Improvements and simulations in PLACET

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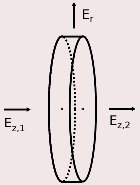
Content

1. Improvement in RF focusing
2. CSR shielding in the Drive beam complex

Motivation

- Default model of rf focusing in PLACET is not accurate when the energy gain is large compared to the particle energy.
- The default model assumes a constant field in the cavity bulk at the particle position.
- Rosenzweig and Serafini[1] give a way of calculating the average force on a particle travelling in a cavity.
- In e.g. the LHeC test facility, the difference is non negligible. In order to simulate CSR shielding here, we need an extension of PLACET.

Gaussian pillbox



Gauss' law in bulk of cavity

- Gauss' law: $\oint \overline{E} \cdot d\overline{A} = \frac{Q}{\epsilon_0} = 0$
- $\Rightarrow \pi r^2 (E_{z,2} - E_{z,1}) + \Delta z 2\pi r E_r = 0$
- $\Rightarrow E_r = -\frac{r}{2}\frac{\Delta E_z}{\Delta z} \rightarrow -\frac{r}{2}\frac{\partial E_z}{\partial z}$

Gauss' law on one of the cavity edges

- $\Rightarrow \pi r^2 E_{z,2} + \Delta z 2\pi r E_r = 0$
- $\Rightarrow E_r = -\frac{r}{2}\frac{E_{z,2}}{\Delta z}$
- $\Delta x' = \frac{eE_r \Delta z}{E} = -\frac{eE_{z,2}}{2\mathcal{E}} r$

[1] J. Rosenzweig and L. Serafini, Transverse particle motion in radio-frequency linear accelerators. Phys. Rev. E **49**, 1599 (1994)

Transport matrices

Prerequisites

- $\gamma' = e\mathcal{E}\cos(\phi)/mc^2 = \frac{\Delta E}{L}/mc^2$,
- $\delta = \frac{\Delta E}{E}$, $\frac{\gamma_f}{\gamma_i} = 1 + \delta$, $\frac{\gamma'}{\gamma_i} = \frac{\delta}{L}$, $\frac{\gamma'}{\gamma_f} = \frac{\delta}{1+\delta}\frac{1}{L}$
- $\alpha = \sqrt{\frac{\eta}{8}}\frac{1}{\cos(\phi)}\ln\left(\frac{\gamma_f}{\gamma_i}\right)$
- $\eta = 1$ for a π-mode cavity.

A solution to the equations of motion

- The cavity bulk has the transport matrix

$$K_{center} = \begin{bmatrix} \cos(\alpha) & \sqrt{\frac{8}{\eta}}\frac{\gamma_i}{\gamma'}\cos(\phi)\sin(\alpha) \\ -\sqrt{\frac{\eta}{8}}\frac{\gamma'}{\gamma_f\cos(\phi)}\sin(\alpha) & \frac{\gamma_i}{\gamma_f}\cos(\alpha) \end{bmatrix} \quad (1)$$

$$= \begin{bmatrix} \cos(\alpha) & \sqrt{\frac{8}{\eta}}\frac{L}{\delta}\cos(\phi)\sin(\alpha) \\ -\sqrt{\frac{\eta}{8}}\frac{\delta}{1+\delta}\frac{1}{L}\frac{\sin(\alpha)}{\cos(\phi)} & \frac{1}{1+\delta}\cos(\alpha) \end{bmatrix} \quad (2)$$

- At the edges are simple angular kicks $K_{initial} = \begin{bmatrix} 1 & 0 \\ -\frac{\gamma'}{2\gamma_i} & 1 \end{bmatrix}$, $K_{final} = \begin{bmatrix} 1 & 0 \\ \frac{\gamma'}{2\gamma_f} & 1 \end{bmatrix}$,

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A solution to the equations of motion

