

Flow fluctuations versus Initial state fluctuations

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Outline

- Selected experimental results on flow fluctuations.
- Are flow fluctuations simply related to initial state fluctuations?
- Recent progress since IS2013

Anisotropic flow

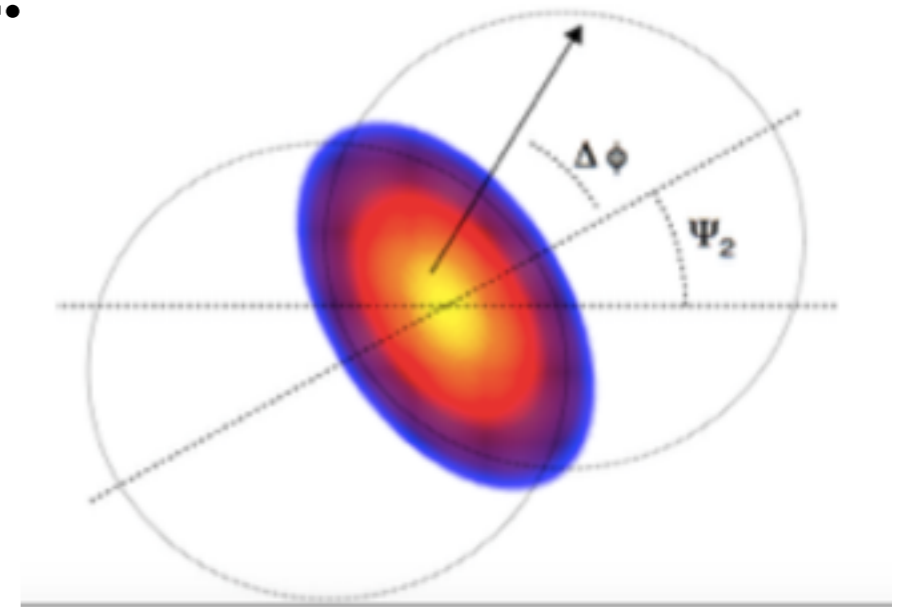
- Particles are emitted independently, with a *probability distribution* that is not isotropic in azimuthal angle

$$P(\phi) = \sum_n V_n \exp(-in\phi)$$

- $v_n \equiv |V_n| \equiv$ **anisotropic flow**
 $v_2 \equiv$ elliptic flow
 $v_3 \equiv$ triangular flow...

Flow fluctuations

- V_n fluctuates event to event.
- V_2 in nucleus-nucleus:
mean V_2 from geometry
+ fluctuations
- V_3 in nucleus-nucleus,
 V_2 in proton-nucleus:
just fluctuations



Alver et al nucl-ex/0610037

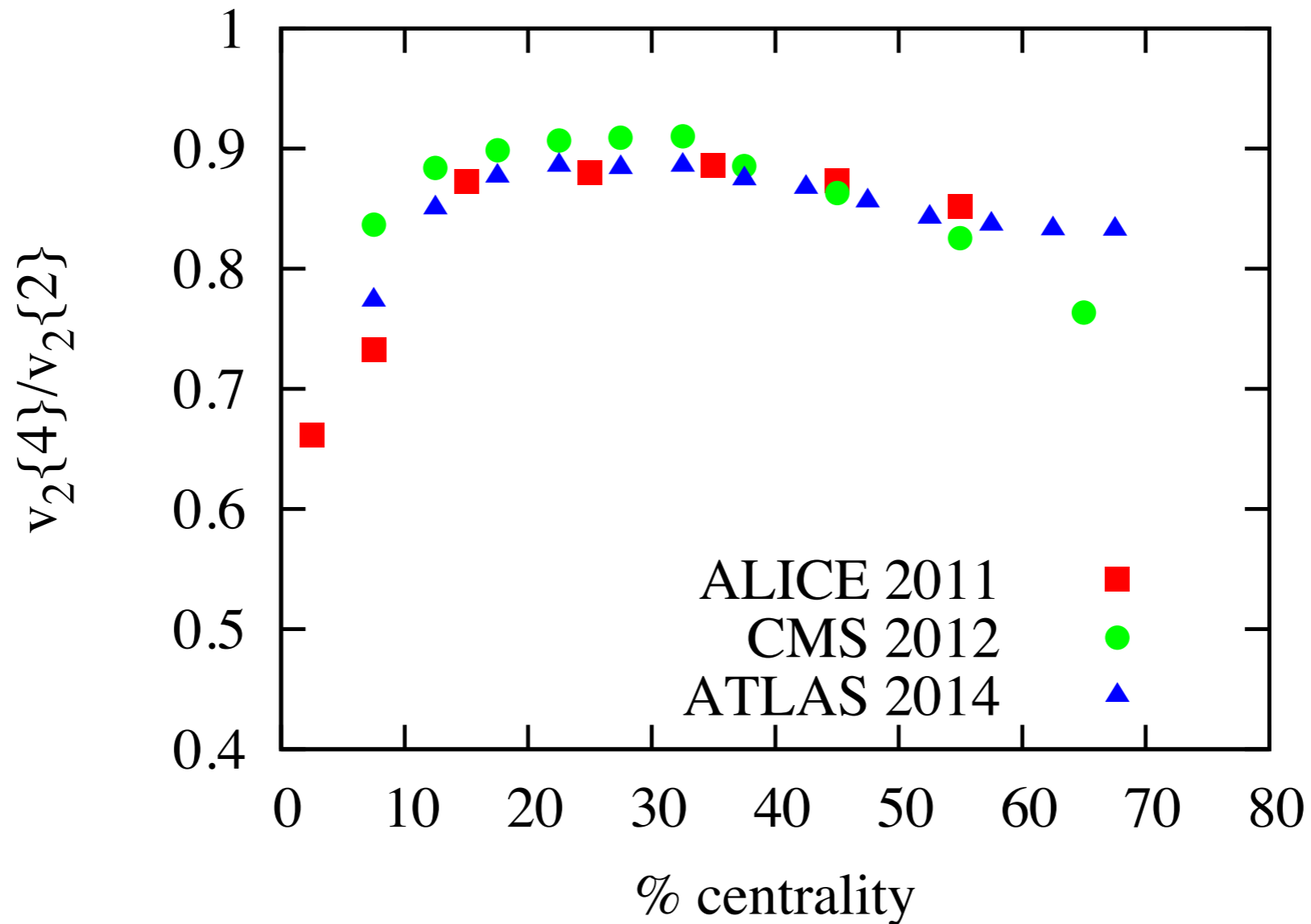
Alver Roland 1003.0194

Moments and cumulants

- $v\{2\} \equiv (\langle v^2 \rangle)^{1/2}$
- $v\{4\} \equiv (2\langle v^2 \rangle^2 - \langle v^4 \rangle)^{1/4}$
- $v\{6\} \equiv ((\langle v^6 \rangle - 9\langle v^4 \rangle \langle v^2 \rangle + 12\langle v^2 \rangle^3)/4)^{1/6}$
- If v does not fluctuate, $v\{2\} = v\{4\} = v\{6\} = v$
- In general $v\{4\} < v\{2\}$
- Event-plane method: $\langle v \rangle < v\{EP\} < v\{2\}$

Alver et al 0711.3724

v_2 fluctuations in Pb-Pb



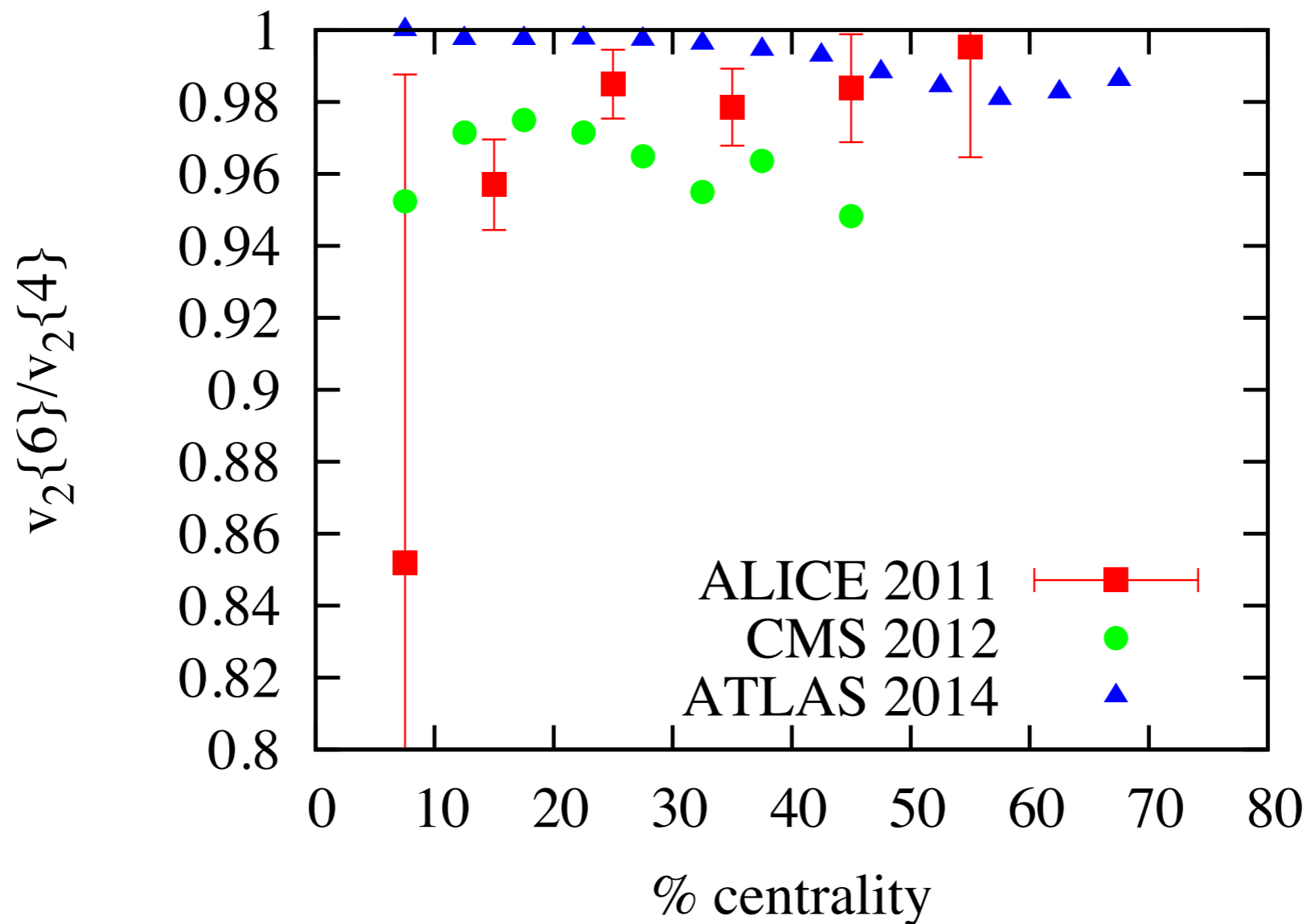
For CMS, I use $v_2\{EP\}$ as an approximation for $v_2\{2\}$:
probably explains the small discrepancy

Gaussian fluctuations?

Voloshin et al. 0708.0800

- Gaussian (aka Bessel-Gaussian) flow fluctuations:
- V_2 in nucleus-nucleus:
 $v_2\{4\}=v_2\{6\}=v_2\{LYZ\}=v_2$ in reaction plane
- V_3 in nucleus-nucleus: $v_3\{4\}=0$
 V_2 in proton-nucleus: $v_2\{4\}=0$

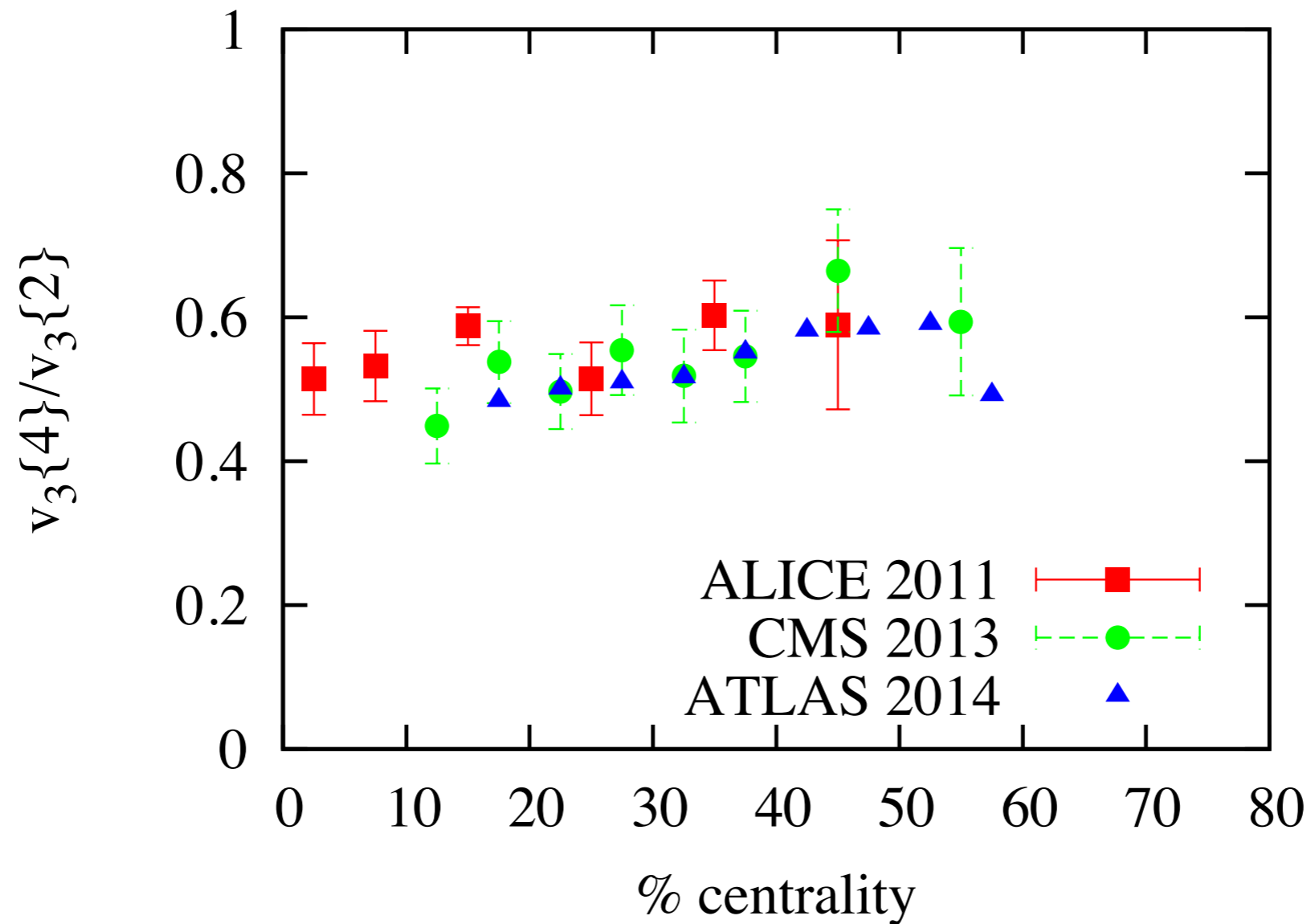
Are v_2 fluctuations Gaussian?



