

# Quark Properties from the Resonance Gas Model

Eugenio Megías<sup>1</sup>, **E. Ruiz Arriola**<sup>2</sup> and L.L. Salcedo<sup>2</sup>

<sup>1</sup>Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)  
Föhringer Ring 6, 80805 Munich, Germany

<sup>2</sup>Departamento de Física Atómica, Molecular y Nuclear,  
Universidad de Granada, Spain.

Excited QCD 2015  
Tatranska Lomnica, Slovakia  
(8-14 March 2015)

- **QCD:** Phys.Lett. B563 (2003) 173-178, Phys.Rev. D69 (2004) 116003,
- **Polyakov-Nambu–Jona-Lasinio:** hep-ph/0410053; AIP Conf.Proc. 756 (2005) 436-438, Phys.Rev. D74 (2006) 065005, Rom.Rep.Phys. 58 (2006) 081-086. PoS JHW2005 (2006) 025. AIP Conf.Proc. 892 (2007) 444-447. Eur.Phys.J. A31 (2007) 553-556.
- **Dim-2 Condensates:**JHEP 0601 (2006) 073, Phys.Rev. D75 (2007) 105019. Indian J.Phys. 85 (2011) 1191-1196. Nucl.Phys.Proc.Suppl. 186 (2009) 256-259. Phys.Rev. D81 (2010) 096009.
- **Hadron Resonance Gas for Polyakov loop:** Phys.Rev.Lett. 109 (2012) 151601. arXiv:1207.4875 [hep-ph]. arXiv:1207.7287 [hep-ph] Acta Phys. Polon. B **45**, 2407 (2014)
- **Polyakov loop Spectroscopy:** Phys.Rev. D89 (2014) 076006 109.

- The Hadron Spectrum
- Quarks and gluons at finite temperature
- Hadron Resonance Gas from Chiral Quark Models
- Conclusions

- The partition function of QCD

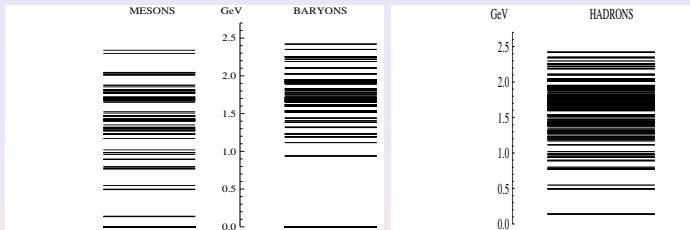
$$Z_{\text{QCD}} = \sum_n e^{-E_n/T} \quad H_{\text{QCD}}\psi_n = E_n\psi_n$$

- Spectrum of QCD  $\rightarrow$  Thermodynamics
- Colour singlet states (hadrons + ....???)
- Do we see quark-gluon substructure BELOW the “phase transition” ?

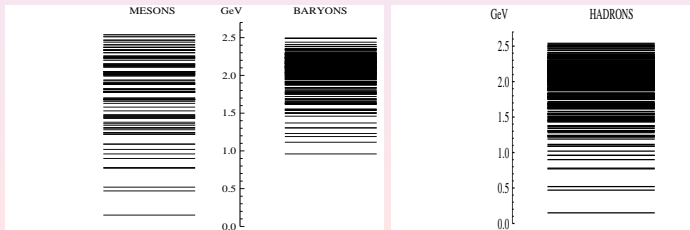
# HADRONIC SPECTRUM AT ZERO TEMPERATURE

# Hadron Spectrum (u,d,s)

- Particle Data Group (PDG) compilation



- Relativized Quark Model (RQM)

