Optics for TME cells with longitudinally varying fields and a new DR design

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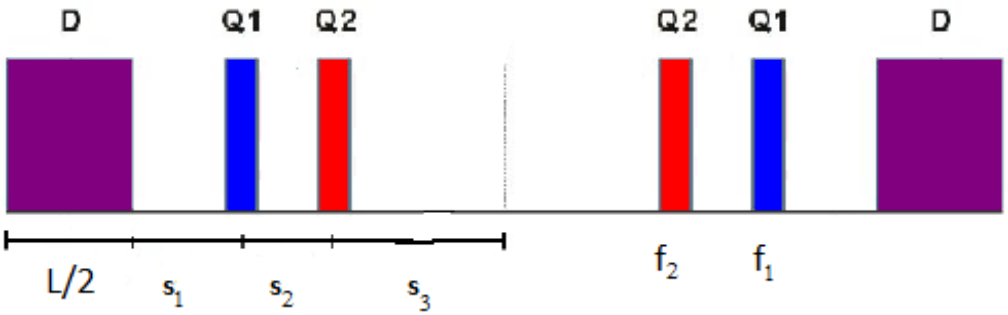
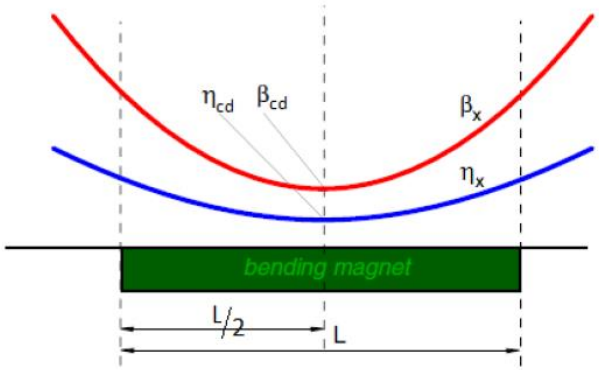
TME cell





$$L_{cell} = 2(s_1 + s_2 + s_3 + 2l_q) + L.$$

The balance between radiation damping and quantum excitation results in the equilibrium betatron emittance. Using a theoretical minimum emittance, TME cell, low emittance values can be achieved. The horizontal emittance of the beam can be generally expressed as:

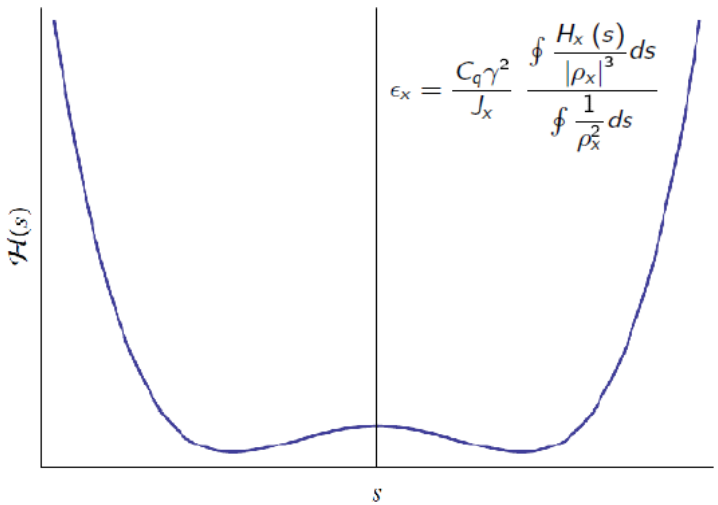
$$\epsilon_x = \frac{C_q\gamma^2}{J_x}\frac{\langle\frac{H_x}{|\rho_x|^3}\rangle}{\langle\frac{1}{\rho_x^2}\rangle} = \frac{C_q\gamma^2}{J_x}\frac{\frac{1}{C}\int_0^C \frac{\mathcal{H}_x}{|\rho_x|^3}ds}{\frac{1}{C}\int_0^C \frac{1}{\rho_x^2}ds}$$

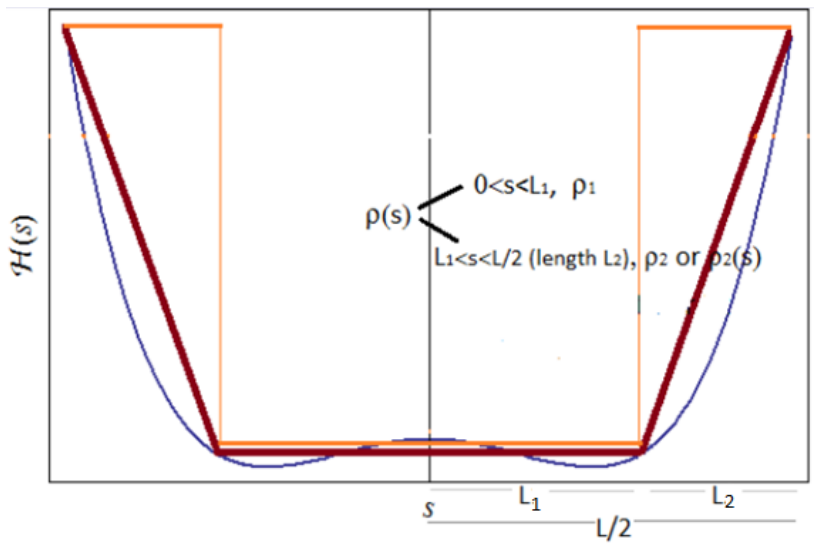
$$\beta(s) = \beta_{cd} - 2\alpha_{cd}s + \gamma_{cd}s^2,\ \alpha(s) = \alpha_{cd} - \gamma_{cd}s,\ \gamma(s) = \gamma_{cd},\ \eta(s) = \eta_{cd} + \eta'_{cd}s + \tilde{\theta}(s),\ \eta'(s) = \eta'_{cd} + \theta(s)$$

