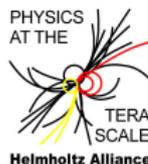


# Vector Boson Scattering at $e^+e^-$ machines & News from WHIZARD

Jürgen R. Reuter

DESY, Hamburg



Alboreanu/Kilian/JRR, **JHEP 0811** (2008) 010;

Beyer/Kilian/Krstonošić/Mönig/JRR/Schmitt/Schröder, **EPJC 48** (2006), 353;

JRR/Kilian/Sekulla, 1307.8170; Kilian/JRR/Ohl/Sekulla, 1408.6207 + in prep.

Kilian/Ohl/JRR, EPJ **C71** (2011) 1742

CLIC 2015 Workshop, CERN, Jan. 29th, 2015

# Motivation

- After discovery of light Higgs boson: what is left to do?
- Mechanism behind generating Higgs vev missing ( $\Rightarrow$  Higgs physics)
- Dynamics of EW interactions:  $\Rightarrow$  **Multiboson Interactions (MBI)**
- **Anomalous Triple Gauge Couplings**: dibosons
- **Anomalous Quartic Gauge Couplings**: tribosons, VV scattering
- Existing studies assume:  $\mathcal{P}(e^-) = 80 - 90\%$ ,  $\mathcal{P}(e^+) = 30 - 60\%$ 
  - Longitudinal polarization of beams: **(V - A) couplings of W/Z**
  - $e_L$  and  $e_R$  different multiplets  $\Rightarrow$  access completely different couplings

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- F. Gianotti, CLIC-Workshop 2014, CERN

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- Not really new: Elementary Particle Physics And Future Facilities. Proceedings, 1982 DPF Summer Study, Snowmass, USA, June 28 - July 16, 1982

## Extensions of the SM

- ▶ Lagrangian of the EW SM (no fermions/QCD here):

$$\mathcal{L}_{EW} = -\frac{1}{2} \text{tr} [W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

with building blocks:

$$D_\mu = \partial_\mu + \frac{i}{2} g \tau^I W_\mu^I + \frac{i}{2} g' B_\mu$$

$$W_{\mu\nu} = \frac{i}{2} g \tau^I (\partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g \epsilon_{IJK} W_\mu^J W_\nu^K)$$

$$B_{\mu\nu} = \frac{i}{2} g' (\partial_\mu B_\nu - \partial_\nu B_\mu)$$

- ▶ Any EFT has higher-dimensional operators:

Weinberg, 1979

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \left[ \frac{a_i}{\Lambda} \mathcal{O}_i^{(5)} + \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \frac{e_i}{\Lambda^4} \mathcal{O}_i^{(8)} \dots \right]$$

- ▶ without more fundamental theory  $\Rightarrow$  no clue on the scale (neither on the coefficients)

# Classification of Operators (I): Dim 6

(always  $v^2$  subtracted)

- Dimension-6 operators (CP-conserving)

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

$$\mathcal{O}_W = (D_{\mu}\Phi)^{\dagger} W^{\mu\nu} (D_{\nu}\Phi)$$

$$\mathcal{O}_B = (D_{\mu}\Phi)^{\dagger} B^{\mu\nu} (D_{\nu}\Phi)$$

$$\mathcal{O}_{\partial\Phi} = \partial_{\mu} (\Phi^{\dagger}\Phi) \partial^{\mu} (\Phi^{\dagger}\Phi)$$

$$\mathcal{O}_{\Phi W} = (\Phi^{\dagger}\Phi) \text{Tr}[W^{\mu\nu} W_{\mu\nu}]$$

$$\mathcal{O}_{\Phi B} = (\Phi^{\dagger}\Phi) B^{\mu\nu} B_{\mu\nu}$$

- Dimension-6 operators (CP-violating)

$$\mathcal{O}_{\widetilde{W}W} = \Phi^{\dagger} \widetilde{W}_{\mu\nu} W^{\mu\nu} \Phi$$

$$\mathcal{O}_{\widetilde{B}B} = \Phi^{\dagger} \widetilde{B}_{\mu\nu} B^{\mu\nu} \Phi$$

$$\mathcal{O}_{\widetilde{W}WW} = \text{Tr}[\widetilde{W}_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

$$\mathcal{O}_{\widetilde{W}} = (D_{\mu}\Phi)^{\dagger} \widetilde{W}^{\mu\nu} (D_{\nu}\Phi)$$

|                                 | ZWW | AWW | HWW | HZZ | HZA | HAA | WWWW | ZZWW | ZAWW | AAWW |
|---------------------------------|-----|-----|-----|-----|-----|-----|------|------|------|------|
| $\mathcal{O}_{WWW}$             | ✓   | ✓   |     |     |     |     | ✓    | ✓    | ✓    | ✓    |
| $\mathcal{O}_W$                 | ✓   | ✓   | ✓   | ✓   | ✓   |     | ✓    | ✓    | ✓    |      |
| $\mathcal{O}_B$                 | ✓   | ✓   |     | ✓   | ✓   |     |      |      |      |      |
| $\mathcal{O}_{\Phi d}$          |     |     | ✓   | ✓   |     |     |      |      |      |      |
| $\mathcal{O}_{\Phi W}$          |     |     | ✓   | ✓   | ✓   | ✓   |      |      |      |      |
| $\mathcal{O}_{\Phi B}$          |     |     |     | ✓   | ✓   | ✓   |      |      |      |      |
| $\mathcal{O}_{\widetilde{W}WW}$ | ✓   | ✓   |     |     |     |     | ✓    | ✓    | ✓    | ✓    |
| $\mathcal{O}_{\widetilde{W}}$   | ✓   | ✓   | ✓   | ✓   | ✓   |     |      |      |      |      |
| $\mathcal{O}_{\widetilde{W}W}$  |     |     | ✓   | ✓   | ✓   | ✓   |      |      |      |      |
| $\mathcal{O}_{\widetilde{B}B}$  |     |     |     | ✓   | ✓   | ✓   |      |      |      |      |

# Classification of Operators (II): Dim 8

(always  $v^2$  subtracted)

- Dimension-8 operators (only  $D_\mu \Phi$ )

$$\mathcal{O}_{S,0} = \left[ (D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[ (D^\mu \Phi)^\dagger D^\nu \Phi \right],$$

$$\mathcal{O}_{S,1} = \left[ (D_\mu \Phi)^\dagger D^\mu \Phi \right] \times \left[ (D_\nu \Phi)^\dagger D^\nu \Phi \right],$$

- Dimension-8 operators (only field strength/mixed)

$$\mathcal{O}_{T,0} = \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \cdot \text{Tr} [W_{\alpha\beta} W^{\alpha\beta}], \quad \mathcal{O}_{M,0} = \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \cdot \left[ (D_\beta \Phi)^\dagger D^\beta \Phi \right]$$

$$\mathcal{O}_{T,1} = \text{Tr} [W_{\alpha\nu} W^{\mu\beta}] \cdot \text{Tr} [W_{\mu\beta} W^{\alpha\nu}], \quad \mathcal{O}_{M,1} = \text{Tr} [W_{\mu\nu} W^{\nu\beta}] \cdot \left[ (D_\beta \Phi)^\dagger D^\mu \Phi \right]$$

$$\mathcal{O}_{T,2} = \text{Tr} [W_{\alpha\mu} W^{\mu\beta}] \cdot \text{Tr} [W_{\beta\nu} W^{\nu\alpha}], \quad \mathcal{O}_{M,2} = [B_{\mu\nu} B^{\mu\nu}] \cdot \left[ (D_\beta \Phi)^\dagger D^\beta \Phi \right],$$

$$\mathcal{O}_{T,5} = \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \cdot B_{\alpha\beta} B^{\alpha\beta}, \quad \mathcal{O}_{M,3} = [B_{\mu\nu} B^{\nu\beta}] \cdot \left[ (D_\beta \Phi)^\dagger D^\mu \Phi \right],$$

$$\mathcal{O}_{T,6} = \text{Tr} [W_{\alpha\nu} W^{\mu\beta}] \cdot B_{\mu\beta} B^{\alpha\nu}, \quad \mathcal{O}_{M,4} = \left[ (D_\mu \Phi)^\dagger W_{\beta\nu} D^\mu \Phi \right] \cdot B^{\beta\nu},$$

$$\mathcal{O}_{T,7} = \text{Tr} [W_{\alpha\mu} W^{\mu\beta}] \cdot B_{\beta\nu} B^{\nu\alpha}, \quad \mathcal{O}_{M,5} = \left[ (D_\mu \Phi)^\dagger W_{\beta\nu} D^\nu \Phi \right] \cdot B^{\beta\mu},$$

$$\mathcal{O}_{T,8} = B_{\mu\nu} B^{\mu\nu} B_{\alpha\beta} B^{\alpha\beta}, \quad \mathcal{O}_{M,6} = \left[ (D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\nu} D^\mu \Phi \right],$$

$$\mathcal{O}_{T,9} = B_{\alpha\mu} B^{\mu\beta} B_{\beta\nu} B^{\nu\alpha}, \quad \mathcal{O}_{M,7} = \left[ (D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\mu} D^\nu \Phi \right],$$

## Classification of Operators (III)

|                           | WWWW | WWZZ | ZZZZ | WWAZ | WWAA | ZZZA | ZZAA | ZAAA | AAAA |
|---------------------------|------|------|------|------|------|------|------|------|------|
| $\mathcal{O}_{S,0/1}$     | ✓    | ✓    | ✓    |      |      |      |      |      |      |
| $\mathcal{O}_{M,0/1/6/7}$ | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    |      |      |
| $\mathcal{O}_{M,2/3/4/5}$ |      | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    |      |      |
| $\mathcal{O}_{T,0/1/2}$   | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    |
| $\mathcal{O}_{T,5/6/7}$   |      | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    |
| $\mathcal{O}_{T,8/9}$     |      |      | ✓    |      |      | ✓    | ✓    | ✓    | ✓    |

- ▶ Dim. 8 operators generate aQGCs, but not aTGCs
- ▶ generate neutral quartics
- ▶ Redundancy of the operators:
  - Equations of motion:  $D_\mu W^{\mu\nu} = \Phi^\dagger (D^\nu \Phi) - (D^\nu \Phi)^\dagger \Phi + \dots$
  - Gauge symmetry structure:  $[D_\mu, D_\nu] \Phi \propto W_{\mu\nu} \Phi$
  - Integration by parts (up to total derivatives)
  - Leads to relations like:

$$\begin{aligned} \mathcal{O}_B &= \mathcal{O}_{\tilde{W}} + \frac{1}{2} \mathcal{O}_{WW} - \frac{1}{2} \mathcal{O}_{BB} \\ \mathcal{O}_{BW} &= -2 \mathcal{O}_W - \mathcal{O}_{WW} \\ \mathcal{O}_{\partial W} &= -4 \mathcal{O}_{WWW} + \text{gauge-fermion operators} \end{aligned}$$

