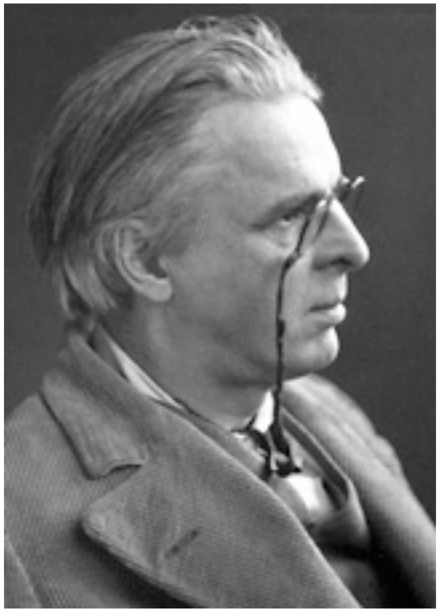
New Insights into Confinement



The Irish poet, William Butler Yeats was prescient. In 1920 he described the Yang Mills phase transition in his apocalyptic poem *Second Coming*:

**"Things fall apart;
the centre cannot hold"**

TDC Phys. Rev. D 90, 047703 (2014)
A. Armoni, TDC, S. Sen ArXive 1502.0136

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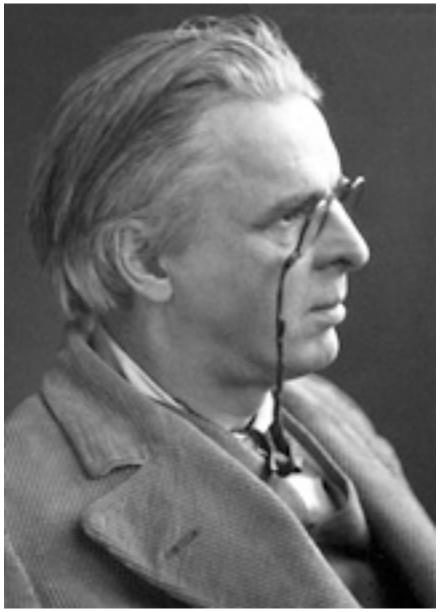

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Yeats won the Nobel Prize in 1923 for his work but, inexplicably, it was in literature rather than physics.

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New Insights into Old Insights into Confinement

An Overview

KAVIAREŇ DEDO
LOMNICKÝ ŠTÍT
2634 m
Kežmarský štít
2558 m
Slavkovský štít
2452 m
LOMNICKÉ SEDLO
2190 m
Freeride zone
Francúzska mulda
SKALNATÉ PLESO
1751 m
Observatórium
Hrebienok - Skalnaté pleso
zimná cesta
Skalnatá chata
RESTAURANT SKALNATÉ PLESO
WC
NEW
Belianske Tatry
ČUČORIEDKY
RESTAURANT PIZZA & PASTA ŠTART
ŠTART
1173m
GRANDHOTEL PRAHA ****
ľahká / easy
stredná / medium
ťažká / difficult
Vodná nádrž
APRÉS SKI BAR
TATRY MOTION
JAMY
ATM
HUMNO - MUSIC PUB
TATRANSKÁ LOMNICA
903m
TATRANSKÁ KOTLINA
STARÝ SMOKOVEC
ŠTRBSKÉ PLESO
POPRAD, KEŽMAROK
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An Overview

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  The focus is on two distinct aspects

  - **Area laws of Wilson loops and the pattern of which ones are equal and also which ones nonzero.**
  - **The emergence of a Hagedorn spectrum at large Nc for confining gauge theories**

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This is a very deep question and it is not completely clear how to even define it.

Instead of trying to tease this out directly, it may be useful look at…

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Of interest mostly to theorists looking at simplified limits or theories other than QCD!