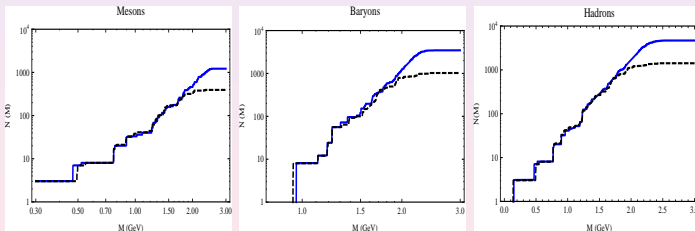


# Cumulative number of states

- Compare  $H_{\text{QCD}}$ ,  $H_{\text{PDG}}$ ,  $H_{\text{RQM}}$  with staircase function

$$N(M) = \sum_n \theta(M - M_n)$$

- Which states count ?
- Is  $N_{\text{QCD}}(M)$  accessible ?



$$N_{q\bar{q}} \sim M^6$$

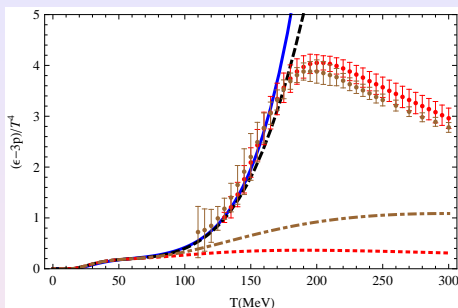
$$N_{qqq} \sim M^{12}$$

$$N_{\bar{q}q\bar{q}q} \sim M^{18}$$

$$N_{\text{hadrons}} \sim e^{M/T_H}$$

$T_H \sim 150\text{MeV} = \text{Hagedorn temperature}$

# QCD Spectrum and Trace anomaly



$$\mathcal{A}_{\text{HRG}}(T) \equiv \frac{\epsilon - 3P}{T^4} = \frac{1}{T^4} \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{E_n(p) - \vec{p} \cdot \vec{\nabla}_p E_n(p)}{e^{E_n(p)/T} + \eta_n},$$
$$E_n(p) = \sqrt{p^2 + M_n^2} \quad \eta_n = \pm 1$$

- Non-interacting Hadron-Resonance Gas works for  $T > 0.8T_c$
- Spectrum  $\rightarrow$  Thermodynamics



# Quark-Hadron duality at zero temperature

- In the confined phase we expect all observables to be represented by hadronic degrees of freedom.

- 1 Gell-Mann–Oakes–Renner relation

$$2 \underbrace{\langle \bar{q}q \rangle}_{\text{quarks}} m_q = - \underbrace{f_\pi^2 m_\pi^2}_{\text{hadrons}}, \quad (1)$$

- 2 Transition form factor of the pion
  - 3 Effective Chiral lagrangians with resonances
  - 4 Deep inelastic scattering
- Are hadrons a complete set of states ?
  - Is the PDG complete or overcomplete ?
  - The “phase transition” is a smooth cross-over, so we expect to see departures from quark-hadron duality below  $T_c$

# Static energies and Casimir scaling

The interaction between heavy sources  $A$  and  $B$  in perturbation theory

$$V_{AB}(r) = \lambda_A \cdot \lambda_B \frac{\alpha_S}{r}$$

The interaction between heavy sources on the lattice

$$E_{AB}(r) = \lambda_A \cdot \lambda_B \left[ \frac{\alpha_S}{r} + \sigma r \right]$$

Casimir scaling requires that the ratio between the fundamental  $Q\bar{Q} \equiv \mathbf{3} \times \bar{\mathbf{3}}$  and adjoint  $GG \equiv \mathbf{8} \times \mathbf{8}$  colour sources are

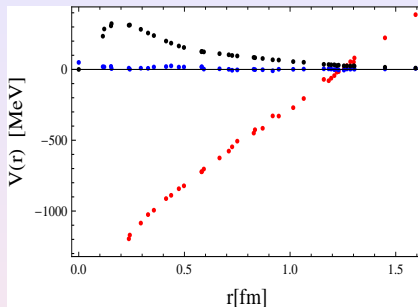
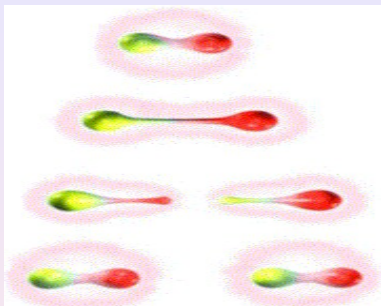
$$V_{Q\bar{Q}}(r) = \sigma_F r - \frac{4\alpha_S}{3r} + \dots \quad (2)$$