For ALICE and CMS, I assumed $v_2\{6\}=v_2\{LYZ\}$: Probably not good.

Small non-Gaussianities seen by ATLAS. Larger for smaller systems.

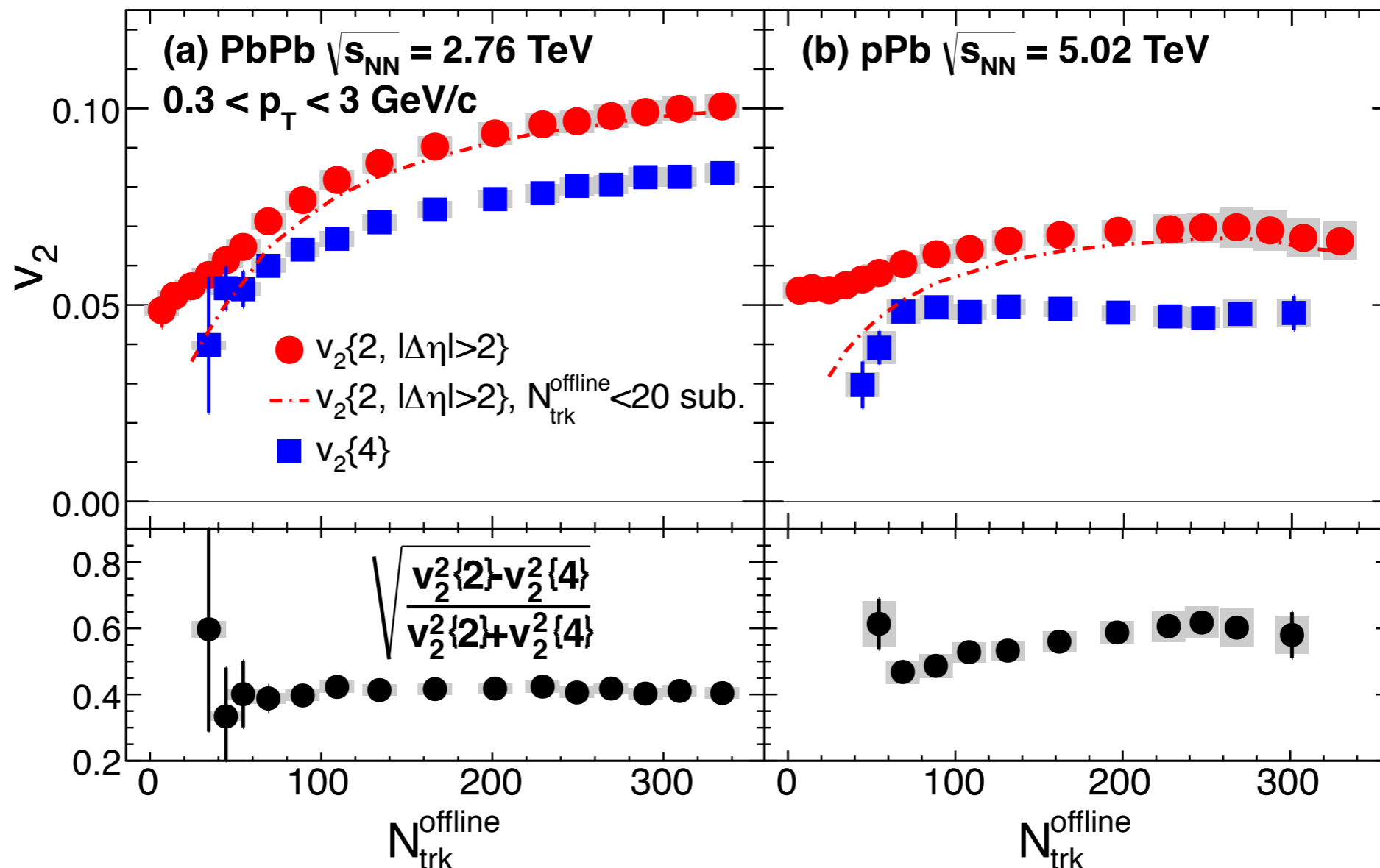
Are v_3 fluctuations Gaussian?



All 3 experiments see $v_3\{4\} \neq 0$:
non-Gaussian fluctuations clearly seen

v_2 fluctuations in p-Pb collisions

CMS 1305.0609



Large $v_2\{4\}$: large non-Gaussianities

From initial state fluctuations to flow fluctuations

- Take a Monte-Carlo model of initial conditions
- Evolve using relativistic hydrodynamics
- Compute particle distribution
- Average over events

Paatelainen et al

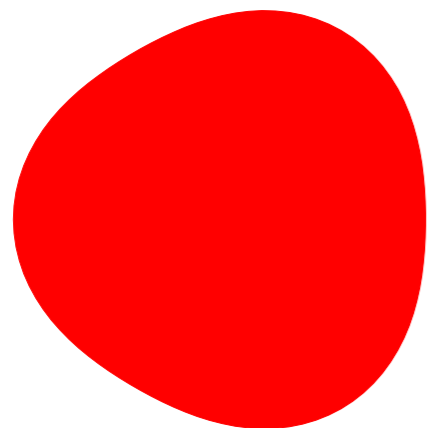
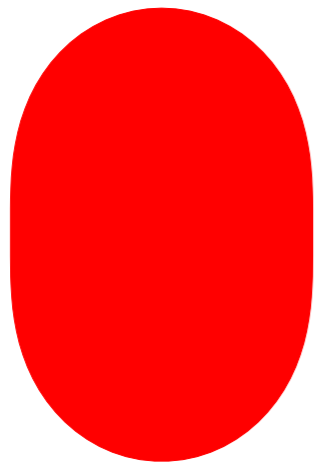
Schenke et al

Bozek et al

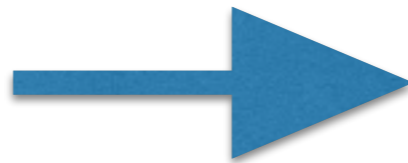
Werner et al

The origin of anisotropic flow

Initial transverse
density profile



Expansion



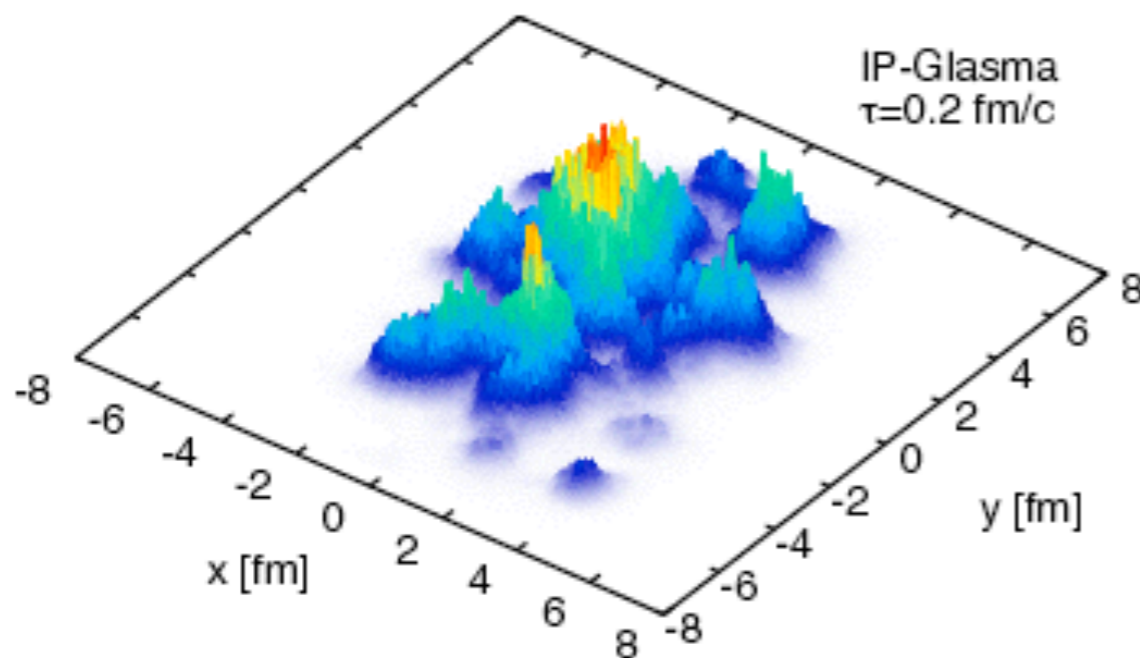
Final distribution

Elliptic flow v_2

Triangular flow v_3

Initial anisotropies

= Fourier decomposition of the initial density profile $\rho(x,y)$



$$\epsilon_n \equiv \frac{\left| \int r^n e^{in\phi} \rho(r,\phi) r dr d\phi \right|}{\int r^n \rho(r,\phi) r dr d\phi}$$

$\epsilon_2 \equiv$ initial eccentricity

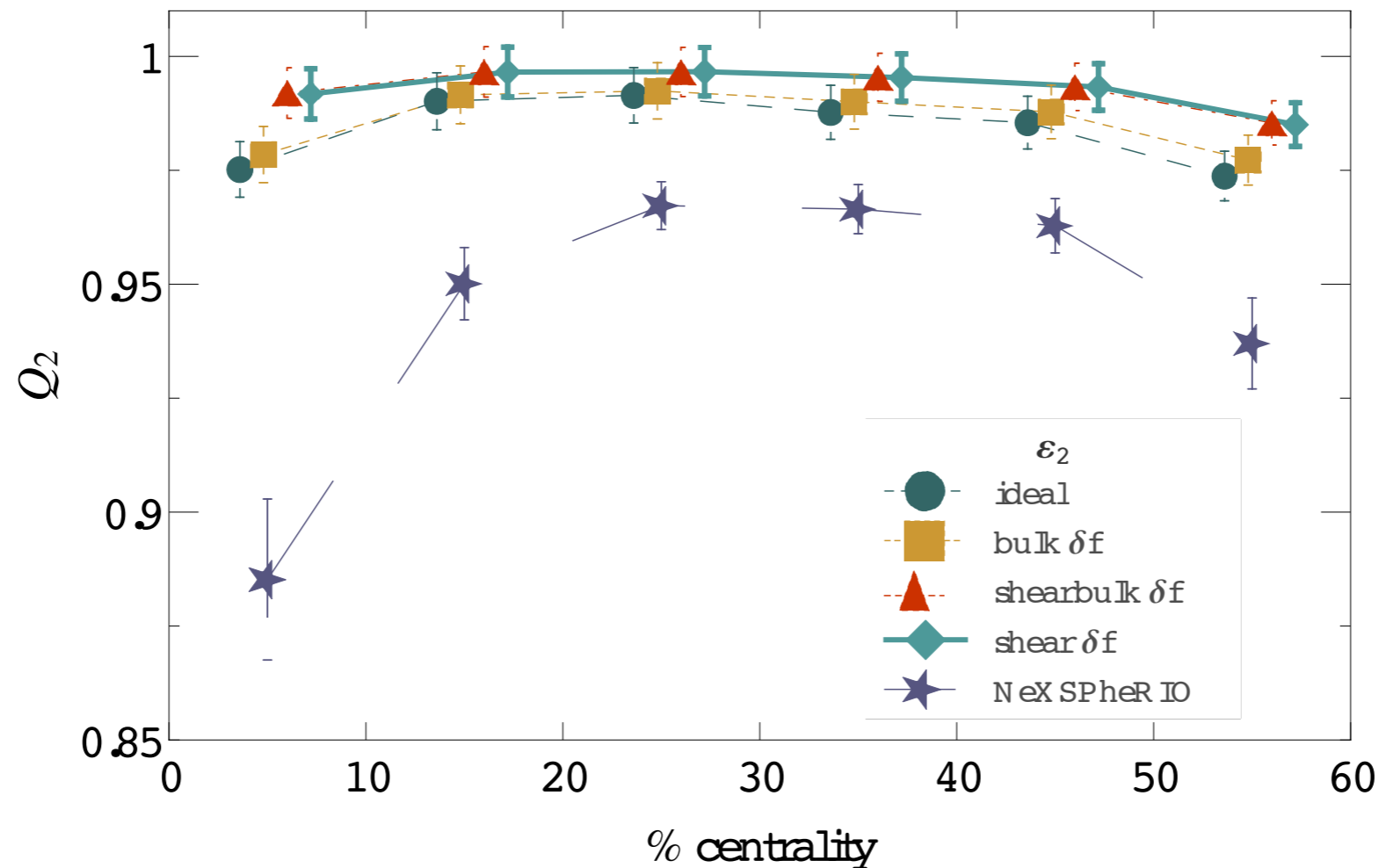
$\epsilon_3 \equiv$ initial triangularity