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These cartoon theories can bring out issues of confinement in a starker form than QCD

PART I—AREA LAWS AND CENTER SYMMETRY

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- **A useful strategy is to concentrate on theories that have area laws for Wilson loops but are not invariant under all transformations associated with the center of the gauge group.**

- **A useful strategy is to concentrate on theories that have area laws for Wilson loops but are not invariant under all transformations associated with the center of the gauge group.**
- **Focus is on SU(Nc) gauge theories with matter in various representations. Theories exist which have area laws but are not invariant under the center of the gauge group. However, they all have *some kind symmetry*—*they are invariant under a nontrival subgroup of the center. Moreover, key aspects of the theories are controled by the nature of this subgroup.***

- **Note that the representations consistent with asymptotic freedom are very limited in 3+1 space time dimensions but in low dimensions many possibilities are allowed. It is worth exploring these to get insight into general problem.**

- **Note that the representations of matter fields consistent with asymptotic freedom are very limited in 3+1 space time dimensions but in low dimensions many possibilities are allowed. It is worth exploring these to get insight into general problem.**
- **For example: an SU(3) Yang Mills theory in 3+1 space-time dimensions, an SU(12) gauge theory with quarks in a 3-index symmetric representation in 2+1space-time dimensions and an SU(15)) gauge theory with quarks in both a 12-index antisymmetric representation and a 9-indexsymmetric representation in 2+1 space-time dimensions all share Z_3 as their maximum center.**

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- Are there properties of Wilson loops that these theories share as a result of this symmetry?

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- **Are there properties of Wilson loops that these theories share as a result of this symmetry?**
- **If so, what are they?**

A New Result

An Su(Nc) gauge theory with matter fields has the Z_{Nc} center symmetry of the gauge theory broken to a maximum subgroup of Z_p by the matter fields. The string tension for representation *R* is given by

$$\sigma_R = \sigma_F \frac{x_R g(x_R)}{(p-1)g(p-1)} \quad \textbf{with}$$

$$x_R = p_R(p - p_R) \quad \textbf{where } p_R = n_R \bmod(p)$$

n_R **is n-ality of the representation**

g **is a function which depends on the theory.**

- **The remainder of this section will explain what that formula means**
- **Key point in this is the relevant thing in determining which string tensions are the same is *p* where Z_p is maximum subgroup of the center preserved in the presence of matter—and not N_c which specifies the gauge group.**

- This rule, says for example that all of those theories which have are invariant under a maximum subgroup of the center given by Z_3 have the property that
  - the string tension for all representations whose n-ality is multiple for three vanish.
  - the string for all other representations are the same.
- This holds regardless of the value of Nc of the gauge group

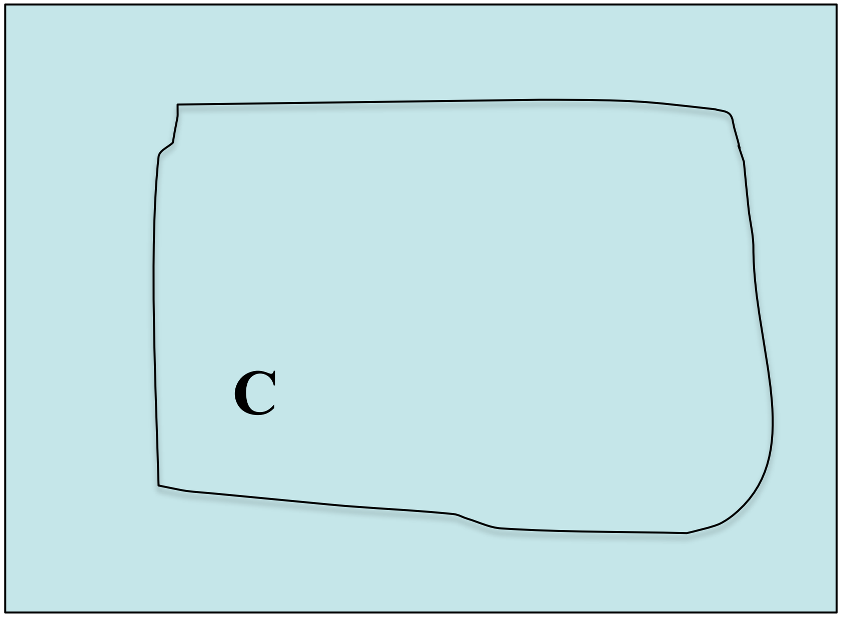
A review:

Some basic and well-known facts about Wilson loops, n-ality, center symmetry and all that

- Wilson Loop is the trace of a Wilson line defined over a closed curve *C*; it is gauge invariant.

$$W \equiv \mathrm{tr}\left(P \exp\left(i \oint_C A_\mu^a \lambda^a \, dx_\mu \right)\right)$$

where P means path-ordered product and λ are Gell-Mann matrices

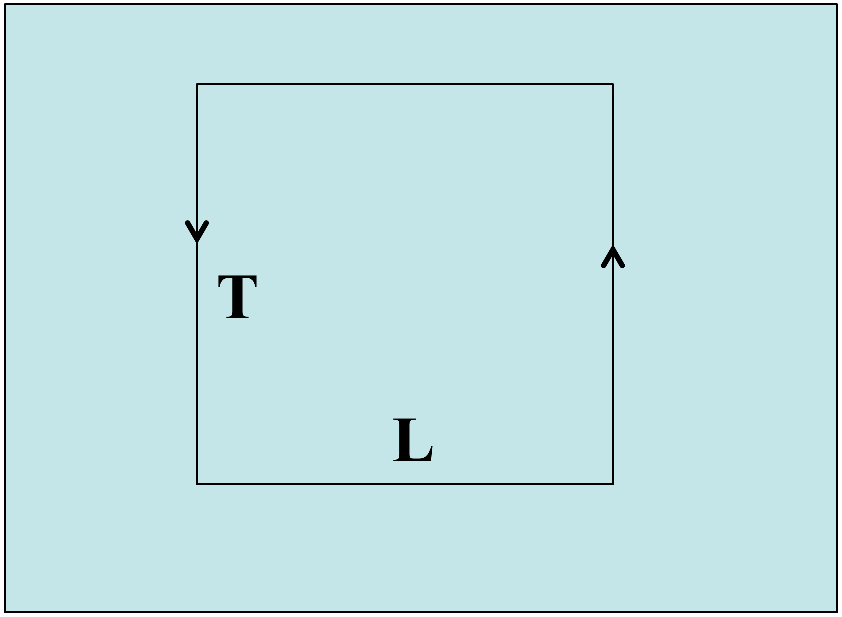


Associated with the exponential of minus the action for separating a fundamental color charge (eg. an infinitely heavy nodynamical quark) from its anticharge, moving it around C in a gauge invariant way and then annihilating against the antquark

- Consider the Wilson loop over a large space-time rectangle; if the theory has linear confinement with a string tension σ, action ~ $\sigma LT \sim \sigma A$ so and we expect

$$W \sim e^{-\sigma A}$$

$$\sigma \equiv \lim_{A\to\infty}\left(-\frac{\log(\langle W \rangle)}{A}\right)$$



An area law for the Wilson loop corresonds to a non-vanishing string tension.

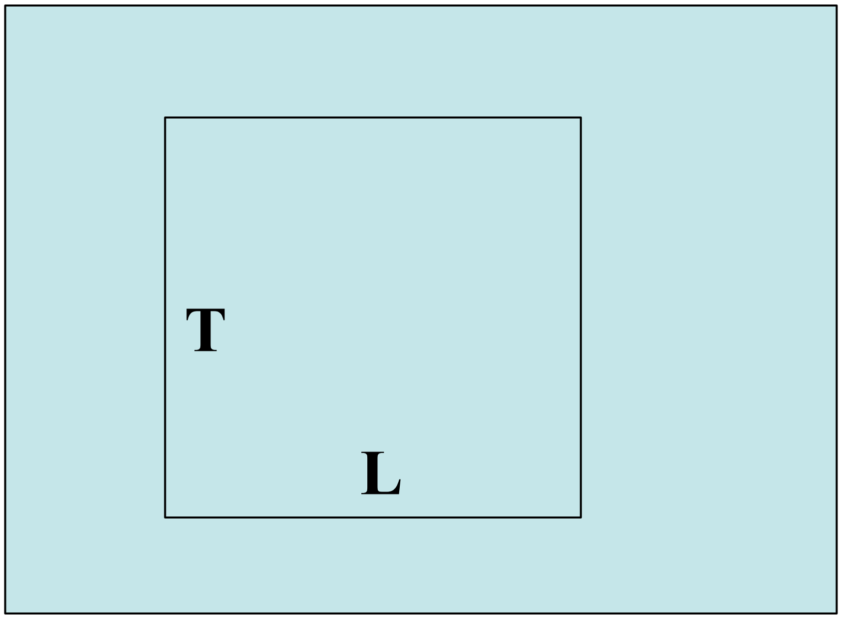
The area law in YM follows from the fact that the flux tube cannot break since there are no fundamental charges in a pure YM theory.