$$V_{GG}(r) = \sigma_A r - \frac{3\alpha_S}{r} + \dots \quad (3)$$

$$\frac{\sigma_A}{\sigma_F} = \frac{9}{4} \quad (4)$$

# Quark potential and string breaking

Transition  $Q\bar{Q} \rightarrow B\bar{B}$



- Energy of two heavy quarks

$$E(r) = m_{\bar{Q}} + m_Q + V(r)$$

- Meson masses

$$M_{\bar{q}Q} = \Delta_{\bar{q}Q} + m_Q \quad M_{q\bar{Q}} = \Delta_{q\bar{Q}} + m_Q$$

- Uncoupled Born-Oppenheimer (diabatic crossings)

$$V_{\bar{Q}Q}(r) = \sigma r, \quad V_{\bar{Q}q\bar{q}Q}(r) = \Delta_{\bar{q}Q} + \Delta_{\bar{Q}q} \equiv 2\Delta_{\bar{q}Q}$$

# Excited states

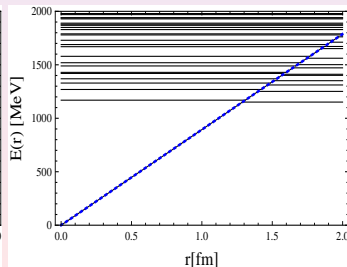
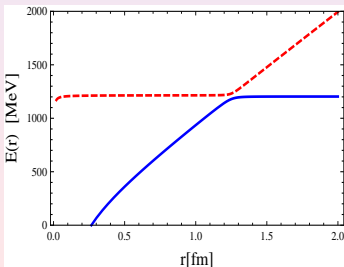
- Estimate of the string breaking distance

$$\sigma r_c = 2M_{q\bar{Q}} - 2m_Q \sim 4M_0$$

(constituent quark mass)  $\rightarrow r_c \sim 1.2\text{fm}$

- In general many excited meson states

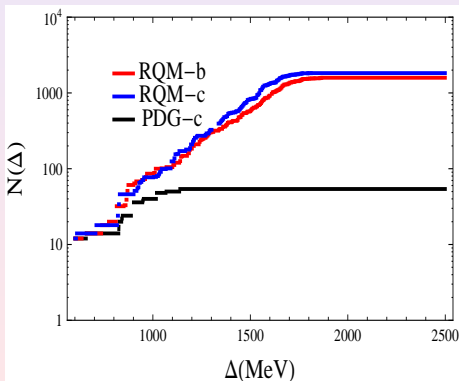
$$V_{\bar{Q}Q}^{(0,0)}(r) = \sigma r, \quad V_{\bar{Q}q, \bar{q}Q}^{(n,m)}(r) = \Delta_{q\bar{Q}}^{(n)} + \Delta_{\bar{q}Q}^{(m)},$$



# Spectrum with one heavy quark

$$N(\Delta) = \sum_n \theta(\Delta - \Delta_n) \sim N_{\bar{q}Q}(\Delta) + N_{Qqq}(\Delta) + \dots \sim e^{\Delta/T_H}$$

$T_H$  Hagedorn temperature for hadrons with ONE heavy quark

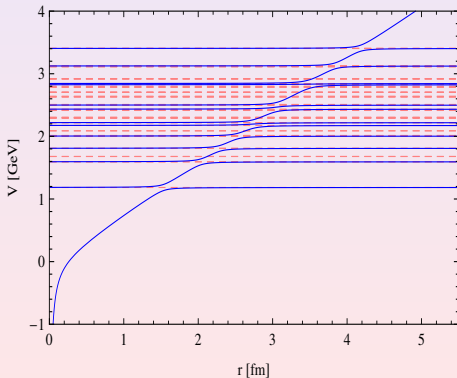


# Avoided crossings

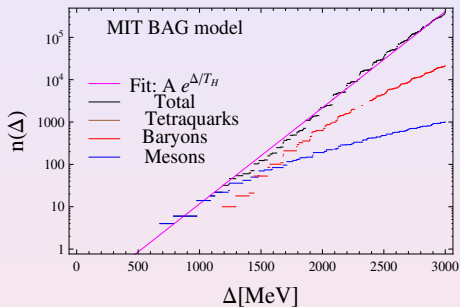
- Transition potential  $V_{\bar{Q}\bar{Q}\rightarrow\bar{B}B}(r)$ . Coupled channels