# EFT coefficients vs. anomalous couplings

- Switch operator bases (vertex-dep.): Snowmass EW White Paper, 1310.6708

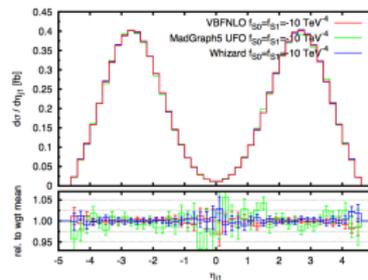
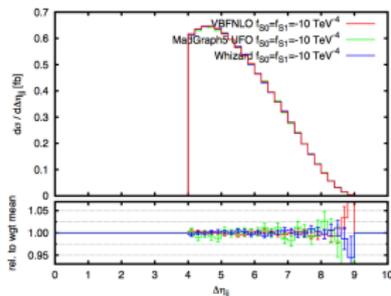
$$\begin{aligned} \text{WWWW-Vertex:} \quad \alpha_4 &= \frac{f_{S,0}}{\Lambda^4} \frac{v^4}{8} \\ \alpha_4 + 2 \cdot \alpha_5 &= \frac{f_{S,1}}{\Lambda^4} \frac{v^4}{8} \end{aligned}$$

$$\begin{aligned} \text{WWZZ-Vertex:} \quad \alpha_4 &= \frac{f_{S,0}}{\Lambda^4} \frac{v^4}{16} \\ \alpha_5 &= \frac{f_{S,1}}{\Lambda^4} \frac{v^4}{16} \end{aligned}$$

ZZZZ-Vertex:

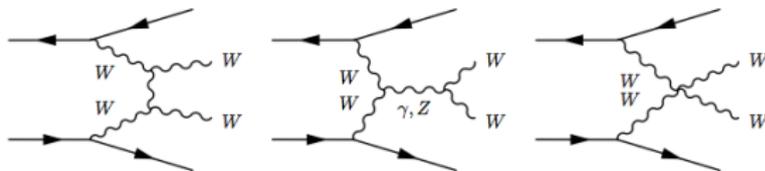
$$\alpha_4 + \alpha_5 = \left( \frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,1}}{\Lambda^4} \right) \frac{v^4}{16}$$

- Full agreement among generators: VBF@NLO, WHIZARD, Madgraph

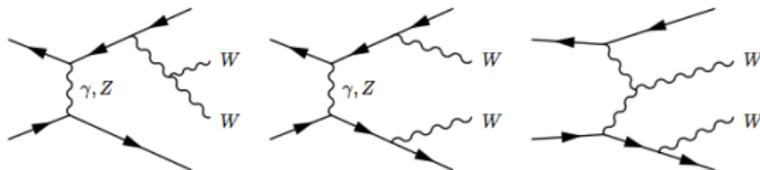


# Vector Boson Scattering at $e^+e^-$ machines

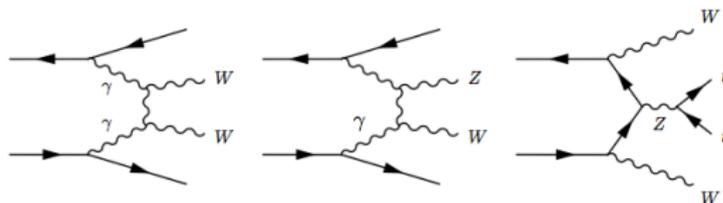
Signal



Irreducible bkgd.



(Partially) reducible bkgd.



# Vector Boson Scattering

Beyer et al., hep-ph/0604048

1 TeV,  $1 \text{ ab}^{-1}$ , full  $6f$  final states, 80 %  $e^-_R$ , 60 %  $e^-_L$  polarization, binned likelihood

Contributing channels:  $WW \rightarrow WW, WW \rightarrow ZZ, WZ \rightarrow WZ, ZZ \rightarrow ZZ$

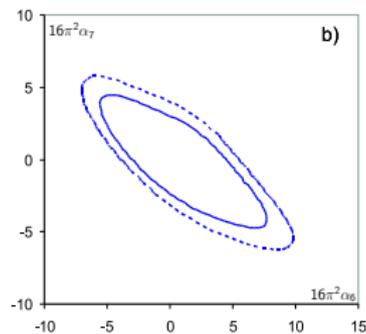
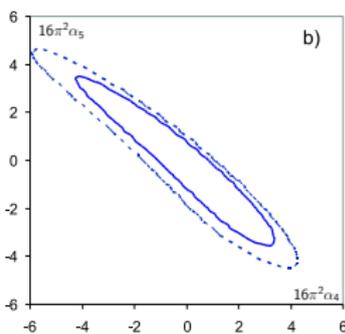
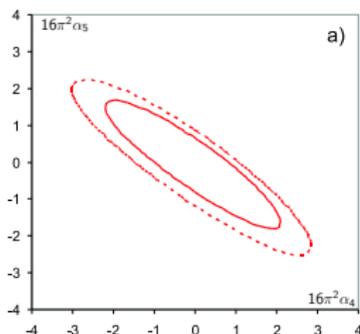
| Process  | Subprocess                     | $\sigma$ [fb] |
|--|--------------------------------|---------------|
| $e^+e^- \rightarrow \nu_e \bar{\nu}_e q \bar{q} q \bar{q}$ | $WW \rightarrow WW$            | 23.19         |
| $e^+e^- \rightarrow \nu_e \bar{\nu}_e q \bar{q} q \bar{q}$ | $WW \rightarrow ZZ$            | 7.624         |
| $e^+e^- \rightarrow \nu \bar{\nu} q \bar{q} q \bar{q}$     | $V \rightarrow VVV$            | 9.344         |
| $e^+e^- \rightarrow \nu e q \bar{q} q \bar{q}$             | $WZ \rightarrow WZ$            | 132.3         |
| $e^+e^- \rightarrow e^+e^- q \bar{q} q \bar{q}$            | $ZZ \rightarrow ZZ$            | 2.09          |
| $e^+e^- \rightarrow e^+e^- q \bar{q} q \bar{q}$            | $ZZ \rightarrow W^+W^-$        | 414.          |
| $e^+e^- \rightarrow b \bar{b} X$                           | $e^+e^- \rightarrow t \bar{t}$ | 331.768       |
| $e^+e^- \rightarrow q \bar{q} q \bar{q}$                   | $e^+e^- \rightarrow W^+W^-$    | 3560.108      |
| $e^+e^- \rightarrow q \bar{q} q \bar{q}$                   | $e^+e^- \rightarrow ZZ$        | 173.221       |
| $e^+e^- \rightarrow e \nu q \bar{q}$                       | $e^+e^- \rightarrow e \nu W$   | 279.588       |
| $e^+e^- \rightarrow e^+e^- q \bar{q}$                      | $e^+e^- \rightarrow e^+e^- Z$  | 134.935       |
| $e^+e^- \rightarrow X$                                     | $e^+e^- \rightarrow q \bar{q}$ | 1637.405      |

$SU(2)_c$  conserved case, all channels

| coupling           | $\sigma^-$ | $\sigma^+$ |
|--------------------|------------|------------|
| $16\pi^2 \alpha_4$ | -1.41      | 1.38       |
| $16\pi^2 \alpha_5$ | -1.16      | 1.09       |

$SU(2)_c$  broken case, all channels

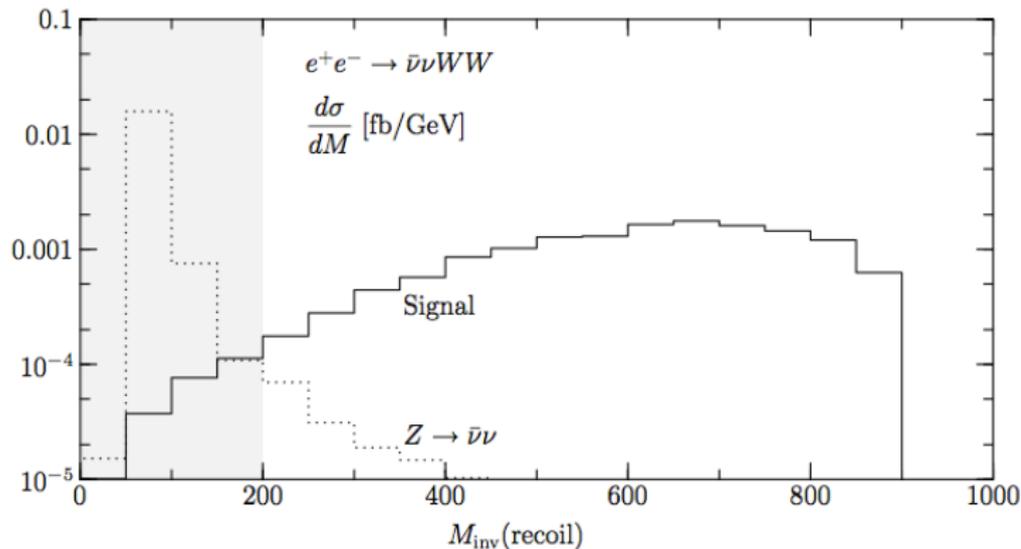
| coupling              | $\sigma^-$ | $\sigma^+$ |
|-----------------------|------------|------------|
| $16\pi^2 \alpha_4$    | -2.72      | 2.37       |
| $16\pi^2 \alpha_5$    | -2.46      | 2.35       |
| $16\pi^2 \alpha_6$    | -3.93      | 5.53       |
| $16\pi^2 \alpha_7$    | -3.22      | 3.31       |
| $16\pi^2 \alpha_{10}$ | -5.55      | 4.55       |



# Vector Boson Scattering: Observables

## Study of $WW$ scattering @ 1.6 TeV

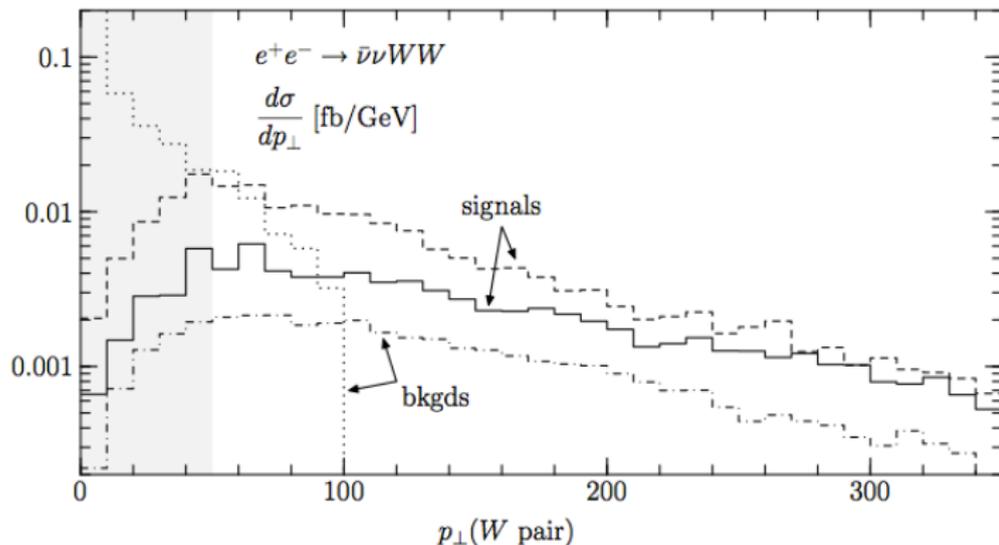
Boos/Kilian/He/Mühlleitner/Pukhov/Zerwas, hep-ph/9708310