- Note the idea can be generalized for color sources of any representation R

$$W_R \equiv \mathrm{tr}\left(P \exp\left(i \oint_C A_\mu^a T_R^a \, dx_\mu \right) \right)$$

T_R* are matrices for the generators in representation *R

$$\sigma_R \equiv \lim_{A \to \infty} \left(- \frac{\log(\langle W_R \rangle)}{A} \right)$$



***σ_R* is the string tension for separating color charges in representation *R* .**

- **For the adjoint representation σ_{Adj} is zero and there is no area law.**
  - **Physically this is because a flux tube with an adjoint source can break: gluons can pop out of the vacuum and can cap the broken ends.**
  - **Thus at long distances it becomes cheaper energetically to make gluons and break the flux tube then to let the flux tube grow.**

N-ality

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  For example:

  - **The fundamental representation (quarks) are n-ality 1.**
  - **The adjoint representation (gluons) are n-ality 0.**
  - **Two index representations are n-ality 2.**

By construction in a Clebsch - Gordan decomposition

$R_1 \otimes R_2 = \sum_i R_i$ the n - alities add mod N_c :

$N[R_i] = \mathrm{mod}[N[R_1] + N[R_2], N_c]$

Thus adding gluons (which have n-ailty of zero) to any configuration cannot change the configuration's n-aility.

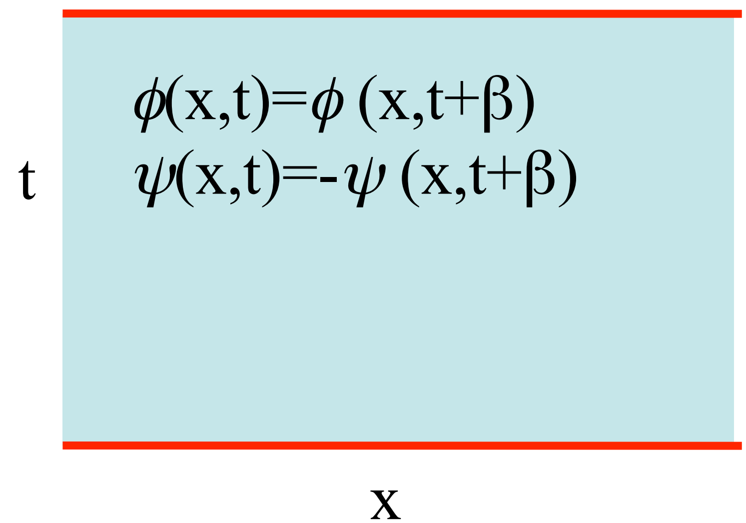
Moreover, it can be shown that by adding gluons one can obtain all representations with the same n-ality

- **An implication:**
  - **The string tension for representation *R* depends only on the n-ality of *R.***
    - ***Key point: by adding gluons popped out of the vacuum one can get from a source in a given representation to any representation with the same n-anilty.***
    - ***Thus at long distance the string tension will be the of the lowest energy represtentation of fixed n-ality***

Center Transformations

- **Center transformations are naturally considered for finite time dimension and some kind of periodic b.c.**

- **Field theories in Euclidean space with periodic b.c. in time for the bosons and anti-periodic for fermion correspond to finite temperature theory with the extent of the time direction, β, being the inverse temperature. We take β to zero at end of problem to get zero temp.**



