Saturation and Geometrical Scaling: from Deep Ineastic Scattering to Heavy Ion Collisions

Michał Praszałowicz
M. Smoluchowski Inst. of Physics
Jagiellonian University, Kraków, Poland

Tatranska Lomnica 1.3.2015.

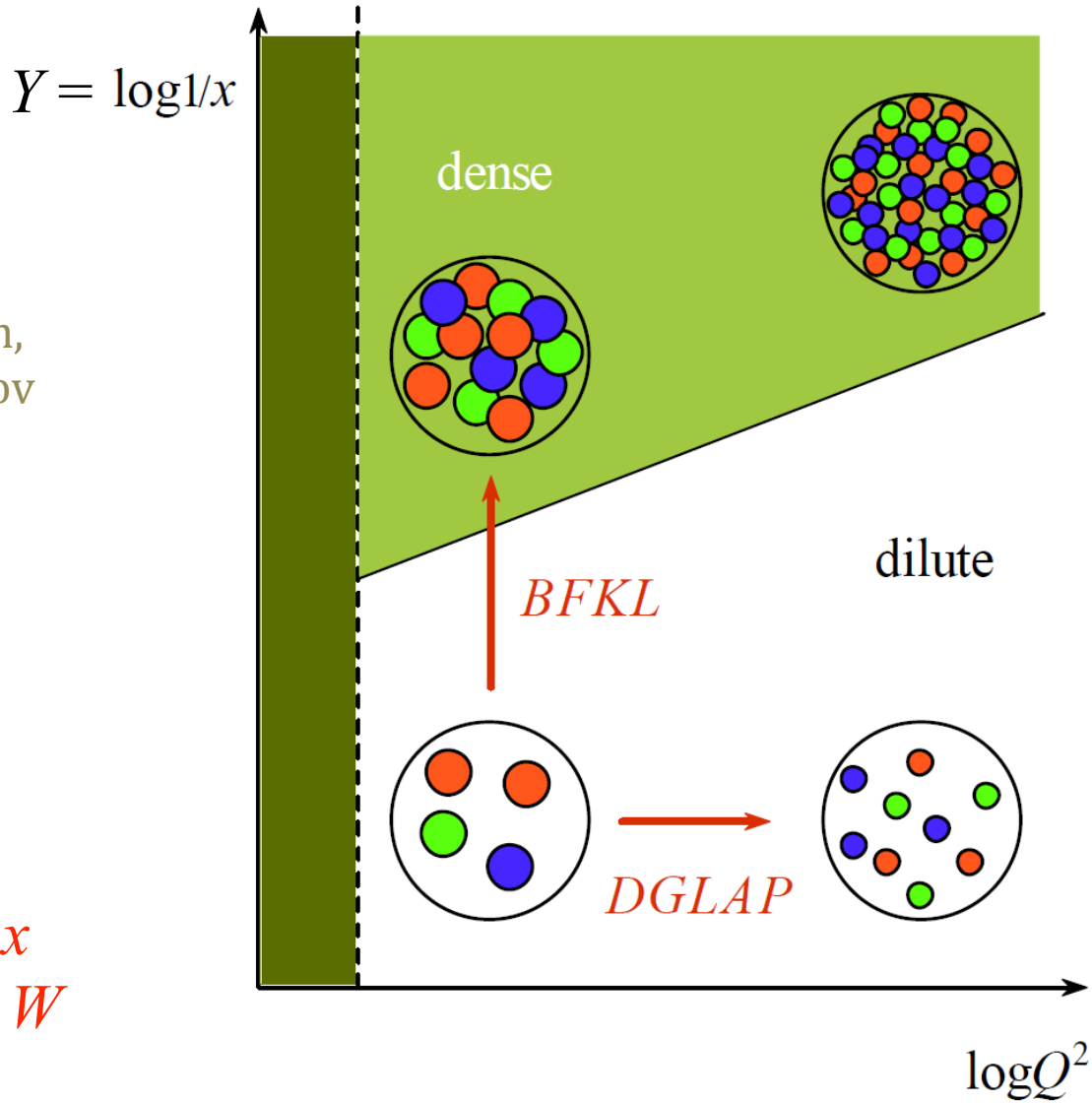
message to take away:

There exits an intermediate energy scale, called *saturation scale*, that, by dimensional arguments, deterimines inclusive and semi--inclusive observables in kinematical regions where no other energy (momentum) scales exist.

DGLAP vs BFKL Evolution

small x
large W

Balitsky, Fadin,
Kuraev, Lipatov

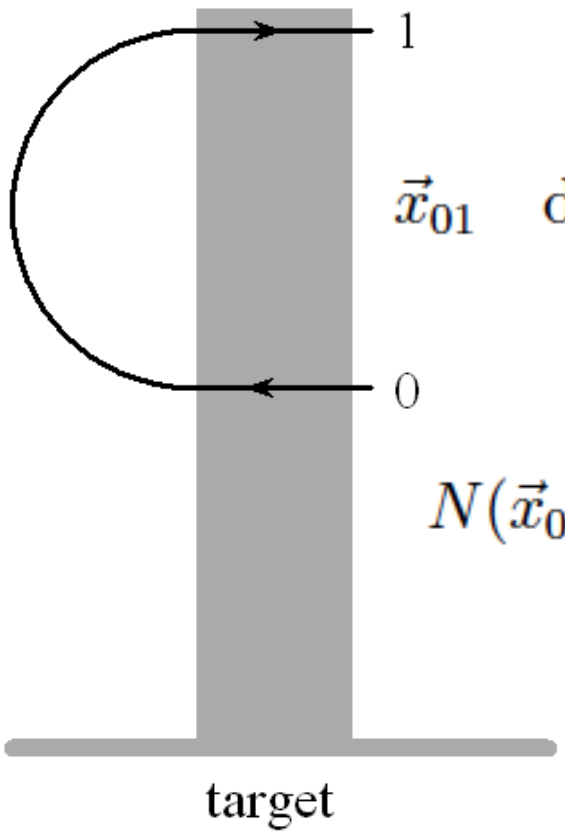


large x
small W

Dipole Picture

BFKL equation has very simple form and interpretation in the dipole picture of A. Mueller

A.H. Mueller and J.-w. Qiu,
Nucl. Phys. B 268 (1986) 427

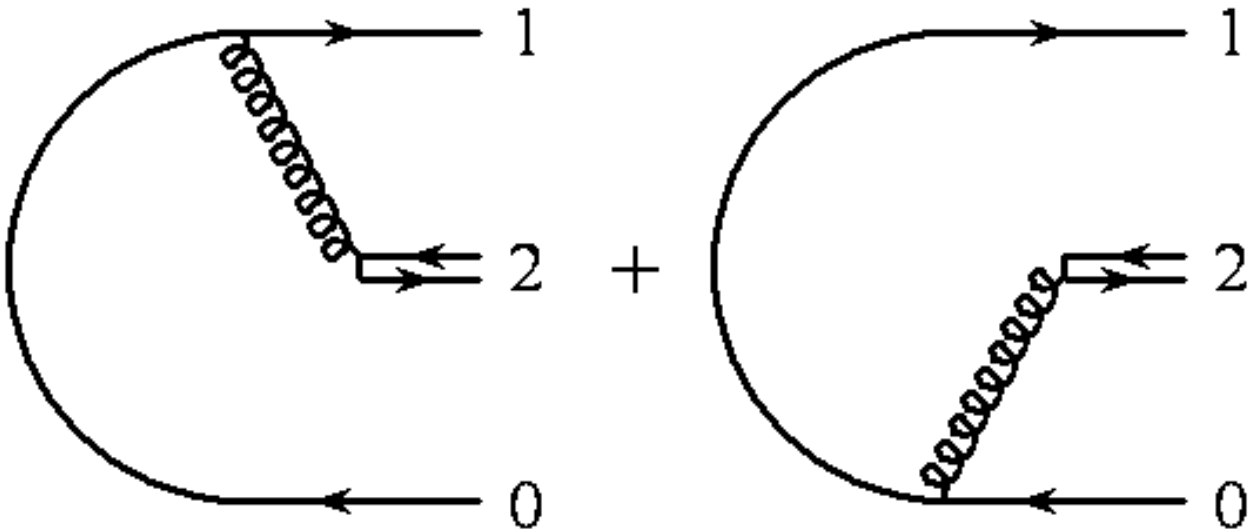


$\vec{x}_{01}$ dipole transverse size

$N(\vec{x}_{01}, Y)$ dipole-target forward amplitude

$$\tilde{N}(\vec{k}, Y) \sim \alpha_s \int F(x, \vec{k}, \vec{l}) \Phi(\vec{l}) \frac{d^2\vec{l}}{\vec{l}^2}$$

Gluon Emission in the Dipole Picture

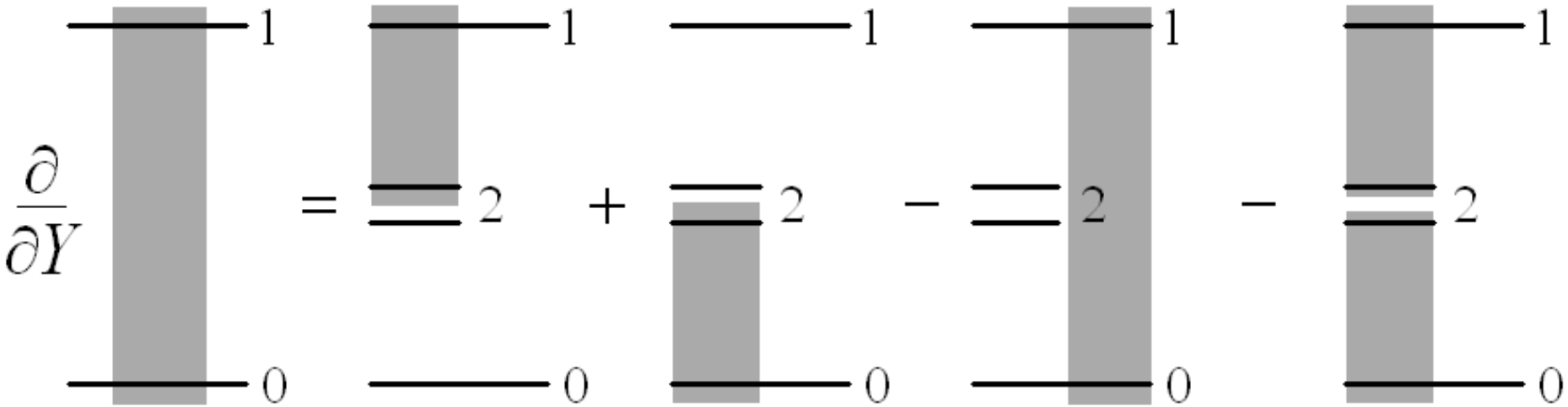


- large N_c
- dipole emission kernel is very simple
- reproduces BFKL equation

$$\int d^2\vec{x}_2 \, \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$

I. Balitsky,
Nucl. Phys. B 463 (1996) 99
Y.V. Kovchegov,
Phys. Rev. D 60 (1999) 034008

BK Equation



$$\frac{\partial}{\partial Y} N(x_{01}, Y) = \frac{\overline{\alpha}_s}{2\pi} \int d^2\vec{x}_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \left[N(x_{02}, Y) + N(x_{12}, Y) - N(x_{01}, Y) - N(x_{02}, Y) N(x_{12}, Y) \right]$$

double scattering stops rapid growth of the amplitude with Y
note that [....] = 0 for $N \rightarrow 1$

BK Equation

$$\frac{\partial}{\partial Y} N(x_{01}, Y) = \frac{\overline{\alpha}_s}{2\pi} \int d^2\vec{x}_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \left[N(x_{02}, Y) + N(x_{12}, Y) - N(x_{01}, Y) - N(x_{02}, Y) N(x_{12}, Y) \right]$$