$$\mathcal{H}(s) = \gamma(s)\eta(s)^2 + 2\alpha(s)\eta(s)\eta(s)' + \beta(s)\eta(s)'^2$$

Longitudinally variable bends

Approaching the evolution of the uniform dipole's dispersion invariant assists in approaching its emittance behaviour in order to reduce it. The evolution of the dispersion invariant along the dipole guides the dipole profile choice for the emittance reduction.





Considering only the half dipole for simplicity (from 0 till L/2) as the other is symmetric and then dividing the dipole into two parts of different bending radii can be expressed as:

$$\text{length } L_1 \text{ with bending radius } \rho_1(s),\ 0 < s < L_1$$

$$\text{length } L_2 \text{ with bending radius } \rho_2(s),\ L_1 < s < L_1 + L_2 = L/2$$

<u>Bending angle of half dipole:</u> $$\theta = \int_0^{L_1} \frac{1}{\rho_1(s)} ds + \int_{L_1}^{L_1+L_2} \frac{1}{\rho_2(s)} ds$$

References: J. Guo and T. Raubenheimer, (EPAC02), Y.Papaphilippou, P. Elleaume, PAC'05, R. Nagaoka, A.F. Wrulich, (NIM A575, 2007), C.-x Wang (PRST-AB, 2009)

Dipole profiles

Bending radii ratio $\rho = \frac{\rho_1}{\rho_2}$ Lengths ratio $\lambda = \frac{L_1}{L_2}$ $(\rho < 1 \; as \; \rho_2 > \rho_1 \; and \; \lambda > 0 \; as \; L_1, L_2 > 0)$

Emittance reduction factor

$$F_{TME} = \frac{\epsilon_{TME_{uni}}}{\epsilon_{TME_{var}}} \qquad F_{TME} > 1$$

F_{TME} depends only on ρ and λ as the bending radii of the uniform and of the chosen profile are the same (as well as their length) and thus are simplified .

If the dipole's characteristics are not fixed F_{TME} is a function of ρ and λ.

Fixing the dipole's characteristics in accordance to the design's **constraints** leads to the dependence of F_{TME} either on ρ or λ . In this way the highest F_{TME} value for a specific design can be found.

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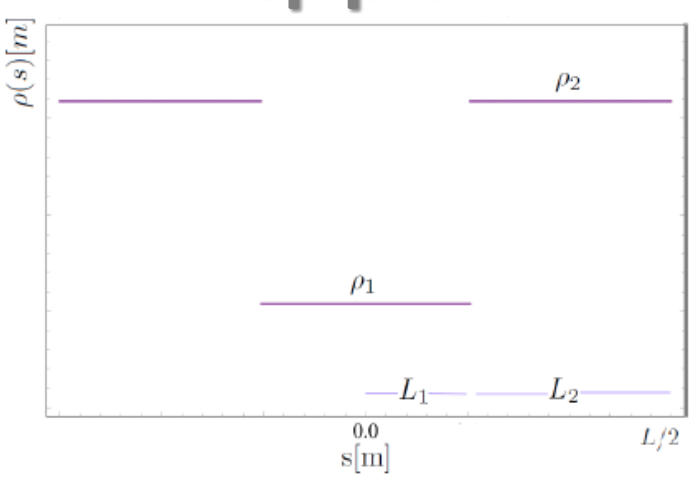
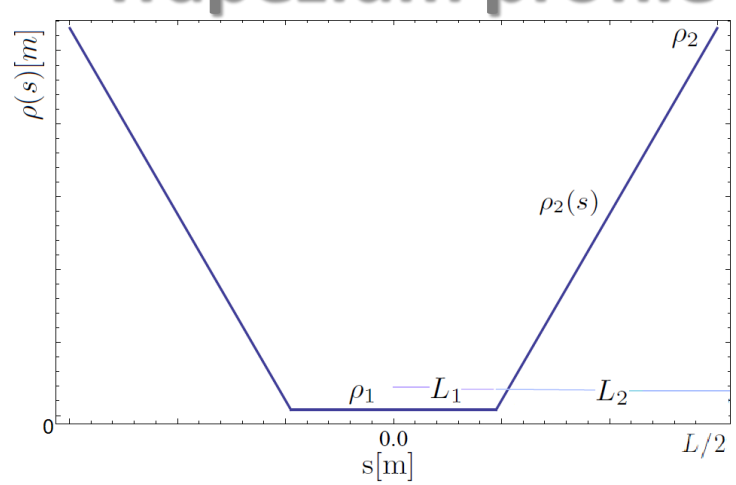
Bending angle of the dipole θ	Dipole's length L	Minimum bending radius ρ_1 (maximum magnetic field 1.77T)
$2\pi/100$	0.58 m	5.4 m