- $K_{i,half} = \begin{bmatrix} \cos(\alpha) - \sqrt{\frac{2}{\eta}}\cos(\phi)\sin(\alpha) & \frac{L/2}{\delta}\sqrt{\frac{8}{\eta}}\sin(\alpha)\cos\phi \\ -\frac{\delta}{1+\delta}\frac{1}{L/2}\left[\frac{\cos(\alpha)}{2} + \sqrt{\frac{\eta}{8}}\frac{\sin(\alpha)}{\cos(\phi)}\right] & \frac{\cos(\alpha)}{1+\delta} \end{bmatrix}$,
- $K_{f,half} = \begin{bmatrix} \cos(\alpha) & \frac{L/2}{\delta}\sqrt{\frac{8}{\eta}}\sin(\alpha)\cos(\phi) \\ \frac{\delta}{1+\delta}\frac{1}{L/2}\left[\frac{\cos(\alpha)}{2} - \sqrt{\frac{\eta}{8}}\frac{\sin(\alpha)}{\cos(\phi)}\right] & \frac{1}{1+\delta}\left[\cos(\alpha) + \sqrt{\frac{2}{\eta}}\cos\phi\sin\alpha\right] \end{bmatrix}$,
- It has been numerically verified that $K_{center} = K_{f,half}K_{i,half}$ for all $\{\delta, \phi\}$ as long as the relative energy change of the first half is $1 + \delta/2$ and the second half $(1+\delta)/(1+\delta/2)$

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A solution to the equations of motion

- Limiting cases, $\cos(\phi) \to 0$, $\delta = \frac{G}{E_i}\cos(\phi)L/2$, $\alpha \to \sqrt{\frac{\eta}{8}}\frac{GL}{2E_i}$. δ is small, but α is not necessarily small.
- $$K_{i,half} \to \begin{bmatrix} \cos(\alpha) & \frac{E_i}{G}\sqrt{\frac{8}{\eta}}\sin(\alpha) \\ -\frac{G}{E_i}\sqrt{\frac{\eta}{8}}\sin(\alpha) & \cos(\alpha) \end{bmatrix},$$
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Checks of the properties of the transport matrix

- Limiting case, $\delta \ll 1, \phi = 0 \rightarrow \sin(\alpha) \approx \alpha$. This is the original model of focusing in PLACET
- $K_{center} = \begin{bmatrix} 1 & \frac{L\ln(1+\delta)}{\delta} \\ 0 & \frac{1}{1+\delta} \end{bmatrix}$
- Reduces to a drift when $\delta = 0$.

Numerical method

Recipe

- Longitudinal wakefields pose a problem when the phase is $\pi/2$ if they are included in δ, e.g if $\delta = \frac{G}{E_i}\cos(\phi)L/2 + \frac{\Delta E_{wf}}{E_i}$.
- This would lead to infinite angular kicks when the phase is zero.
- Thus, the following method is imployed (order of operations is chosen to match existing implementation of cavity focusing):
  1. Calculate wake ΔE_{wf} for first half of the magnet at the magnet entrance.
  2. Apply transport matrix to center of magnet excluding the effect of the wake.
  3. Apply longitudinal wakes and accelerating field for the first half of cavity.
  4. Calculate and apply all transverse wakes.
  5. Apply any dipolar kicks.
  6. Calculate wake ΔE_{wf} for second half of the magnet.
  7. Apply transport matrix to end of magnet excluding the effect of the wake.
  8. Apply longitudinal wakes and accelerating field for the second half of cavity.

Alternate focusing switched on with the boolean `pi_mode` in PLACET.

Twiss parameters for the LHeC test facility

- Compare the twiss parameters of various programs.
- PLACET beam parameters are calculated on basis of the beam (dispersion subtracted).
- PTC agrees qualitatively with the original PLACET focusing.
- ELEGANT and PLACET with `pi_mode true` give similar results.
- Energy 5.511MeV → 306MeV.