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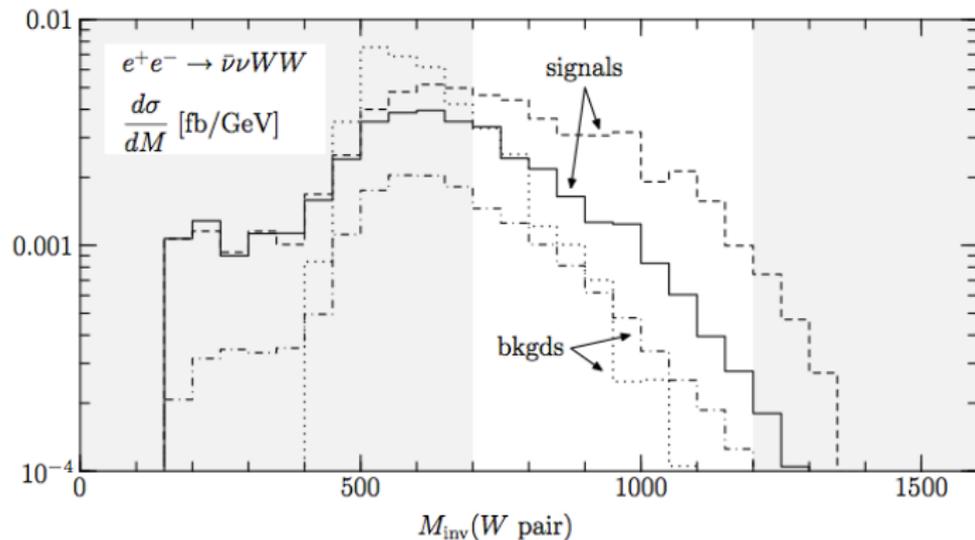
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# Simplified Models for VBS (and VVV): Resonances

- ▶ Resonances in all accessible spin/isospin channels
- ▶ Couplings to the Higgs and gauge sectors are unrelated and arbitrary
- ▶ Still include anomalous couplings
- ▶ Unitarization (later)

## New physics in electroweak sector:

- ▶ Narrow resonances  $\Rightarrow$  particles (**weakly interacting model**)
- ▶ Wide resonances  $\Rightarrow$  continuum (**strongly interacting model**)

$SU(2)_c$  custodial symmetry (weak isospin, broken by hypercharge  
 $g' \neq 0$  and fermion masses)

|         | $J = 0$   | $J = 1$                         | $J = 2$                  |
|---------|---|---------------------------------|--------------------------|
| $I = 0$ | $\sigma^0$ (Higgs ?)                                | $\omega^0$ ( $\gamma'/Z'$ ?)    | $f^0$ (Graviton ?)       |
| $I = 1$ | $\pi^\pm, \pi^0$ (2HDM ?)                           | $\rho^\pm, \rho^0$ ( $W'/Z'$ ?) | $a^\pm, a^0$             |
| $I = 2$ | $\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?) | —                               | $t^{\pm\pm}, t^\pm, t^0$ |

- ▶  $I = 0$ : resonant in  $W^+W^-$  and  $ZZ$  scattering
- ▶  $I = 1$ : resonant in  $W^+Z$  and  $W^-Z$  scattering
- ▶  $I = 2$ : resonant in  $W^+W^+$  and  $W^-W^-$  scattering

## Example: a Scalar Resonance [Not counting $\phi$ with $M = 126$ GeV.]

- ▶ Mass  $M_\sigma$ .
- ▶ Coupling to the Higgs sector (Higgs and longitudinal  $W/Z$ ):

$$g_L^\sigma (D_\mu \Phi)^\dagger (D^\mu \Phi) \sigma$$

- ▶ Coupling to the gauge sector (transversal  $W/Z$ ):

$$g_T^\sigma \text{tr} [\mathbf{W}^{\mu\nu} \mathbf{W}_{\mu\nu}] \sigma$$

### Possible Origin: 2HDM isosinglet (renormalizable)

$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}, \quad g_T^\sigma = O\left(\frac{1}{4\pi M_\sigma}\right) \quad \text{[loop]}$$

### Possible Origin: new strong interactions

$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}, \quad g_T^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}$$

# Interpretation as limits on resonances

Beyer et al., hep-ph/0604048

Consider the width to mass ratio,  $f_\sigma = \Gamma_\sigma/M_\sigma$

1 TeV  $e^+e^-$

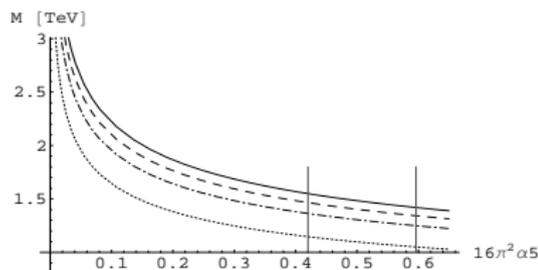
$SU(2)$  conserving scalar singlet

$SU(2)$  broken vector triplet

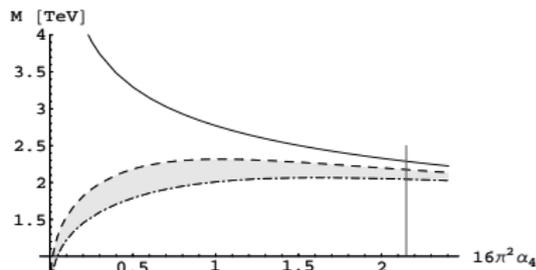
needs input from TGC covariance matrix

$$M_\sigma = v \left( \frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$

$$M_{\rho^\pm} = v \left( \frac{12\pi\alpha_4 f_{\rho^\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2 (\alpha_4^\lambda)^2 / (2c_w^2)} \right)^{\frac{1}{4}}$$



$f = 1.0$  (full),  $0.8$  (dash),  $0.6$  (dot-dash),  $0.3$  (dot)



upper/lower limit from  $\lambda_Z$ , grey area: magnetic moments

**Final  
result:**

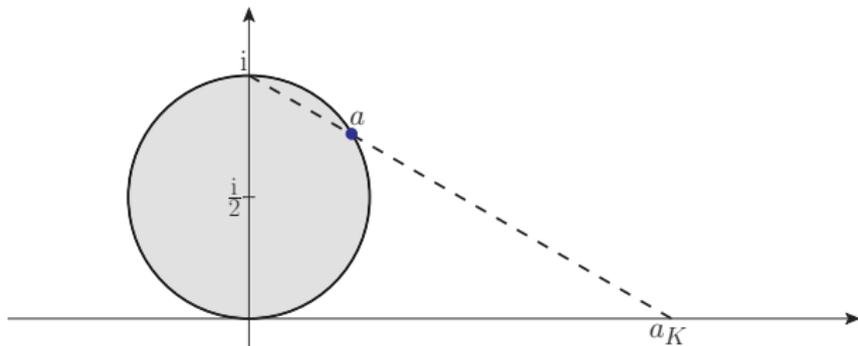
| Spin | $I = 0$ | $I = 1$ | $I = 2$ |
|------|---------|---------|---------|
| 0    | 1.55    | –       | 1.95    |
| 1    | –       | 2.49    | –       |
| 2    | 3.29    | –       | 4.30    |

| Spin | $I = 0$ | $I = 1$ | $I = 2$ |
|------|---------|---------|---------|
| 0    | 1.39    | 1.55    | 1.95    |
| 1    | 1.74    | 2.67    | –       |
| 2    | 3.00    | 3.01    | 5.84    |

# Unitarizing $S$ matrices

- ▶ **Cayley transform of  $S$  matrix:**  $S = \frac{1+iK/2}{1-iK/2}$  Heitler, 1941; Schwinger, 1948
- ▶ “ $K$ ” matrix: translates to transition operator:  $T = \frac{K}{1-iK/2}$
- ▶ Works beyond perturbation theory, but allows perturbative expansion
- ▶ Diagonalize  $S$  matrix (partial waves):  

$$\mathcal{M}(s, t, u) = 32\pi \sum_{\ell} (2\ell + 1) \mathcal{A}_{\ell}(s) P_{\ell}(\cos \theta)$$
 (“Power spectrum”)
- ▶ Complex eigenvalues:  $t = 2a$        $k = 2a_K \Rightarrow a_K = \frac{a}{1+ia}$
- ▶ Corresponds to stereographic projection:



- ▶ **Coulomb singularities** Bloch/Nordsieck, 1937; Yennie/Frautschi/Suura, 1961

# Unitarization Primer

Kilian/JRR/Ohl/Sekulla, 1408.6207

- ▶ Unitarization prescription not unique
- ▶ Padé (reordering pert. series) introduces artificial poles
- ▶ Form factors parameterize close-by new physics (additional parameters)

$$\frac{1}{\left(1 + \frac{s}{\Lambda_{FF}^2}\right)^n}$$

unphysical form factor scale  $\Lambda_{FF}$  and multipole order  $n$

- ▶ minimal version ( $K$  or  $T$  matrix)  $\Rightarrow$  just saturation no new parameters, does not rely on pert. expansion, stable against small perturbations
- ▶ Additional known features (resonances) should be implemented before unitarization

# Unitarity Bound for $\alpha_4$ AQQC

## Bounds for $\alpha_4$

$$\ell = 0 : \sqrt{s} \leq \left( \frac{6\pi}{\alpha_4} \right)^{\frac{1}{4}} v \approx \frac{0.5 \text{ TeV}}{\sqrt[4]{\alpha_4}}$$

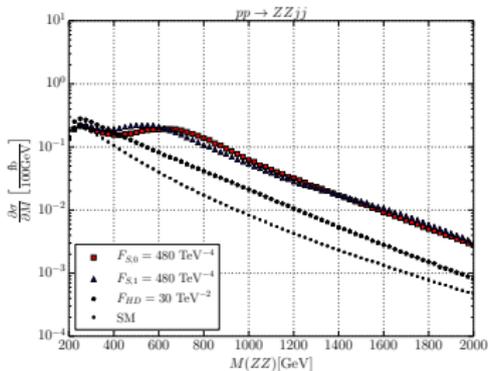
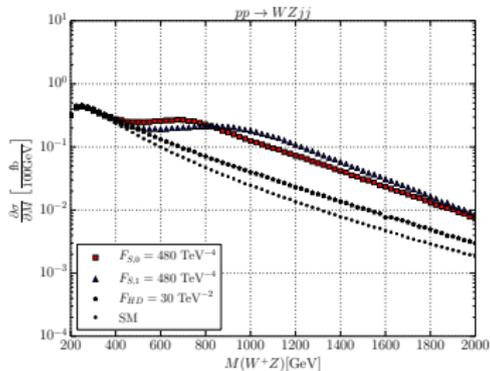
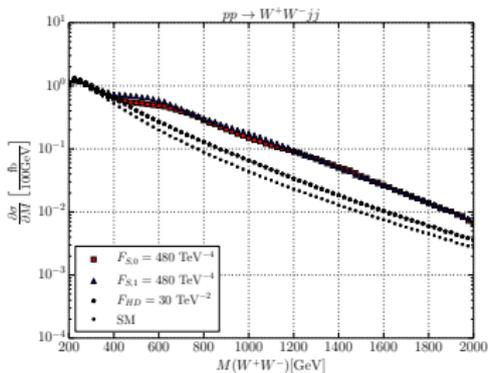
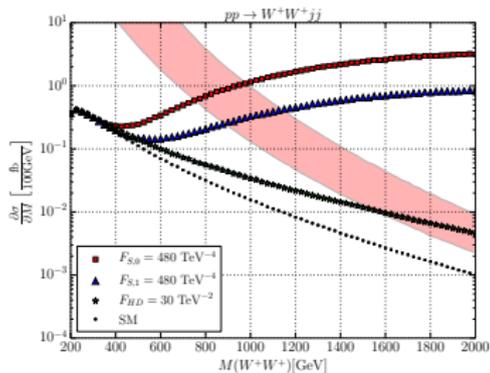
$$\ell = 2 : \sqrt{s} \leq \left( \frac{60\pi}{\alpha_4} \right)^{\frac{1}{4}} v \approx \frac{0.9 \text{ TeV}}{\sqrt[4]{\alpha_4}}$$