Step profile



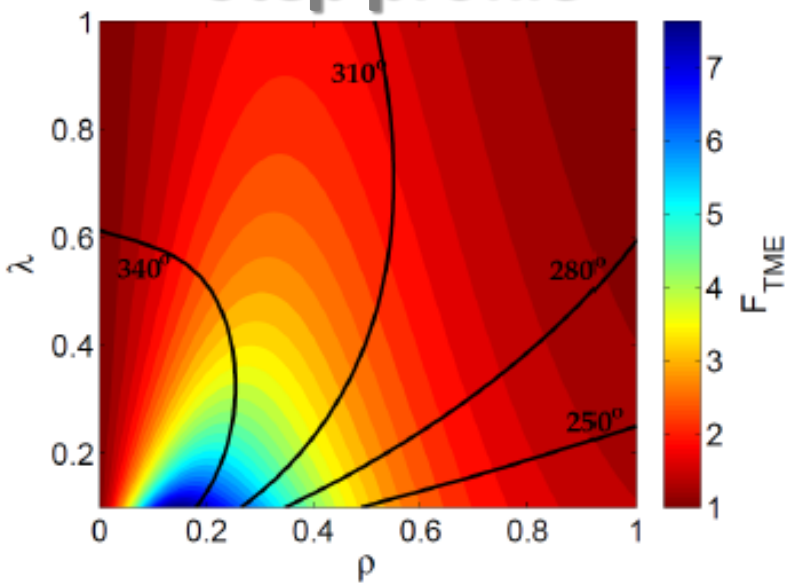
$$\rho(s) = \begin{cases} |\rho_1|, \; 0 < s < L_1 \\ \rho_2, \; L_1 < s < L_1 + L_2 \end{cases}$$

Trapezium profile

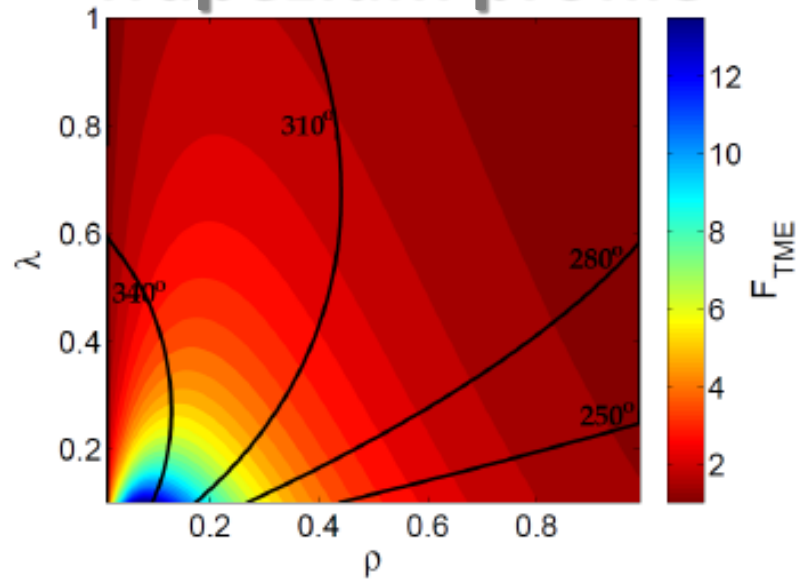


$$\rho(s) = \begin{cases} \rho_1, \; 0 < s < L_1 \\ \rho_2(s) = \rho_1 + (L_1 - s)(\rho_1 - \rho_2)/L_2, \; L_1 < s < L_1 + L_2 \end{cases}$$

