Saturation and Geometrical Scaling: from Deep Ineastic Scattering to Heavy Ion Collisions

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message to take away:

There exists an intermediate energy scale, called *saturation scale*, that, by dimensional arguments, determines inclusive and semi-inclusive observables in kinematical regions where no other energy (momentum) scales exist.
DGLAP vs BFKL Evolution

small $x$  
large $W$

Balitsky, Fadin, Kuraev, Lipatov

large $x$  
small $W$

$Y = \log 1/x$

BFKL

DGLAP

log $Q^2$
Dipole Picture

BFKL equation has very simple form and interpretation in the dipole picture of A. Mueller

\[ N(\vec{x}_{01}, Y) \text{ dipole-target forward amplitude} \]

\[ \tilde{N}(\vec{k}, Y) \sim \alpha_s \int F(x, \vec{k}, \vec{l}) \Phi(\vec{l}) \frac{d^2\vec{l}}{\vec{l}^2} \]

Gluon Emission in the Dipole Picture

- large $N_c$
- dipole emission kernel is very simple
- reproduces BFKL equation

$$\int d^2\vec{x}_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$
\[
\frac{\partial}{\partial Y} N(x_{01}, Y) = \frac{\alpha_s}{2\pi} \int d^2 \vec{x}_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} [N(x_{02}, Y) + N(x_{12}, Y) - N(x_{01}, Y) \\
- N(x_{02}, Y)N(x_{12}, Y)]
\]

double scattering stops rapid growth of the amplitude with \( Y \)

note that \([...]=0\) for \( N \rightarrow 1 \)
BK Equation

\[ \frac{\partial}{\partial Y} N(x_{01}, Y) = \frac{\bar{\alpha}_s}{2\pi} \int d^2 \bar{x}_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \left[ N(x_{02}, Y) + N(x_{12}, Y) - N(x_{01}, Y) \right] - N(x_{02}, Y) N(x_{12}, Y) \]


rewrite in terms of a Fourier transform:

\[ N(x, Y) = x^2 \int \frac{d^2 \vec{k}}{2\pi} e^{i\vec{k} \cdot \vec{x}} \tilde{N}(k, Y) \]

\[ \frac{\partial}{\partial Y} \tilde{N}(k, Y) = \bar{\alpha}_s \chi(-\partial/\partial \ln k^2) \tilde{N}(k, Y) - \bar{\alpha}_s \tilde{N}^2(k, Y) \]

here \( \chi \) is a BFKL characteristic function related to the kernel \( K(k_1, k_2) \)

\[ \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma) \]

there exists a theorem from the ‘30 (Fisher, Kolmogorov, Petrovsky, Piscounov) that non-linear equations of this sort have asymptotically travelling wave solutions
Travelling waves

identify

\[
\text{time: } t = Y, \quad \text{position: } x = \ln k^2
\]

\[
\partial_t u(x, t) = \partial^2_x u(x, t) + u(x, t) - u^2(x, t)
\]

Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP) 1937

Asymptotic solution: travelling wave

\[
u(x, t) = u(x - v_c t)
\]

Position: \( X(t) = X_0 + v_c t \)
Travelling waves in QCD

\[ \frac{\partial}{\partial Y} \tilde{N}(k, Y) = \tilde{\alpha}_s \chi(-\partial/\partial \ln k^2) \tilde{N}(k, Y) - \tilde{\alpha}_s \tilde{N}^2(k, Y) \]

translate travelling wave to QCD

\[ u(x, t) = u(x - v_c t) \]

minimal velocity:

\[ v = \min \frac{\tilde{\alpha}_s \chi(\gamma)}{\gamma} \quad \rightarrow \quad \gamma_c \chi'(\gamma_c) = \chi(\gamma_c) \]

\[ \gamma_c = 0.6275 \quad v_c = 4.8834 \tilde{\alpha} \]

travelling wave condition:

\[ \tilde{\alpha}_s \chi(\gamma_c) Y - \gamma_c \ln\left(\frac{k^2}{k_0^2}\right) = -\gamma_c \ln\left[\left(\frac{1}{x}\right)^{-v_c} \frac{k^2}{k_0^2}\right] = -\gamma_c \ln\left[\frac{k^2}{Q_s^2(x)}\right] \]