- ▶ Bound **depends** on coupling  $\alpha_4$
  
- ▶ Use strongest bound

$\alpha_4$  AQQC contribution to  
 $WW \rightarrow ZZ$

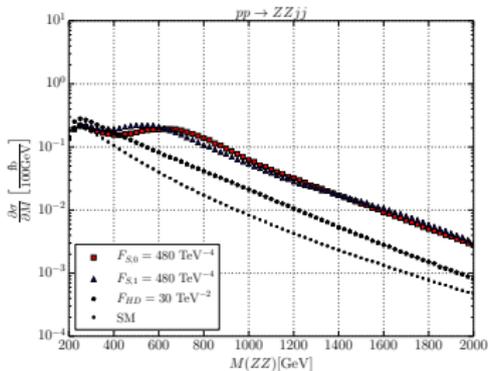
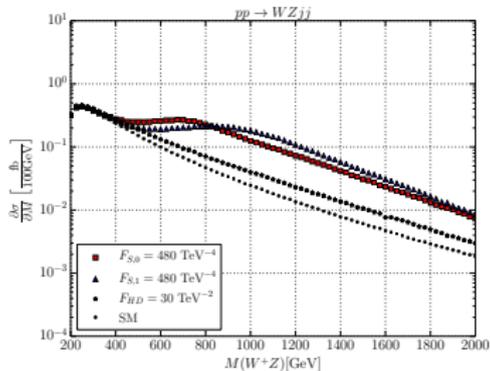
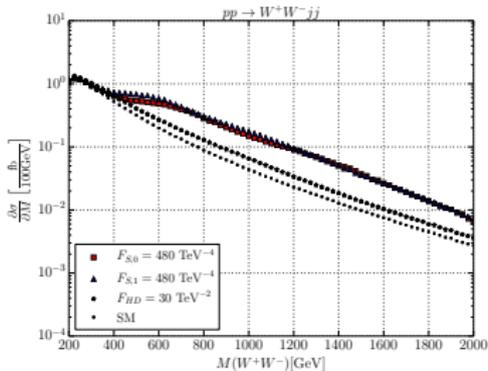
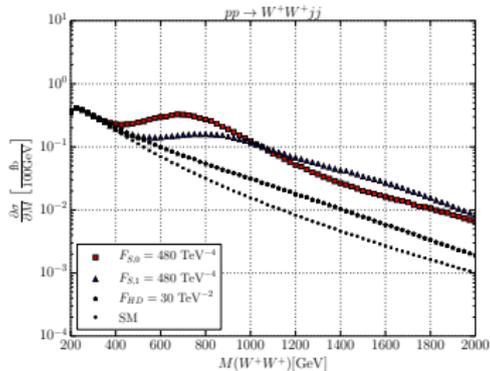
$$\mathcal{A}(s, t, u) = 4\alpha_4 \frac{t^2 + u^2}{v^4}$$

# Diboson invariant masses



General cuts:  $M_{jj} > 500 \text{ GeV}$ ;  $\Delta\eta_{jj} > 2.4$ ;  $p_T^j > 20 \text{ GeV}$ ;  $|\eta_j| < 4.5$

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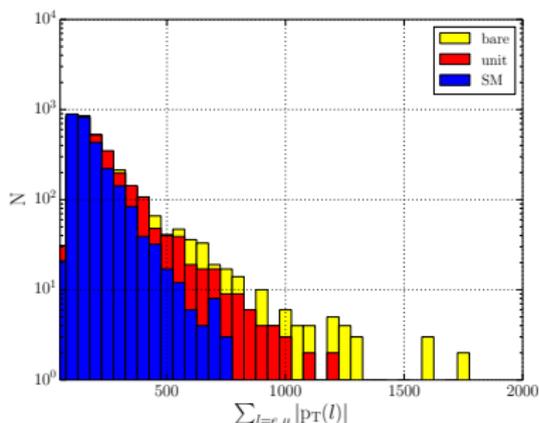
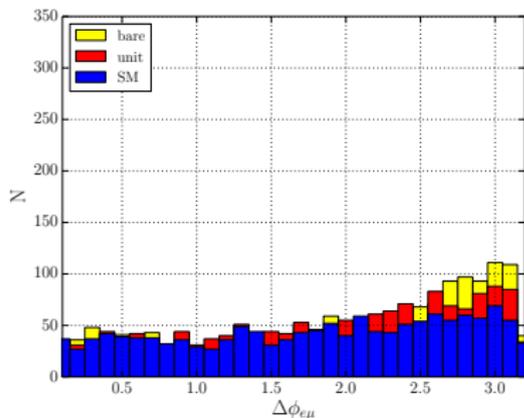
## $p_T$ and angular distributions

$$pp \rightarrow e^+ \mu^+ \nu_e \nu_\mu jj, \sqrt{s} = 14 \text{ TeV}, \mathcal{L} = 1000 \text{ fb}^{-1}$$

Simulations with WHIZARD →

Not possible to use automated tool due to  $s$ -channel prescription

$$F_{HD} = 30 \text{ TeV}^{-2}$$



General cuts:  $M_{jj} > 500 \text{ GeV}$ ;  $\Delta\eta_{jj} > 2.4$ ;  $p_T^j > 20 \text{ GeV}$ ;  $|\eta_j| < 4.5$ ,  $p_T^\ell > 20 \text{ GeV}$

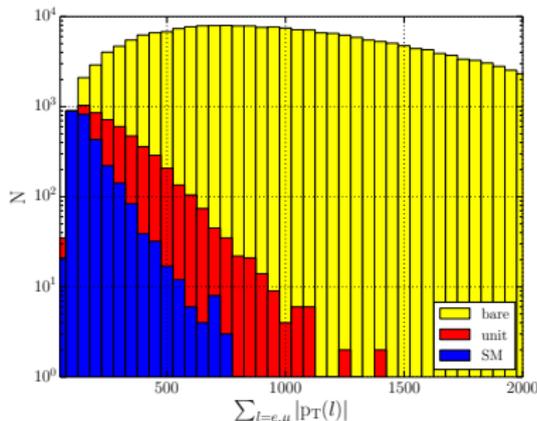
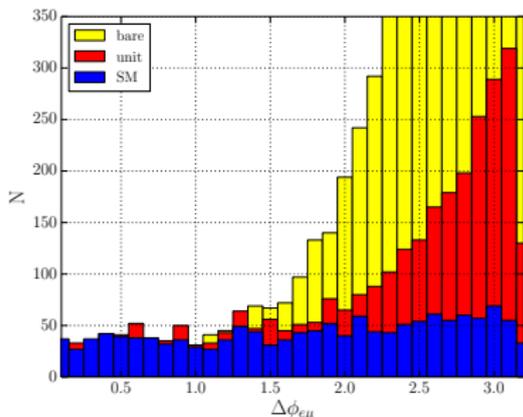
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$$F_{S,0} = 480 \text{ TeV}^{-4}$$



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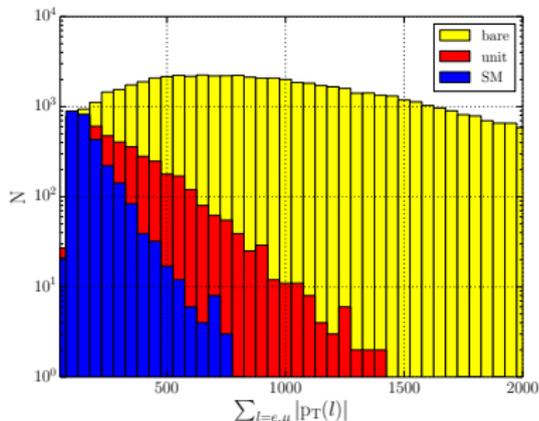
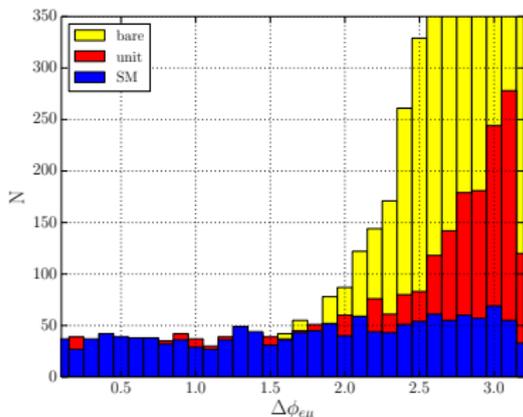
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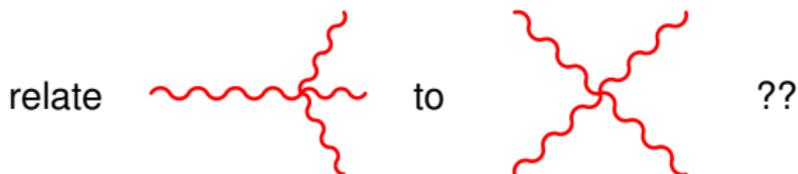
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# And Triple Vector Boson Production?



Yes, the same Feynman graphs (in the SM), but. . . Tribosons:

- one external  $W/Z/\gamma$  is always far off-shell
- Unitarization formalism not available
- different (anom.) couplings contribute (**particularly for resonances**)

$$\sigma(e^+e^- \rightarrow VVV) \propto \frac{1}{s} \quad \begin{array}{l} \text{Limits usefulness to subprocess energies} \\ \text{in the lower range where cross section} \\ \text{of fusion process still small} \end{array}$$

$$\sigma_{\text{VBS}}(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-) \propto \log(s)$$

$$\left. \begin{array}{l} e^+e^- \rightarrow ZZZ \\ \rightarrow WWZ \end{array} \right\} \begin{array}{l} ZH \\ \hookrightarrow WW \\ \hookrightarrow ZZ \end{array} \quad \text{Present in spectrum}$$

$$\rightarrow WW\gamma \quad \text{Complementary (and present at lower energies)}$$

⇒ Important physics **independent** w.r.t. VBS. Don't just combine results!

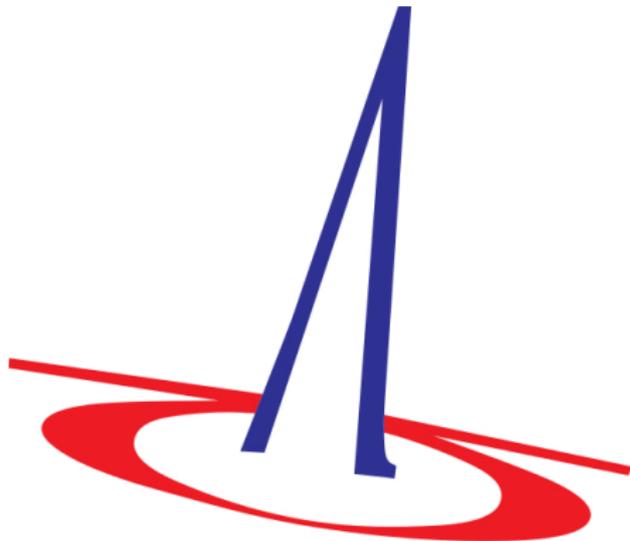


# Advertisement: MBI 2015 @ DESY

2.-4. Sept. 2015, DESY, Hamburg



# Switching gears: WHIZARD



# WHIZARD in a Nutshell

WHIZARD universal event generator for processes at colliders:  $e^+e^-$ ,  $pp$ ,  $p\bar{p}$ ,  $\gamma\gamma$ ,  $ep$  etc.