Schenke Tribedy Venugopalan 1202.6646

$|\epsilon_n| < 1$ by definition

v_2 is strongly correlated with ϵ_2

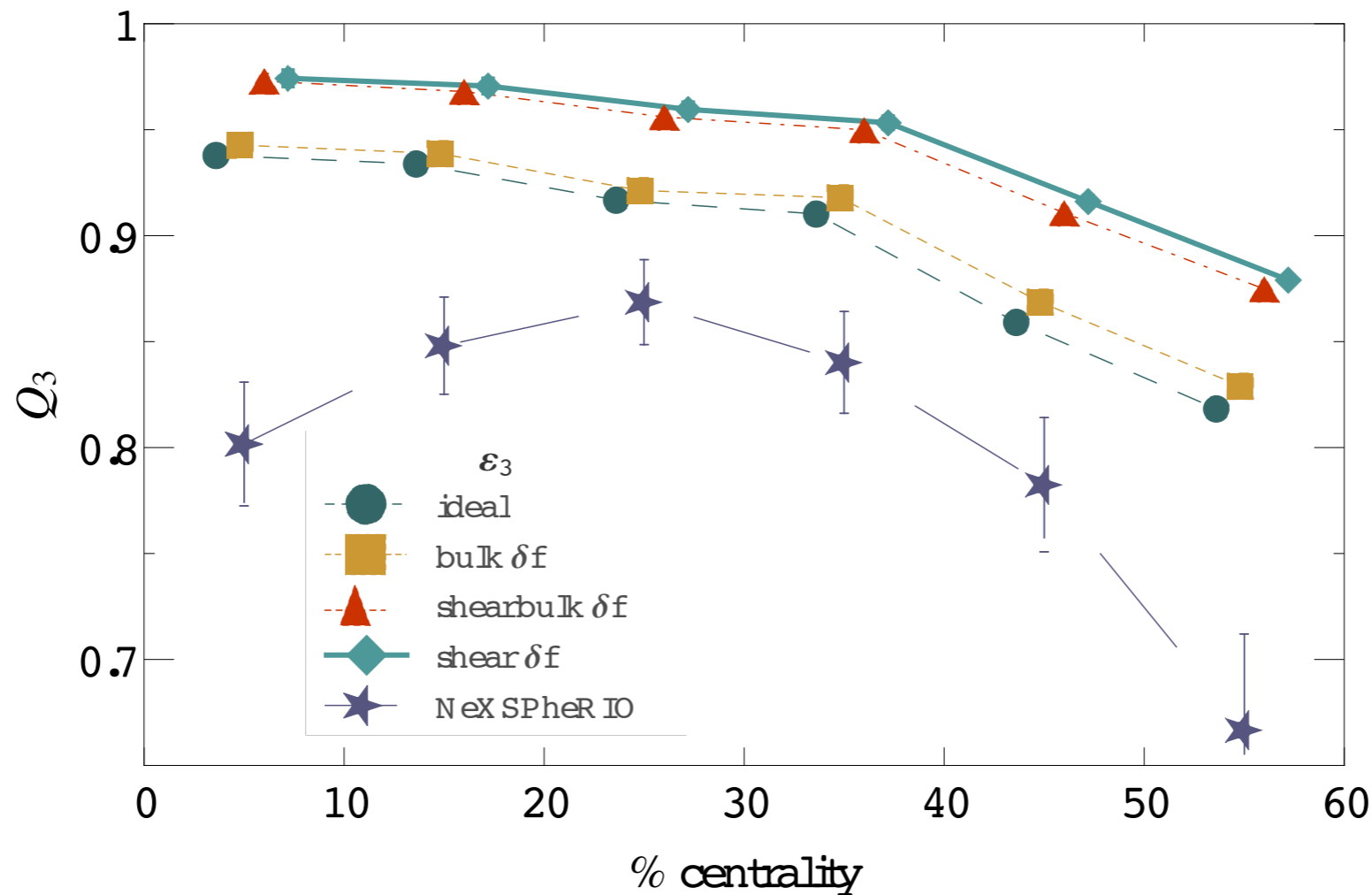
linear (Pearson) correlation coefficient
in event-by-event viscous hydro



Gardim Noronha-Hostler Luzum Grassi [1411.2574](#)

v_3 is strongly correlated with ε_3

linear (Pearson) correlation coefficient
in event-by-event viscous hydro



Gardim Noronha-Hostler Luzum Grassi 1411.2574

Flow as linear response

$$v_2 \approx K_2 \epsilon_2$$

$$v_3 \approx K_3 \epsilon_3$$

response coefficients

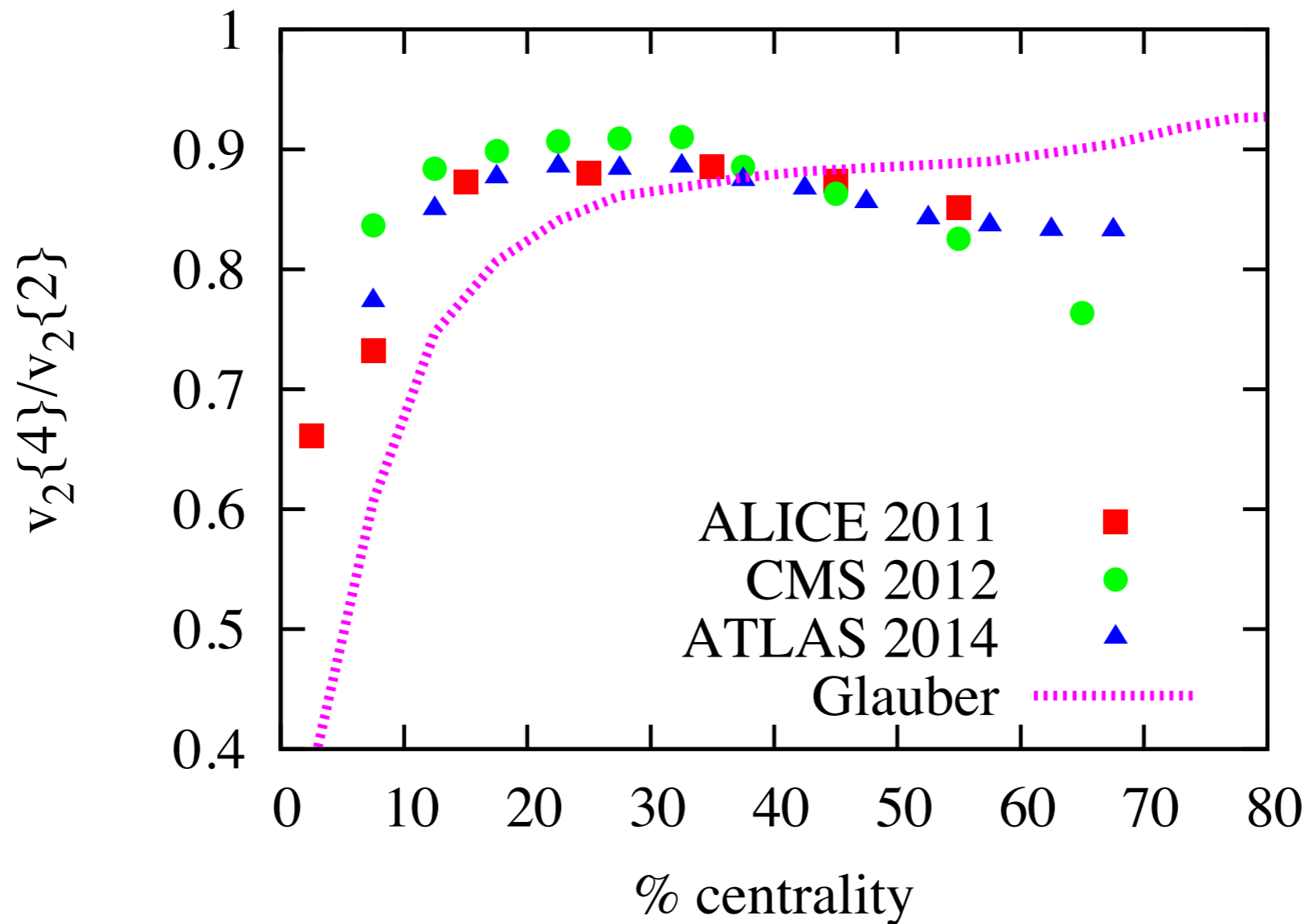
fluctuate event to event.

*depend on system and centrality, not
on details of initial conditions*

v_n fluctuations are due to ϵ_n fluctuations

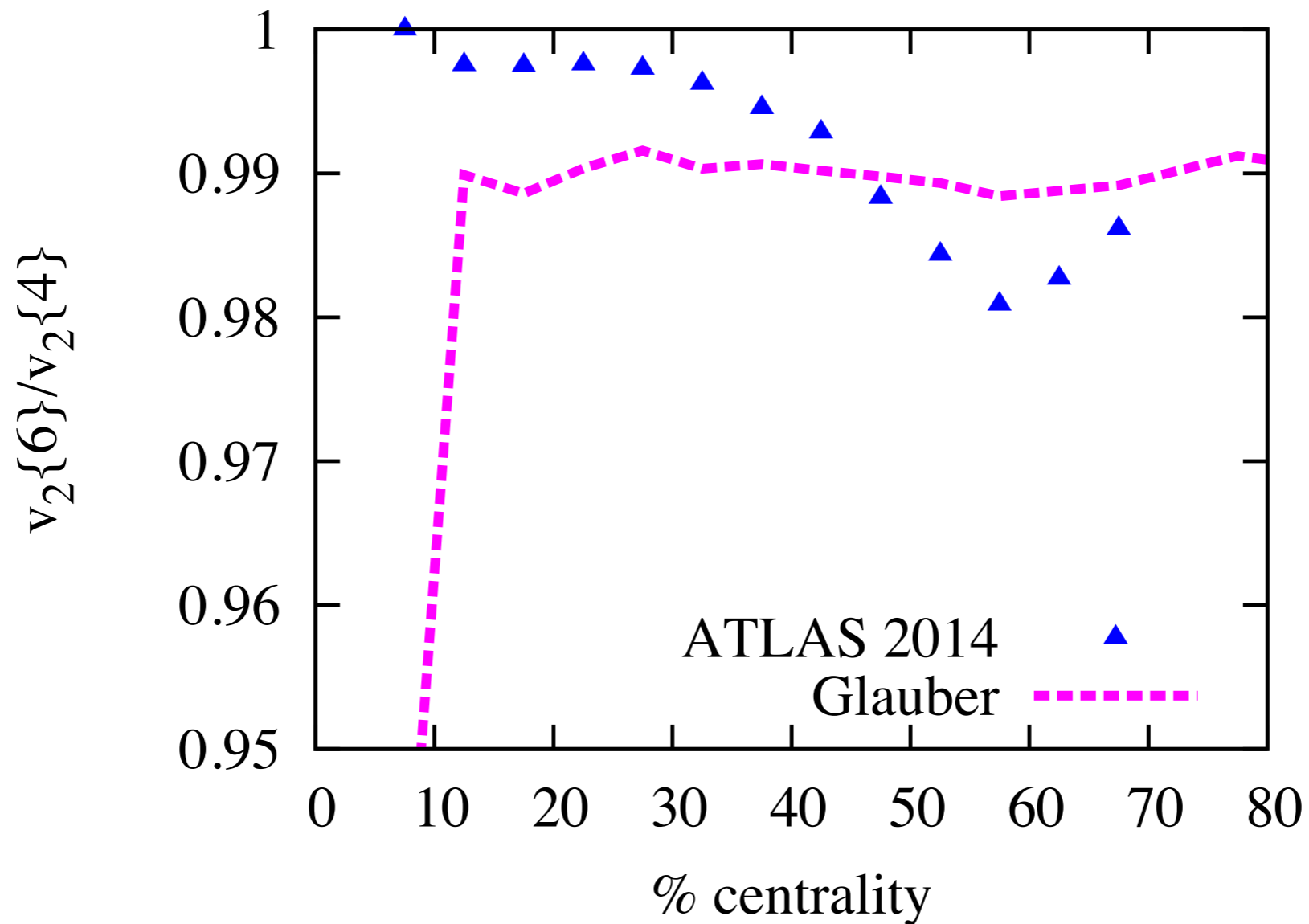
$$v_n\{4\}/v_n\{2\} = \epsilon_n\{4\}/\epsilon_n\{2\}$$