Y.V. Kovchegov, Phys. Rev. D 61 (2000) 074018

rewrite in terms of a Fourier transform: $N(x, Y) = x^2 \int \frac{d^2\vec{k}}{2\pi} e^{i\vec{k}\cdot\vec{x}} \tilde{N}(k, Y)$

$$\frac{\partial}{\partial Y} \tilde{N}(k, Y) = \overline{\alpha}_s \, \chi(-\partial/\partial \ln k^2) \, \tilde{N}(k, Y) - \overline{\alpha}_s \, \tilde{N}^2(k, Y)$$

here χ is a BFKL characteristic function related to the kernel $K(k_1, k_2)$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

there exists a theorem from the '30 (Fisher, Kolmogorov, Petrovsky, Piscounov) that non-linear equations of this sort have asymptotically travelling wave solutions

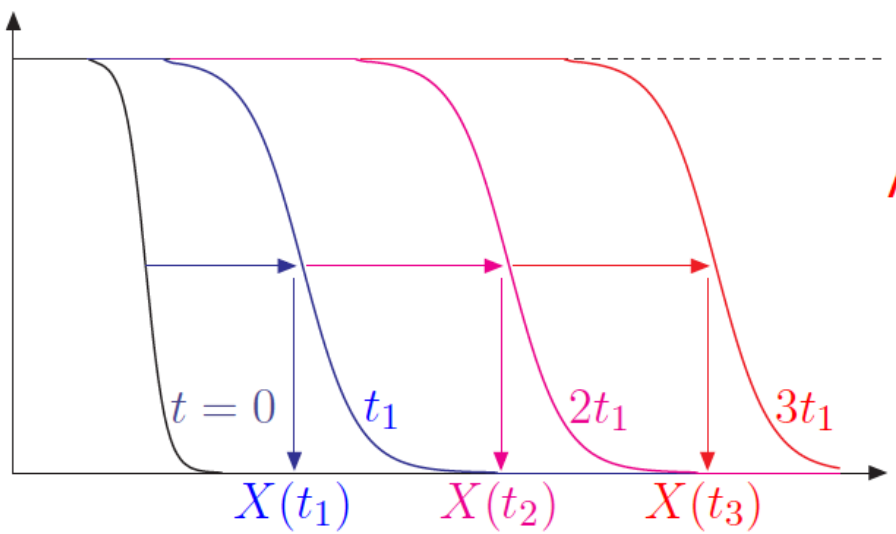
S. Munier, R.B. Peschanski
PRL 91 (2003) 232001
PRD 69 (2004) 034008

Travelling waves

identify time : $t = Y$, position : $x = \ln k^2$

$$\boxed{\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)}$$

Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP) 1937



Asymptotic solution:
travelling wave

$$u(x,t) = u(x - v_c t)$$

Position: $X(t) = X_0 + v_c t$

Travelling waves in QCD

$$\frac{\partial}{\partial Y}\tilde{N}(k,Y) = \bar{\alpha}_s\,\chi(-\partial/\partial\ln k^2)\,\tilde{N}(k,Y) - \bar{\alpha}_s\,\tilde{N}^2(k,Y)$$

translate travelling wave to QCD $\quad u(x,t) = u(x - v_c t)$

minimal velocity: $\quad v = \min \frac{\bar{\alpha}_s\,\chi(\gamma)}{\gamma} \quad \rightarrow \quad \gamma_c\chi'(\gamma_c) = \chi(\gamma_c)$

$$\gamma_c = 0.6275$$
$$v_c = 4.8834\bar{\alpha}$$

travelling wave condition:

$$\underline{\bar{\alpha}_s\chi(\gamma_c)Y - \gamma_c\ln(k^2/k_0^2)} = -\gamma_c\ln\left[\left(\frac{1}{x}\right)^{-v_c}\frac{k^2}{k_0^2}\right] = -\gamma_c\ln\left[\frac{k^2}{Q_s^2(x)}\right]$$

saturation scale: $\quad Q_s^2(x) = k_0^2\left(\frac{1}{x}\right)^{v_c}$

scaling variable

Travelling waves in QCD imply Geometrical Scaling

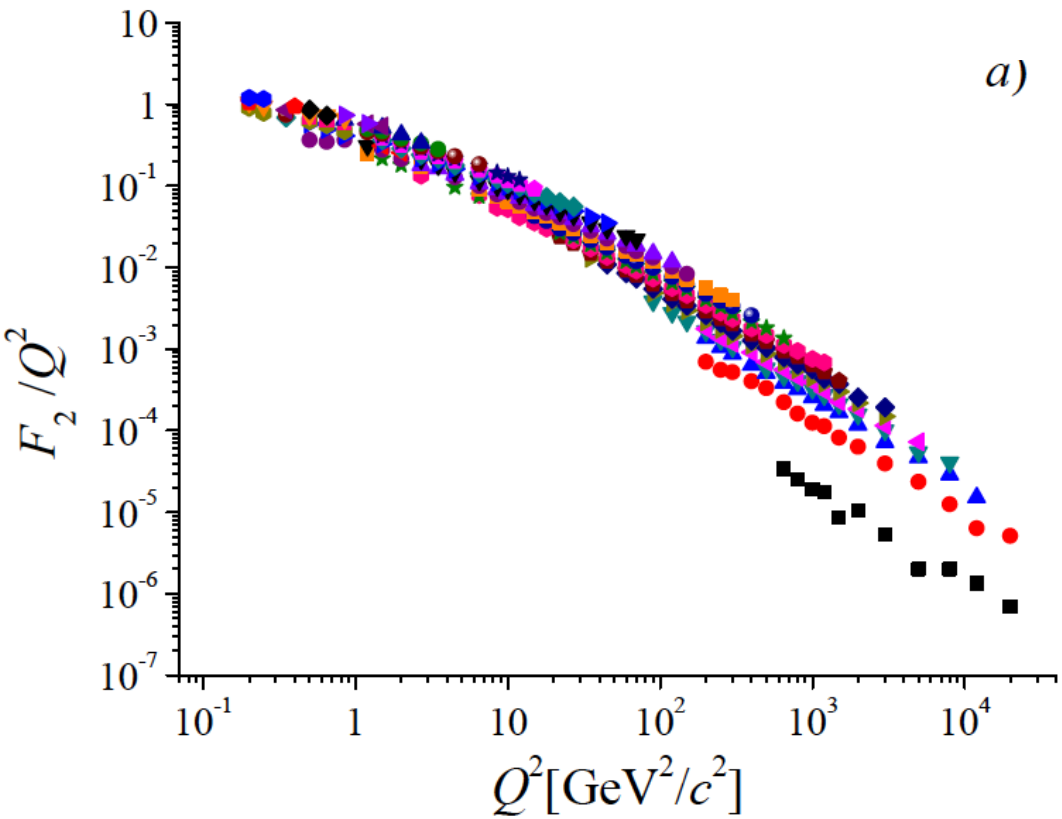
$$f(x, k^2) = \mathcal{F}\left(\frac{k^2}{Q_s^2(x)}\right)$$

$$Q_s(x) = Q_0 \left(\frac{x_0}{x}\right)^{\lambda/2}$$

Deep Inealstic Scattering

Saturation scale: energy and x dependence

$$Q^2_{\mathrm{sat}}(x) = Q^2_0 \left(\frac{x}{x_0}\right)^{-\lambda}$$

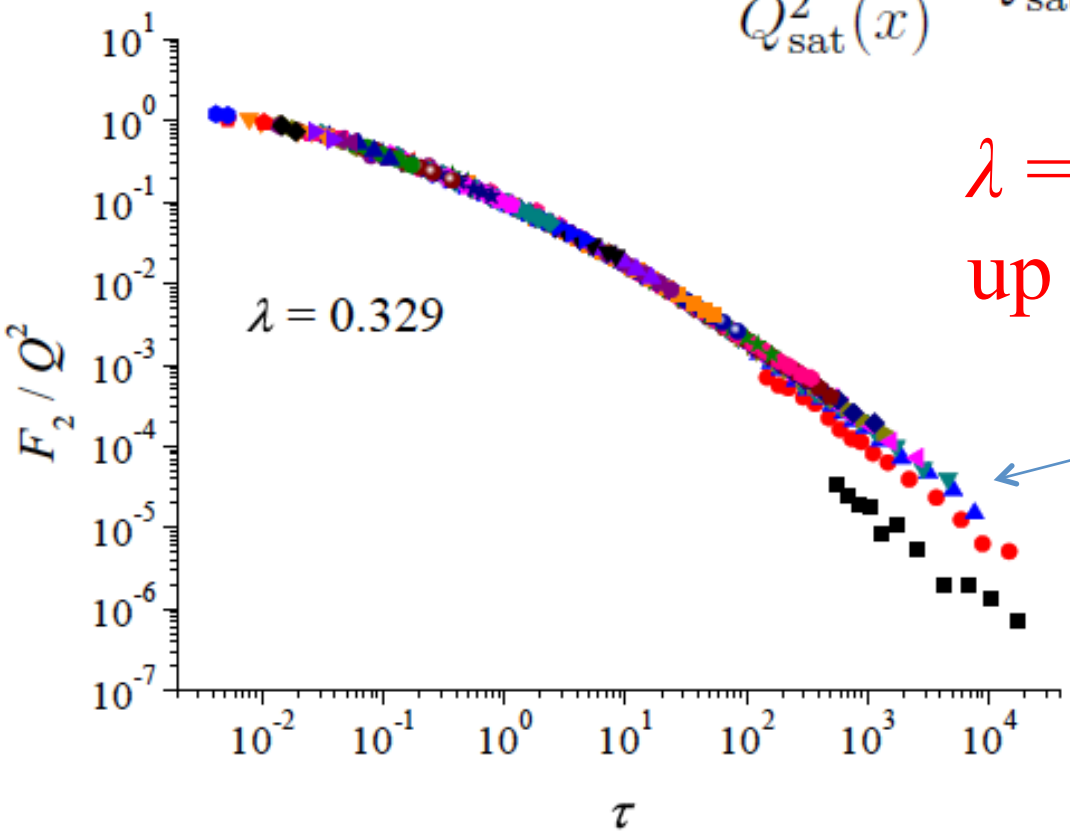


A.M. Stasto, K. J. Golec-Biernat, J. Kwiecinski
PRL 86 (2001) 596-599

M.Praszalowicz and T.Stebel
JHEP 1303, 090 (2013)
arXiv:1211.5305 [hep-ph]
and
JHEP 1304, 169 (2013)
arXiv:1302.4227 [hep-ph]

Saturation scale: energy and x dependence

$$\tau = \frac{Q^2}{Q^2_{\rm sat}(x)} \quad Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$



$\lambda = 0.329 \pm 0.005$

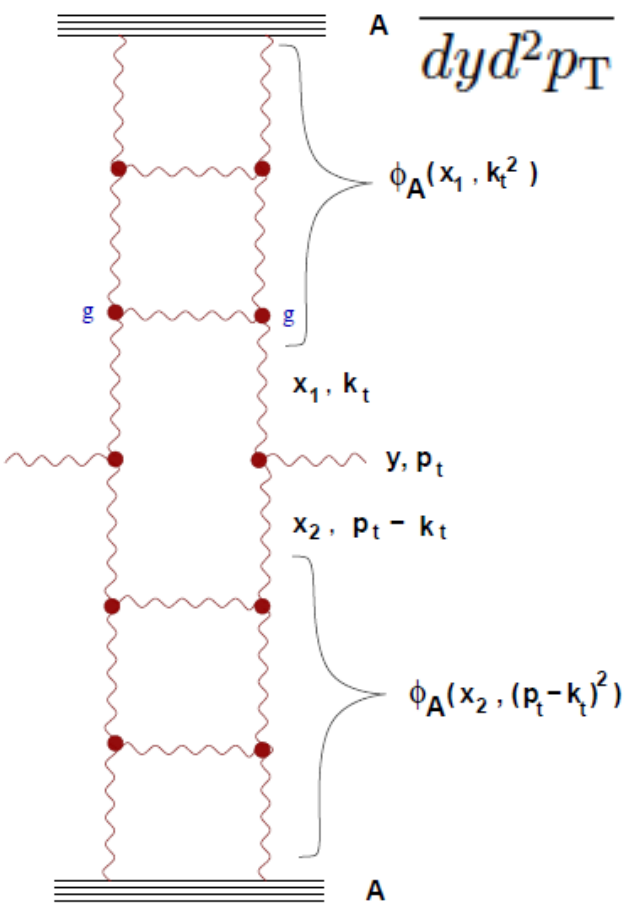
up to $x = 0.08$ (!)

large x

proton-proton @ LHC

Basics of geometrical scaling

Gribov, Levin Ryskin, *High p_T Hadrons In The Pionization Region In QCD*. Phys.Lett.B100:173-176,1981.