$$\begin{pmatrix} V_{\bar{Q}\bar{Q}}(r) & V_{\bar{Q}\bar{Q}\rightarrow\bar{B}B}(r) \\ V_{\bar{Q}\bar{Q}\rightarrow\bar{B}B}(r) & V_{\bar{B}\bar{B}}(r) \end{pmatrix}$$

- Avoided crossing with states having the same quantum numbers as  $QQ$



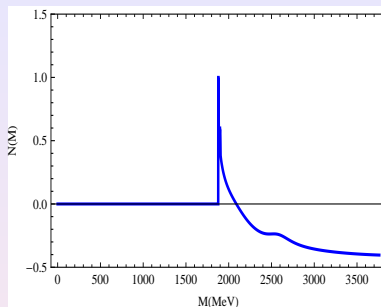
# Hagedorn and The bootstrap



Which are the complete set of states in the PDG ?

Should X,Y,Z's or the deuteron or  $^{208}\text{Pb}$  enter as multiquark states ?

# Who counts ?



- The cumulative number in a given channel in the continuum with threshold  $M_{\text{th}}$

$$N(M) = \sum_n \theta(M - M_n) + [\delta(M) - \delta(M_{\text{th}})]/\pi$$

- Levinson's theorem

$$N(\infty) = n_B + [\delta(\infty) - \delta(M_{\text{th}})]/\pi = 0$$

- Deuteron doesn't count



# QUARKS AND GLUONS AT FINITE TEMPERATURE

# QCD at finite temperature

QCD Lagrangian:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_f \bar{q}_f^a (i\gamma_\mu D_\mu - m_f) q_f^a;$$

Partition function

$$\begin{aligned} Z_{\text{QCD}} &= \text{Tre}^{-H/T} = \sum_n e^{-E_n/T} \\ &= \int \mathcal{D}A_{\mu,a} \exp \left[ -\frac{1}{4} \int d^4x (G_{\mu\nu}^a)^2 \right] \text{Det}(i\gamma_\mu D_\mu - m_f) \end{aligned}$$

Boundary conditions and Matsubara frequencies

$$q(\vec{x}, \beta) = -q(\vec{x}, 0) \quad A_\mu(\vec{x}, \beta) = A_\mu(\vec{x}, 0) \quad \beta = 1/T$$

$$\int \frac{dp_0}{2\pi} f(p_0) \rightarrow T \sum_n f(w_n)$$

$$w_n = (2n + 1)\pi T \quad w_n = 2n\pi T$$

# Thermodynamic relations

- Statistical mechanics of non-interacting particles

$$\log Z = V \eta g_i \int \frac{d^3 p}{(2\pi)^3} \log \left[ 1 + \eta e^{-E_p/T} \right] \quad E_p = \sqrt{p^2 + m^2}$$

$\eta = -1$  for bosons ;  $\eta = +1$  for fermions ;  $g_i$ -number of species

$$F = -T \log Z \quad P = -T \frac{\partial F}{\partial V}$$

$$S = -\frac{\partial(TF)}{\partial T} \quad E = F + TS$$

- High temperature limit  $\rightarrow$  Free gas of gluons and quarks

$$P = \left[ 2(N_c^2 - 1) + 4N_c N_f \frac{7}{8} \right] \frac{\pi^2}{90} T^4$$

Interaction measure (trace anomaly)

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} \rightarrow 0 \quad (T \rightarrow \infty)$$