v_2 fluctuations vs ϵ_2 fluctuations



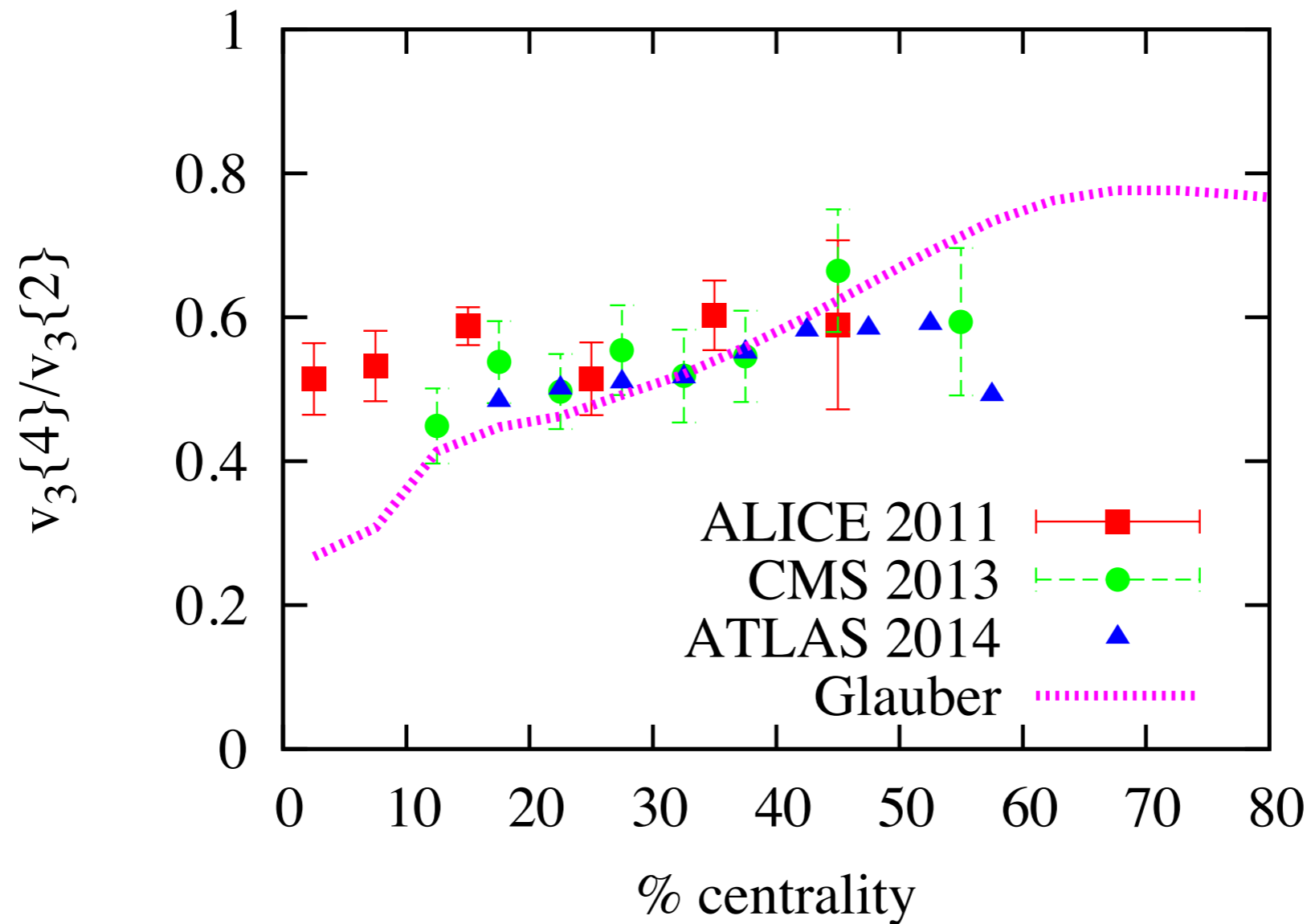
ϵ_2 fluctuations from Glauber: in the ballpark

v_2 fluctuations vs ε_2 fluctuations



Non-Gaussianities from Glauber: in the ballpark

v_3 fluctuations vs ϵ_3 fluctuations



Non-Gaussianities from Glauber: in the ballpark

A discussion at IS2013

- Non-Gaussian flow fluctuations are seen both in Pb-Pb (v_3) and in pPb (v_2)
- Similar non-Gaussianities are seen in initial state models
- Do we understand their origin?

Alver et al. [0711.3724](#)

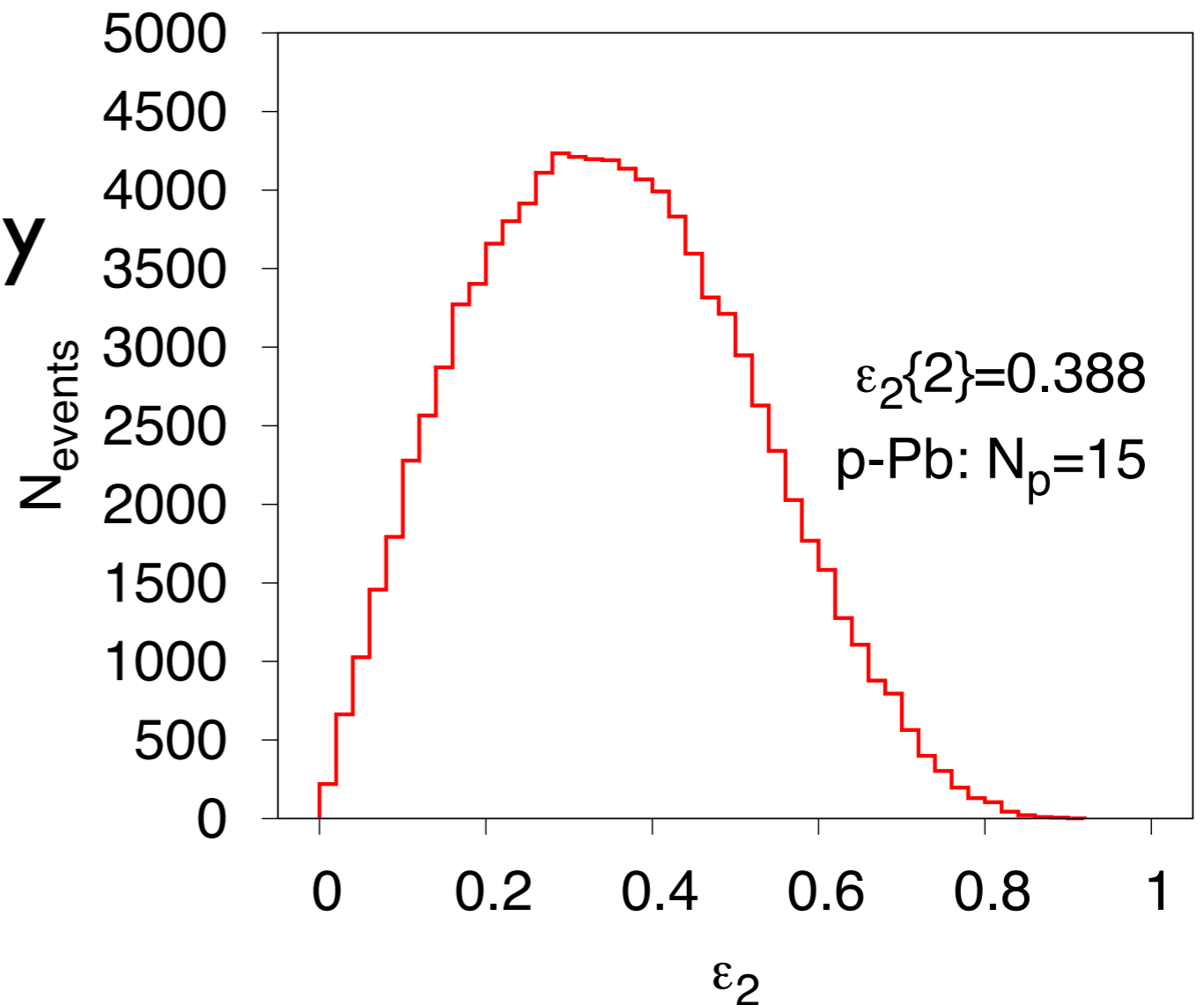
Bhalerao Luzum JYO [1107.5485](#)

Distribution of initial anisotropy

Central p+Pb collision:
 ϵ_2 from fluctuations only

small system:
large fluctuations &
anisotropies

Monte-Carlo Glauber simulation



Is there a simple law that describes this distribution?

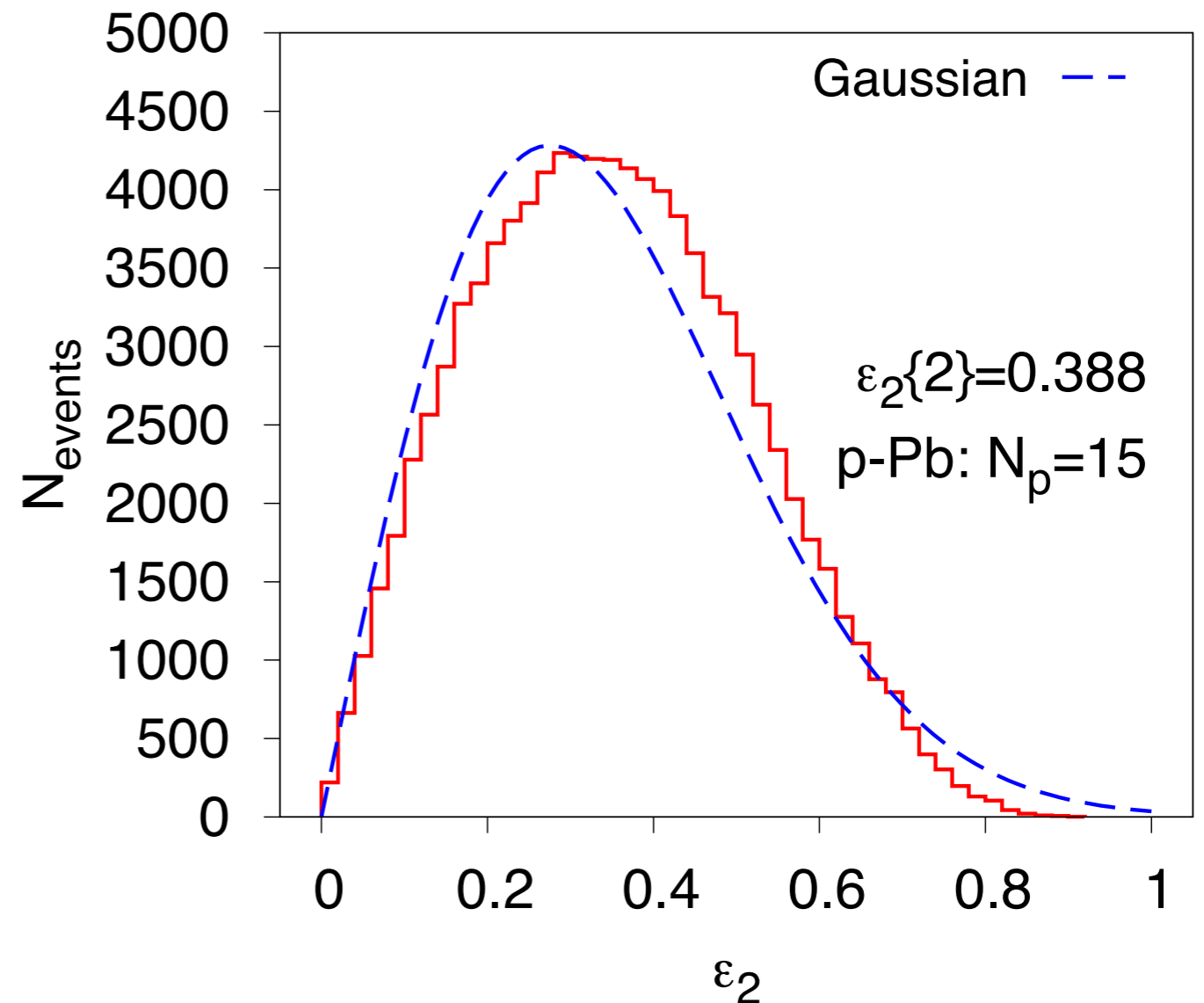
Gaussian?

Central limit theorem

$$P(\varepsilon_2) = 2(\varepsilon_2/\sigma^2) \exp(-\varepsilon_2^2/\sigma^2)$$

Not a good fit.

