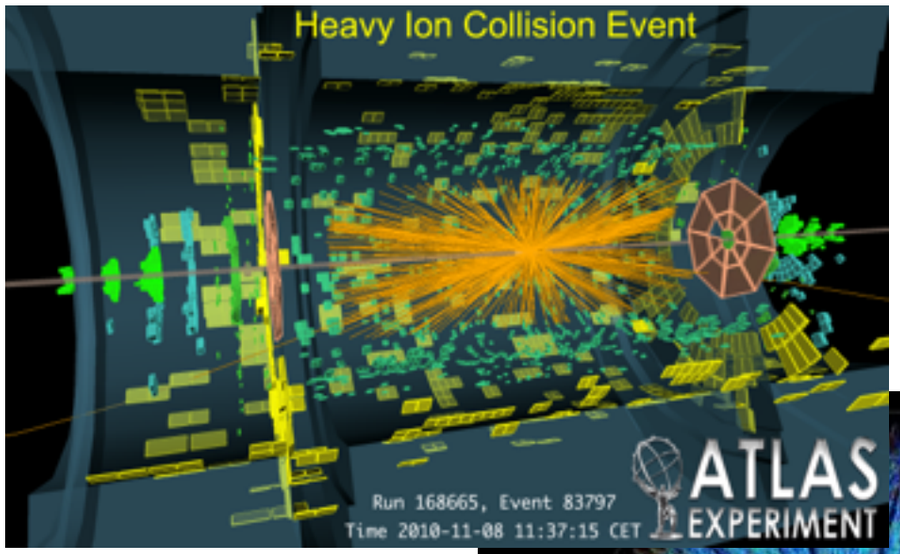
before LHC turned on:

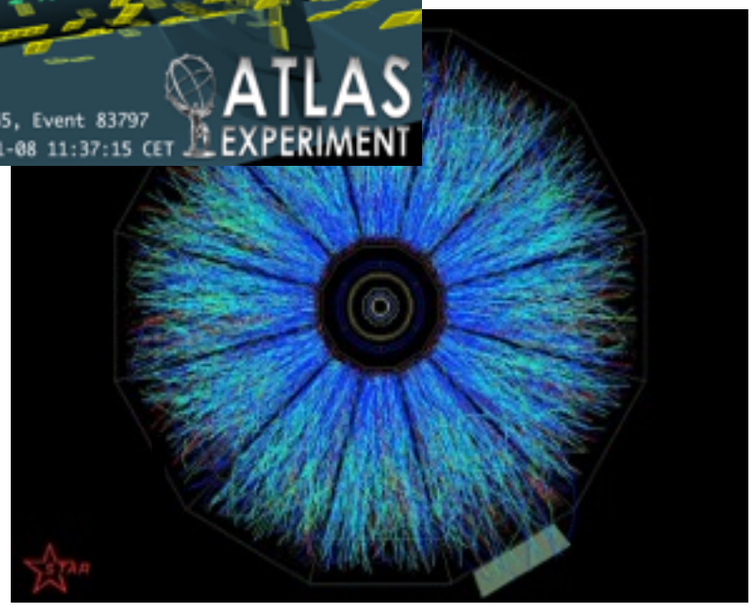
after LHC turned on:

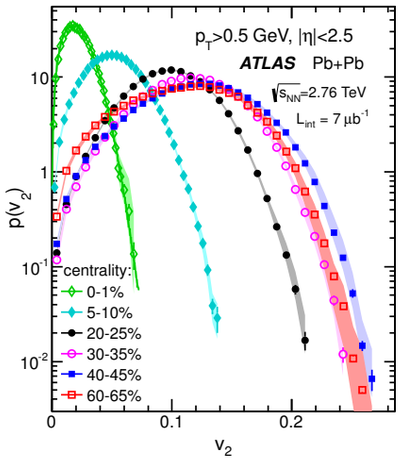
- LHC data has been extremely successful in falsifying theoretical models and/or constraining their parameter-space
- quantitative progress in high energy-density QCD will require a likewise effort
- **resilient heavy-ion models: need to perform a global quantitative analysis**