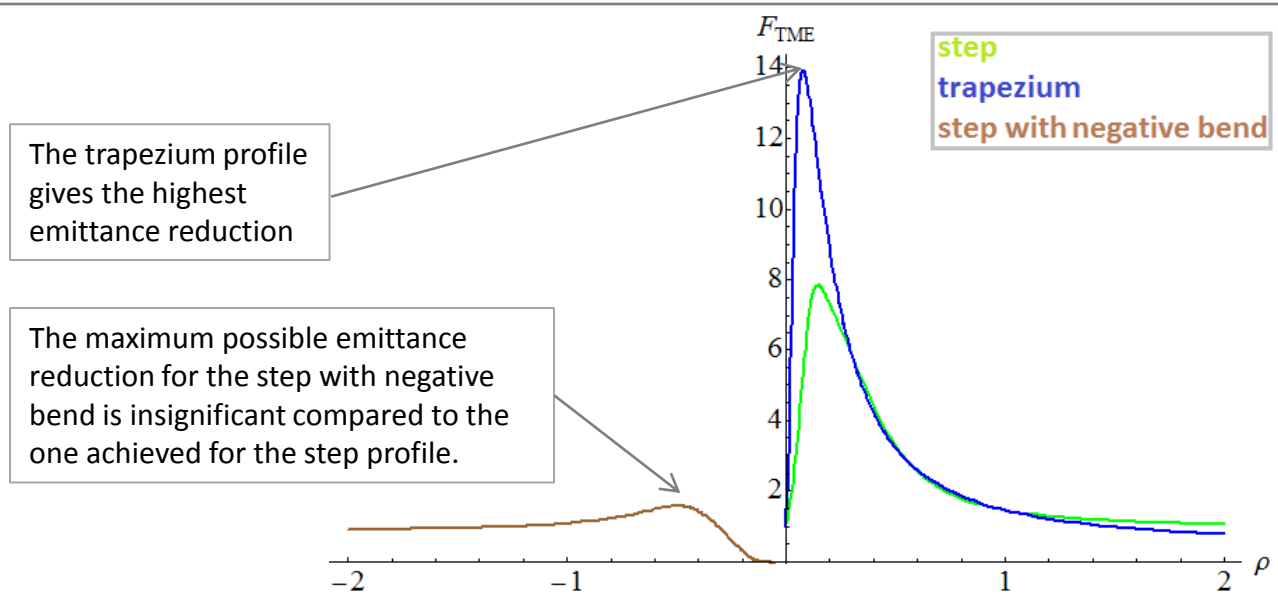
Step profile



Trapezium profile



The parameterization of the emittance reduction factor F_{TME} with the bending radii ratio ρ and the lengths ratio λ, always for λ>0.1 so that the lengths L_1, L_2 are comparable.
The black contour lines correspond to different values of horizontal phase advances (for the uniform dipole it is $\mu_{x,TME}$=284.5°).



Comparison of the non-uniform dipole profiles' reduction factors when λ=0.1 (there the highest reductions are localized)

Comparison of non-uniform dipole profiles when fixing the dipole's characteristics (bending angle, length and minimum bending radius)

step

$$\theta = \frac{L(\lambda+\rho)}{\rho_1(1+\lambda)}$$

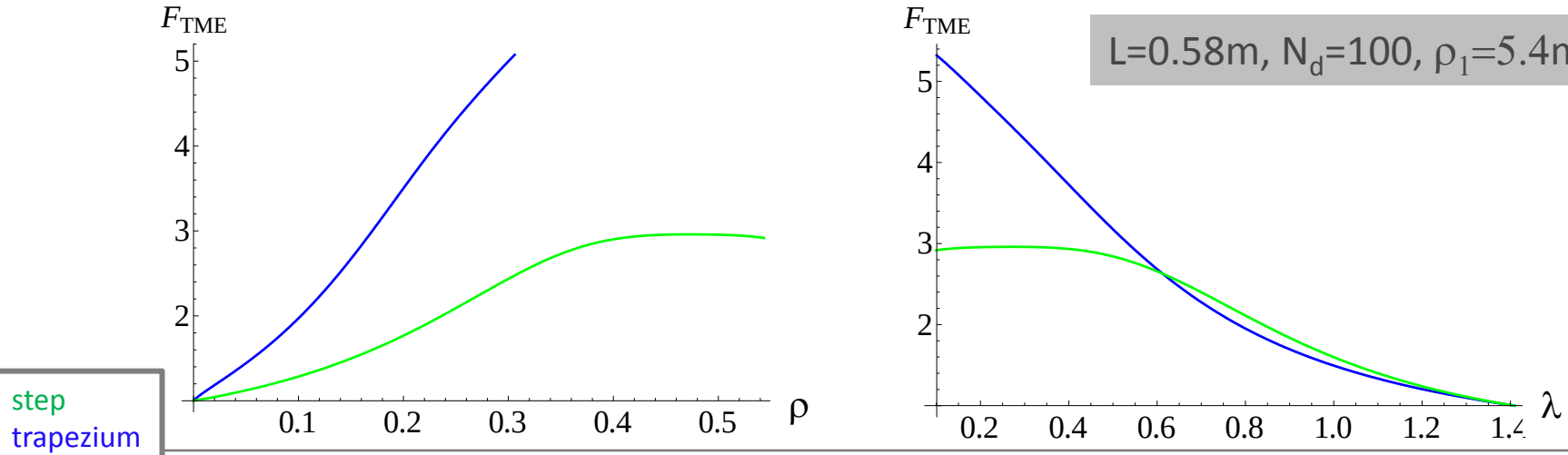
trapezium

$$\theta = \frac{L(\lambda(-1+\rho)+\rho\log\rho)}{\rho_1(-1+\rho)(1+\lambda)}$$

$\lambda(\rho)$ or $\rho(\lambda)$ $\xrightarrow{F_{TME}(\rho,\lambda)}$ $F_{TME}(\rho)$ or $F_{TME}(\lambda)$