$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2p_T^2} \int d^2\vec{k}_T \, \varphi_1(x_1, \vec{k}_T^2) \varphi_2(x_2, (\vec{k} - \vec{p})_T^2)$$

$$x_{1,2} = \frac{p_T}{\sqrt{s}} e^{\pm y}$$

gluon distribution unintegrated glue

$$xG(x, Q^2) = \int^{Q^2} dk_T^2 \, \varphi(x, k_T^2)$$

Kharzeev, Levin
Phys.Lett.B523:79-87,2001.

Basics of geometrical scaling

gluon distribution unintegrated glue

$$xG(x,Q^2) = \int^{Q^2} dk_T^2\, \varphi(x,k_T^2)$$

Golec-Biernat – Wuesthoff (DIS)

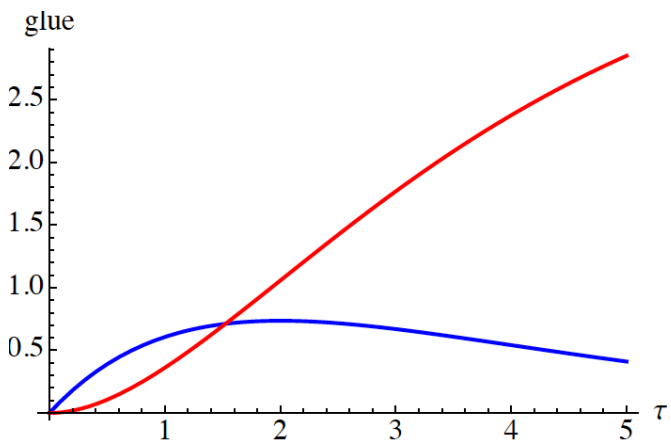
$$\varphi(x,k_T^2) = S_\perp \frac{3}{4\pi^2}\frac{k_T^2}{Q_s(x)^2}\exp\left(-k_T^2/Q_s(x)^2\right)$$

$$S_\perp = \sigma_0$$

Kharzeev – Levin (AA)

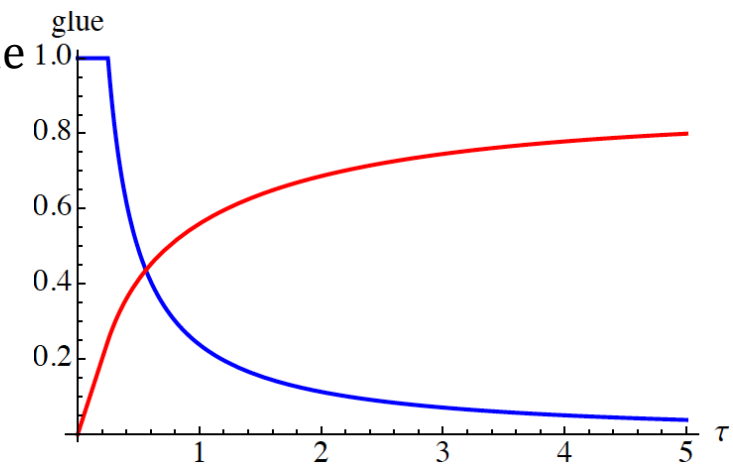
$$\varphi(x,k_T^2) = S_\perp \begin{cases} 1 & \text{for} \quad k_T^2 < Q_s(x)^2 \\ Q_s(x)^2/k_T^2 & \text{for} \quad Q_s(x)^2 < k_T^2 \end{cases}$$

$S_\perp$ is the transverse size given by geometry



scaling variable

$$\tau = \frac{p_T^2}{Q_s^2(x)}$$



Michal Praszalowicz

Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_{\mathrm{T}}} = \frac{3\pi\alpha_{\mathrm{s}}}{2} \frac{Q_s^2(x)}{p_{\mathrm{T}}^2} \int \frac{d^2\vec{k}_{\mathrm{T}}}{Q_s^2(x)} \varphi_1\left(\vec{k}_{\mathrm{T}}^2/Q_s^2(x)\right) \varphi_2\left((\vec{k}-\vec{p})_{\mathrm{T}}^2/Q_s^2(x)\right)$$

Basics of geometrical scaling

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$$\frac{dN}{dyd^2p_{\mathrm{T}}} = S_\perp\mathcal{F}(\tau) \quad \tau = \frac{p_{\mathrm{T}}^2}{Q_s^2(x)} \qquad dp_{\mathrm{T}}^2 = \frac{2}{2+\lambda}\bar{Q}_{\mathrm{s}}^2(W)\,\tau^{-\lambda/(2+\lambda)}d\tau$$

$$\bar{Q}_s(W) = Q_0\left(\frac{W}{Q_0}\right)^{\lambda/(2+\lambda)}$$

Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dyd^2p_{\rm T}} = \frac{3\pi\alpha_{\rm s}}{2}\frac{Q_s^2(x)}{p_{\rm T}^2}\int\frac{d^2\vec{k}_{\rm T}}{Q_s^2(x)}\varphi_1\left(\vec{k}_{\rm T}^2/Q_s^2(x)\right)\varphi_2\left((\vec{k}-\vec{p})_{\rm T}^2/Q_s^2(x)\right)$$

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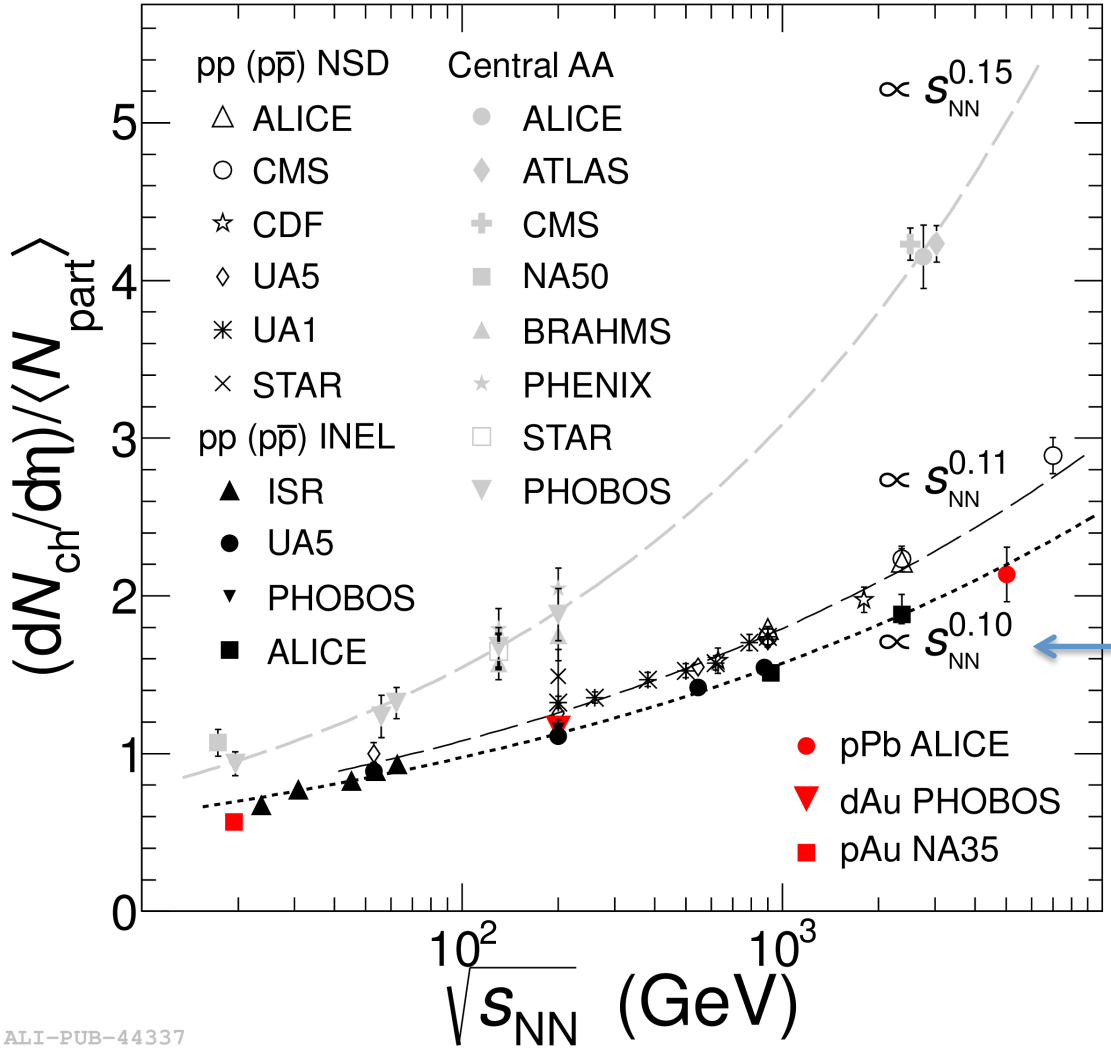
$$\frac{dN}{dy} = S_\perp\int\mathcal{F}(\tau)d^2p_{\rm T} = S_\perp\bar{Q}_s^2(W)\int\mathcal{F}(\tau)\ldots d\tau = \frac{1}{\kappa}S_\perp\bar{Q}_s^2(W)$$

Power-like growth of multiplicity

http://th-www.if.uj.edu.pl/school/2014/talks/braun-munzinger_1.pdf



plot: P. Braun-Munzinger, 54 Cracow School of Theoretical Physics (from ALICE-PUB-44337)

$$\frac{dN_{ch}}{dy} \sim S_\perp \bar{Q}_s^2(W)$$

$$\sim S_\perp Q_0^2 \left(\frac{W^2}{Q_0^2}\right)^{\lambda/(2+\lambda)}$$

inelastic

transverse area is energy indpendent

is power correct?