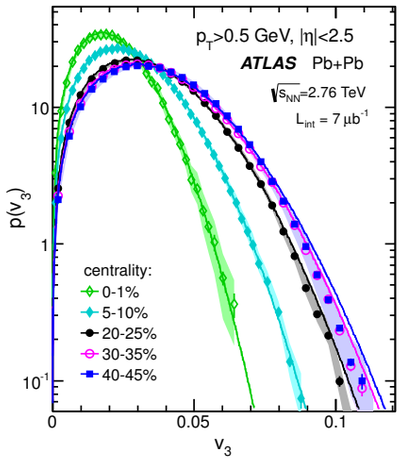
Probing QCD in Heavy-Ion Collisions

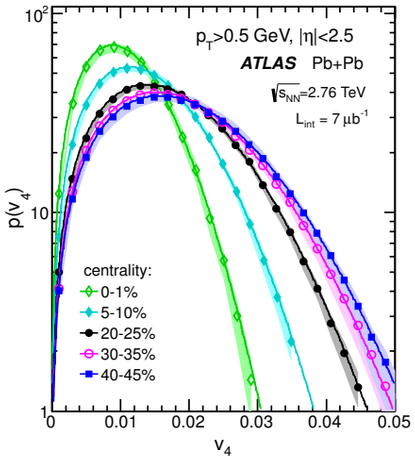
Data:





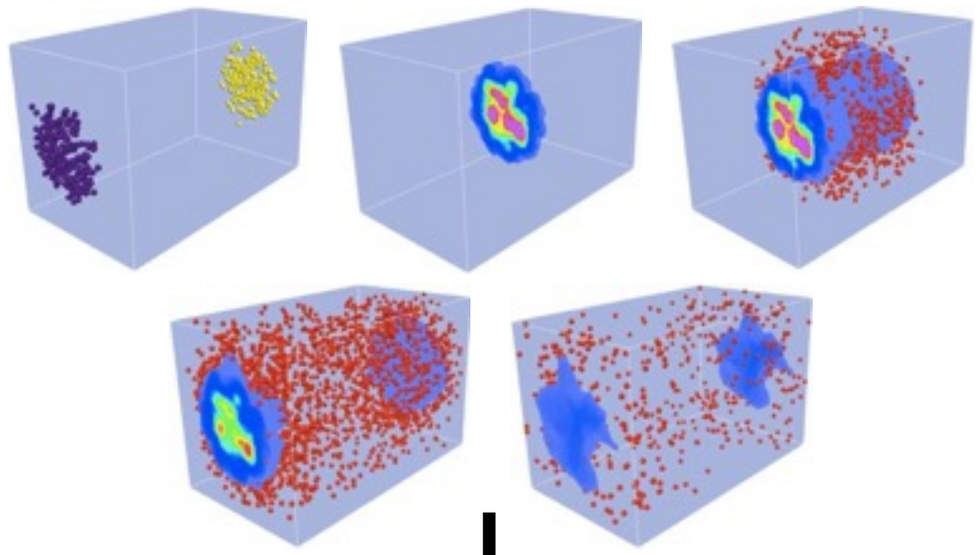


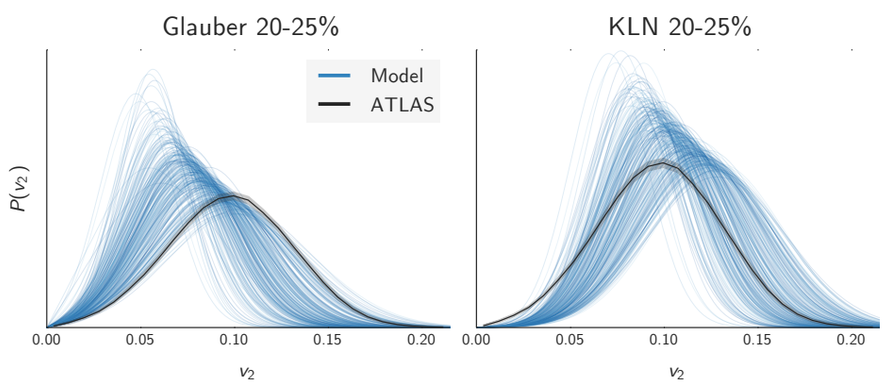




Model:

initial conditions, τ_0, η/s, ζ/s, ….





extracted QGP properties: η/s, …

Transport Models for RHIC & LHC

Transport Models for RHIC & LHC

(viscous) relativistic fluid dynamics:

- transport of macroscopic degrees of freedom
- based on conservation laws:

$$\partial_\mu T^{\mu\nu} = 0$$

$$\begin{aligned} T_{ik} &= \varepsilon u_i u_k + P\left(\delta_{ik} + u_i u_k\right) \\ &- \eta\left(\nabla_i u_k + \nabla_k u_i - \frac{2}{3}\delta_{ik}\nabla\cdot u\right) \\ &+ \varsigma\,\delta_{ik}\nabla\cdot u \end{aligned}$$

(plus an additional 9 eqns. for dissipative flows)

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diffusive transport models based on the **Langevin Equation**:

- transport of a system of microscopic particles in a thermal medium
- interactions contain a drag term related to the properties of the medium and a noise term representing random collisions

$$\vec{p}(t+\Delta t) = \vec{p}(t) - \frac{\kappa}{2T}\vec{v}\cdot\Delta t + \vec{\xi}(t)\Delta t$$

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- combine microscopic & macroscopic degrees of freedom
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Each transport model relies on roughly a dozen physics parameters to describe the time-evolution of the collision and its final state. These physics parameters act as a representation of the information we wish to extract from RHIC & LHC.