The reduction factor F_{TME} as a function of ρ (left) and as a function of λ (right)

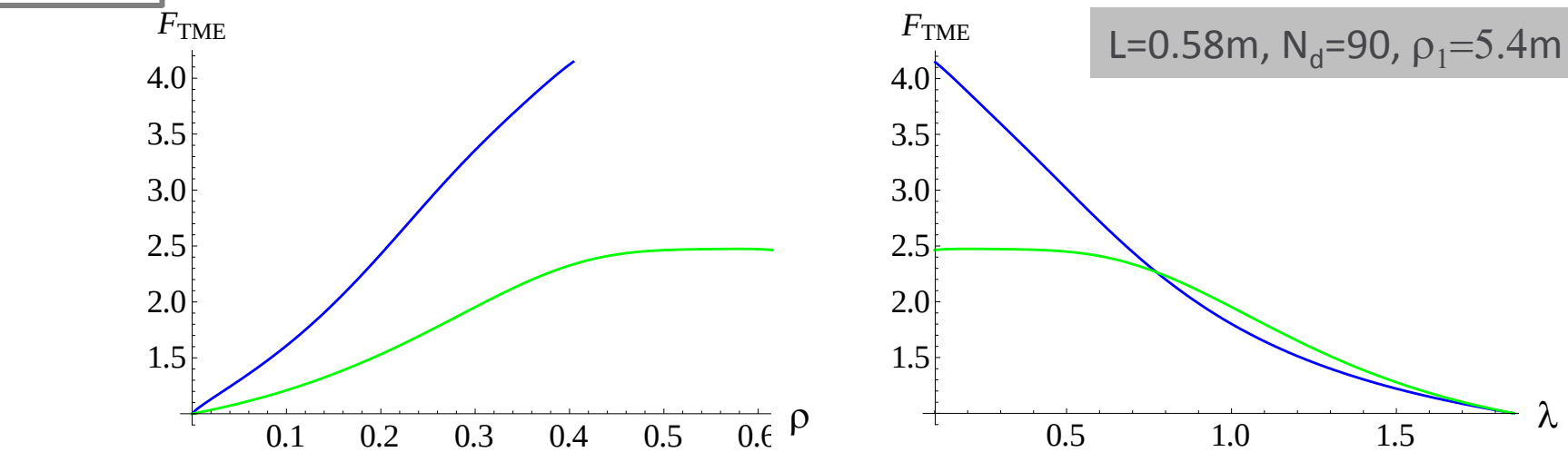
L=0.58m, N_d=100, ρ_1=5.4m



step

trapezium

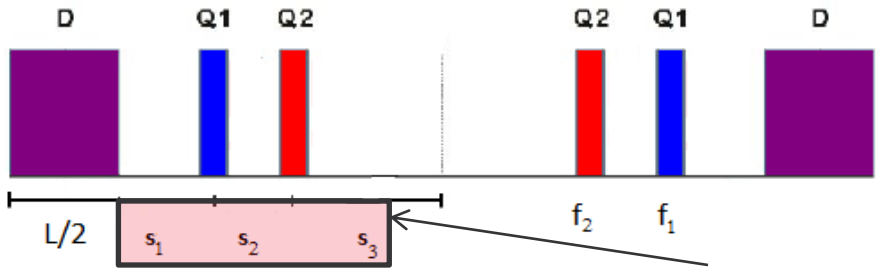
L=0.58m, N_d=90, ρ_1=5.4m



Analytical parameterization of a variable bend TME cell

Knowing the dipole's characteristics it is important to fix some more parameters in order to produce the numerical results for the CLIC DR lattice design:

- The quadrupoles' length is set to l_q = 0.2m.
- The maximum dipole field is set to 1.77 T (minimum bending radius = 5.4m)
- The maximum pole tip field of the quadrupoles and the sextupoles is Bmaxq = 1.1T and Bmaxs = 0.8T respectively.
- The required output normalized emittance for N_d = 100 dipoles is 500nm and the operational energy of the CLIC Damping Rings complex of 2.86 GeV.



Fixing those parameters the free parameters left are the drift space lengths S1, S2, S3 and the emittance.

The stability criterion is governing every result and is included in the feasibility constraints:

$$|cos\varphi_{x,y}| < 1$$

$$k = \frac{1}{fl_q} =< \frac{1}{(B\rho_x)} \frac{B_q^{\max}}{R_{\min}}$$

$$S \leq \frac{2B_s^{\max}}{R_{\min}^2} \frac{1}{(B\rho_x)}$$

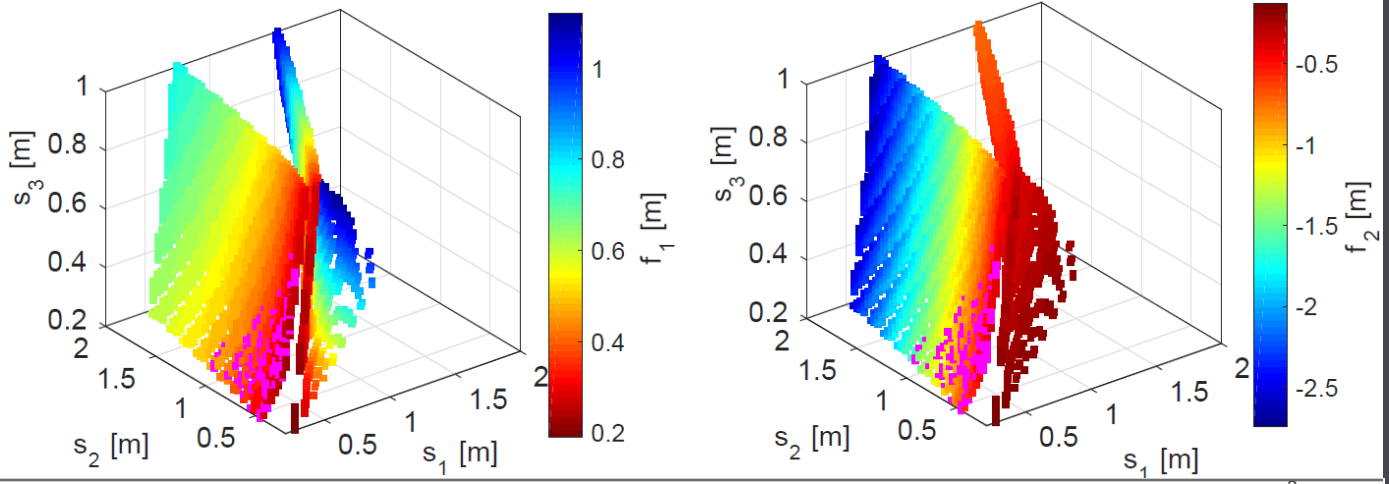
Emittance detuning factor: Emittance deviation from the absolute TME $\epsilon_r = \frac{\epsilon_x}{\epsilon_{TME}}$

Dipole profiles (L=0.58 m, B_{max}=1.77 T, N=100)	F_{TMEmax} (CLIC design)	$\lambda(F_{TMEmax})$ (CLIC design)
Step	2.96	0.27
Trapezium	5.32	0.1

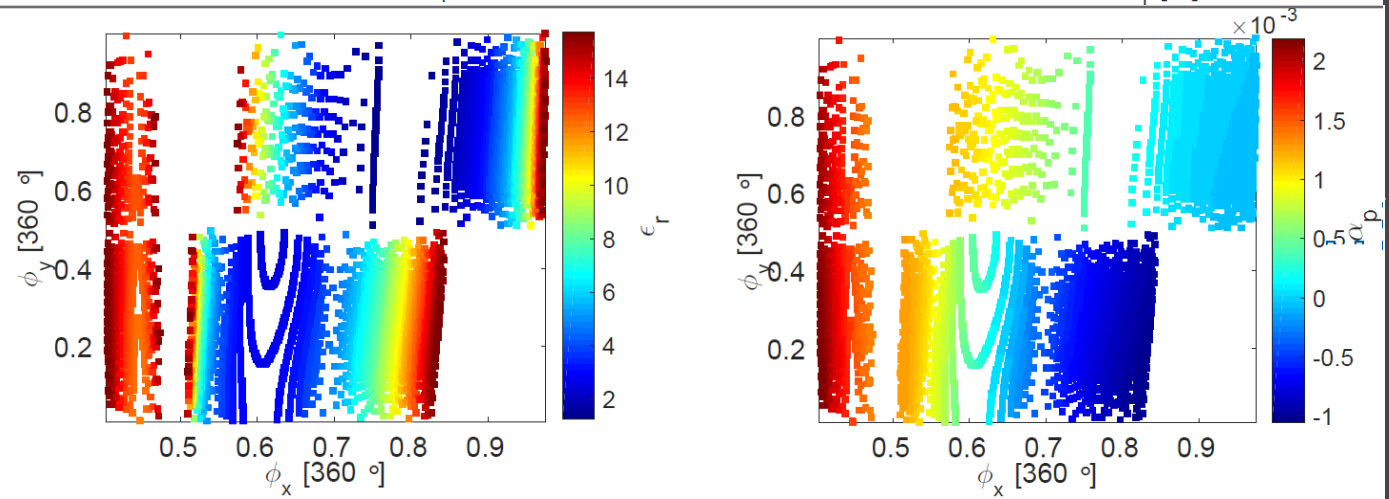
Reference: F. Antoniou and Y. Papaphilippou, *PRSTAB*, **17**, 064002 , 23 June 2014

Parameterization with the drift lengths and with the emittance (only for the step profile)