Does not implement
the condition $\varepsilon_2 < 1$



New “Power” distribution

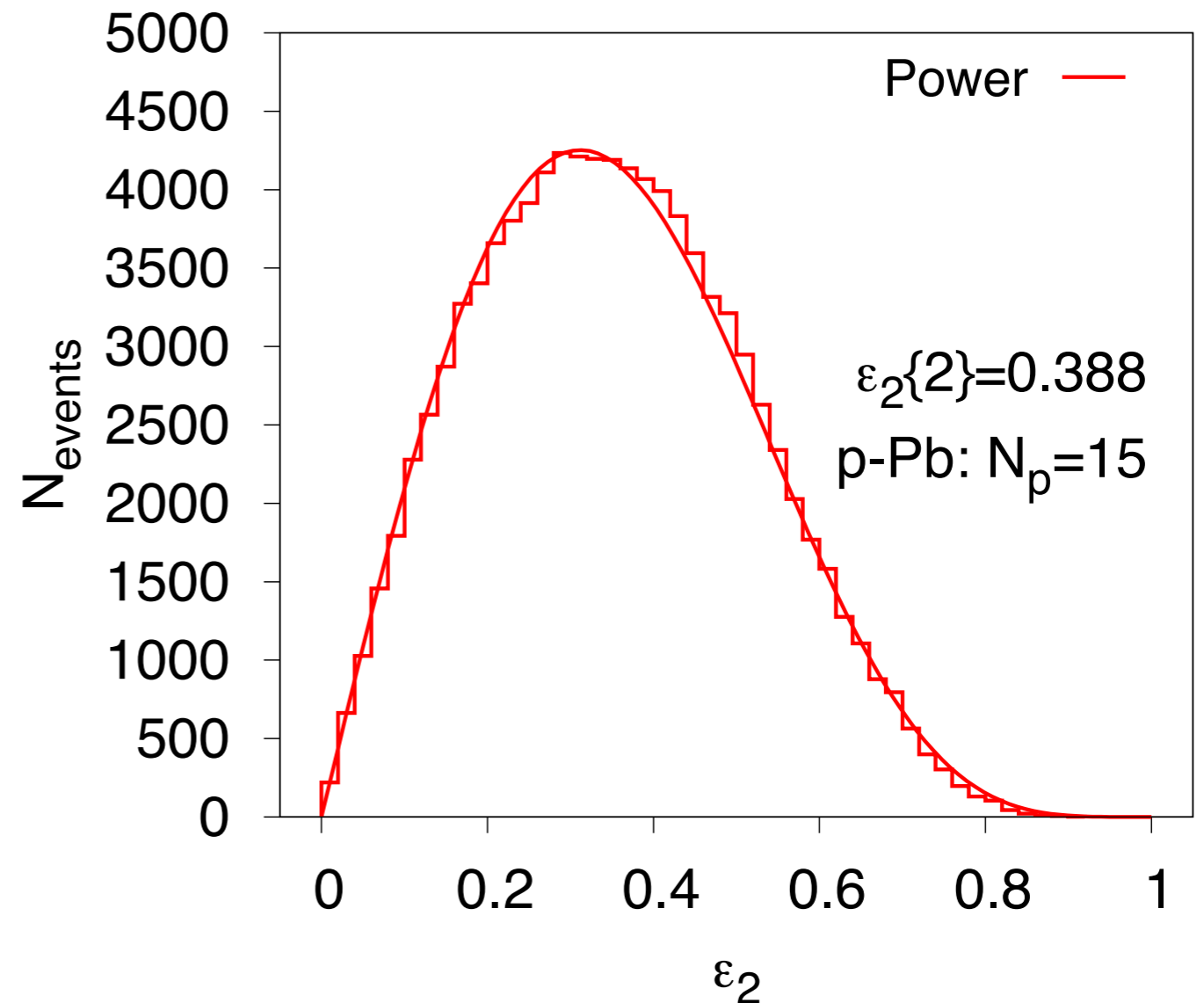
Li Yan, JYO, PRL 112 (2014) 082301

$$P(\varepsilon_2) = 2\alpha\varepsilon_2(1-\varepsilon_2^2)^{\alpha-1}$$

Equivalent to Gaussian
for $\alpha \gg 1$

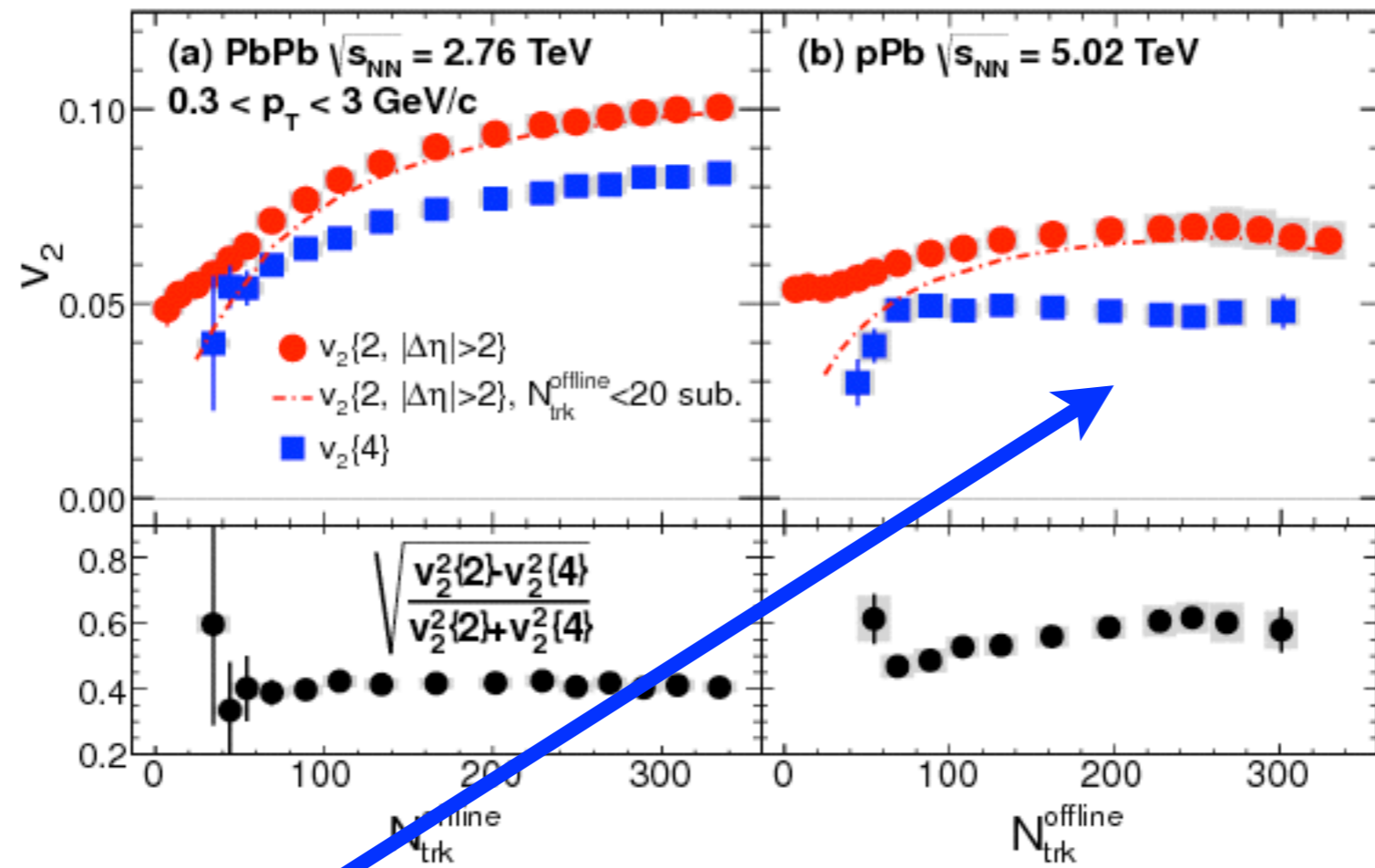
Naturally implements
the condition $\varepsilon_2 < 1$.

Predicts $\varepsilon_2\{4\} > 0$



Much better fit to Monte-Carlo results!

Natural explanation for $v_2\{4\}$ in pPb

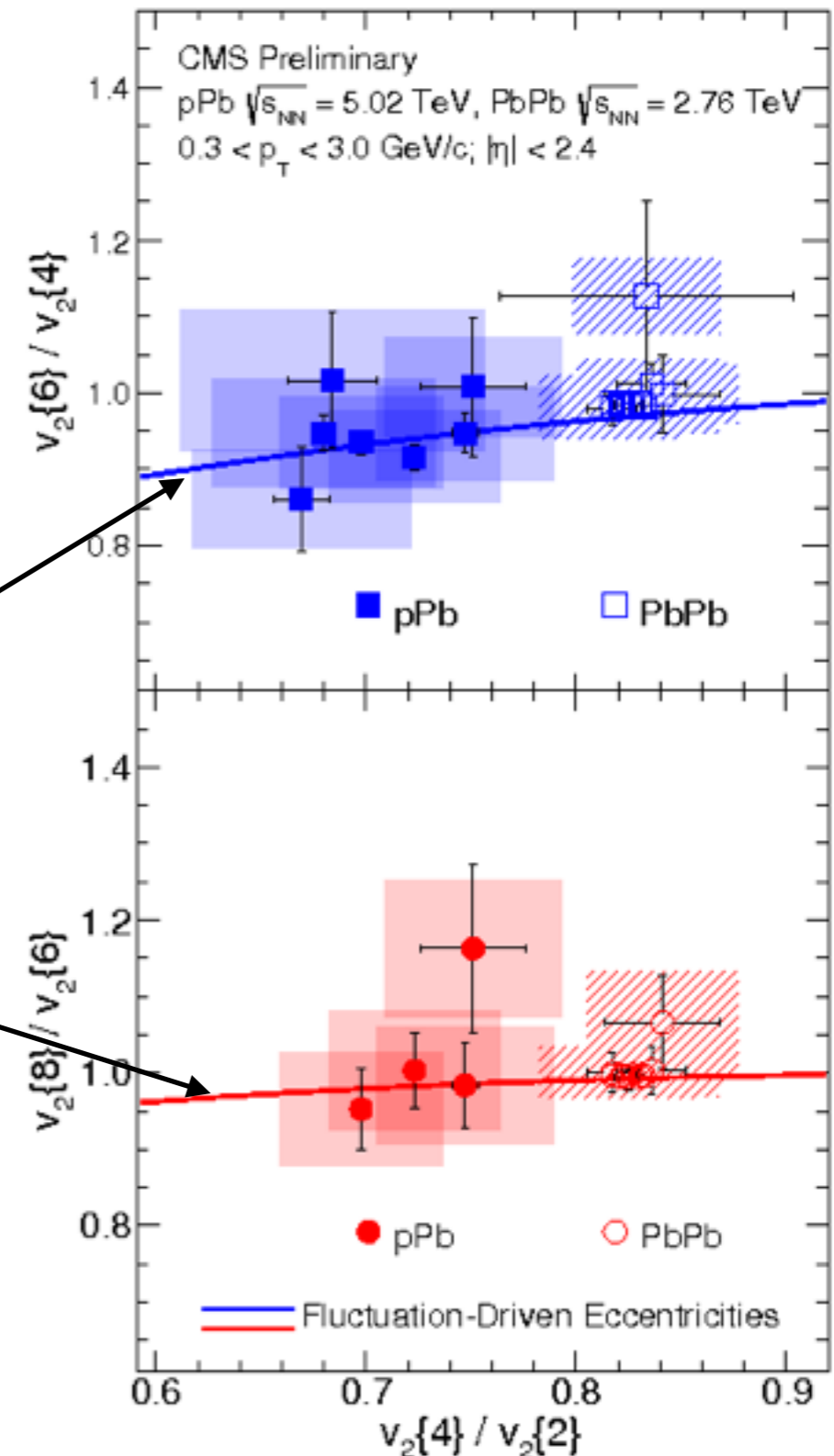


CMS 1305.0609

Small system: large fluctuations: large $v_2\{4\}/v_2\{2\}$

Predictions: higher-order cumulants

- Using as input the experimentally measured ratio $v_2\{4\}/v_2\{2\}$
- Quantitative prediction for higher-order cumulants $v_2\{6\}$ and $v_2\{8\}$
- New CMS data (QM2014) in good agreement with our prediction



Conclusions

- Direct evidence from experimental data that anisotropic flow in p-Pb and Pb-Pb collisions is driven by **large anisotropies** in the initial state: the statistics of ϵ_n *hits the boundary* $\epsilon_n < 1$
- The statistics of large fluctuations is not described by the central limit theorem but nevertheless **universal** to a good approximation.
- Flow fluctuations reflect to a large extent fluctuations in initial anisotropies. Corrections to this picture?