Machine layout

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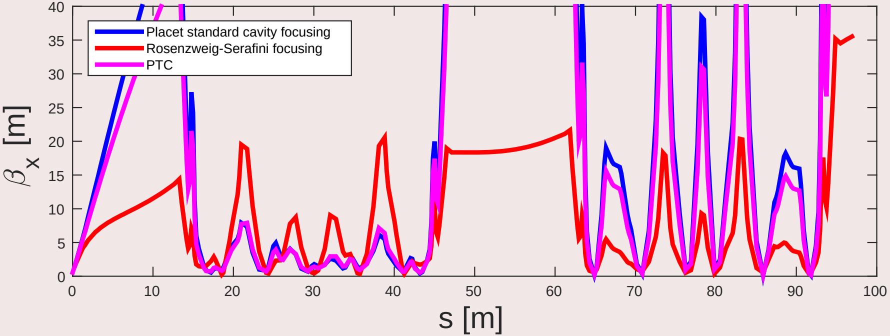
Horizontal Beta



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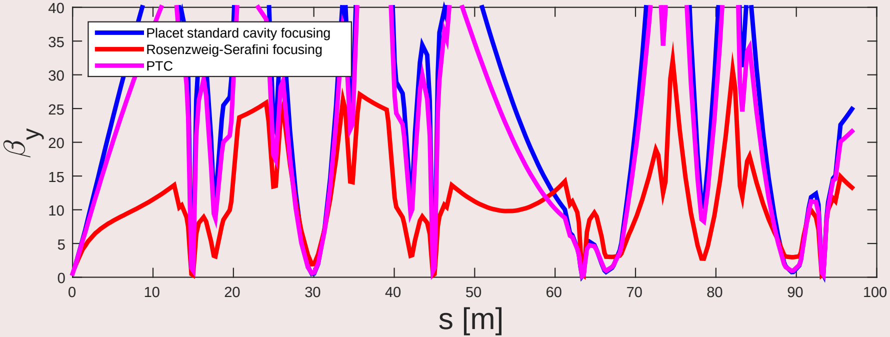
Vertical Beta



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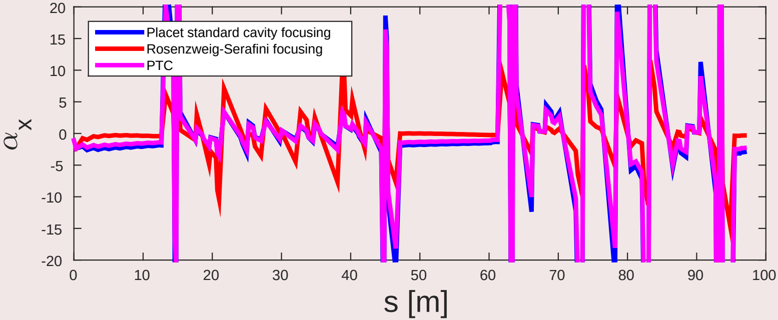

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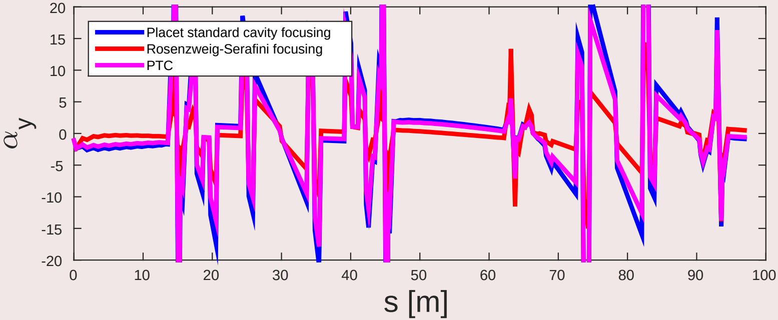


Extension to higher order modes (and TM cavities)