The Challenge of a rigorous Model to Data Comparison

Model Parameter:	**experimental data:**
Eq. of state	π/K/P spectra
Viscosity	yields vs. centrality & beam
Saturation	elliptic flow
Pre-equilibrium state	HBT
Hadronization dynamics	charge correlations & BFs
Quark chemistry	density correlations
Jet Quenching	

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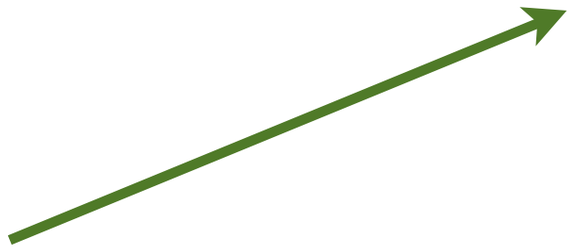

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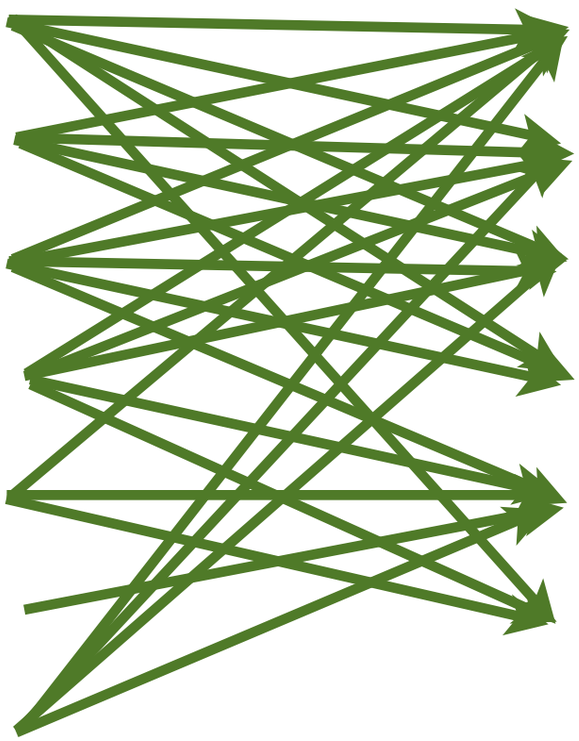

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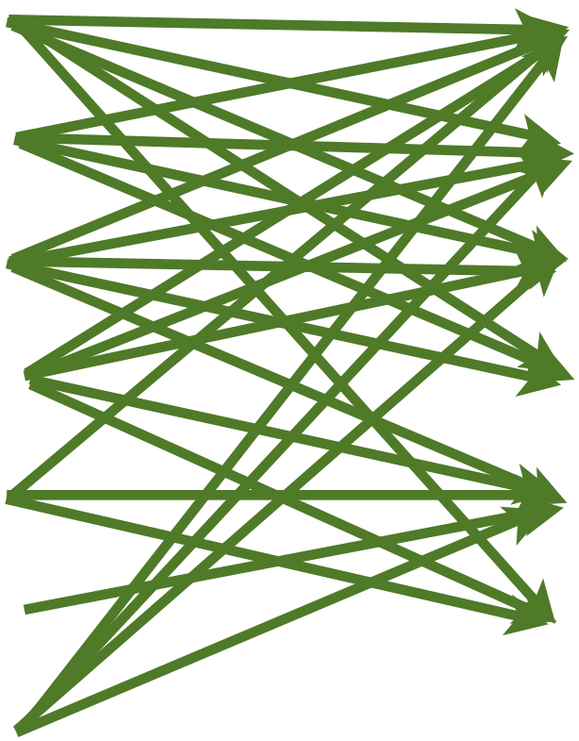

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- large number of interconnected parameters w/ non-factorizable data dependencies
- data have correlated uncertainties
- develop novel optimization techniques: Bayesian Statistics and MCMC methods
- transport models require too much CPU: need new techniques based on emulators
- general problem, not restricted to RHIC Physics