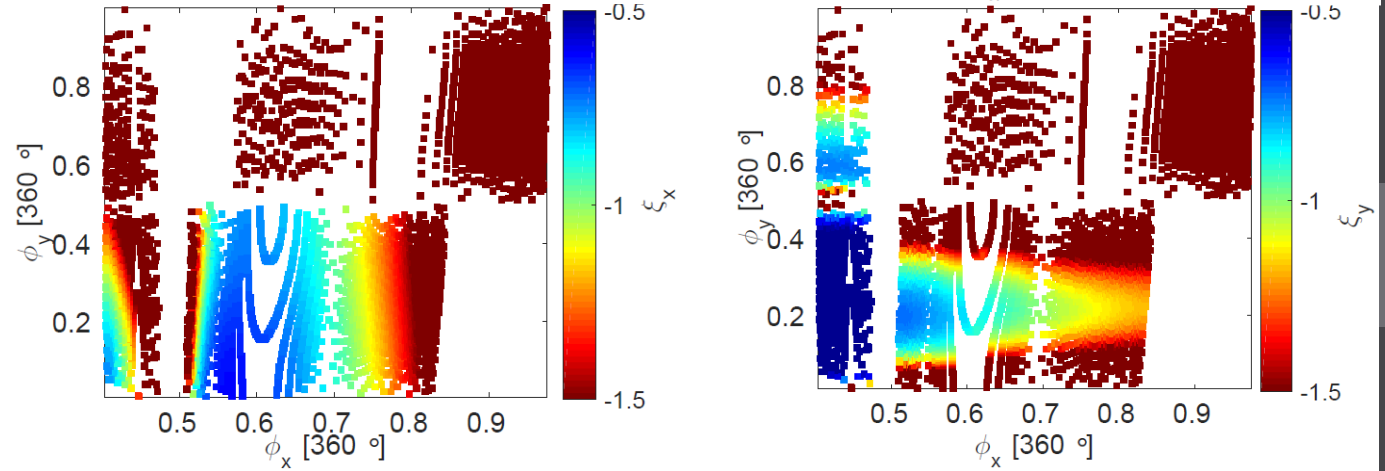
Parameterization of the quadrupoles' focal lengths with the drift space lengths (low chromaticity solutions).



Parameterization of the det. factor and of the momentum compaction factor with the horizontal and vertical phase advances.



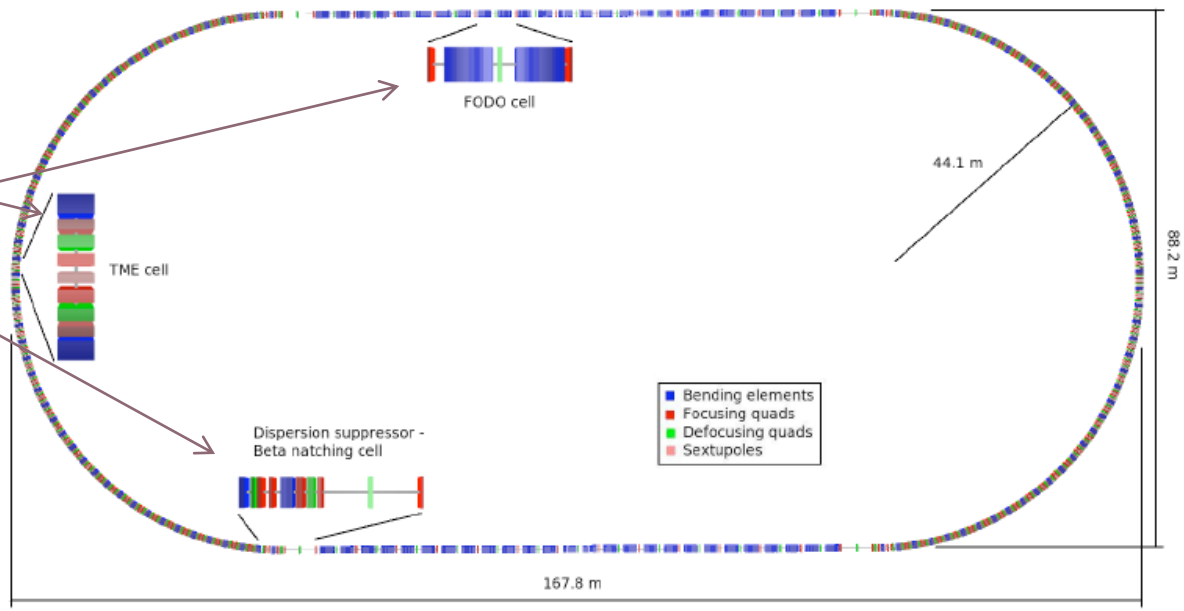
Parameterization of the horizontal and vertical chromaticities with the horizontal and vertical phase advances.



New DR design

Optimization of :

- the arc TME cell
- the wiggler FODO cell
- the dispersion suppression



Replacing the current TME cell with the one having a variable bend, while keeping the rest of the ring unchanged, gives an emittance reduction ($F_{TME} \approx 2$)- as expected. However, the resulted chromaticities are high and the mom. compaction factor is very small.

Reoptimization of the arc TME cell is needed (number of dipoles N_d, dipoles' length L, drift space lengths s_1, s_2, s_3).

When matching the lattice functions between the arcs and the straight sections, the optical functions get exceedingly high values. So after the optimization of the arc TME cell, an optimization of the disp. suppressor may also be needed. The current drift space lengths and magnets' strengths should be reconsidered.