Geometrical scaling of p_T distribution

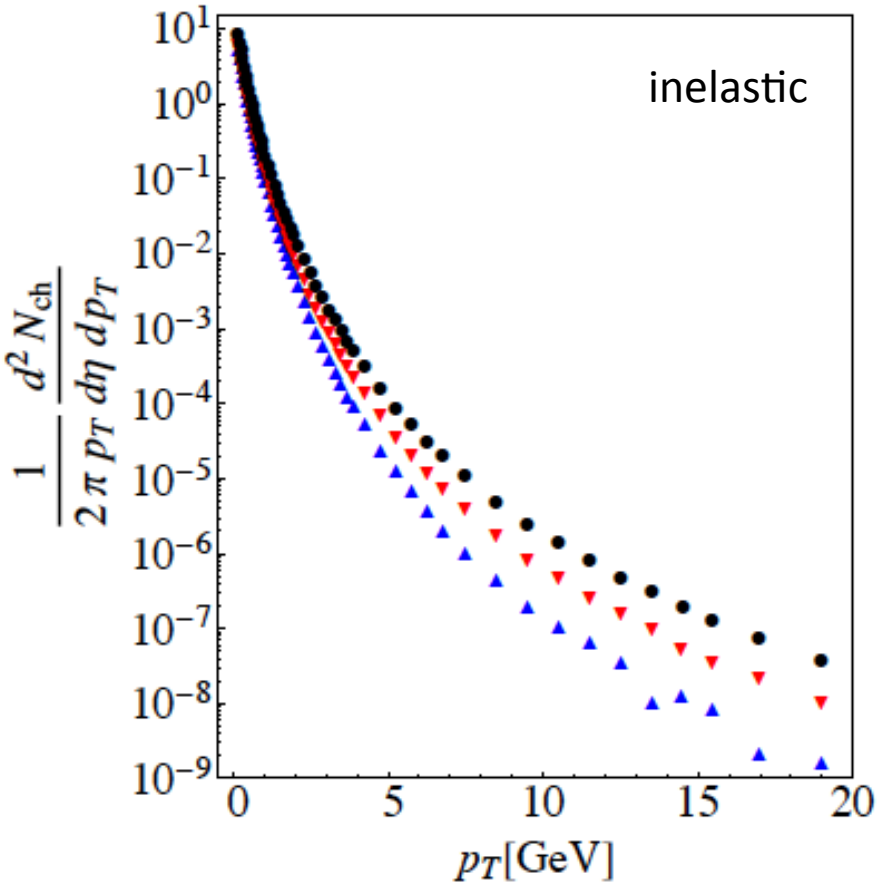
L. McLerran, M. P. Acta Phys.Polon.B41:1917,2010, B42:99,2011
M. P. Phys.Rev.Lett.106:142002,2011, Acta Phys.Pol. B42 (2011) 1557-1566
Phys.Rev. D87 (2013) 071502(R)

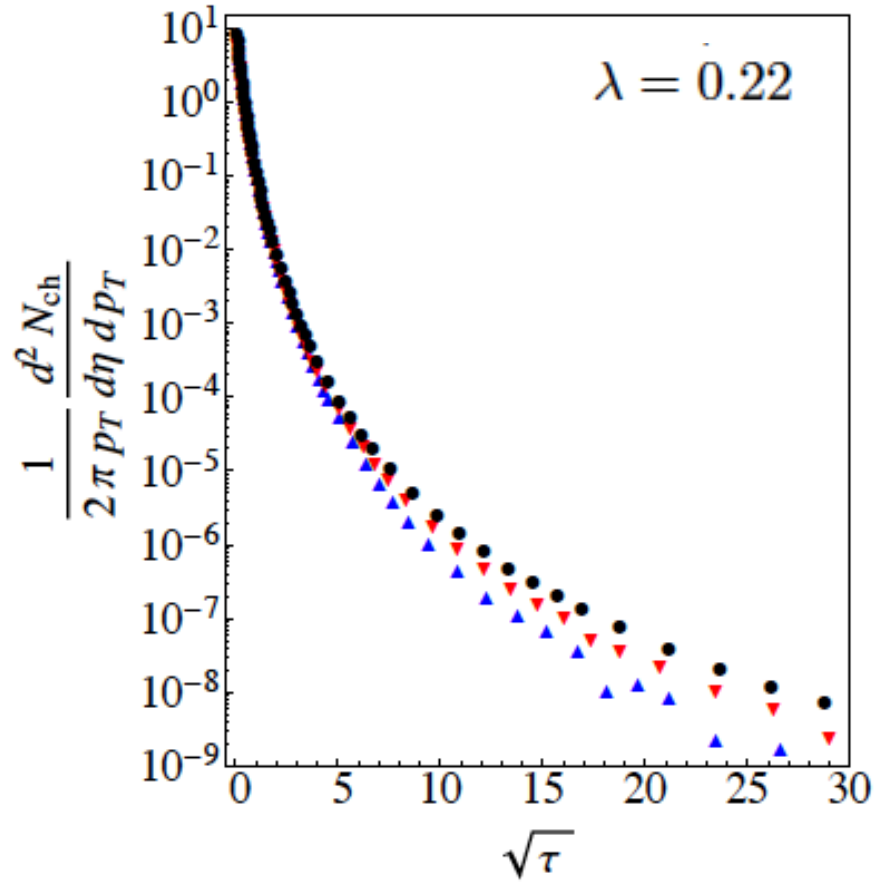
$$\tau = \frac{p_{\mathrm{T}}^2}{Q_{\mathrm{sat}}^2(p_{\mathrm{T}}/\sqrt{s})} = \frac{p_{\mathrm{T}}^2}{1\,\mathrm{GeV}^2}\left(\frac{p_{\mathrm{T}}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$

Determination of lambda

$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_\perp \mathcal{F}(\tau) \qquad \tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^\lambda$$

ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662





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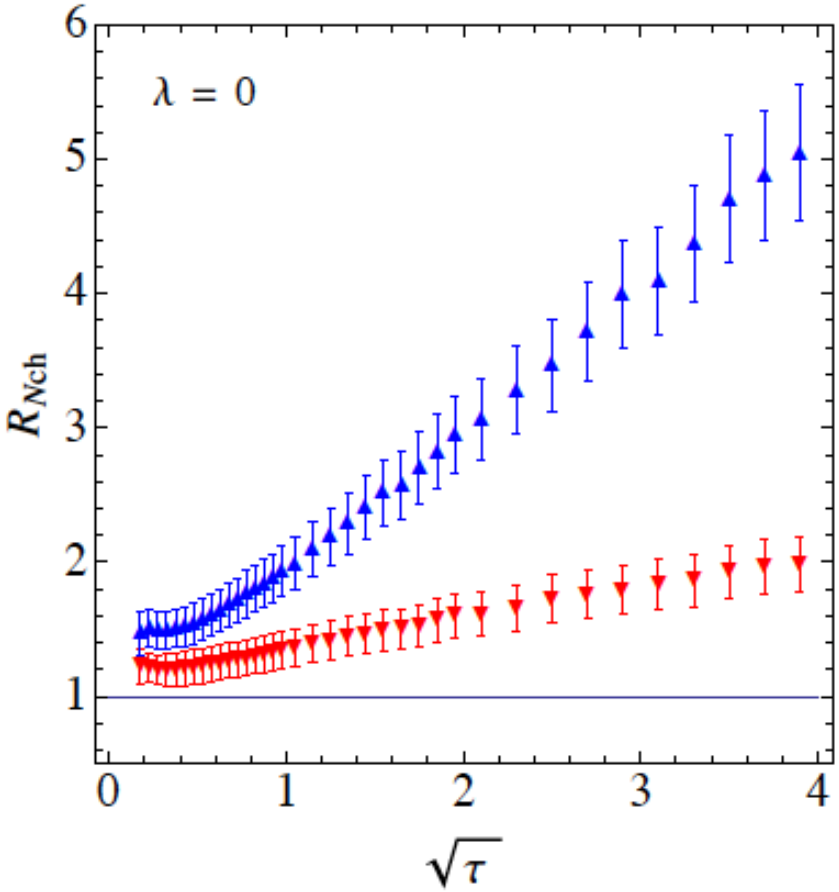
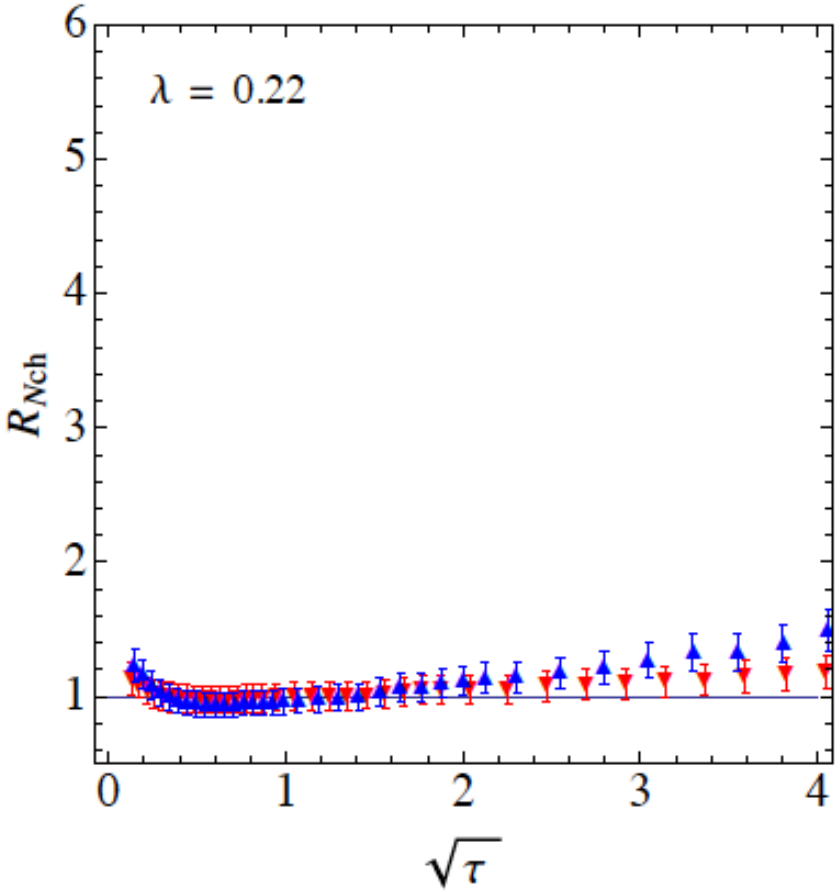
$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_\perp \mathcal{F}(\tau) \qquad \tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^\lambda$$

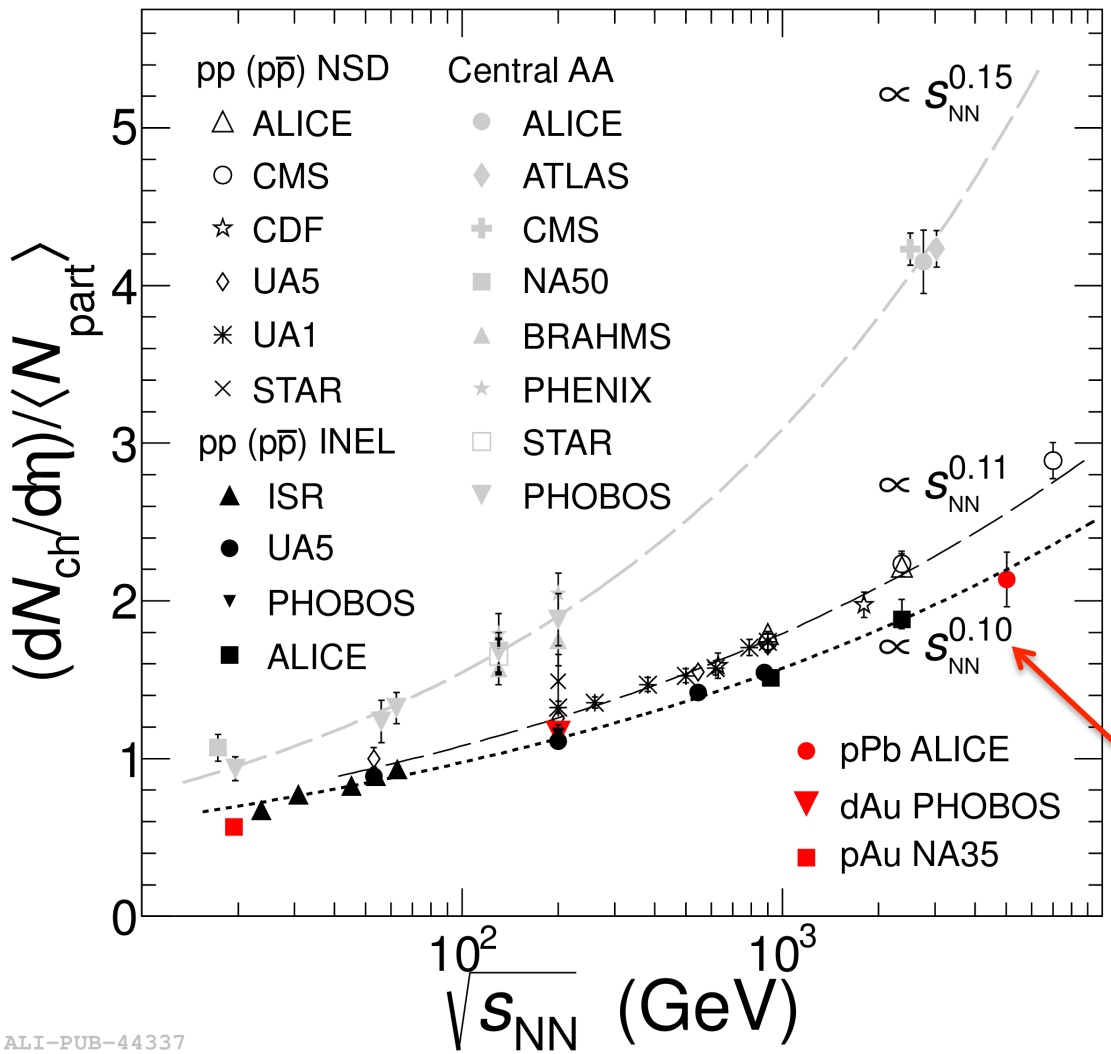
ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662