1. O'Mega: **Optimized automatic matrix elements** for arbitrary elementary processes, supports SM and many BSM extensions
2. **Phase-space** parameterization module (**very efficient PS**)
3. **VAMP: Generic adaptive Monte Carlo integration and (unweighted) event generation**
4. **CIRCE1/2**: Lepton/[photon] collider beam spectra
5. Intrinsic support or external interfaces for: Feynman rules, beams cascade decays, shower, hadronization, analysis, event file formats, etc.
6. Free-format steering language **SINDARIN**

**WHIZARD 2.2.3**

release: Nov. 30, 2014



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**WHIZARD 2.2.4**

release: Febr. 06, 2015

**The WHIZARD team:** F. Bach, B. Chokouf , **W. Kilian**, **T. Ohl**, **JRR**, M. Sekulla, F. Staub, C. Weiss, DESY summer students



**Web address:** <http://projects.hepforge.org/whizard>

**Standard Reference:** Kilian/Ohl/JRR, EPJ **C71** (2011) 1742, arXiv:0708.4233

# WHIZARD 2: Status 2010-15 – Technical Features

- Fortran2003/2008 (gfortran 4.7.4 or newer) and OCaml (for MEs)
- WHIZARD core: separate interface from implementation **Complete object orientation**
  - ▶ **Replaceable modules** with well-defined interface: matrix-elements, beam structure, phase space, integration, decays, shower, ...
  - ▶ Much easier to outsource small(er) projects
  - ▶ **Much better self checks, regression testing and maintainability**
- OpenMP **parallelization**
- Operation modes: dynamic linking, static linking, library mode, shell mode
- **Standard conformance**: uses autotools: automake/autoconf/libtool  
Installation: ./configure, make, make check, make install
- Large self test suite: unit tests, feature tests, run tests
- Version control (svn) at [HepForge](#): use of **ticket system** and **bug tracker**
- Continuous integration system (jenkins) linked with svn repository

# WHIZARD Manual

with distribution and online: <http://whizard.hepforge.org/manual>

• WHIZARD

• HOME

- Main Page

• MANUAL, WIKI, NEWS

- Manual
- Wiki Page
- News
- ChangeLog

• REPOSITORY, BUG TRACKER

- Subversion Repository
- SVN Browser
- Bug Tracker

• DOWNLOADS

- Download Page

• CONTACT

- Contact us

• INTERNAL WHIZARD PAGE

- You Shall Not Pass!

## WHIZARD 2.2

### A generic Monte-Carlo integration and event generation package for multi-particle processes

## MANUAL

Wolfgang Kilian,<sup>1</sup> Thorsten Ohl,<sup>1</sup> Jürgen Reuter,<sup>1</sup> with contributions from Fabian Bach,<sup>1</sup> Sebastian Schmidt,<sup>1</sup> Christian Speckner,<sup>1</sup> Florian Staub.<sup>1</sup>

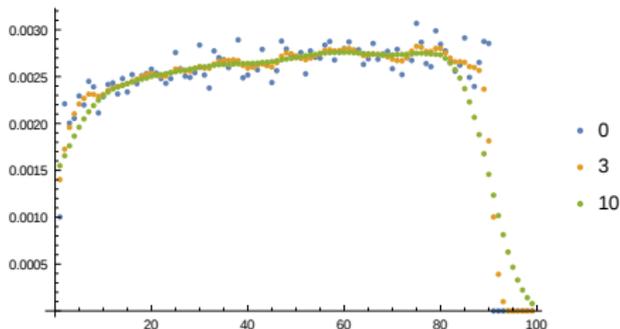
- Contents
- Chapter 1 Introduction
  - 1.1 Disclaimer
  - 1.2 Overview
  - 1.3 Historical remarks
  - 1.4 About examples in this manual
- Chapter 2 Installation
  - 2.1 Package Structure
  - 2.2 Prerequisites
  - 2.3 Installation
  - 2.4 Working With WHIZARD
  - 2.5 Troubleshooting
- Chapter 3 Getting Started
  - 3.1 Hello World
  - 3.2 A Simple Calculation
- Chapter 4 Steering WHIZARD: SINDARIN Overview
  - 4.1 The command language for WHIZARD
  - 4.2 SINDARIN scripts
  - 4.3 Errors
  - 4.4 Statements
  - 4.5 Control Structures
  - 4.6 Expressions
  - 4.7 Variables

## Correlated lepton beam spectra with Circe2

- ▶ Guinea-Pig++ event files too short for high lumi simulations
- ▶ Fixed width histogramming struggles with steep distributions
- ▶ Circe1 too restrictive, assumes
  - ▶ factorized beam spectra:  $D_{p_1 p_2}(x_1, x_2) = D_{p_1}(x_1)D_{p_2}(x_2)$
  - ▶ power laws in continuum:  $D(x) = d \cdot \delta(1 - x) + c \cdot x^\alpha(1 - x)^\beta$
- ▶ Circe2 algorithm:
  - ▶ Adapt 2D factorized variable width histogram (à la VEGAS) to steep part of distribution
  - ▶ smooth the correlated fluctuations with a moderate gaussian filter to suppress artifacts from limited Guinea-Pig++ statistics
  - ▶ smooth separately continuum/boundary bins (avoid artificial beam energy spread)

Smoothing  $x_{e^+} = 1$  boundary bin with Gaussian filters of width 3 and 10 bins, resp. 5 bins reasonable compromise for histograms with 100 bins.

[bins are *not* equidistant, shrink with power law towards the  $x_{e^-} = 1$  boundary on RHS!]



# Workflow Guinea-Pig++/Circe2/WHIZARD

## 1. Run Guinea-Pig++ with

```
do_lumi=7;num_lumi=100000000;num_lumi_eg=100000000;num_lumi_gg=100000000;
```

to produce lumi.[eg][eg].out with  $(E_1, E_2)$  pairs.

[Large event numbers, as Guinea-Pig++ will produce only a small fraction!]

## 2. Run circe2\_tool.opt with steering file

```
{ file="ilc500/beams.circe" # to be loaded by WHIZARD
  { design="ILC" roots=500 bins=100 scale=250 # E in [0,1]
    { pid/1=electron pid/2=positron pol=0 # unpolarized e-/e+
      events="ilc500/lumi.ee.out" columns=2 # <= Guinea-Pig
      lumi = 1564.763360 # <= Guinea-Pig
      iterations = 10 # adapting bins
      smooth = 5 [0,1] [0,1] # Gaussian filter 5 bins
      smooth = 5 [1] [0,1] smooth = 5 [0,1] [1] } } }
```

to produce correlated beam description

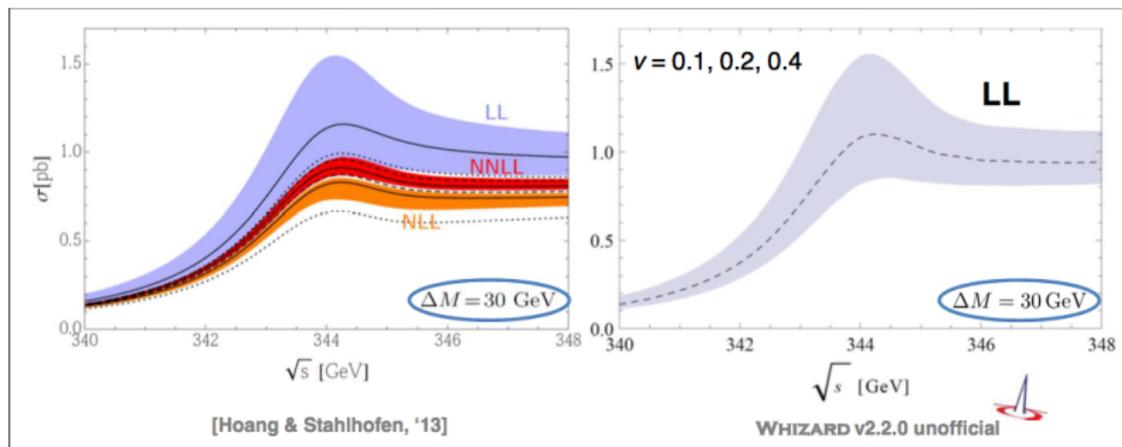
## 3. Run WHIZARD with SINDARIN input:

```
beams = e1, E1 => circe2
$circe2_file = "ilc500.circe"
$circe2_design = "ILC"
?circe2_polarized = false
```

- Soon also files for polarized beams within distribution

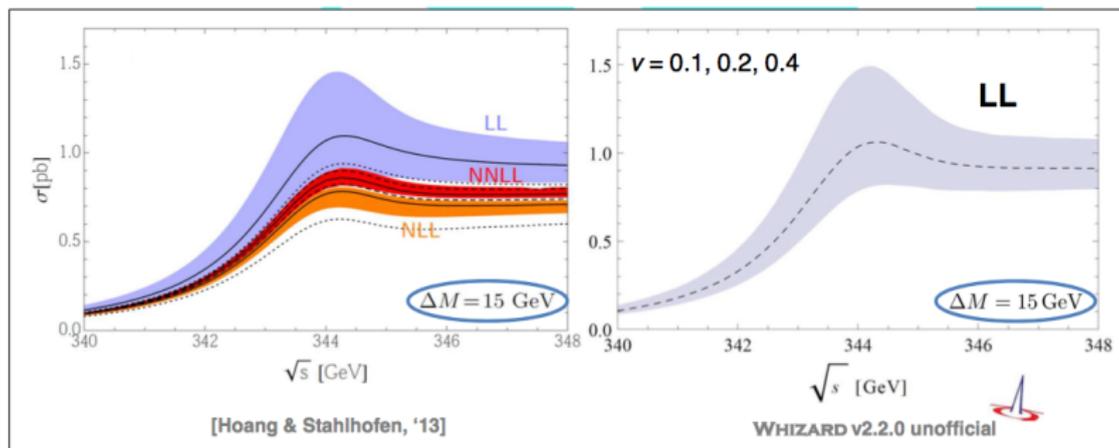
# Top quark threshold in $e^+e^-$

- ▶  $e^+e^-$  top threshold scan offers best option for  $m_t$
- ▶ now: analytic LL  $ttV$  form factor implemented Bach/JRR/Stahlhofen
- ▶ default parameters:  $M^{1S} = 172$  GeV,  $\Gamma_t = 1.5$  GeV,  $\alpha_s(M^{1S}) = 0.1077$
- ▶ analytic LL unstable far off-shell: top mass cut  $\Delta M_t \leq 30$  GeV