# Symmetries in QCD

Colour gauge invariance

$$q(x) \rightarrow e^{i \sum_a (\lambda_a)^c \alpha_a(x)} q(x) \equiv g(x) q(x)$$
$$A_\mu^g(x) = g^{-1}(x) \partial_\mu g(x) + g^{-1}(x) A_\mu(x) g(x)$$

Only **periodic gauge transformations** are allowed:

$$g(\vec{x}, x_0 + \beta) = g(\vec{x}, x_0), \quad \beta = 1/T.$$

In the static gauge  $\partial_0 A_0 = 0$

$$g(x_0) = e^{i 2\pi x_0 \lambda / \beta}, \quad \text{where } \lambda = \text{diag}(n_1, \dots, n_{N_c}), \quad \text{Tr} \lambda = 0.$$

**Large Gauge Invariance:**  $\Rightarrow$  periodicity in  $A_0$  with period  $2\pi/\beta$

$$A_0 \rightarrow A_0 + 2\pi T \text{diag}(n_j) \quad \text{Gribov copies}$$

**Explicitly Broken in perturbation theory** (non-perturbative finite temperature gluons)

# Symmetries in QCD

In the limit of massless quarks ( $m_f = 0$ ),

- **Invariant under scale**

$$(\mathbf{x} \longrightarrow \lambda \mathbf{x})$$

Broken by quantum corrections regularization (Trace anomaly)

$$\epsilon - 3p = \frac{\beta(g)}{2g} \langle (G_{\mu\nu}^a)^2 \rangle \neq 0,$$

- **Chiral Left  $\leftrightarrow$  Right transformations.**

$$q(x) \rightarrow e^{i \sum_a (\lambda_a)^f \alpha_a} q(x) \quad \bar{q}(x) \rightarrow e^{i \sum_a (\lambda_a)^f \alpha_a \gamma_5} \bar{q}(x)$$

Broken by chiral condensate in the vacuum

$$\langle \bar{q}q \rangle \neq 0$$

# Symmetries in QCD

Gluodynamics: In the limit of heavy quarks ( $m_f \rightarrow \infty$ )

$$Z \rightarrow \int \mathcal{D}A_{\mu,a} \exp \left[ -\frac{1}{4} \int d^4x (G_{\mu\nu}^a)^2 \right] \text{Det}(-m_f)$$

Larger symmetry ('t Hooft) Center Symmetry  $\mathbb{Z}(N_c)$

$$g(\vec{x}, x_0 + \beta) = z g(\vec{x}, x_0), \quad z^{N_c} = 1, \quad (z \in \mathbb{Z}(N_c)).$$

$$g(x_0) = e^{i2\pi x_0 \lambda / (N_c \beta)}, \quad A_0 \rightarrow A_0 + \frac{2\pi T}{N_c} \text{diag}(n_j)$$

The Polyakov loop

$$L_T = \frac{1}{N_c} \langle \text{tr}_c e^{iA_0/T} \rangle = e^{-F_q/T} = e^{i2\pi/N_c} L_T = 0$$

$F_q = \infty$  means CONFINEMENT

At high temperatures  $A_0/T \ll 1$

$$L_T = 1 - \frac{\langle \text{tr}_c A_0^2 \rangle}{2N_c T^2} + \dots = e^{-\frac{\langle \text{tr}_c A_0^2 \rangle}{2N_c T^2} + \dots}$$

In full QCD  $L_T = \mathcal{O}(e^{-m_q/T}) \neq 0$  Large Violation of center sym.