→collaboration with Statistical Sciences

Multi-Parameter Analysis

general strategy:

- collect data & quantify uncertainties
- construct model(s) & choose parameters to vary
- sweep parameter space & calculate observables
- utilize GP emulators for increased resolution
- compare model(s) to data
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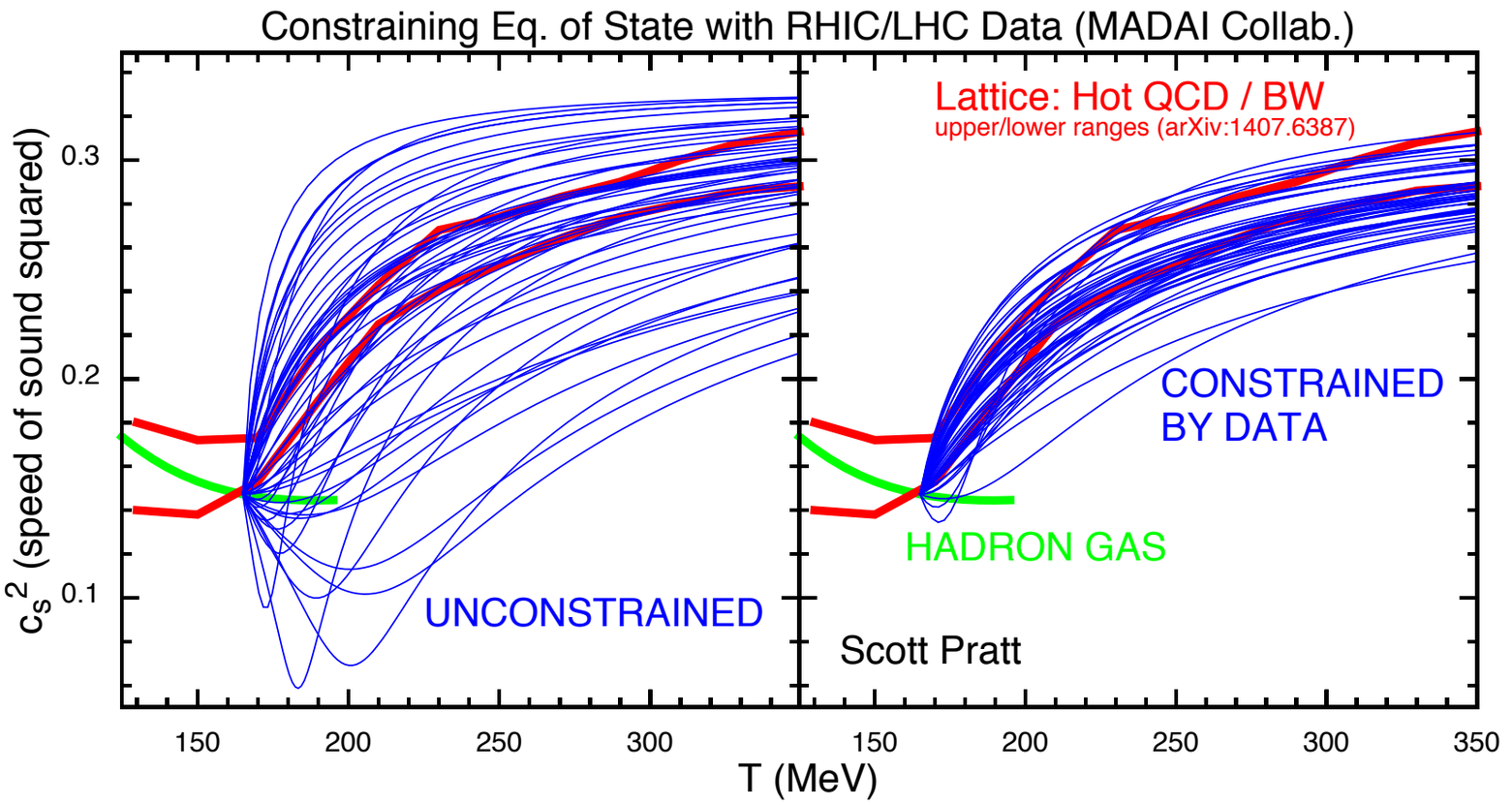
- create set of QCD Equations of State (aka the *prior*)
- run model with each EoS
- use comparison with RHIC/ LHC data to determine which Equations of State are consistent with data (i.e. the *posterior*)
- posterior is very similar to Lattice EoS!!