saturation scale:

\[ Q_s^2(x) = k_0^2 \left(\frac{1}{x}\right)^{v_c} \]

scaling variable
Travelling waves in QCD imply Geometrical Scaling

\[ f(x, k^2) = \mathcal{F} \left( \frac{k^2}{Q_s^2(x)} \right) \]

\[ Q_s(x) = Q_0 \left( \frac{x_0}{x} \right)^{\lambda/2} \]
Deep Inealstic Scattering
Saturation scale: energy and $x$ dependence

$$Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{x}{x_0} \right)^{-\lambda}$$

A.M. Stasto, K. J. Golec-Biernat, J. Kwiecinski
PRL 86 (2001) 596-599

M. Praszalowicz and T. Stebel
and
Saturation scale: energy and $x$ dependence

$$\tau = \frac{Q^2}{Q_{sat}^2(x)}$$

$$Q_{sat}^2(x) = Q_0^2 \left( \frac{x}{x_0} \right)^{-\lambda}$$

$\lambda = 0.329 \pm 0.005$ up to $x = 0.08$ (!)
proton-proton @ LHC
Basics of geometrical scaling


\[
\frac{d\sigma}{dy d^2p_T} = \frac{3\pi \alpha_s}{2p_T^2} \int d^2k_T \varphi_1(x_1, k_T^2) \varphi_2(x_2, (k - p)^2_T)
\]

\[
x_{1,2} = \frac{p_T}{\sqrt{s}} e^{\pm y}
\]

Kharzeev, Levin

Michal Praszalowicz
Basics of geometrical scaling

gluon distribution

\[ xG(x, Q^2) = \int dk_T^2 \varphi(x, k_T^2) \]

unintegrated glue

Golec-Biernat – Wuesthoff (DIS)

\[ \varphi(x, k_T^2) = S_\perp \frac{3}{4\pi^2} \frac{k_T^2}{Q_s(x)^2} \exp \left(-\frac{k_T^2}{Q_s(x)^2}\right) \]

\[ S_\perp = \sigma_0 \]

Kharzeev – Levin (AA)

\[ \varphi(x, k_T^2) = S_\perp \begin{cases} 
1 & \text{for } k_T^2 < Q_s(x)^2 \\
\frac{Q_s(x)^2}{k_T^2} & \text{for } Q_s(x)^2 < k_T^2
\end{cases} \]

\[ S_\perp \text{ is the transverse size given by geometry} \]

scaling variable

\[ \tau = \frac{p_T^2}{Q_s^2(x)} \]

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Basics of geometrical scaling

for $y \sim 0$ (central rapidity) i.e. for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dyd^2p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2k_T}{Q_s^2(x)} \varphi_1 \left( \frac{k_T^2}{Q_s^2(x)} \right) \varphi_2 \left( \frac{(\vec{k} - \vec{p})_T^2}{Q_s^2(x)} \right)$$
Basics of geometrical scaling

for $y \sim 0$ (central rapidity) \textit{i.e.} for $x_1 \sim x_2 = x$ and for symmetric systems

\[
\frac{d\sigma}{dyd^2p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2\vec{k}_T}{Q_s^2(x)} \varphi_1 \left( \frac{\vec{k}_T^2}{Q_s^2(x)} \right) \varphi_2 \left( \frac{(\vec{k} - \vec{p})^2}{Q_s^2(x)} \right)
\]

\[
\frac{dN}{dyd^2p_T} = S_\perp \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)}
\]

\[
dp_T^2 = \frac{2}{2 + \lambda} \bar{Q}_s^2(W) \tau^{-(\lambda/(2+\lambda))} d\tau
\]

\[
\bar{Q}_s(W) = Q_0 \left( \frac{W}{Q_0} \right)^{\lambda/(2+\lambda)}
\]
Basics of geometrical scaling

for $y \sim 0$ (central rapidity) i.e. for $x_1 \sim x_2 = x$ and for symmetric systems