# Lattice results in full QCD

The chiral-deconfinement cross over is a unique prediction of lattice QCD

- Order parameter of chiral symmetry breaking ( $m_q = 0$ )  
Quark condensate  $SU(N_f) \otimes SU(N_f) \rightarrow SU_V(N_f)$

$$\langle \bar{q}q \rangle \neq 0 \quad T < T_c \quad \langle \bar{q}q \rangle = 0 \quad T > T_c$$

- Order parameter of deconfinement ( $m_q = \infty$ )  
Polyakov loop: Center symmetry  $Z(N_c)$  broken

$$L_T = \frac{1}{N_c} \langle \text{tr}_c e^{iA_0/T} \rangle = 0 \quad T < T_c \quad L_T = \frac{1}{N_c} \langle \text{tr}_c e^{iA_0/T} \rangle = 1 \quad T > T_c$$

- In the real world  $m_q$  is finite. The chiral-deconfinement crossover (connected) crossed correlator (never computed on lattice),

$$\langle \bar{q}q \text{tr}_c e^{igA_0/T} \rangle - \langle \bar{q}q \rangle \langle \text{tr}_c e^{igA_0/T} \rangle = \frac{\partial L_T}{\partial m_q}, \quad (5)$$

# Polyakov loop correlators in higher representations

Polyakov line in the fundamental representation

$$\Omega_F(\vec{x}) = e^{iA_0(\vec{x})/T} \quad A_0 = \sum_{a=1}^{N_c^2-1} \lambda_a A_0^a$$

The interaction between heavy sources  $A$  and  $B$  (Yang-Mills)

$$\langle \text{Tr}_F \Omega(\vec{x}_1) \text{Tr}_F \Omega(\vec{x}_2)^\dagger \rangle \rightarrow e^{-\sigma_F |\vec{x}_1 - \vec{x}_2|/T}$$

In general (Casimir scaling)

$$\langle \text{Tr}_R \Omega(\vec{x}_1) \text{Tr}_R \Omega(\vec{x}_2)^\dagger \rangle \rightarrow e^{-\sigma_R |\vec{x}_1 - \vec{x}_2|/T} \quad \sigma_R = (C_R/C_F)\sigma_F$$



# Quark free energy in QCD

- Spectral decomposition with integral weights  $w_n$  and positive energies  $E_n(|\vec{x}_1 - \vec{x}_2|) > 0$ ,

$$\langle \text{Tr}_F \Omega(\vec{x}_1) \text{Tr}_F \Omega(\vec{x}_2)^\dagger \rangle = \sum_n w_n e^{-E_n(|\vec{x}_1 - \vec{x}_2|)/T} = e^{-F(r,T)/T},$$

At large distances

$$\langle \text{Tr}_F \Omega(\vec{r}) \text{Tr}_F \Omega(0)^\dagger \rangle \rightarrow |\langle \text{Tr}_F \Omega \rangle|^2 \equiv L_T^2. \quad (6)$$

Neglect avoided crossings  $\rightarrow w_0 = 1$  and

$$E_0(r) = V_{\bar{Q}Q}(r) = \sigma r - \pi/12r,$$

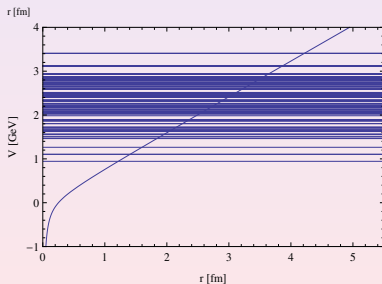
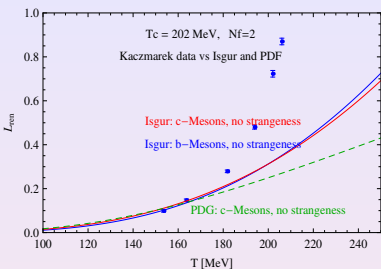
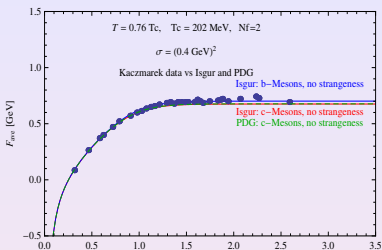
$$\begin{aligned} e^{-F(r,T)/T} &= \langle \text{Tr}_F \Omega(\vec{r}) \text{Tr}_F \Omega(0)^\dagger \rangle = \sum_{n,m} e^{-V_{\bar{Q}Q}^{(n,m)}(r)/T} \\ &= e^{-V_{\bar{Q}Q}(r)/T} + L_T^2 \end{aligned}$$

$$\Delta_n = \Delta_{\bar{q}Q}^{(n)} = \Delta_{q\bar{Q}}^{(n)} \text{ (charge conjugation)}$$