Power-like growth of multiplicity

http://th-www.if.uj.edu.pl/school/2014/talks/braun-munzinger_1.pdf



plot: P. Braun-Munzinger, 54 Cracow School of Theoretical Physics (from ALICE-PUB-44337)

$$\frac{dN_{ch}}{dy} \sim S_\perp \bar{Q}_s^2(W)$$

$$\sim S_\perp Q_0^2 \left(\frac{W^2}{Q_0^2}\right)^{\lambda/(2+\lambda)}$$

transverse area is energy indpendent

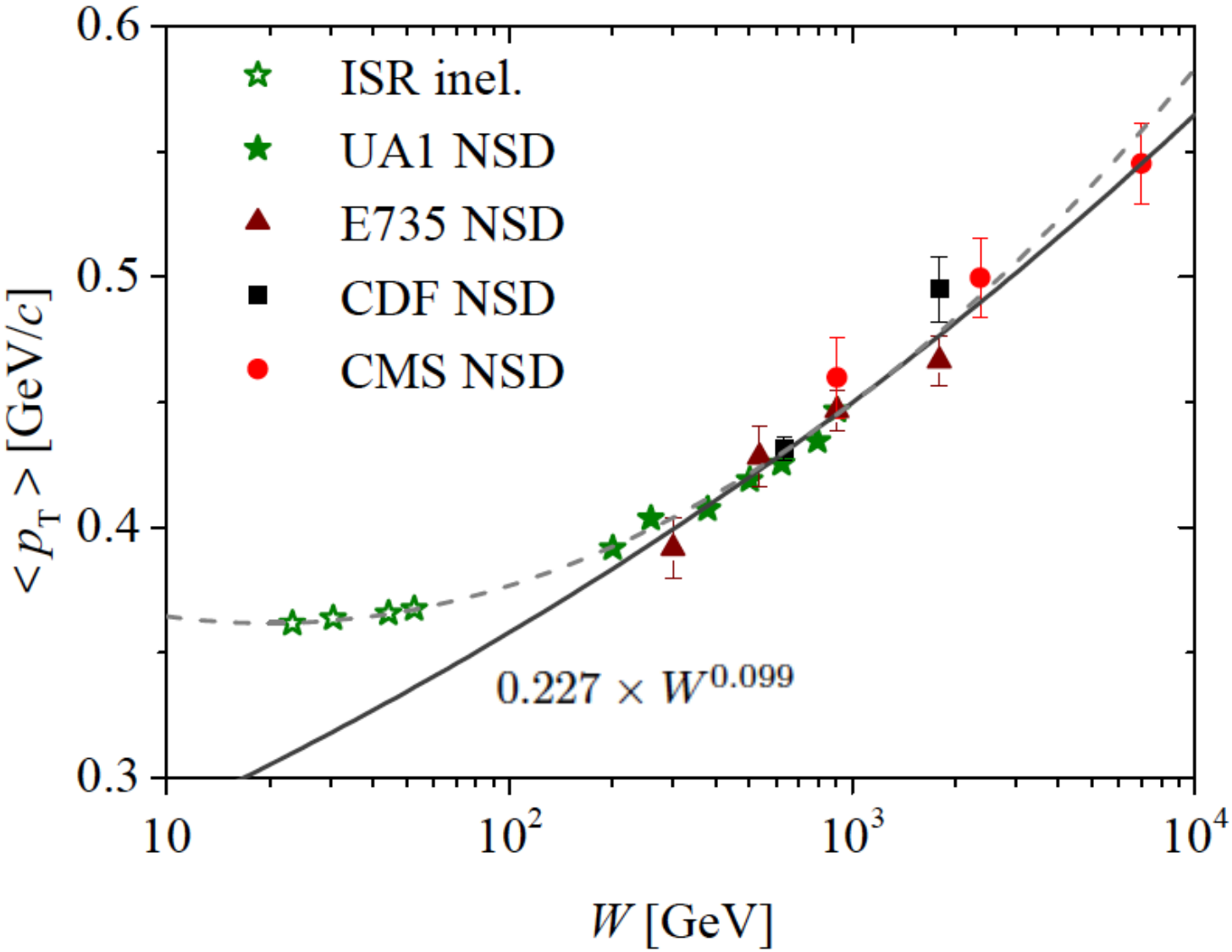
$$\lambda/(2+\lambda) \simeq 0.099$$

Average transverse momentum

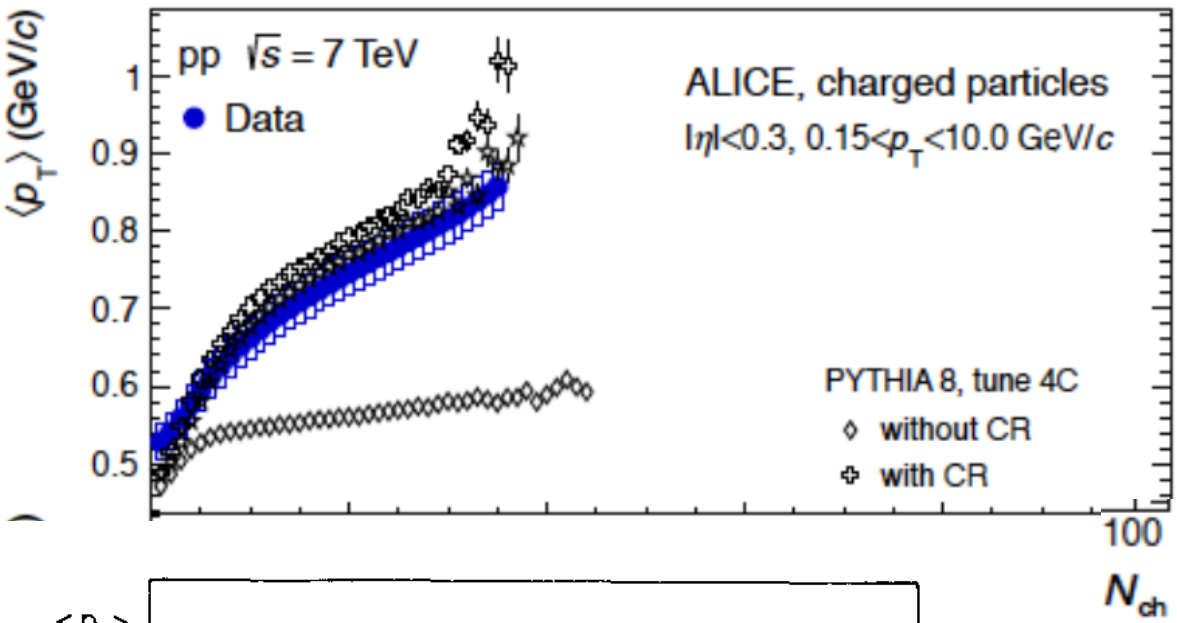
$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_\perp \mathcal{F}(\tau)$$

$$\langle p_{\rm T} \rangle = \frac{\int p_T \frac{dN_{\rm g}}{dy d^2 p_{\rm T}} d^2 p_T}{\int \frac{dN_{\rm g}}{dy d^2 p_{\rm T}} d^2 p_T} \sim \bar{Q}_{\rm s}(W) \sim Q_0 \left(\frac{W}{Q_0}\right)^{\lambda/(2+\lambda)}$$

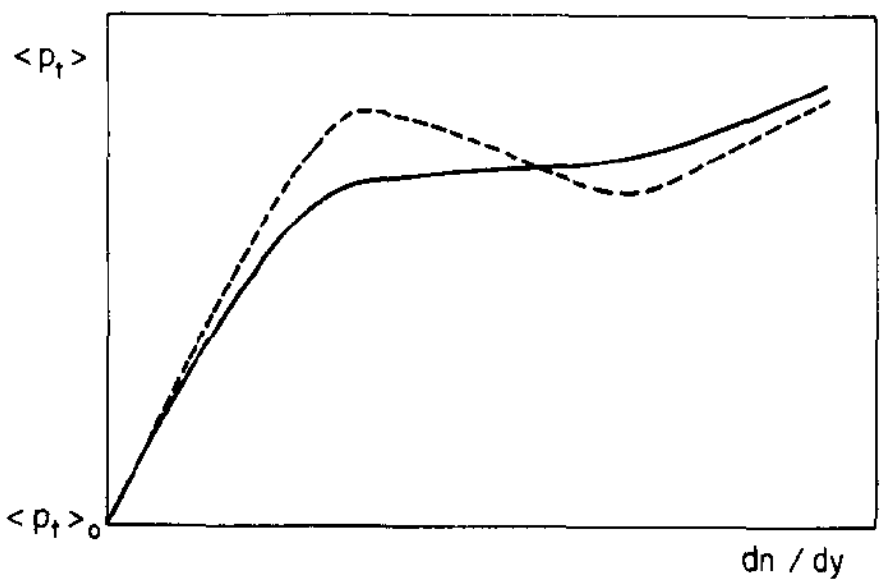
Average transverse momentum



Mean p_T as a function of N_{ch}



- $\langle p_T \rangle (N_{ch})$ – correlations are sensitive to the fine details of dynamics
- difficult to describe by untuned MonteCarlos
- possible sign of phase transition



MULTIPLICITY DEPENDENCE OF p_t SPECTRUM AS A POSSIBLE SIGNAL FOR A PHASE TRANSITION IN HADRONIC COLLISIONS

L. VAN HOVE
CERN, Geneva, Switzerland

Received 25 August 1982

Mean p_T as a function of N_{ch}

$$\langle p_{\mathrm{T}} \rangle \sim \bar{Q}_{\mathrm{s}}(W)$$

Mean p_T as a function of N_{ch}

$$\langle p_{\mathrm{T}} \rangle \sim \bar{Q}_{\mathrm{s}}(W) \sim \sqrt{\frac{dN/dy}{S_\perp}} \sim \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

interaction radius

Mean p_T as a function of N_{ch}

$$\langle p_{\mathrm{T}} \rangle \sim \bar{Q}_{\mathrm{s}}(W) \sim \sqrt{\frac{dN/dy}{S_\perp}} \sim \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

interaction radius

phenomenological formula:

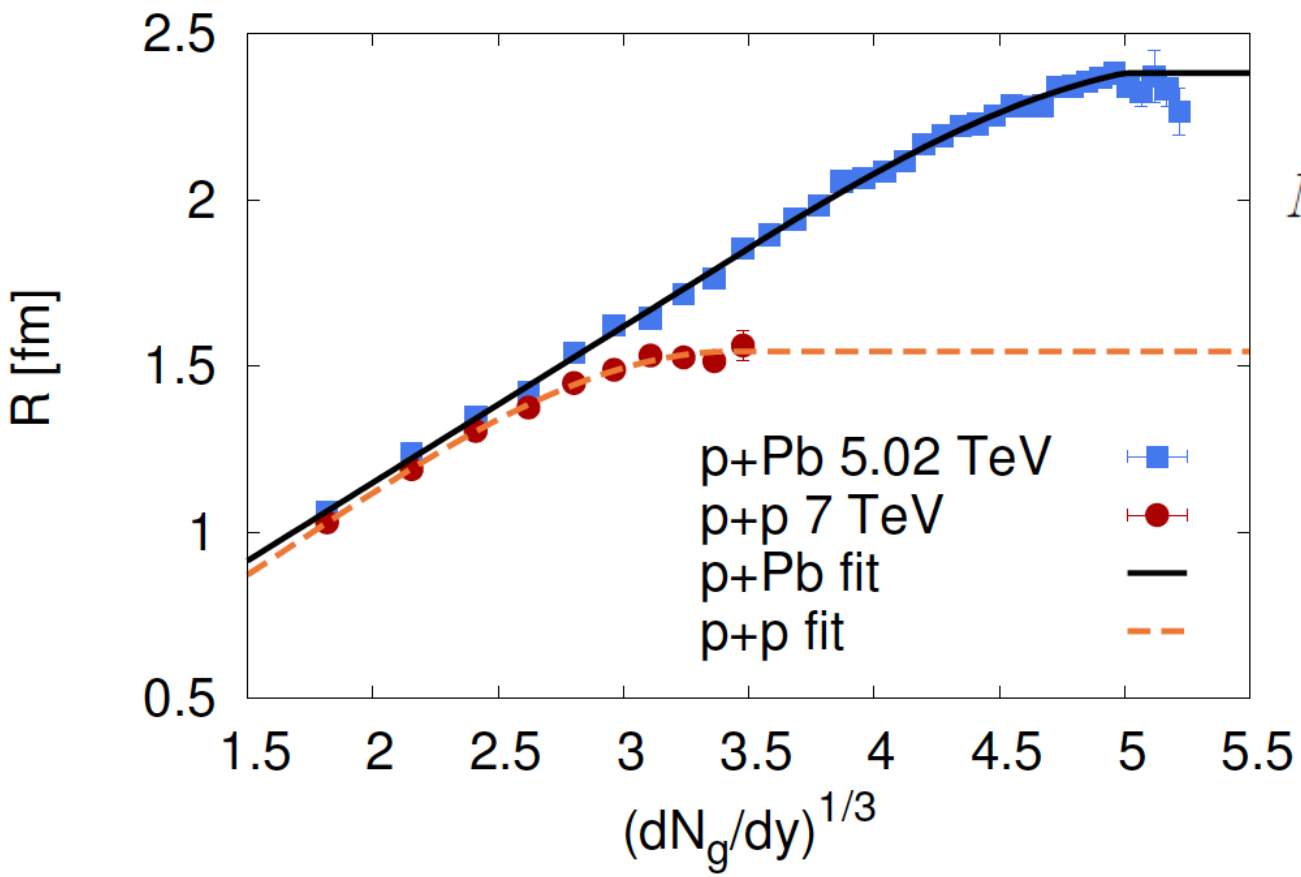
$$\langle p_{\mathrm{T}} \rangle = \alpha + \beta \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

nonperturbaitive
coefficient

α, β do not depend on energy, do depend on particle species

Interaction radius

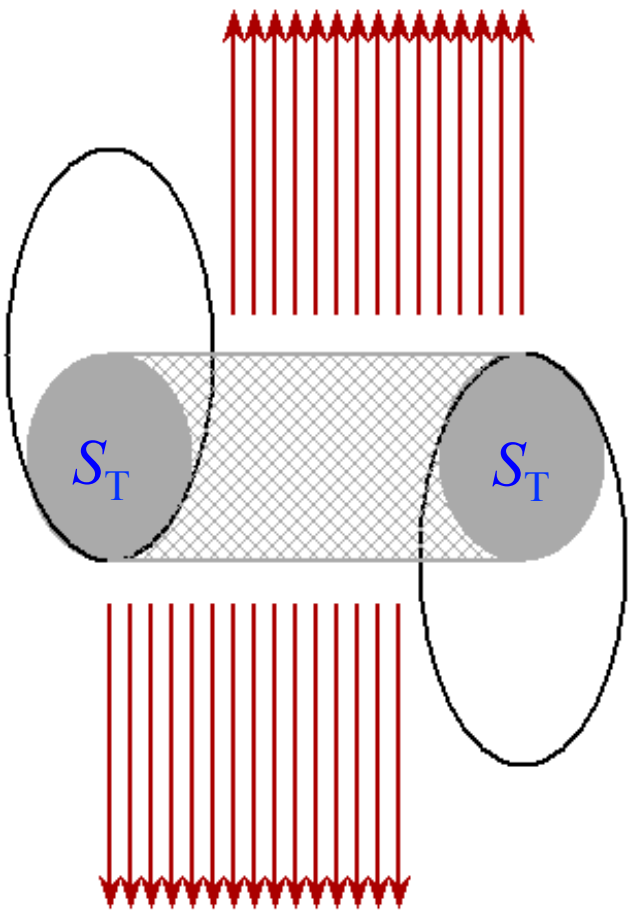
A. Bzdak, B. Schenke, P. Tribedy and R. Venugopalan,
Initial state geometry and the role of hydrodynamics in proton-proton, proton-nucleus and deuteron-nucleus collisions,
Phys. Rev. C 87 (2013) 064906, [arXiv:1304.3403 [nucl-th]].



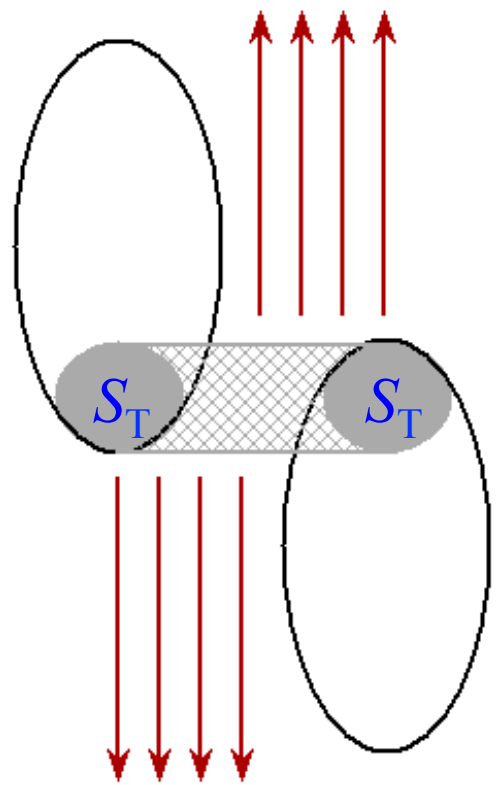
$$N_{\rm ch} = \frac{1}{\gamma \Delta y} \int_{\Delta y} \frac{dN_{\rm g}}{dy} dy$$

Interaction radius

Transverse size and expansion time (longitudinal size) are proportional for fixed multiplicity



large multiplicity



small multiplicity

similar effect in multipomeron model, where string tension is growing with multiplicity

M. A. Braun, C. Pajares
Phys. Lett. B 287, 154(1992)
Nucl. Phys. B 390 , 542, 559 (1993)
N. Armesto, D.A. Derkach, G.A. Feofilov
Phys. of At. Nuclei 71, 2087 (2008)

Energy dependence of mean p_T

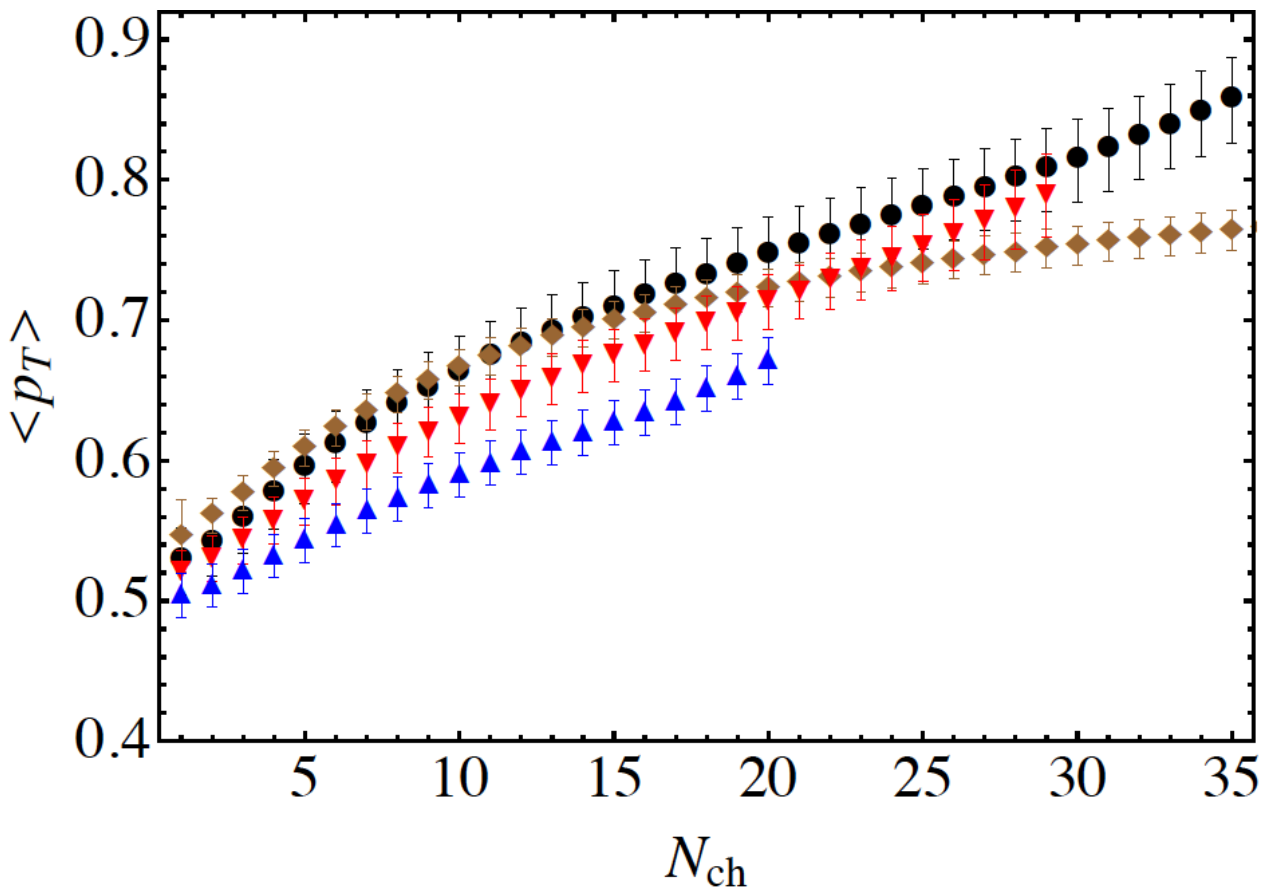
$$\langle p_{\mathrm{T}} \rangle|_W = \alpha + \beta \frac{\sqrt{N_{\mathrm{ch}}}}{R(\sqrt[3]{\gamma N_{\mathrm{ch}}})|_W} = \alpha + \beta \left(\frac{W}{W_0}\right)^{\lambda/(2+\lambda)} \frac{\sqrt{N_{\mathrm{ch}}}}{R(\sqrt[3]{\gamma N_{\mathrm{ch}}})|_{W_0}}$$

Energy dependence of mean p_T

$$\langle p_{\mathrm{T}} \rangle|_W = \alpha + \beta \frac{\sqrt{N_{\mathrm{ch}}}}{R(\sqrt[3]{\gamma N_{\mathrm{ch}}})|_W} = \alpha + \beta \left(\frac{W}{W_0}\right)^{\lambda/(2+\lambda)} \frac{\sqrt{N_{\mathrm{ch}}}}{R(\sqrt[3]{\gamma N_{\mathrm{ch}}})|_{W_0}}$$