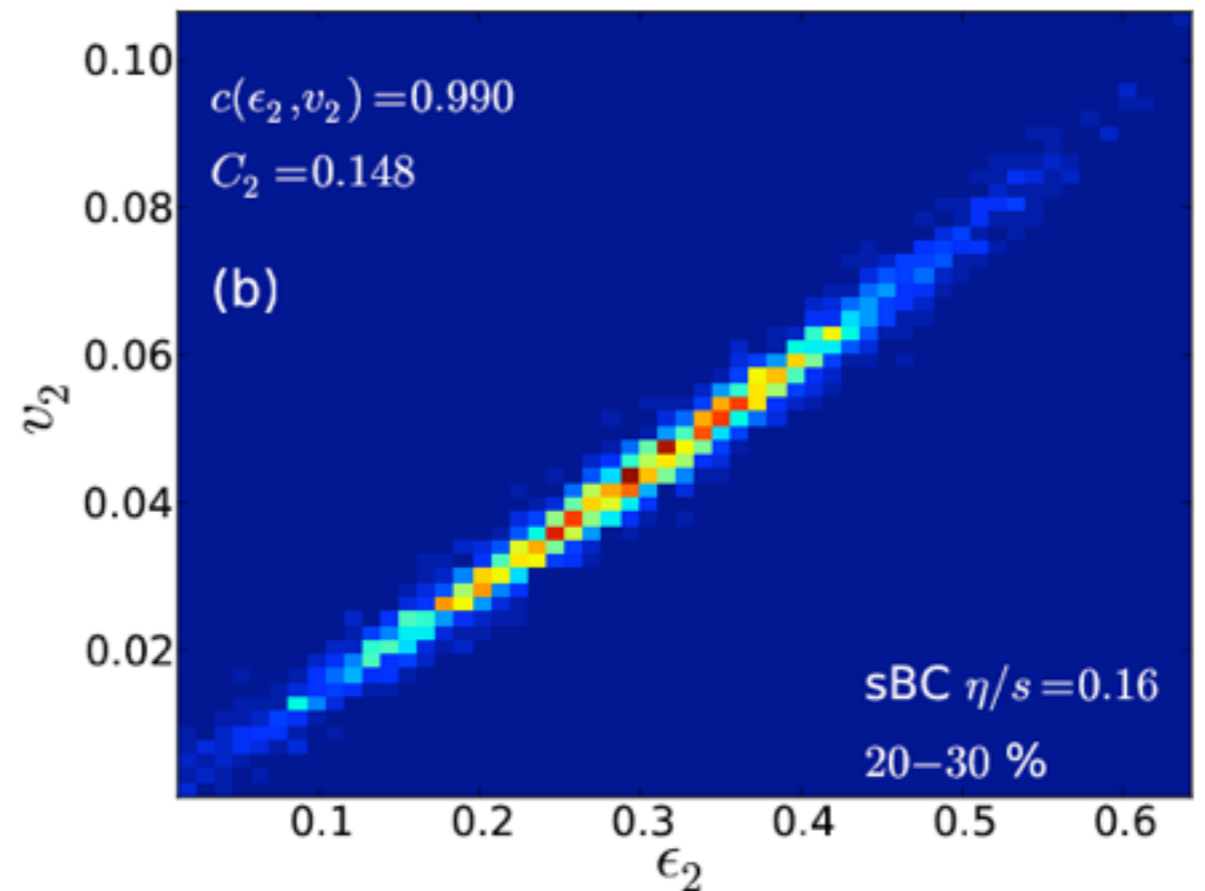
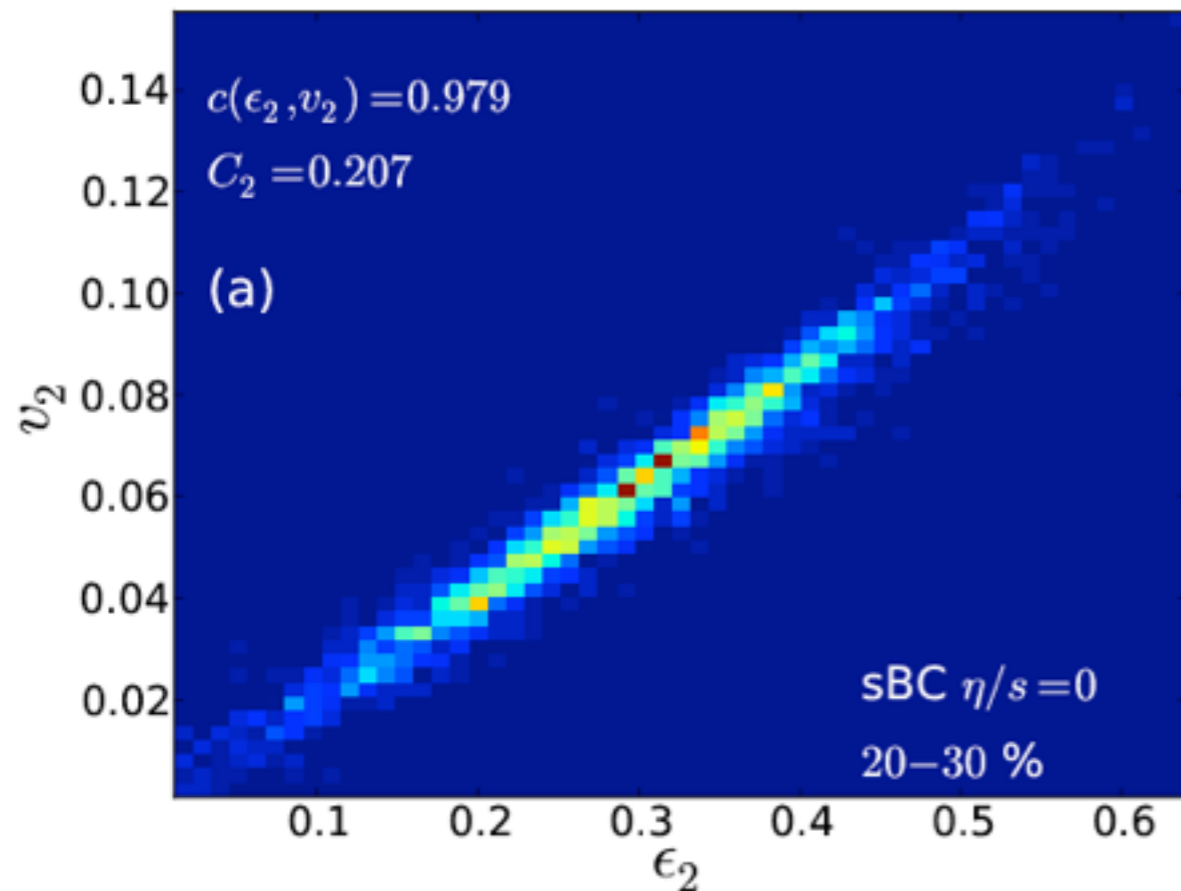
More in the next talk by Art Poskanzer

Perspectives

- Experiments: explore flow fluctuations through the double-differential structure of pair correlations *Talks by Wei Li, Rajeev Bhalerao*
- Hydro: do we understand the response to initial fluctuations beyond simple eccentricity scaling? *More in the next talk by Art Poskanzer*
- Initial state: understand on general grounds the initial anisotropies and their statistical properties. *Talk by Jean-Paul Blaizot*

Backup

Elliptic flow v_2 versus initial eccentricity ϵ_2

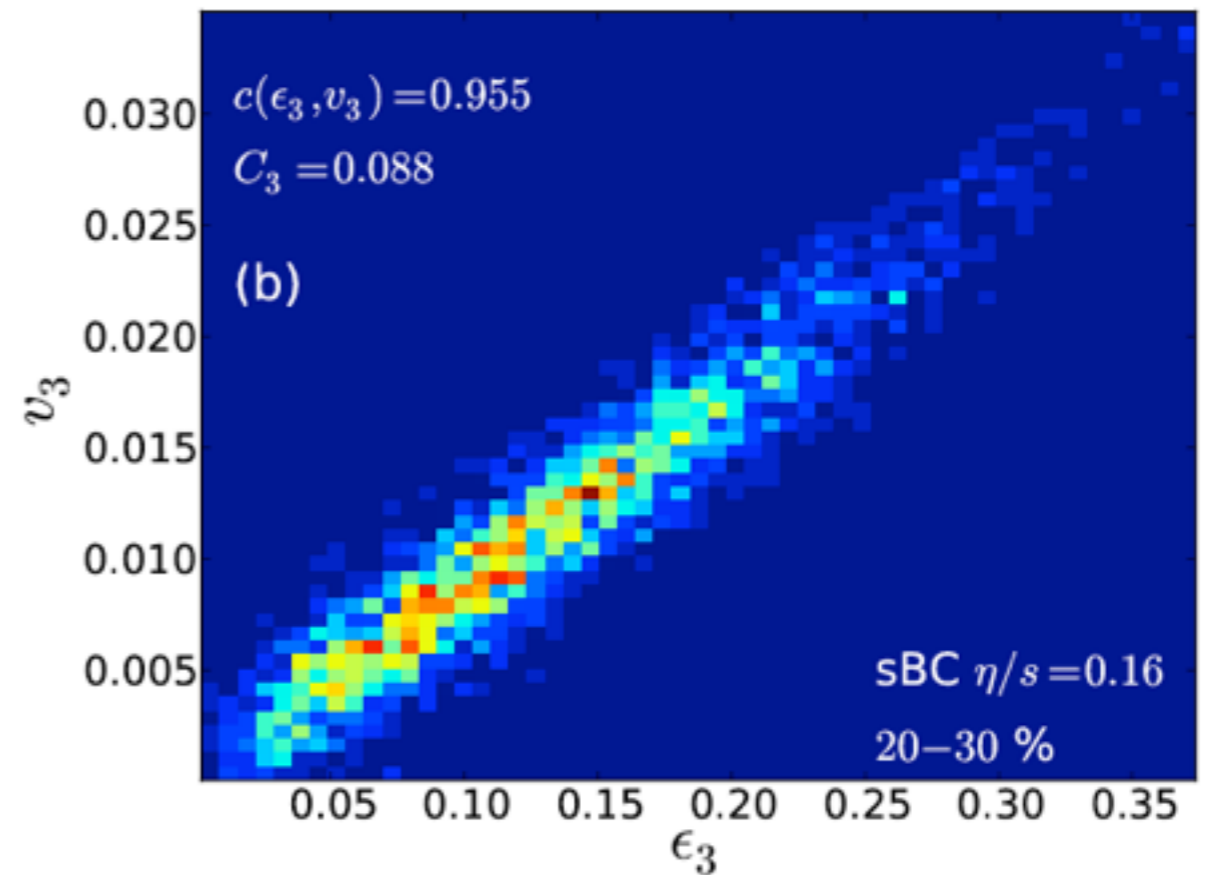
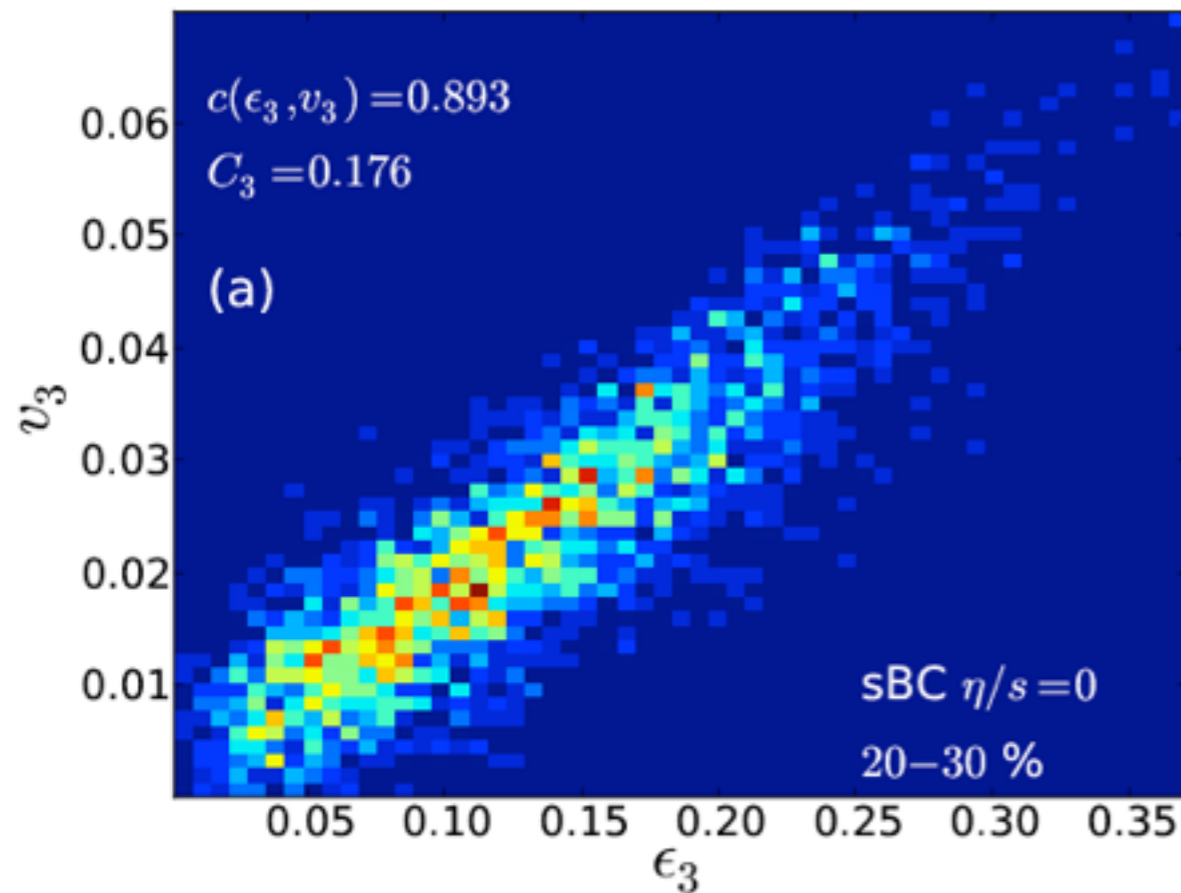


Niemi Denicol Holopainen Huovinen 1212.1008

Each point=different initial density profile.