Conclusions and next steps

- The highest emittance reduction is given by the trapezium profile, concurrently it provides feasible-low chromaticity solutions for low detuning factors .
- The agreement with the simulation code MADX validates the analytical solutions for both profiles, specially for the thin lens approximation.
- Studies on the fringe fields of the individual parts of the non-uniform dipoles will provide a better understanding of their behaviour.
- A further improvement of the final emittance values can be achieved when taking into consideration the collective effects, such as the Intrabeam scattering IBS that in the regime of ultralow emittances with high bunch charge has a significant impact on the emittance limits.
- The alternative design of the CLIC DRs will be based on TME cells with longitudinally variable bends and on a high-field wiggler.

Paper in preparation: "Emittance reduction with variable bending magnet strengths: Analytical optics considerations"

Thank you!

Special thanks to F. Antoniou for her valuable help.

Dispersion invariant (1,2 for the individual dipole parts)

$$\mathcal{H}_{1,2}(s) = \gamma_{1,2}\eta_{1,2}{}^2 + 2\alpha_{1,2}\eta_{1,2}\eta'_{1,2} + \beta_{1,2}\eta'_{1,2}{}^2$$
$$(\alpha_{cd} = 0, \eta'_{cd} = 0)$$

Horizontal Emittance

$$\epsilon_x = G\left(\frac{1}{L_1}\int_0^{L_1}\frac{\mathcal{H}_1}{|\rho_1|^3}ds + \frac{1}{L_2}\int_{L_1}^{L_1+L_2}\frac{\mathcal{H}_2}{|\rho_2|^3}ds\right), \; where\; G = \frac{C_q\gamma^2}{J_x}\left(\frac{1}{L_1}\int_0^{L_1}\frac{1}{\rho_1^2}ds + \frac{1}{L_2}\int_{L_1}^{L_1+L_2}\frac{1}{\rho_2^2}ds\right)^{-1}$$

$$\epsilon_x = G\frac{(I_7 + I_8\lambda + (I_1 + I_2\lambda)\beta_{cd}^2 + \eta_{cd}(I_5 + I_6\lambda + (I_3 + I_4\lambda)\eta_{cd}))}{L_1\beta_{cd}}$$

$$I_1 = \int_0^{L_1}\frac{\theta_1^2}{|\rho_1|^3}ds\ 1,\quad I_2 = \int_{L_1}^{L_1+L_2}\frac{(\theta_2 + \theta_{L_1})^2}{|\rho_2|^3}ds\ ,\quad I_3 = \int_0^{L_1}\frac{1}{|\rho_1|^3}ds\ ,\quad I_4 = \int_{L_1}^{L_1+L_2}\frac{1}{|\rho_2|^3}ds\ ,\quad I_5 = \int_0^{L_1}2\frac{-s\theta_1 + \tilde{\theta}_1}{|\rho_1|^3}ds$$

$$I_6 = \int_{L_1}^{L_1+L_2}2\frac{-s\theta_2 + \tilde{\theta}_2 - L_1\theta_{L_1} + \tilde{\theta}_{L_1}}{|\rho_2|^3}ds\ ,\quad I_7 = \int_0^{L_1}\frac{(-s\theta_1 + \tilde{\theta}_1)^2}{|\rho_1|^3}ds\ ,\quad I_8 = \int_{L_1}^{L_1+L_2}\frac{(-s\theta_2 + \tilde{\theta}_2 - L_1\theta_{L_1} + \tilde{\theta}_{L_1})^2}{|\rho_2|^3}ds$$

Beta and dispersion functions for the TME

$$\beta_{TME} = \frac{\sqrt{-I_5^2 - 2I_5I_6\lambda + 4I_3(I_7 + I_8\lambda) + \lambda(-I_6^2\lambda + 4I_4(I_7 + I_8\lambda))}}{2\sqrt{(I_1 + I_2\lambda)(I_3 + I_4\lambda)}}\quad and\quad \eta_{TME} = -\frac{I_5 + I_6\lambda}{2(I_3 + I_4\lambda)}$$

$$\epsilon_{TME} = G\frac{(I_1 + I_2\lambda)\sqrt{-I_5^2 - 2I_5I_6\lambda + 4I_3(I_7 + I_8\lambda) + \lambda(-I_6^2\lambda + 4I_4(I_7 + I_8\lambda))}}{L_1\sqrt{(I_1 + I_2\lambda)(I_3 + I_4\lambda)}}$$

The relation between the reduction and the detuning factor.

$$\frac{\epsilon_{var}}{\epsilon_{uni}} = \frac{\epsilon_{r_{var}}\epsilon_{TME_{var}}}{\epsilon_{r_{uni}}\epsilon_{TME_{uni}}} = \frac{\epsilon_{r_{var}}}{\epsilon_{r_{uni}}}\frac{1}{F_{TME}}$$

$$\frac{\epsilon_{var}}{\epsilon_{uni}} < 1$$

$$\Rightarrow \quad \frac{\epsilon_{r_{var}}}{\epsilon_{r_{uni}}} < F_{TME}$$