- Polyakov loop  $M_0 \sim 350 \text{ MeV}$  (chiral symmetry breaking)

$$L_T = \sum_n e^{-\Delta_n/T} \sim e^{-2M_0/T} \ll 1,$$

# Quark free energy



$$F(r, T) = -T \log \left[ e^{-V_{\bar{a}a}(r)/T} + e^{-F(\infty, T)/T} \right], \quad (7)$$

# FROM CHIRAL QUARK MODELS TO THE HADRON RESONANCE GAS

# Minimal coupling of the Polyakov loop to Chiral Quark models

Constituent Quark model:

$$\mathcal{L}_{\text{QC}} = \bar{q} \mathbf{D} q, \quad \mathbf{D} = \not{\partial} + \not{V}^f + \not{A} + MU\gamma^5 + \hat{m}_0$$

Consider the minimal coupling of the gluons in the model:

$$V_\mu^f \longrightarrow V_\mu^f + gV_\mu^c, \quad V_\mu^c = \delta_{\mu 0} V_0^c$$

- We introduce a colour source (Polyakov loop).
- We obtain the (Peirls-Yoccoz) projection onto the color neutral states by integrating over the  $A_0$  field.
- In Quenched approximation: Group integration in  $SU(N_c)$ .

$$Z = \int DUD\Omega e^{-\Gamma_G[\Omega]} e^{-\Gamma_Q[U,\Omega]}$$

# Quark-Hadron Duality at Finite Temperature (Polyakov loop)

Partition function for  $N_f$ -flavours

$$Z_{\text{HRG}}(N_f) \equiv \int D\Omega e^{-S(N_f)} \quad S(N_f) = S_q(N_f) + S_G$$

Quark contribution

$$S_q(N_f) = -2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left( \text{tr}_c \log [1 + \Omega(x) e^{-E_p/T}] + \text{c.c.} \right)$$

One extra HEAVY QUARK (not anti-quark) with flavour  $a$

$$S_q(N_f + 1) - S_q(N_f) = -2 \log(1 + \Omega_{aa} e^{-E_h/T}) \approx -2 e^{-m_H/T} \Omega_{aa}$$

$$\frac{1}{N_c} \langle \text{tr}_c \Omega \rangle = \lim_{m_H \rightarrow \infty} \frac{1}{2} \left[ \frac{Z_{\text{HRG}}(N_f + 1)}{Z_{\text{HRG}}(N_f)} - 1 \right] e^{m_H/T} = \frac{1}{2N_c} \sum_{\alpha} g_{\alpha} e^{-\Delta_{\alpha}/T}$$

$$\Delta_{\alpha} = \lim_{m_H \rightarrow \infty} (M_{H,\alpha} - m_H)$$

# Quark-Hadron Duality at Finite Temperature (Quark free energy)

Four point correlator (cluster decomposition)

$$\begin{aligned} \langle \text{tr}_c \Omega(\vec{x})^\dagger \text{tr}_c \Omega(0) \text{tr}_c \Omega(\vec{x}_1)^\dagger \text{tr}_c \Omega(\vec{x}_2) \rangle_G &= e^{-\sigma r/T} e^{-\sigma r_{12}/T} \\ &+ e^{-\sigma |\vec{x} - \vec{x}_2|/T} e^{-\sigma r_1/T} \end{aligned} \quad (8)$$

This yields

$$\begin{aligned} \langle \text{tr}_c \Omega(\vec{x}) \text{tr}_c \Omega^\dagger(0) \rangle &= \frac{e^{-\sigma r/T} [1 + Z_{\bar{q}q} + \dots] + |\langle \text{tr}_c \Omega \rangle|^2}{1 + Z_{\bar{q}q} + \dots} \\ &= e^{-\sigma r/T} + |\langle \text{tr}_c \Omega \rangle|^2. \end{aligned} \quad (9)$$