v_2 is almost perfectly linear in ϵ_2

Triangular flow v_3 versus initial triangularity ϵ_3

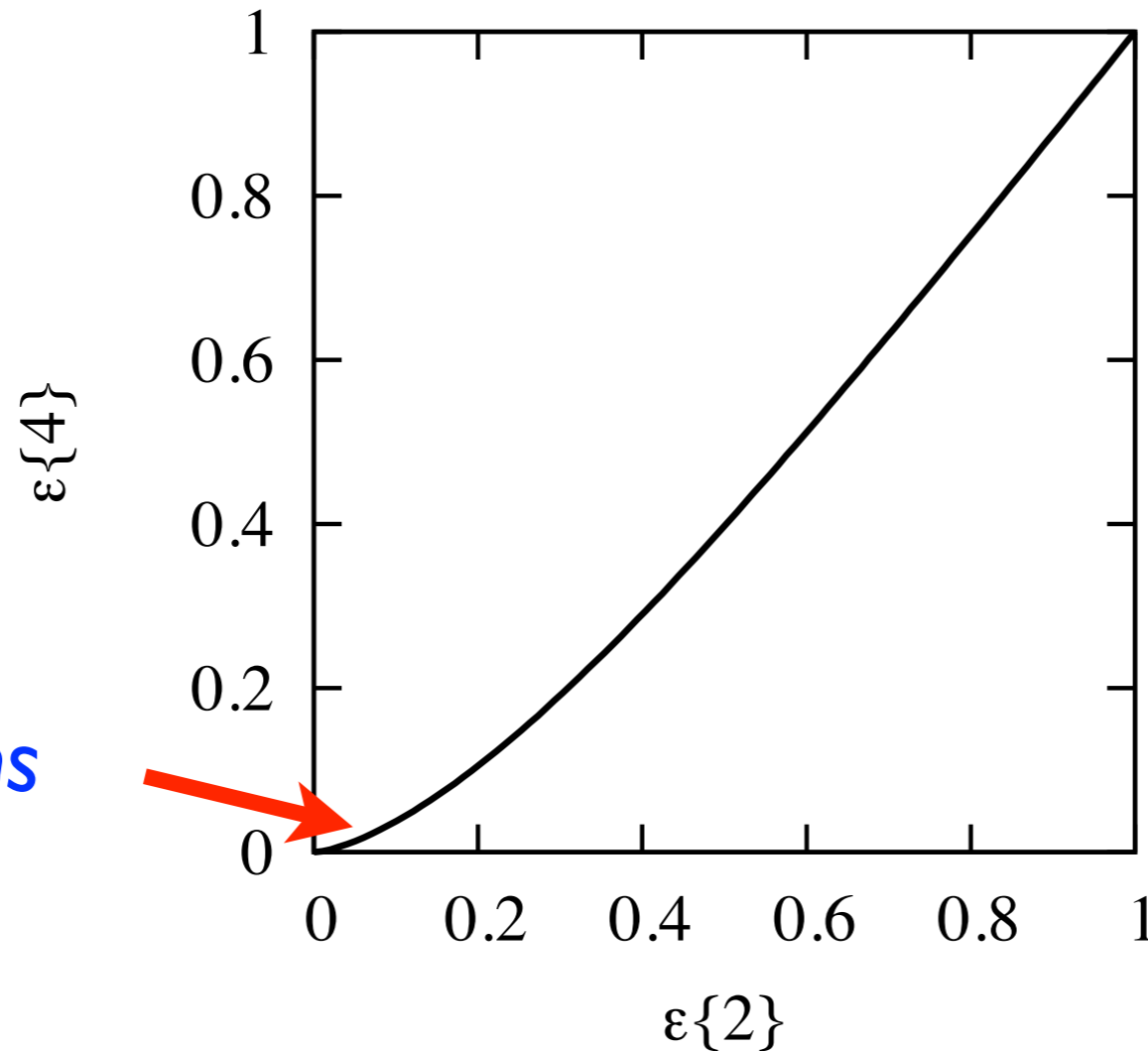


Niemi Denicol Holopainen Huovinen 1212.1008

v_3 is also strongly correlated with ϵ_3

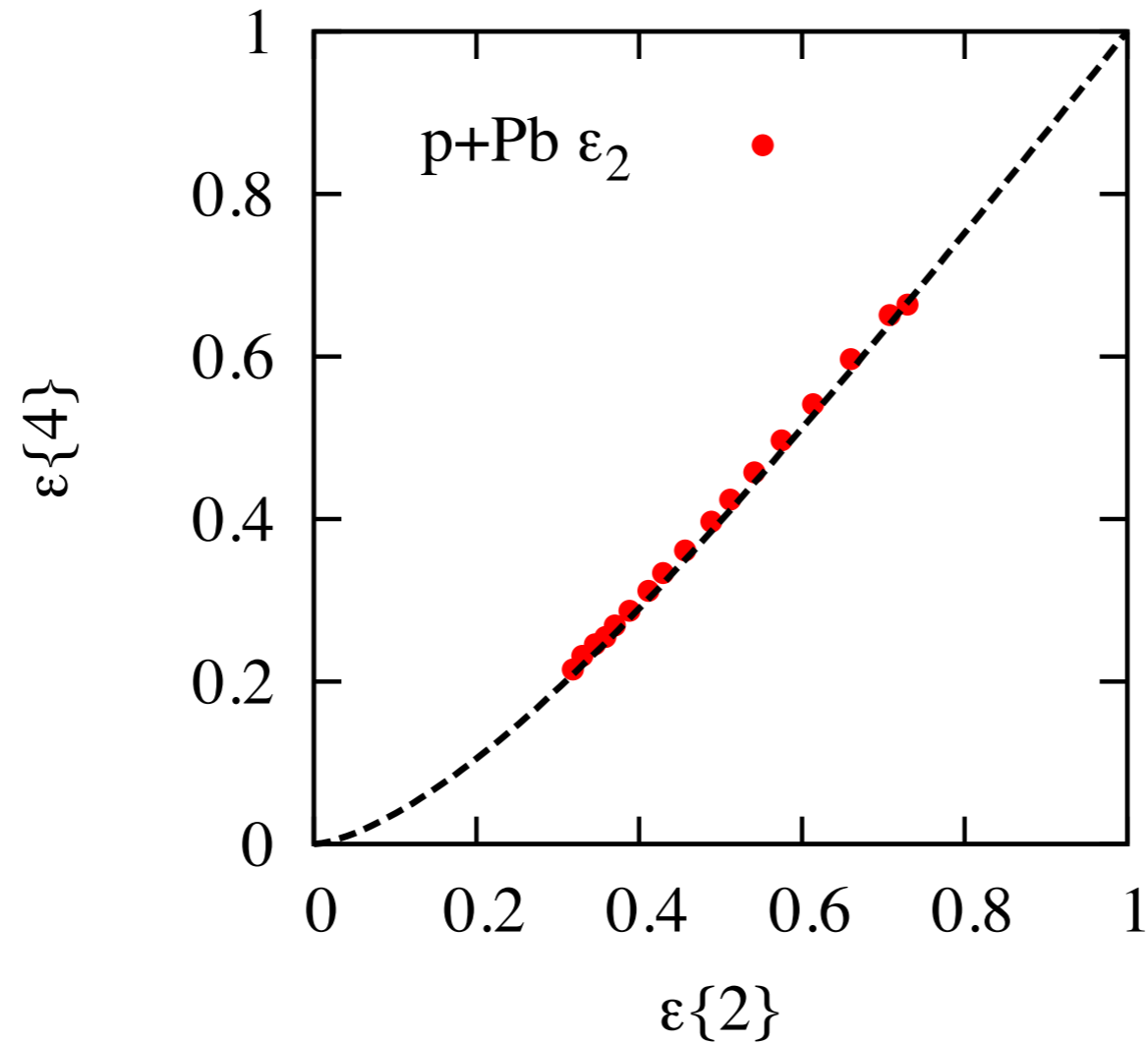
Natural explanation for $\varepsilon_2\{4\}$ in pPb

*Central limit:
large system,
small fluctuations
 $\varepsilon\{2\} \ll 1$ and
 $\varepsilon\{4\} \ll \varepsilon\{2\}$*



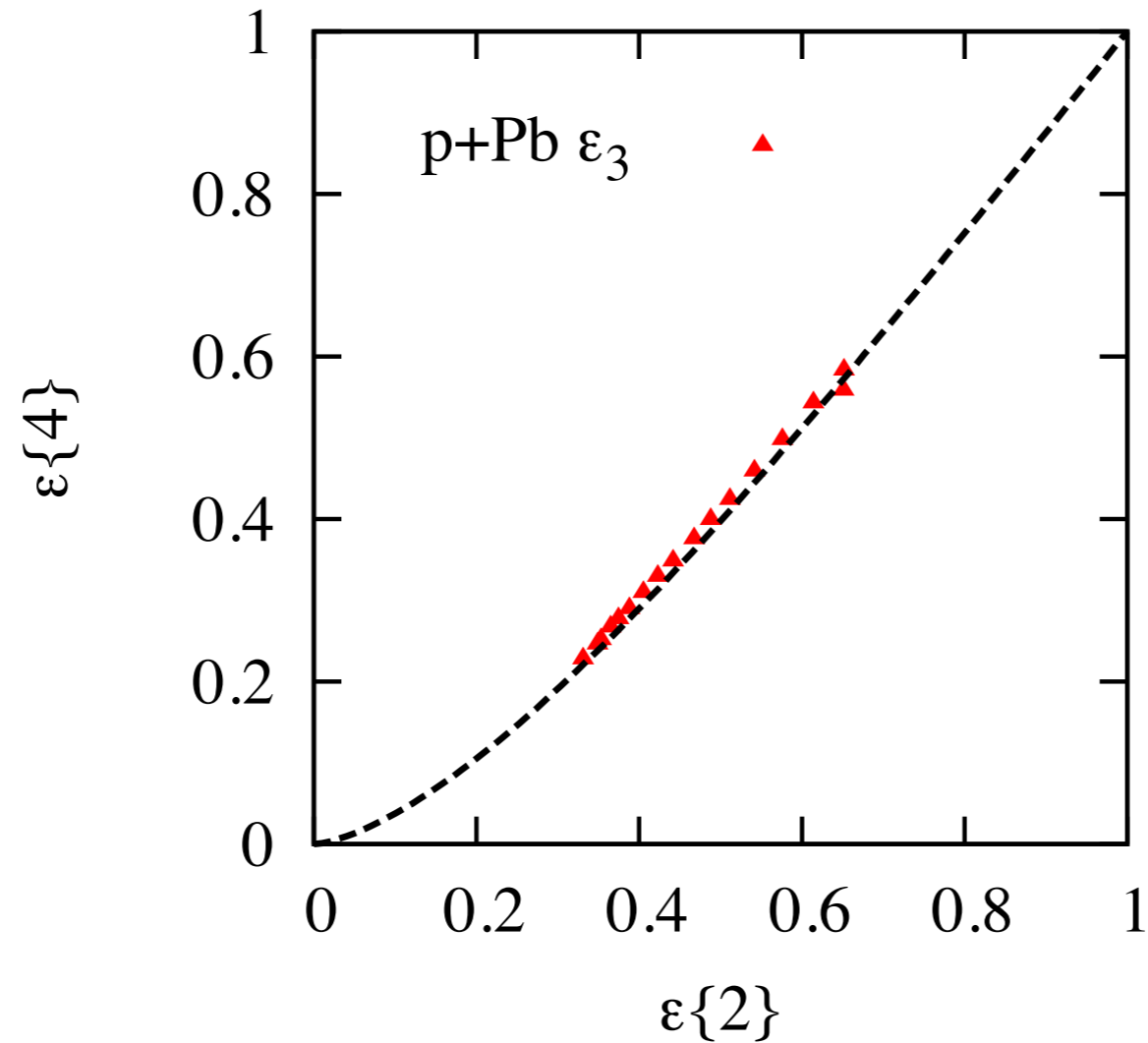
Prediction of the *power* distribution

Testing universality with cumulants



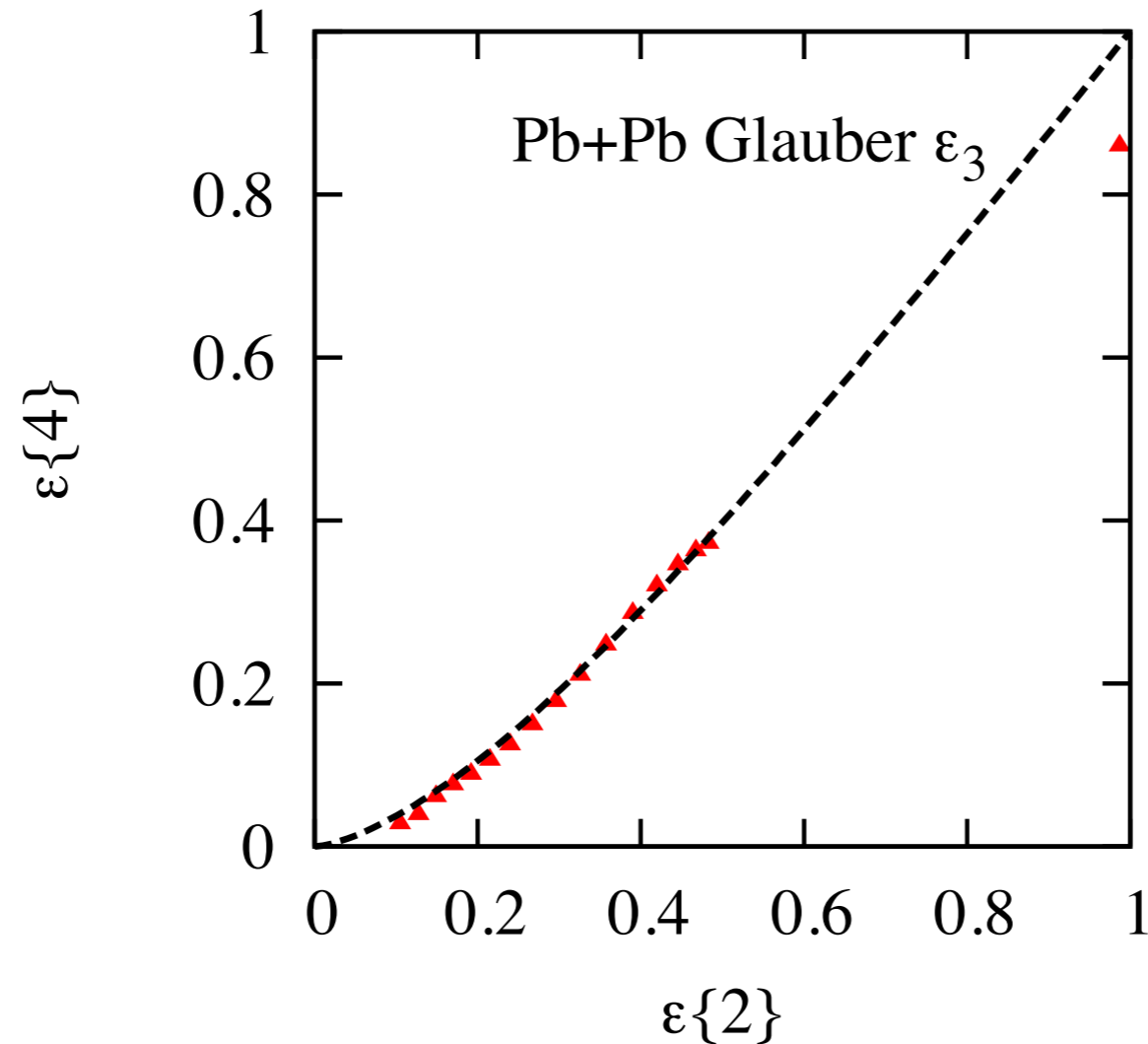
Each point: different number of hit nucleons in target