\[
\frac{d\sigma}{dyd^2p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2\vec{k}_T}{Q_s^2(x)} \varphi_1 \left( \frac{\vec{k}_T^2}{Q_s^2(x)} \right) \varphi_2 \left( \frac{(\vec{k} - \vec{p})^2}{Q_s^2(x)} \right)
\]

\[
\frac{dN}{dyd^2p_T} = S_\perp \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad dp_T^2 = \frac{2}{2 + \lambda} \bar{Q}_s^2(W) \tau^{-\lambda/(2+\lambda)} d\tau
\]

\[
\frac{dN}{dy} = S_\perp \int \mathcal{F}(\tau)d^2p_T = S_\perp \bar{Q}_s^2(W) \int \mathcal{F}(\tau) \ldots d\tau = \frac{1}{\kappa} S_\perp \bar{Q}_s^2(W)
\]
Power-like growth of multiplicity


plot: P. Braun-Munzinger, 54 Cracow School of Theoretical Physics (from ALICE-PUB-44337)

\[
\frac{dN_{ch}}{dy} \sim S_{NN}^{0.15} \quad S_{NN}^{0.11} \quad S_{NN}^{0.10}
\]

transverse area is energy independent

is power correct?
Geometrical scaling of $\rho_T$ distribution

Phys.Rev. D87 (2013) 071502(R)

$$\tau = \frac{p_T^2}{Q_{sat}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1\text{ GeV}^2} \left( \frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^{\lambda}$$
Determination of lambda

\[ \frac{dN_{ch}}{dy d^2 p_T} = S_\perp \mathcal{F}(\tau) \]

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Power-like growth of multiplicity


plot: P. Braun-Munzinger, 54 Cracow School of Theoretical Physics (from ALICE-PUB-44337)

\[
\frac{dN_{ch}}{dy} \sim S_{\perp} Q_s^2(W) \sim S_{\perp} Q_0^2 \left( \frac{W^2}{Q_0^2} \right)^{\lambda/(2+\lambda)}
\]

transverse area is energy independent

\[\lambda/(2 + \lambda) \approx 0.099\]
Average transverse momentum

\[
\frac{dN_{\text{ch}}}{dy d^2p_T} = S_\perp F(\tau)
\]

\[
\langle p_T \rangle = \frac{\int p_T \frac{dN_g}{dy d^2p_T} d^2p_T}{\int \frac{dN_g}{dy d^2p_T} d^2p_T} \sim \bar{Q}_s(W) \sim Q_0 \left( \frac{W}{Q_0} \right)^{\lambda/(2+\lambda)}
\]
Average transverse momentum

\[ \langle p_T \rangle \text{ [GeV/c]} \]

- ISR inel.
- UA1 NSD
- E735 NSD
- CDF NSD
- CMS NSD

\[ 0.227 \times W^{0.099} \]

\( W \) [GeV]
Mean $\rho_T$ as a function of $N_{\text{ch}}$

- $\langle \rho_T \rangle(N_{\text{ch}})$ – correlations are sensitive to the fine details of dynamics
- difficult to describe by untuned MonteCarlos
- possible sign of phase transition

**MULTICLITY DEPENDENCE OF $\rho_T$ SPECTRUM AS A POSSIBLE SIGNAL FOR A PHASE TRANSITION IN HADRONIC COLLISIONS**

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Received 25 August 1982
Mean $\rho_T$ as a function of $N_{\text{ch}}$

$\langle p_T \rangle \sim \tilde{Q}_s(W)$
Mean $p_T$ as a function of $N_{ch}$

$$\langle p_T \rangle \sim \bar{Q}_s(W) \sim \sqrt{\frac{dN/dy}{S_{\perp}}} \sim \frac{1}{R} \sqrt{\frac{dN}{dy}}$$

interaction radius
Mean $p_T$ as a function of $N_{ch}$

phomenological formula:

$$\langle p_T \rangle \sim \frac{dN}{dy} \sqrt{S_\perp} \sim \frac{1}{R} \sqrt{\frac{dN}{dy}}$$

nonperturbative coefficient

$\alpha, \beta$ do not depend on energy, do depend on particle species
Interaction radius