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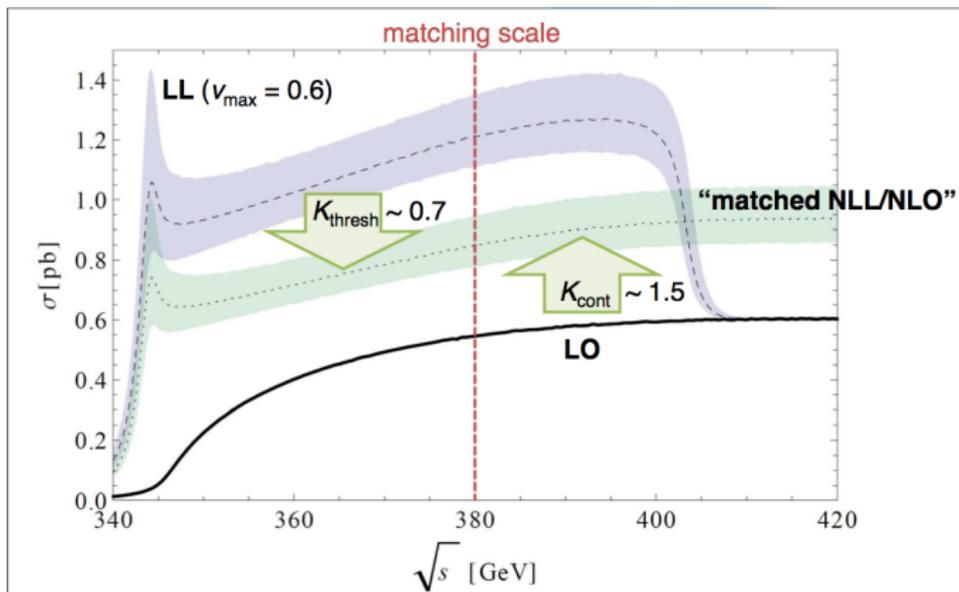


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- ▶ Proper NLO/NLL matched implementation
- ▶ TOPPIK code ships with WHIZARD
- ▶ Own model: `SM_tt_threshold`
- ▶ Parameters: `wtop, m1S, vsoft, match`

Bach/Hoang/JRR/Stahlhofen/Teubner

courtesy to T. Teubner

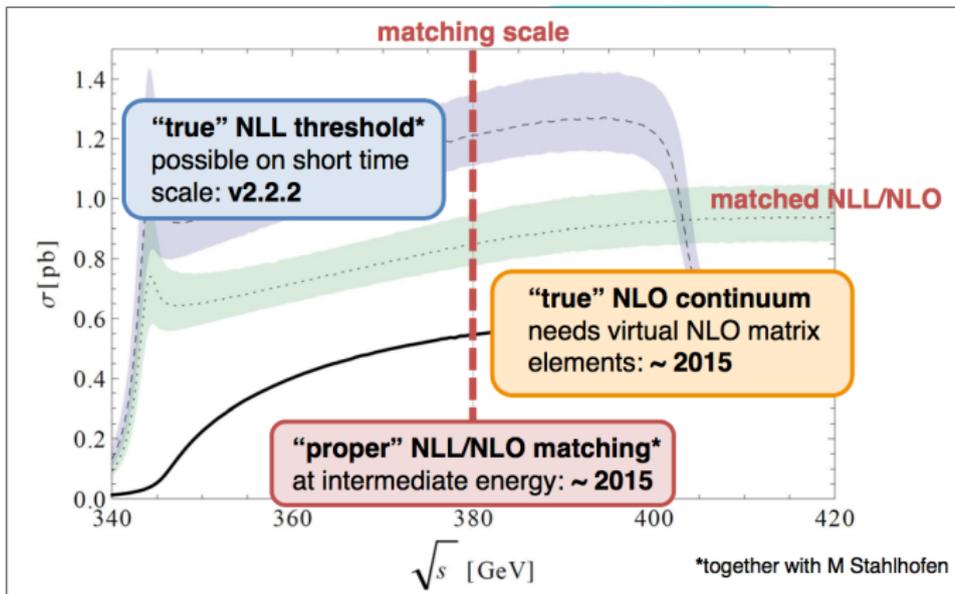


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# Status of NLO automation in WHIZARD

- ▶ **BLHA(2) interface** MC / OLP programs Speckner/JRR/Weiss, 2014 ✓
- ▶ First implementation: GoSAM (also FeynArts/FormCalc,OpenLoops)
- ▶ **Work flow / Plans**
  - Automatic generation of subtraction terms Speckner, 2012; Kilian/JRR/Weiss, 2014
  - proof-of-concept code in WHIZARD 2.2

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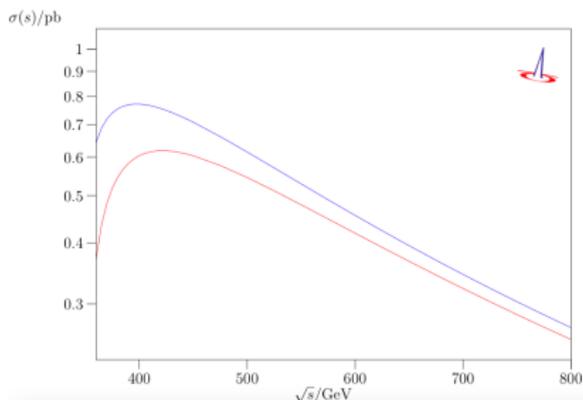
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- Plan: PowHeg box formalism for NLO processes (w/ Matching)
- Release: WHIZARD 3.0



# WHIZARD Outlook: upcoming releases 2.3 – 2.4 – 3.0

## ► New features in production version 2.2

- LHAPDF 6 support, FastJet interface ✓
- ILC TDR beam spectra, CLIC (correlated) spectra (CIRCE1/2) ✓
- Direct Guinea-Pig interface ✓
- LCIO support test phase (✓)
- Complete Reweighting of Event Samples (incl. LHEF 2013) ✓
- Process containers: inclusive production samples (e.g. SUSY) ✓
- Automatic generation of decays, depending on the model ✓
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## ► Features in preparation: 2.3 – 2.4 – 3.0

- BSM: general Lorentz structures in matrix-element generator (O'Mega)
- Performance: parallelization, flavor sums, MC over helicities/colors, PS, etc.
- (N)LL/(N)LO matched  $e^+e^-$  top threshold Bach/Hoang/JRR/Stahlhofen/Teubner
- New syntax/features decays and chains (steering unstable particles):

```
process higgsstr = e1, E1 => (Z => e2, E2), (H => b, bbar)
```

- Improved matching/merging for jets/photons Chokouf /JRR/Kilian/Weiss, ca. 2015
- Specification of QCD and electroweak order
- Automatic QCD NLO corrections (test phase)
- Matched  $e^+e^- \rightarrow X$  at LO/NLO, POWHEG box Chokouf /JRR/Weiss, 2015

# 2nd International WHIZARD Forum 16.-18.3.2015

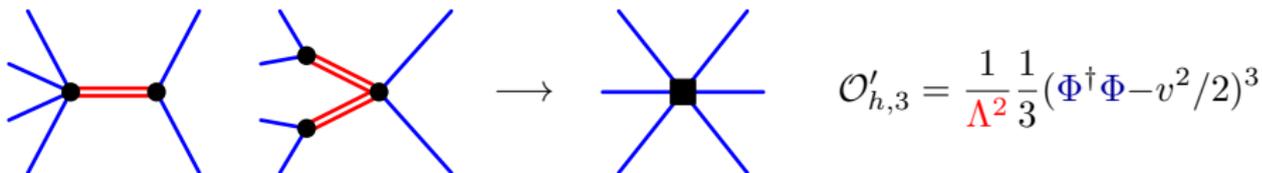
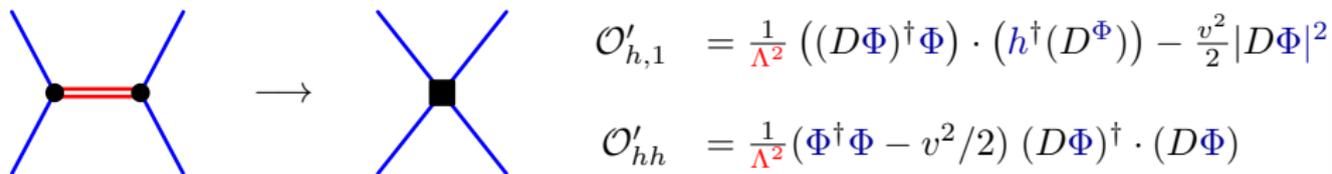
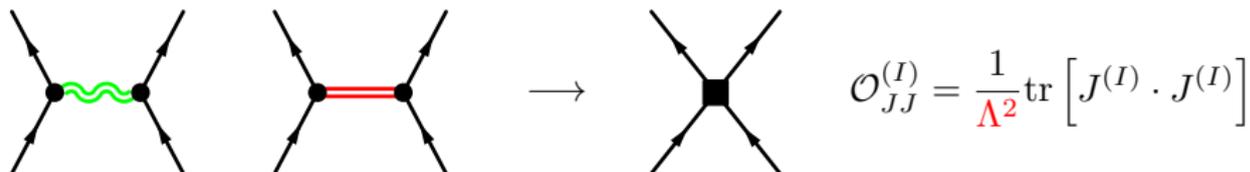
- SM and BSM physics
- LHC, ILC, CLIC, FCC collider physics
- Matrix elements, models and effective theories
- QED/QCD/Weak Radiation and Merging
- Higher Orders: Automation and Interfacing
- User interfaces, computing and performance & Event formats

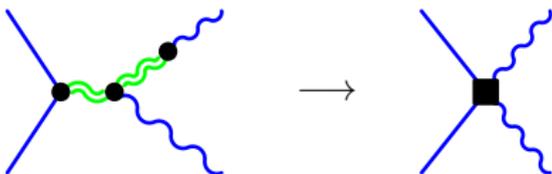


BACKUP SLIDES:

# Effective EW Dim. 6 Operators

Hagiwara/Hikasa/Peccei/Zeppenfeld, 1987; Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993

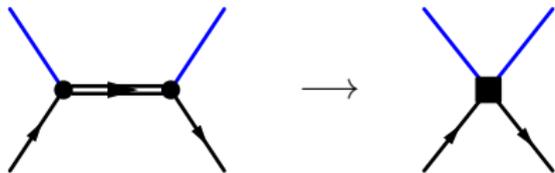




$$\mathcal{O}_{\Phi W} = -\frac{1}{\Lambda^2} \frac{1}{2} (\Phi^\dagger \Phi - v^2/2) \text{tr} [W_{\mu\nu} W^{\mu\nu}]$$

$$\mathcal{O}_B = \frac{1}{\Lambda^2} \frac{i}{2} (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)$$

$$\mathcal{O}_{\Phi B} = -\frac{1}{\Lambda^2} \frac{1}{4} (\Phi^\dagger \Phi - v^2/2) B_{\mu\nu} B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{\Lambda^2} \bar{q} h (\not{D} h) q$$

# Integrating out resonances

- ▶ Simplest example: scalar singlet  $\sigma$ :

$$\mathcal{L}_\sigma = -\frac{1}{2} [\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v \sigma \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] - h_\sigma \text{tr} [\mathbf{T} \mathbf{V}_\mu] \text{tr} [\mathbf{T} \mathbf{V}^\mu]]$$

- ▶ Effective Lagrangian  $\mathcal{L}_\sigma^{\text{eff}} = \frac{v^2}{8M_\sigma^2} \left[ g_\sigma \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] + h_\sigma \text{tr} [\mathbf{T} \mathbf{V}_\mu] \text{tr} [\mathbf{T} \mathbf{V}^\mu] \right]^2$