Possible extension

- If one knows the Floquet form of the longitudinal field (at the position of the particle)
- $$E_z = E_0 Re\left[\sum_{n=-\infty}^{\infty} b_n \exp(i(\omega t - k_n z))\right] = E_0 Re\left[\sum_{n=-\infty}^{\infty} b_n \exp\left[i\left(2\pi n\frac{z}{d} + \phi\right)\right]\right]$$

  where d is the cell length.
- One can determine the effect on the focusing through the parameter η.
- $$\eta(\phi) = \sum_{n=1}^{\infty} b_n^2 + b_{-n}^2 + 2b_n b_{-n}\cos(2\phi)$$
- Simple for a π-mode cavity: In this case $b_0 = b_{-1} = 1$, all other coefficients are 0.
- Travelling wave cavity: $b_0 = 0$, all other coefficients are $0 \rightarrow \eta = 0 \rightarrow$, the matrix reduces to the default PLACET focusing.
- HOMs are not implemented in PLACET, but could readily be added.

Content

CSR shielding in the DB complex

- Since CSR shielding in PLACET, we can apply it to simulations of the drive beam.
- Beam energy loss becomes smaller because it is travelling between parallel plates.
- An approximation, but gives a good idea about the behavior of the process.
- Let us apply the shielding to a section of the recombination complex as shown below.

Schematic view of the simulated lattice

Figure: Three turns in the first combiner ring.

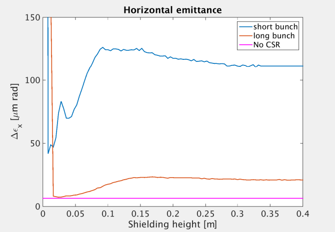
Some results

- With a factor 2 decompressed realistic bunch bunch the beneficial effects of shielding begins with larger plate separations.
- With a short bunch, one can almost recover the emittance growth due to csr, but this occurs at very small plate separations.

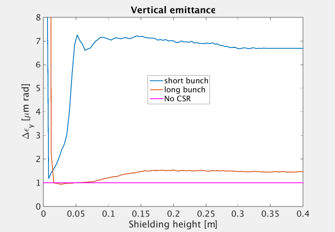
Variation with shielding height

- Emittance growth budget for the entire recombination complex is approximately 100nm rad.
- Use a realistic beam and see the behavior with respect to varying the distance between the parallel plates.

Horizontal ϵ growth



Vertical ϵ growth



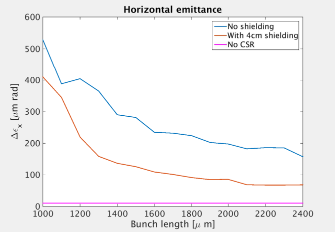
Some results

- Set the parallel plate separation to 4cm.
- Increasing the Gaussian bunch length by a factor 2 decreases the emittance growth by around a factor 4, in the shielded case.
- The decrease in emittance growth for the non-shielded CSR wake is slightly less steep.
- The emittance growth is approximately linear in the incoming emittance.

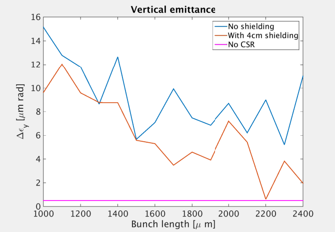
Variation with bunch length

- Emittance growth budget for the entire recombination complex is approximately 100nm rad.
- Observe the effect of increasing the Gaussian bunch length with and without shielding.

Horizontal ϵ growth



Vertical ϵ growth



Conclusions

- A model that efficiently and accurately models cavity focusing has been implemented in placet
- Works in cases where the energy gain is comparable to the particle energy.
- Also simulates off phase particles.
- Only models π-mode standing wave cavities accurately.
- The model does however allow for higher order modes. Possibility to model travelling wave cavities and cavities with several modes if needed.

- CSR shielding decreases the emittance growth in the drive beam complex by a significant amount.
- An increase of at least a factor 2 in bunch length from the DBL seems to be necessary even with shielding.