A. Bzdak, B. Schenke, P. Tribedy and R. Venugopalan, 
Interaction radius

Transverse size and expansion time (longitudinal size) are proportional for fixed multiplicity

Similar effect in multipomeron model, where string tension is growing with multiplicity

M. A. Braun, C. Pajares

N. Armesto, D.A. Derkach, G.A. Feofilov
Energy dependence of mean $p_T$

$$\langle p_T \rangle_W = \alpha + \beta \left( \frac{\sqrt{N_{ch}}}{R\left(\frac{3}{\sqrt{\gamma N_{ch}}}\right)} \right)_W = \alpha + \beta \left( \frac{W}{W_0} \right)^{\lambda/(2+\lambda)} \frac{\sqrt{N_{ch}}}{R\left(\frac{3}{\sqrt{\gamma N_{ch}}}\right)} \bigg|_{W_0}$$
Energy dependence of mean $p_T$

\[
\langle p_T \rangle_W = \alpha + \beta \frac{\sqrt{N_{ch}}}{R(\sqrt[3]{\gamma N_{ch}})} |_W = \alpha + \beta \left( \frac{W}{W_0} \right)^{\frac{\lambda}{2(2+\lambda)}} \frac{\sqrt{N_{ch}}}{R(\sqrt[3]{\gamma N_{ch}})} |_{W_0}
\]
Mean $p_T$ scaling


$\langle p_T \rangle$ vs $N_{ch}$

- pp 7 TeV
- pp 2.76 TeV
- pPb 5.02 TeV
- pp 0.9 TeV
Mean $\langle p_T \rangle$ scaling


\[ \langle p_T \rangle \quad (W/W_0)^{\lambda/(2+\lambda)} \sqrt{N_{ch}/S_T} \]

pp 7 TeV
pp 2.76 TeV
pPb 5.02 TeV
pp 0.9 TeV
Geometrical Scaling in Heavy Ion Collisions
GS in HI: centrality

small centrality

large centrality

number of participants

$N_{\text{part}} \sim V$
GS in HI: centrality dependence

\[ S_\perp \sim N_{\text{part}}^{2/3} \]
\[ \frac{dN}{dy} \sim N_{\text{part}} \]

Geometrical Scaling of Direct-Photon Production in Hadron Collisions from RHIC to the LHC

Scaling of the saturation scale:

\[ Q_s^2(x) = \frac{\kappa}{S_\perp} \frac{dN}{dy} \sim N_{\text{part}}^{1/3} \left( \frac{\sqrt{s}}{p_T} \right)^\lambda \]

\[ \frac{Q_0^2}{N_{\text{part}}^{2/3}} \frac{dN_{\text{ch}}}{2\pi p_T d\eta dp_T} = F(\tau) \]

\[ \tau = \frac{1}{N_{\text{part}}^{1/3} Q_0^2} \left( \frac{p_T}{W} \right)^\lambda \]
GS in HI

![Graphs showing the relationship between $p_T$ and $\tau^{1/2}$ with data from ALICE PbPb@2.76 TeV, STAR AuAu@200 GeV, PHENIX AuAu@200 GeV, and PHENIX AuAu@130 GeV.]
energy scaling works quite well, why?
Centrality Scaling in HI

![Graph showing centrality scaling in HI](image)
Summary

• QCD evolution equations lead to overabundance of gluons
• Nonlinear evolution introduces new scale: saturation momentum
• GS should emerge if no other scales are present
• GS in DIS works for rather high Bjorken $x$
• GS works also for charged particles in pp
• GS for mean $p_T$ and for $<p_T>(N_{ch})$
• Energy and centrality dependence of GI in HI
Summary

• QCD evolution equations lead to overabundance of gluons
• Nonlinear evolution introduces new scale: *saturation momentum*
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• GS in DIS works for rather high Bjorken $x$
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• GS for mean $p_T$ and for $<p_T>(N_{ch})$
• Energy and centrality dependence of GS in HI
• Is GS a real sign of saturation?
• Why in pp GS is not washed out by FSI?
• Why in HI hydro preserves (at least partially) GS?
• Nonuniversality: different values of $\lambda$