- ▶ leads to **anomalous quartic couplings (aQGCs)**

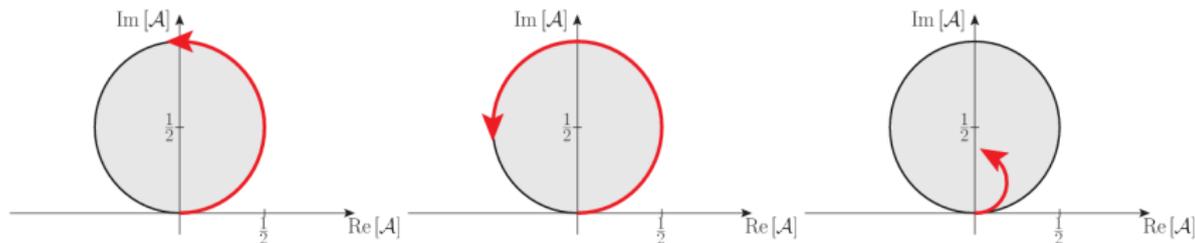
$$\alpha_5 = g_\sigma^2 \left( \frac{v^2}{8M_\sigma^2} \right) \quad \alpha_7 = 2g_\sigma h_\sigma \left( \frac{v^2}{8M_\sigma^2} \right) \quad \alpha_{10} = 2h_\sigma^2 \left( \frac{v^2}{8M_\sigma^2} \right)$$

| Resonance                                  | $\sigma$       | $\phi$          | $\rho$                                       | $f$            | $a$            |
|--|----------------|-----------------|--|----------------|----------------|
| $\Gamma[g^2 M^2 / (64\pi v^2)]$            | 6              | 1               | $\frac{4}{3} \left( \frac{v^2}{M^2} \right)$ | $\frac{1}{5}$  | $\frac{1}{30}$ |
| $\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$ | 0              | $\frac{1}{4}$   | $\frac{3}{4}$                                | $\frac{5}{2}$  | $-\frac{5}{8}$ |
| $\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$ | $\frac{1}{12}$ | $-\frac{1}{12}$ | $-\frac{3}{4}$                               | $-\frac{5}{8}$ | $\frac{35}{8}$ |

# Unitary Description of EW interactions

## ► Five possible cases:

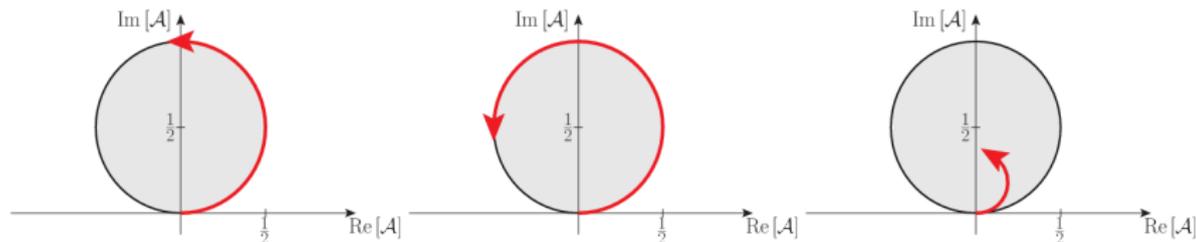
- Amplitude perturbative, close to zero, small imag. part (SM)
- Amplitude rises, gets imag. part, strongly interacting regime (presence of at least one dim. 8 operator)
- Amplitude approaches maximum absolute value asymptotically
- Turn over: new resonance
- New inelastic channels open: eff. form factor, extra channels observable in multi-vector boson processes



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- Interpretation of EFT operator coefficients changes: formally still low-energy coefficients of Taylor expansion  $\Rightarrow$  threshold parameters
- Complete description necessary (only) beyond threshold

# Unitarity of Amplitudes

**UV-incomplete theories could violate unitarity**

Cross section: 
$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2$$

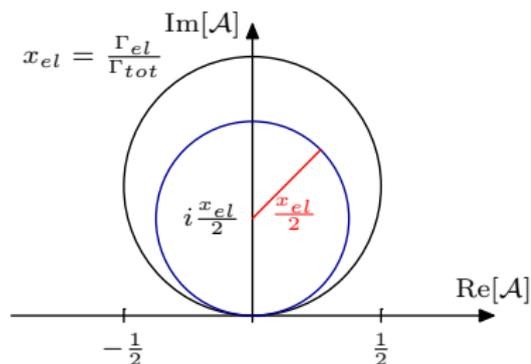
**Optical Theorem** (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im} [\mathcal{M}_{ii}(t=0)] / s \quad t = -s(1 - \cos\theta)/2$$

Partial wave amplitudes: 
$$\mathcal{M}(s, t, u) = 32\pi \sum_{\ell} (2\ell + 1) \mathcal{A}_{\ell}(s) P_{\ell}(\cos\theta)$$

Assuming only elastic scattering:

$$\sigma_{\text{tot}} = \sum_{\ell} \frac{32\pi(2\ell+1)}{s} |\mathcal{A}_{\ell}|^2 \stackrel{!}{=} \sum_{\ell} \frac{32\pi(2\ell+1)}{s} \text{Im} [\mathcal{A}_{\ell}] \Rightarrow \boxed{|\mathcal{A}_{\ell}|^2 = \text{Im} [\mathcal{A}_{\ell}]}$$



**Argand circle**

$$\boxed{|\mathcal{A}(s) - \frac{i}{2}| = \frac{1}{2}}$$

Resonance: 
$$\mathcal{A}(s) = \frac{-M\Gamma_{\text{el}}}{s - M^2 + iM\Gamma_{\text{tot}}}$$

Counterclockwise circle, **radius**  $\frac{x_{el}}{2}$

Pole at  $s = M^2 - iM\Gamma_{\text{tot}}$

# Unitarization Prescriptions

- ▶  **$K$ -matrix unitarization prescription** Gupta, 1950; Berger/Chanowitz, 1991
  - Hermitian  $K$ -matrix interpreted as incompletely calculated approximation to true amplitude
  - $\Rightarrow$  Unitary  $S, T$  as a non-perturbativ completion of this approximation
  - Insert pert. expansion into expansion:

$$a = \frac{a_K}{1 - ia_K} \Rightarrow a^{(n)} = \frac{a_0^{(1)} + \text{Re}a_0^{(2)} + \dots}{1 - i(a_0^{(1)} + \text{Re}a_0^{(2)} + \dots)}$$

- Prescription does a partial resummation of perturbative series
- Example Dyson resummation:  $a_K^{(0)}(s) = \frac{\lambda}{s - m^2} \rightarrow a^{(0)}(s) = \frac{\lambda}{s - m^2 - i\lambda}$
- ▶ **Drawbacks of (original)  $K$ -matrix:**
  - Needs to construct self-adjoint  $K$ -matrix as intermediate step
  - Problem if  $S$ -matrix is not diagonal, or ...  
there are non-perturbative contributions

- ▶  **$T$ -matrix unitarization**

- $a_0$  complex approximation to eigenvalue of true  $T$  matrix
- use again pseudo-stereographic projection (intersection of Argand circle with line  $\overline{a_0} i$ )
- Results in:  $a = \frac{\text{Re}a_0}{1 - ia_0^*} \Rightarrow a^{(n)} = \frac{a_0^{(1)} + \text{Re}a_0^{(2)} + \dots}{1 - i(a_0^{(1)} + \text{Re}a_0^{(2)} - i\text{Im}a_0^{(2)} + \dots)}$

# Form Factor

## Form Factor

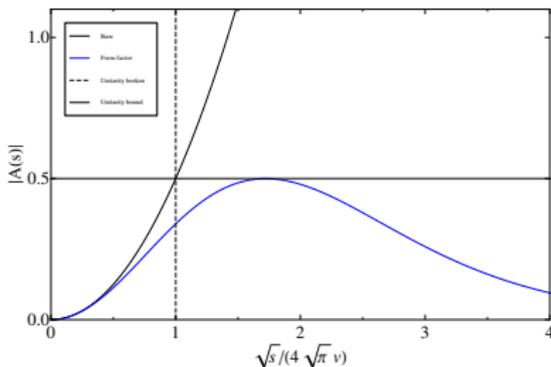
$$\frac{1}{\left(1 + \frac{s}{\Lambda_{FF}^2}\right)^n}$$

- ▶ Use Form Factor to suppress breaking of unitarity
- ▶ Can be generally used for arbitrary anomalous operator
- ▶ Need "Fine Tuning"

## Parameters

$n$  Chosen to prevent breaking of Unitarity

$\Lambda_{FF}$  Calculate highest possible value that satisfy real Unitarity bound (0th partial wave)

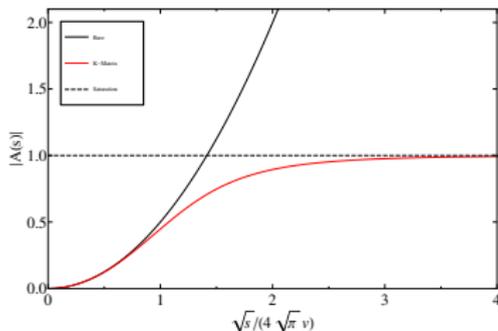
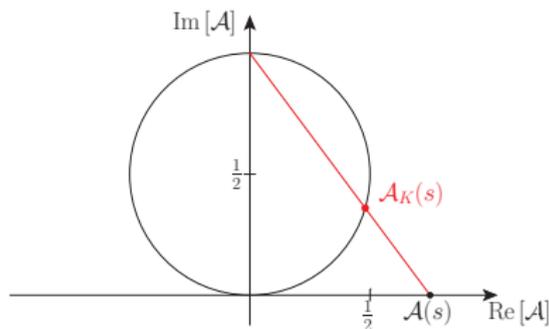


# K-Matrix

## K-Matrix Unitarisation

$$\begin{aligned} \mathcal{A}_K(s) &= \frac{1}{\operatorname{Re}\left(\frac{1}{\mathcal{A}(s)}\right) - i} \\ &= \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} \quad \text{if } \mathcal{A}(s) \in \mathbb{R} \end{aligned}$$

- ▶ Projection of elastic amplitudes onto Argand-Circle
- ▶ At high energies the amplitude saturates
- ▶ Is usable for complex amplitudes
- ▶ Not dependent on additional parameters

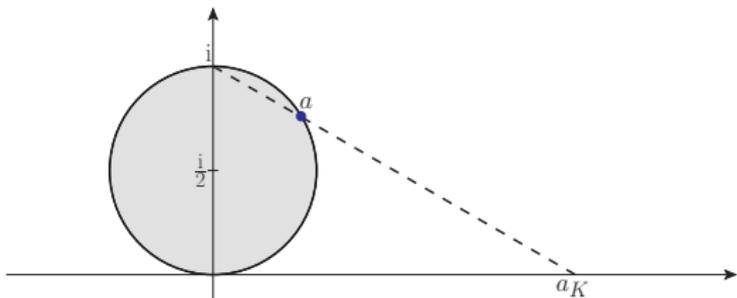


# Alternative Unitarization Prescriptions

## ► Comparison of $T$ -matrix and (original) $K$ -matrix:

- $T$ -matrix does not rely on perturbation theory
  - Special treatment for non-normal  $T$  matrices (eigenvalues having imaginary parts larger than  $i$ ; Riesz-Dunford operator calculus)
1.  $T$  matrix description leads to point on the Argand circle
  2. For real  $a \Rightarrow$  (original)  $K$ -matrix case
  3.  $a_0$  on Argand circle  $\Rightarrow$  left invariant