Mean p_T scaling

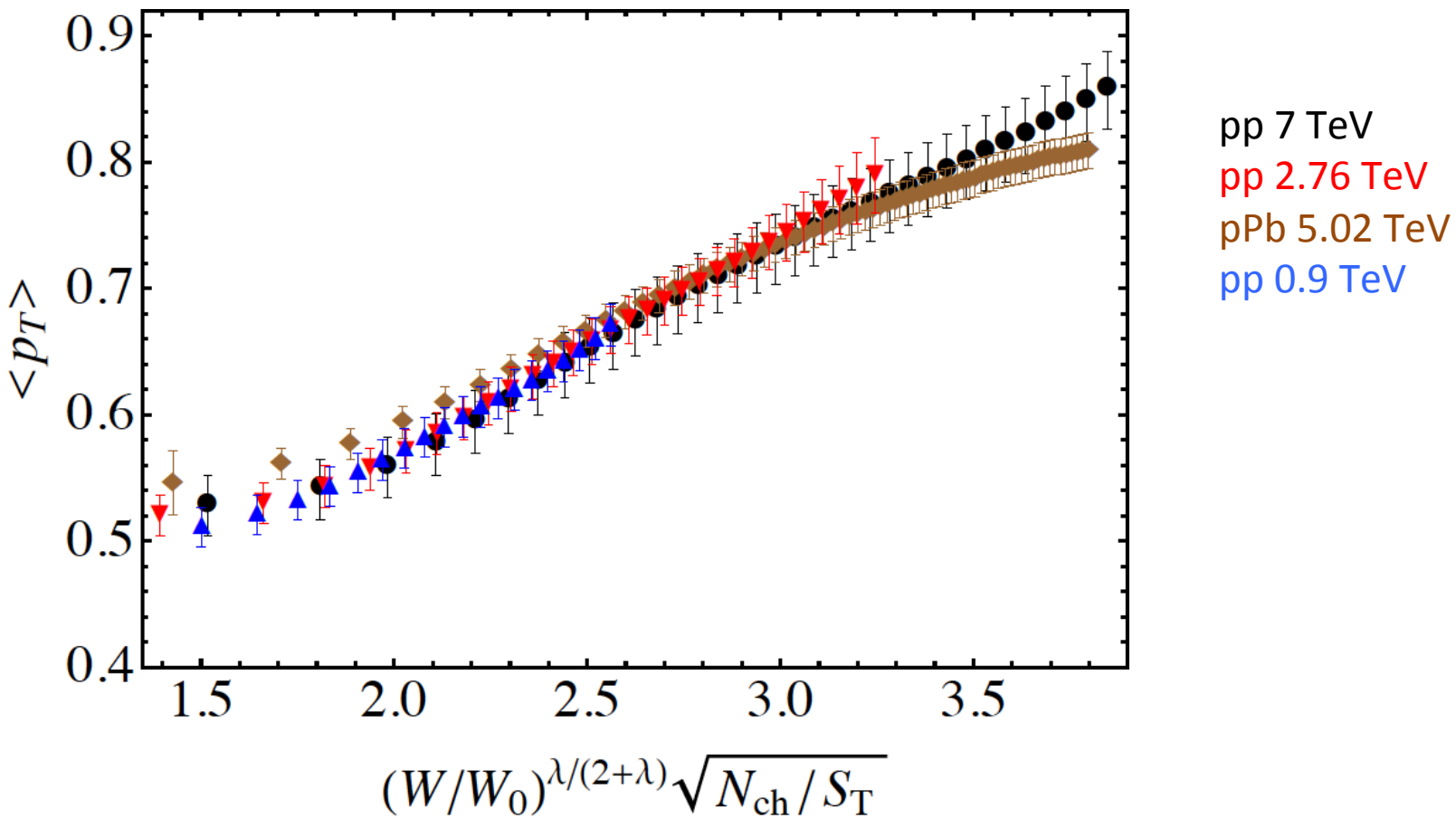
ALICE Collaboration, Phys. Lett. B727 (2013) 371 [arXiv:1307.1094 [nucl-ex]]



pp 7 TeV
pp 2.76 TeV
pPb 5.02 TeV
pp 0.9 TeV

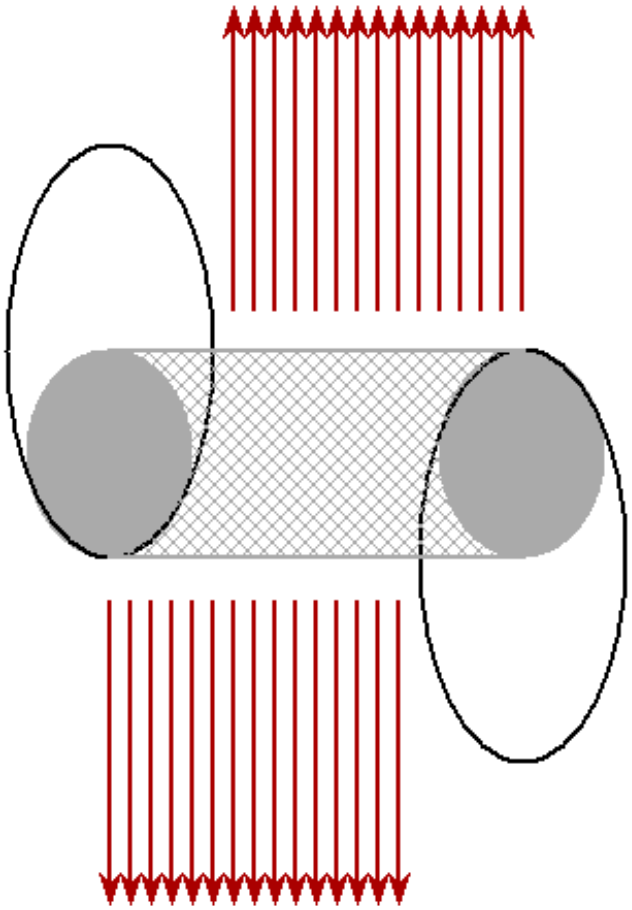
Mean p_T scaling

ALICE Collaboration, Phys. Lett. B727 (2013) 371 [arXiv:1307.1094 [nucl-ex]]

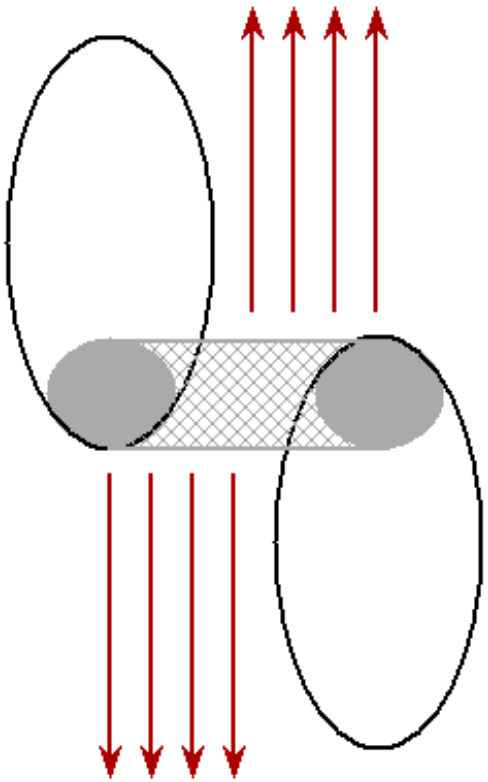


Geometrical Scaling in Heavy Ion Collisions

GS in HI: centrality



small centrality



large centrality

number of participants

$$N_{\text{part}} \sim V$$

GS in HI: centrality dependence

$$S_\perp \sim N_{\rm part}^{2/3}$$
$$\frac{dN}{dy} \sim N_{\rm part}$$

Geometrical Scaling of Direct-Photon Production in Hadron Collisions from RHIC to the LHC
Christian Klein-Bösing, Larry McLerran arXiv:1403.1174

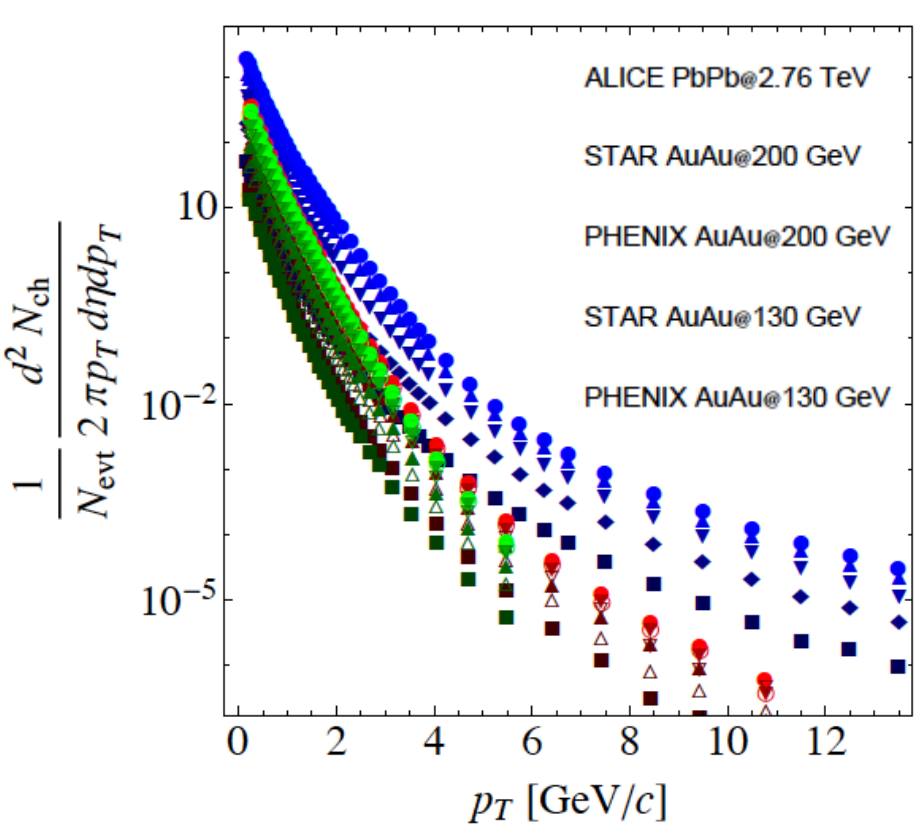
Scaling of the saturation scale:

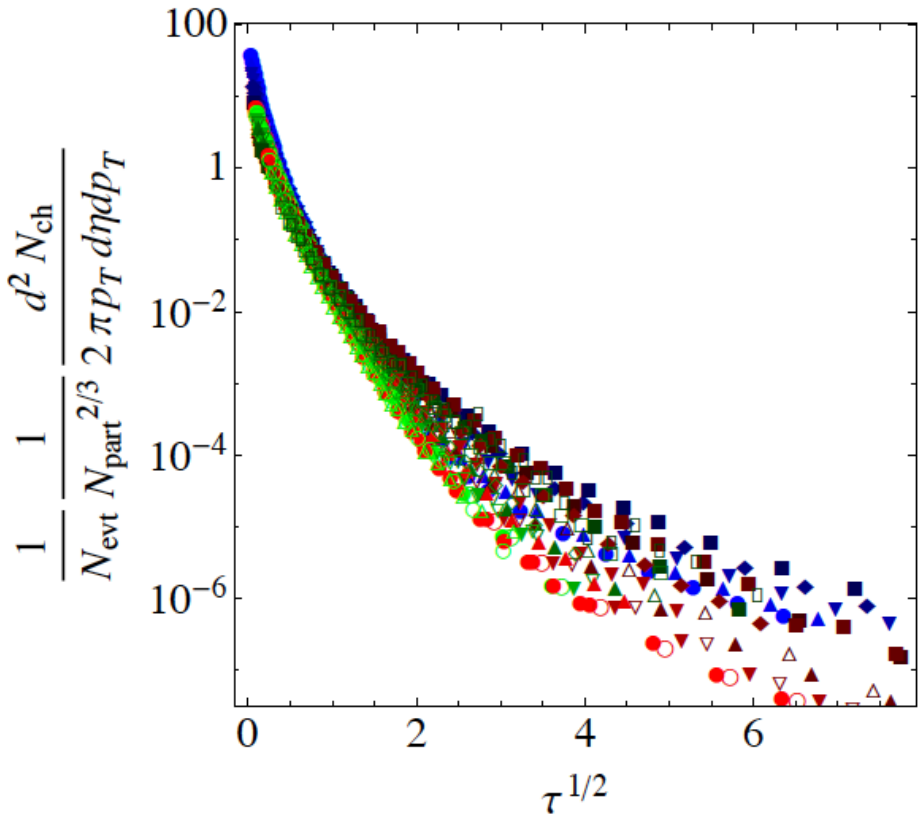
$$Q_s^2(x) = \frac{\kappa}{S_\perp}\frac{dN}{dy} \sim N_{\rm part}^{1/3}\left(\frac{\sqrt{s}}{p_{\rm T}}\right)^\lambda$$

$$\frac{Q_0^2}{N_{\rm part}^{2/3}}\frac{dN_{\rm ch}}{2\pi p_{\rm T}\, d\eta dp_{\rm T}} = F(\tau)$$