Testing universality with cumulants



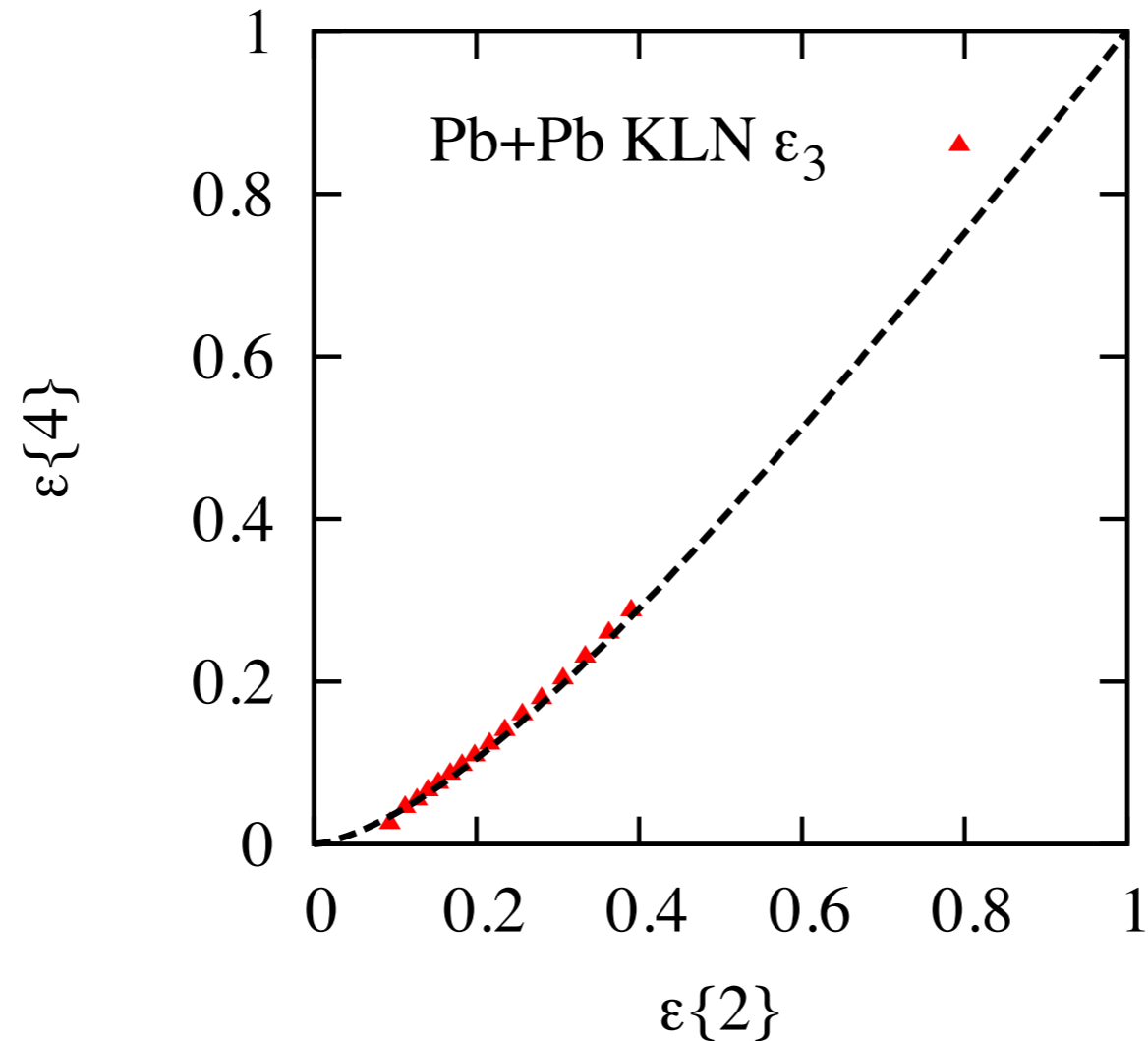
Each point: different number of hit nucleons in target

Testing universality with cumulants



Each point: different centrality
Pb-Pb: Larger system: smaller anisotropies

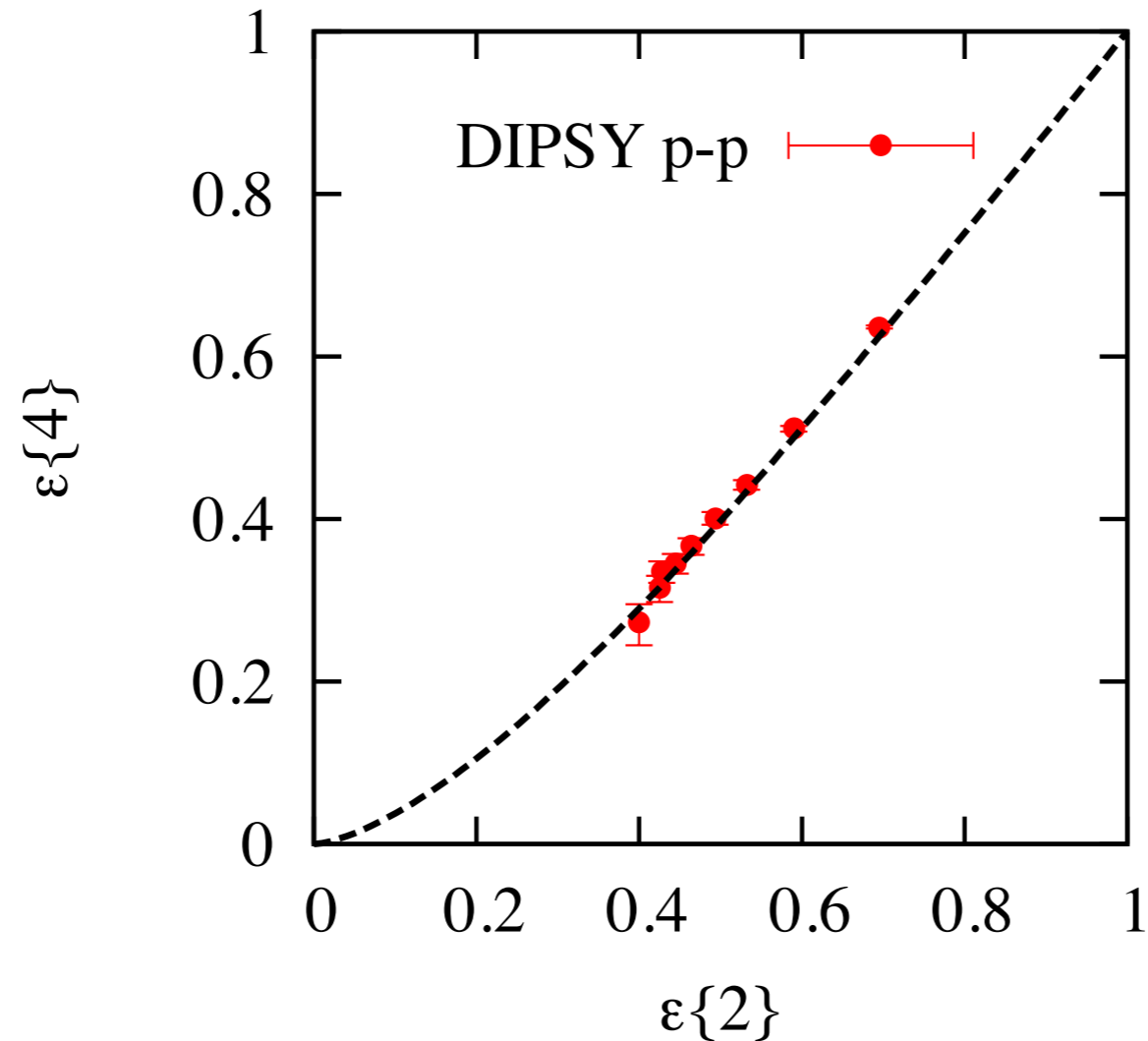
Testing universality with cumulants



Each point: different centrality

Pb-Pb: Larger system: smaller anisotropies

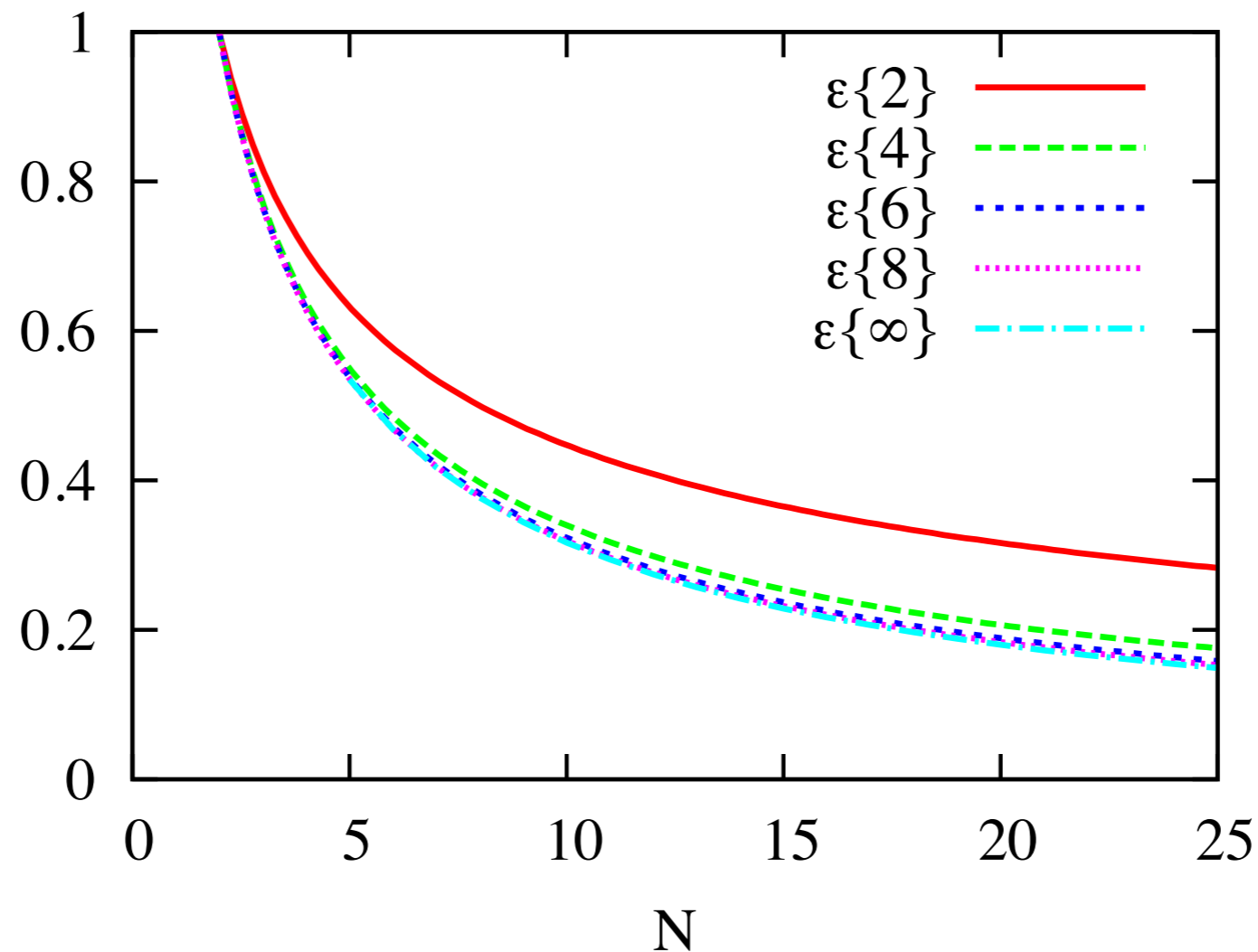
Testing universality with cumulants



data from Avsar Flensburg Hatta JYO Ueda 1009.5643

Each point: different parton multiplicity

Higher-order cumulants (predicted by the power distribution)



$\varepsilon\{n\}$ quickly converges as order n increases