## ► Thales circle construction:



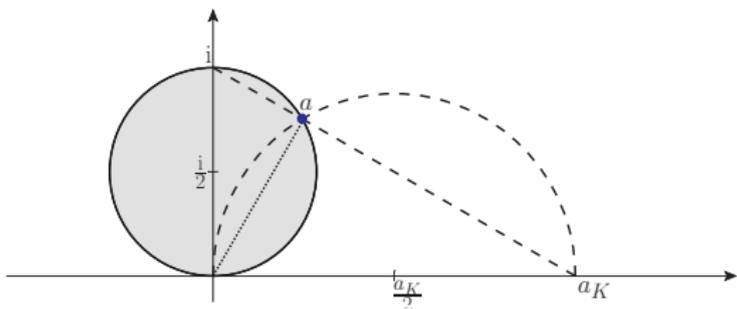
- Defined via  $\left| a - \frac{a_K}{2} \right| = \frac{a_K}{2} \Rightarrow a = \frac{1}{\operatorname{Re}\left(\frac{1}{a_0}\right) - i}$
- avoids non-normal matrices, but not single-valued around  $a = 0$

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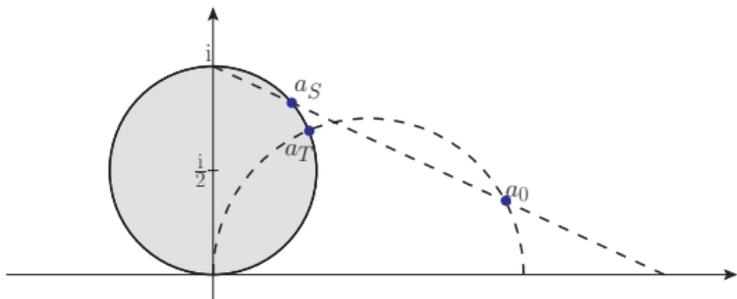
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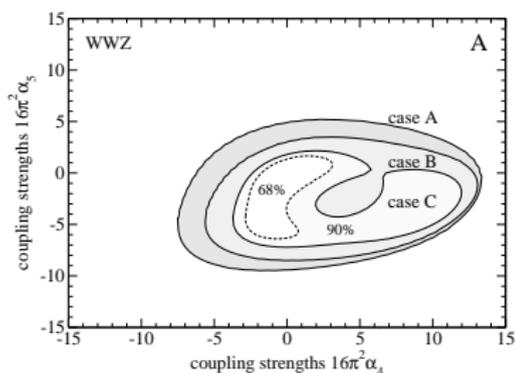
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# ILC Results: Triboson production

Beyer et al., hep-ph/0604048

$e^+e^- \rightarrow WWZ/ZZZ$ , dep. on  $(\alpha_4 + \alpha_6)$ ,  $(\alpha_5 + \alpha_7)$ ,  $\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JRR

1 TeV,  $1 \text{ ab}^{-1}$ , full 6-fermion final states, SIMDET fast simulation

Observables:  $M_{WW}^2$ ,  $M_{WZ}^2$ ,  $\sphericalangle(e^-, Z)$

A) unpol., B) 80%  $e_R^-$ , C) 80%  $e_R^-$ , 60%  $e_L^+$

| $16\pi^2 \times$   | WWZ     |            |           | ZZZ     | best  |
|--------------------|---------|------------|-----------|---------|-------|
|                    | no pol. | $e^-$ pol. | both pol. | no pol. |       |
| $\Delta\alpha_4^+$ | 9.79    | 4.21       | 1.90      | 3.94    | 1.78  |
| $\Delta\alpha_4^-$ | -4.40   | -3.34      | -1.71     | -3.53   | -1.48 |
| $\Delta\alpha_5^+$ | 3.05    | 2.69       | 1.17      | 3.94    | 1.14  |
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32 % hadronic decays

Durham jet algorithm

Bkgd.  $t\bar{t} \rightarrow 6 \text{ jets}$

Veto against  $E_{\text{mis}}^2 + p_{\perp, \text{mis}}^2$

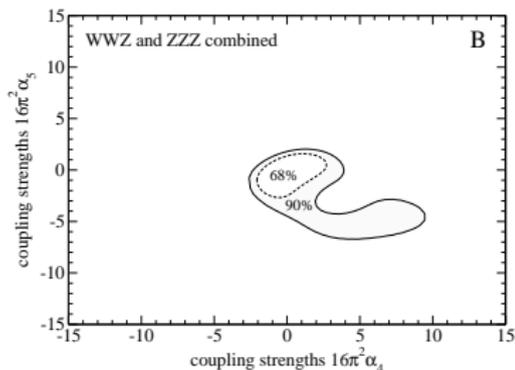
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# The WHIZARD Event Generator – Release 2.2

- 1.0 Project started around 1999: Studies for electroweak multi-particle processes at TESLA (W, Higgs, Z)
  - 1.5 Event samples for LC studies at SLAC
  - 1.9 Full SM w/ QCD, beam properties, SUSY/BSM, event formats
  - 2.1 QCD shower+matching, FeynRules support, internal density-matrix formalism (cascade decays), SINDARIN as user interface, OpenMP, ...
  - 2.2 Major refactoring, event reweighting, inclusive processes and selective decay chains **(production version)**
- Plan** Improve  $e^+e^-$  support; NLO + matching; improve user interface  $\Rightarrow$  adapt to specific needs of user groups

# Beams and hard matrix elements

## ▶ Hadron Colliders structured beams

- LHAPDF interface, most prominent PDFs directly included
- QCD ISR and FSR (2 diff. own implementations, interface to PYTHIA)
- Matching/merging matrix elements/showers
- Underlying event/multiple interactions (proof of principle)

## ▶ Hadronic events/hadronic decays + hadronic (QED) FSR (ext.)

## ▶ Lepton Colliders structured beams

- Beam structure (CIRCE1/2 module) **more later**
- arbitrarily polarized beams (density matrices)
- QED ISR (Skrzypek/Jadach, Kuraev/Fadin, incl.  $p_T$  distributions [caveat!])
- [Photon collider spectra (CIRCE2 module)]

## ▶ Hard matrix elements:

- Particle spins:  $0, \frac{1}{2}, 1, \frac{3}{2}, 2$
- Lorentz structures: high set of hard-coded structures
- Fully general Lorentz structures foreseen for 2.3.0
- Color structures:  $\mathbf{3}, \bar{\mathbf{3}}, \mathbf{8}, [\mathbf{6}]$
- Color flow formalism
- General color structures  $\mathbf{6}, \mathbf{10}, \epsilon_{ijk} \phi^i \phi^j \phi^k$

Stelzer/Willenbrock, 2003; Kilian/Ohl/JRR/Speckner, 2011

# WHIZARD – Overview over Physics Models

| MODEL TYPE                                      | with CKM matrix | trivial CKM     |
|---|-----------------|-----------------|
| QED with $e, \mu, \tau, \gamma$                 | –               | QED             |
| QCD with $d, u, s, c, b, t, g$                  | –               | QCD             |
| <b>Standard Model</b>                           | SM_CKM          | SM              |
| <b>SM with anomalous gauge coupl.</b>           | SM_ac_CKM       | SM_ac           |
| <b>SM with anomalous top coupl.</b>             | SMtop_CKM       | SMtop           |
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| SM with anom. Higgs coupl.                      | –               | SM_rx / NoH     |
| SM ext. for VV scattering                       | –               | SSC / Alth      |
| SM with $Z'$                                    | –               | Zprime          |
| 2HDM  | 2HDM_CKM        | 2HDM            |
| MSSM  | MSSM_CKM        | MSSM            |
| MSSM with gravitinos                            | –               | MSSM_Grav       |
| NMSSM   | NMSSM_CKM       | NMSSM           |
| extended SUSY models                            | –               | PS/E/SSM        |
| Littlest Higgs                                  | –               | Littlest        |
| Littlest Higgs with ungauged $U(1)$             | –               | Littlest_Eta    |
| Littlest Higgs with $T$ parity                  | –               | Littlest_Tpar   |
| Simplest Little Higgs (anomaly-free/univ.)      | –               | Simplest[_univ] |
| 3-site model                                    | –               | Threshl         |
| UED   | –               | UED             |
| SM with gravitino and photino                   | –               | GravTest        |
| Augmentable SM template                         | –               | Template        |

new models easily: FeynRules interface [Christensen/Duhr/Fuks/JRR/Speckner, 1010.3251](#)

Interface to SARAH in the SUSY Toolbox [Staub, 0909.2863; Ohl/Porod/Speckner/Staub, 1109.5147](#)

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JRR et al. 1408.6207  
Talk LCWS14 EW session 7.10.

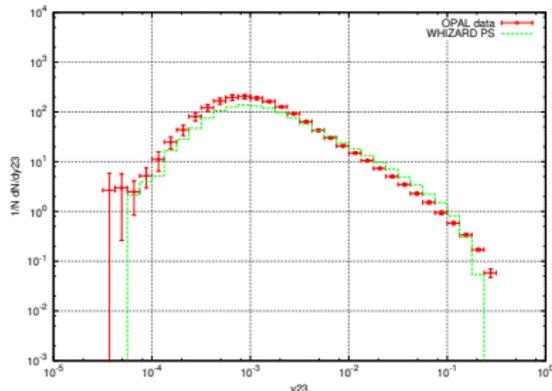
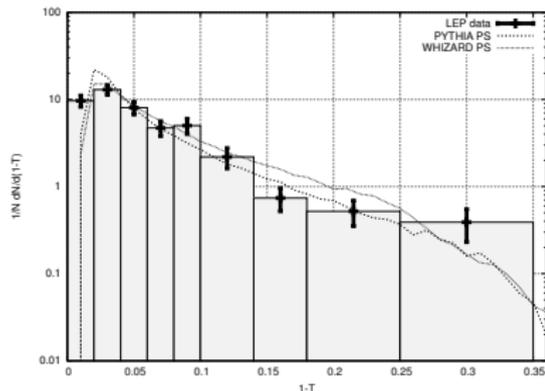
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# Analytic Parton Shower

Kilian/JRR/Schmidt/Wiesler, JHEP **1204** 013 (2012)

- ▶ **Analytic Parton Shower:**
  - no shower veto: shower history is exactly known
  - allows reweighting and maybe more reliable error estimate
- ▶ new algorithm for initial state QCD radiation

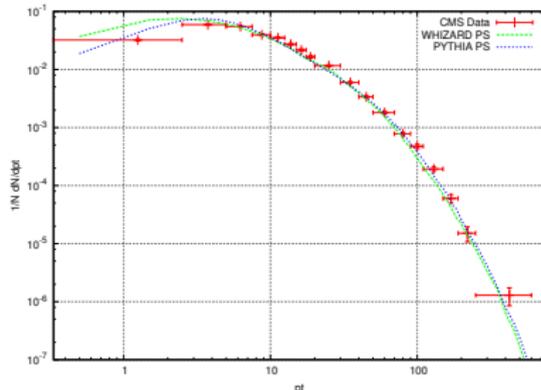
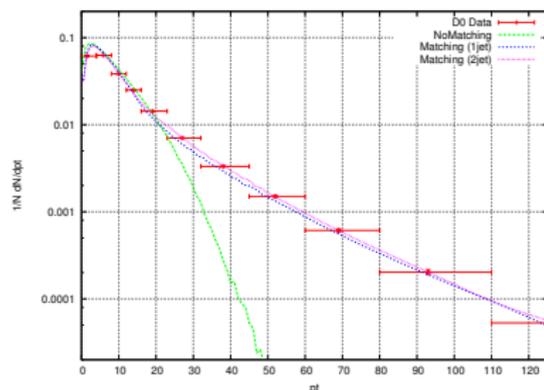


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- ▶ matching with hard matrix elements, no "power-shower"
- ▶ Improvement/Tuning/Merging with higher-order matrix elements