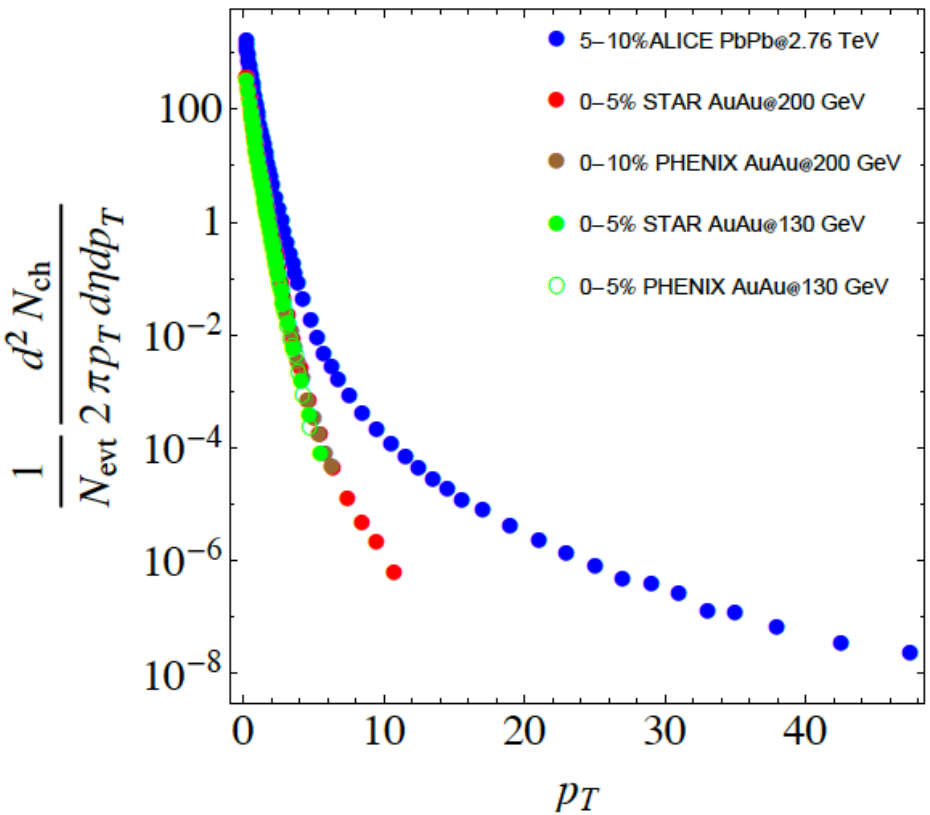
$$\tau = \frac{1}{N_{\rm part}^{1/3}}\frac{p_{\rm T}^2}{Q_0^2}\left(\frac{p_{\rm T}}{W}\right)^\lambda$$

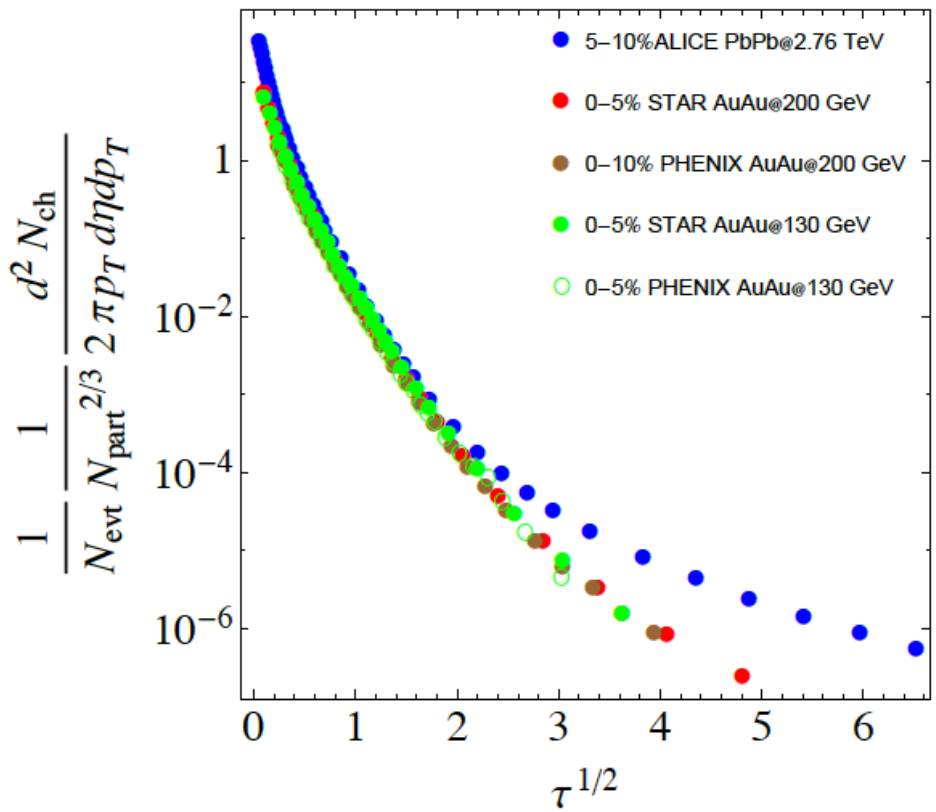
GS in HI


ALICE PbPb@2.76 TeV
STAR AuAu@200 GeV
PHENIX AuAu@200 GeV
STAR AuAu@130 GeV
PHENIX AuAu@130 GeV
$\frac{1}{N_{evt}} \frac{d^2 N_{ch}}{2\pi p_T d\eta dp_T}$
p_T [GeV/c]


$\frac{1}{N_{evt}} \frac{1}{N_{part}^{2/3}} \frac{d^2 N_{ch}}{2\pi p_T d\eta dp_T}$
$\tau^{1/2}$

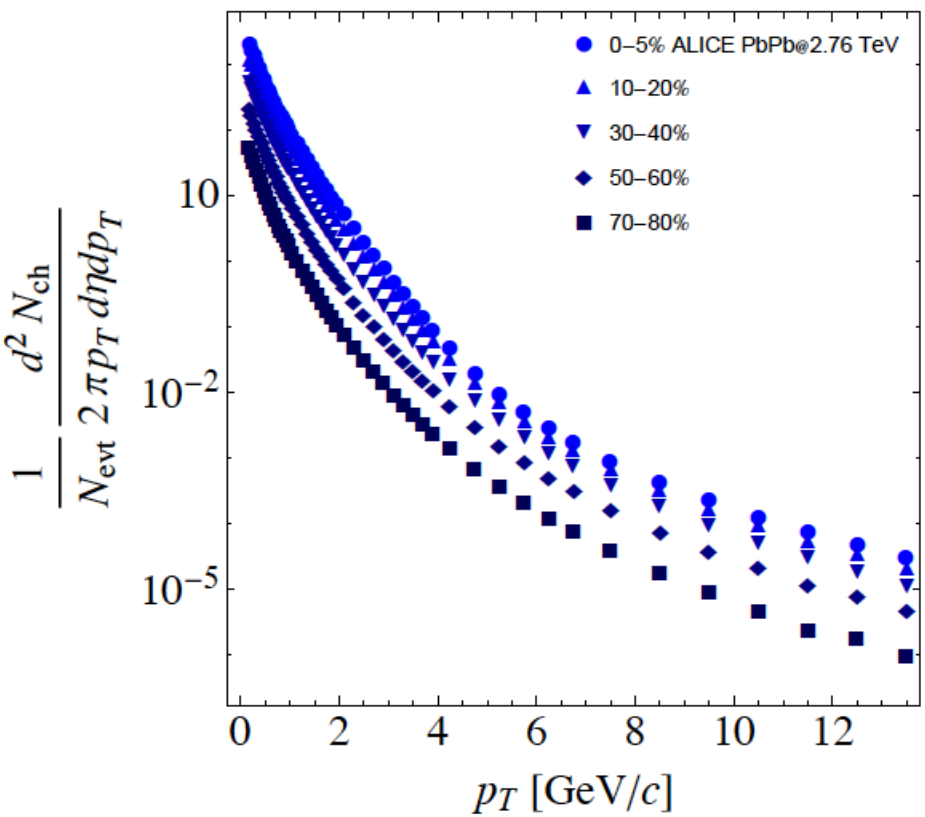
Energy Scaling in HI

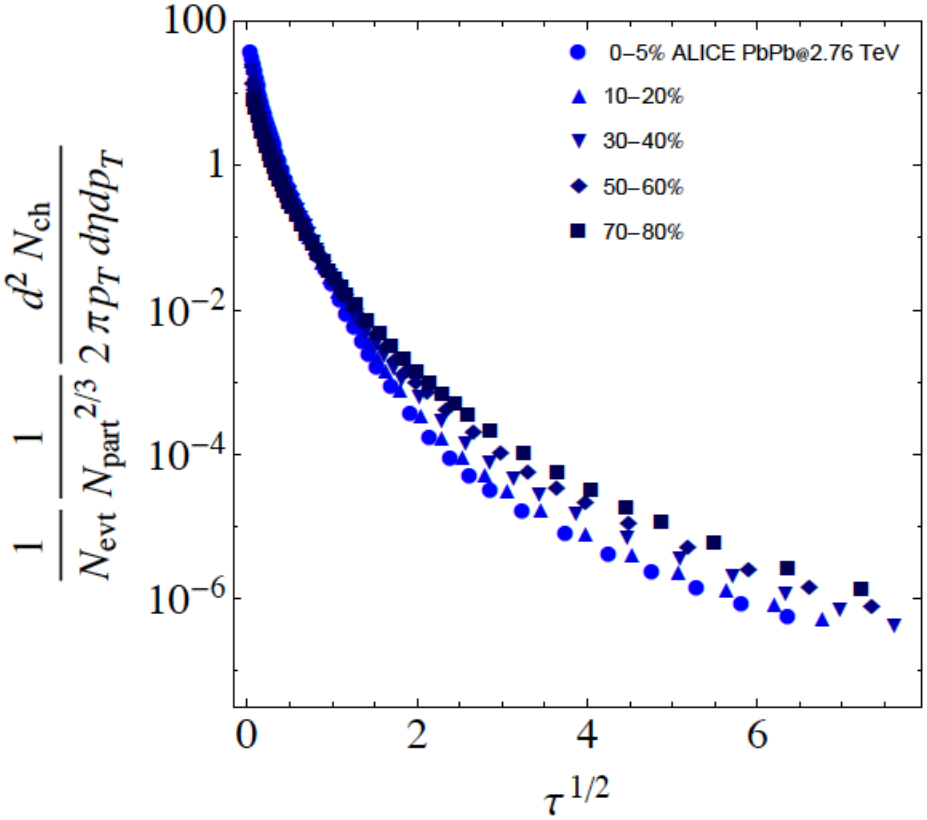




energy scaling works quite well, why?

Centrality Scaling in HI





Summary

- QCD evolution equations lead to overabundance of gluons
- Nonlinear evolution introduces new scale: *saturation momentum*
- GS should emerge if no other scales are present
- GS in DIS works for rather high Bjorken x
- GS works also for charged particles in pp
- GS for mean p_T and for $<p_T>(N_{ch})$
- Energy and centrality dependence of GI in HI

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- GS in DIS works for rather high Bjorken x
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- GS for mean p_T and for $\langle p_T \rangle (N_{ch})$
- Energy and centrality dependence of GS in HI
- Is GS a real sign of saturation?
- Why in pp GS is not washed out by FSI?
- Why in HI hydro preserves (at least partially) GS?
- Nonuniversality: different values of λ