This is the SAME as

$$F(r, T) = -T \log \left[ e^{-V_{\text{had}}(r)/T} + e^{-F(\infty, T)/T} \right], \quad (10)$$

# Quantization of multiquark states

- Quantum and local Polyakov loop (PNJL models)
- Multiquark states: Create/Anihilate a quark at point  $\vec{x}$  and momentum  $p$

$$\Omega(x)e^{-E_p/T} \quad \Omega(x)^+ e^{-E_p/T}$$

- At low temperatures quark Boltzmann factor small  $e^{-E_p/T} < 1$ .  
The action becomes small

$$S_q[\Omega] = 2N_f \int \frac{d^3x d^3p}{(2\pi)^3} [\text{tr}_c \Omega(x) + \text{tr}_c \Omega(x)^+] e^{-E_p/T} + \dots$$

$$Z = \int D\Omega e^{-S[\Omega]} = \int D\Omega \left( 1 - S[\Omega] + \frac{1}{2} S[\Omega]^2 + \dots \right)$$

- $\bar{q}q$  contribution

$$Z_{\bar{q}q} = (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \int \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-E_1/T} e^{-E_2/T} \underbrace{\langle \text{tr}_c \Omega(\vec{x}_1) \text{tr}_c \Omega^\dagger(\vec{x}_2) \rangle}_{e^{-\sigma|\vec{x}_1 - \vec{x}_2|/T}}$$

$$= (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-H(x_1, p_1; x_2, p_2)/T}$$

$\bar{q}q$  Hamiltonian

$$H(x_1, p_1; x_2, p_2) = E_1 + E_2 + V_{12}.$$

- Quantization in the CM frame  $p_1 = -p_2 \equiv p$

$$\left( 2\sqrt{p^2 + M^2} + V_{q\bar{q}}(r) \right) \psi_n = M_n \psi_n.$$

- Boosting the CM to any frame with momentum  $P$

$$Z_{\bar{q}q} \rightarrow \sum_n \int \frac{d^3R d^3P}{(2\pi)^3} e^{-E_n(P)/T}$$

A GAS OF NON INTERACTING MESONS ! (valid to  $\bar{q}q\bar{q}q$ )



# Polyakov loop in the quark model

$$\begin{aligned} L_T &= 2N_f \int \frac{d^3x d^3p}{(2\pi)^3} e^{-E_p/T} \frac{1}{N_c} \underbrace{\langle \text{tr}_c \Omega(\vec{x}_0) \text{tr}_c \Omega^\dagger(\vec{x}) \rangle}_{e^{-\sigma|\vec{x}_0 - \vec{x}|/T}} + \dots \\ &= \frac{2N_f}{N_c} \int \frac{d^3x d^3p}{(2\pi)^3} e^{-H(\vec{x}, \vec{p})/T} \end{aligned}$$

Heavy-light ground state system

$$(\sqrt{p^2 + m_q^2} + \sigma r)\psi_n = \Delta_n \psi_n$$

In the limit  $m_q \rightarrow 0$  we make  $p \sim 1/r$  and  $\Delta \sim 2\sqrt{\sigma} \sim 900\text{MeV}$

$$N_c L(T) \sim 2N_f e^{-\Delta_M/T} + (2N_f^2 + N_f) e^{-\Delta_B/T} + \dots = 21 e^{-\bar{\Delta}/T} \quad (N_f = 3)$$

# CONCLUSIONS

# Conclusions:

- Quark Hadron Duality suggests that at low temperatures Hadrons can be considered as a complete basis of states in terms of a hadron resonance gas. The HRG works up to relatively large temperatures.
- PDG states incorporate currently just  $q\bar{q}$  or  $qqq$  states which fit into the quark model. What states are needed when approaching the crossover from below ?
- Saturating at subcritical temperatures requires many hadronic states, so the excited spectrum involves relativistic effects even for heavy quarks.
- Polyakov loops in fundamental and higher representations allow to deduce multiquark quark states, gluelumps etc. containing one or several heavy quark states.