- **What is center the of a group?**
  - **The center of a group is the set of elements in the group which commute with all the elements of the group.**
  - **For SU(Nc) the center is Z_{Nc}. Namely C is an element of the center iff $C=z_j \mathbb{I}_{Nc \times Nc}$ with $z_j^{Nc}=1$ *so that* $z_j=e^{i(2\pi j/Nc)}$**

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- **What is a center transformation in a gauge theory?**

$$A_\mu \longrightarrow A'_\mu \equiv \Omega A_\mu \Omega^+ - \Omega \partial_\mu \Omega^+$$

$$\text{with} \quad \Omega(\vec{x}, t+\beta) = C\Omega(\vec{x})$$

where C is an element of the center and Ω is an arbitrary matrix-valued function subject to these boundary conditions.

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  - **This looks just like a gauge transformation (and hence unphysical) but in a periodic theory, true gauge transformations are periodic whereas this is not unless C is the unit element.**
  - **Key thing for Yang-Mills: easy to show that if *A* is periodic so is *A* thus center transformations are consistent with b.c.**

Facts about Yang-Mills Theory:

- Pure SU(Nc) Yang-Mills theory, is invariant under a Z_{Nc} center transformations. Since action is unchanged and gluon fields after center transformation satisfy boundary conditions
- Pure Yang-Mills has an area law for Wilson loops in the fundamental representation. There is also an area law for all other representations with non-zero n-ality

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Is the area law connected to the center?

Note that theories with matter fields with n-ality other than zero are NOT invariant under center transformations. Consider quark fields with n-ality n_q. Under center transformations they pick up a factor of $(z_i)^{nq}$

Why are these theories not center invariant?

Recall the transformation of gluons under a center transformation: $A_\mu \to A'_\mu \equiv \Omega A_\mu \Omega^+ - g\Omega\partial_\mu\Omega^+$

with $\Omega(\vec{x}, t+\beta) = C\Omega(\vec{x})$

Except for the boundary conditions this looks like a gauge transformation. This will leave the action invariant only if the matter fields transform via the same "gauge transformation"

$q(\vec{x},t) \to q'(\vec{x},t)$ **under** Ω

so if $q(\vec{x},t+\beta) = -q'(\vec{x},t)$

then $q'(\vec{x},t+\beta) = -\left(z_j\right)^{n_q} q'(\vec{x},t)$ **with** $z_j = e^{i(2\pi j/Nc)}$

But this means that for $(z_i)^{n_q} \neq 1$ **, q' does not satisfy the boundary conditions and the transformation is not allowed.**

- While for $n_q \neq 0$, such theories are not invariant under transformations of the full center, they may be invariant under transformations associated with a subgroup of the center
  - For example, in theories in which all matter fields have the same n-ality the subgroup of the center preserved is Z_p with p = gcf (n_q, Nc) where gcf is greatest common factor.
    - *Eg. For n_q =3 and Nc=15, p=3*

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  - For example, in theories in which all matter fields have the same n-ality the subgroup of the center preserved is Z_p with $p = \mathrm{gcf}\,(n_q, Nc)$ where gcf is greatest common factor.
    - *Eg. For $n_q = 3$ and $Nc = 15$, $p = 3$*
- Note that the matter also act to screen color charges in various representations causing some color charges to vanish and others to be equal.

- The inclusion of the matter fields in a guage theory both
  - reduces the allowable center transformations to a subgroup of the center.
  - by screening the color charge,act to alter the pattern of which representations string tensions are the identical (and also which are zero)

- **The inclusion of the matter fields in a guage theory both**
  - **reduces the allowable center transformations to a subgroup of the center.**
  - **by screening the color charge,act to alter the pattern of which representations string tensions are the identical (and also which are zero)**
- **The nontrivial point derived via elementary number theoretic consideration in** Phys. Rev. D 90, 047703 (2014) **is that the *way* these two things happen are strongly connected so that all theories with same maximum subgroup of the center share patterns for string tensions regardless of the Nc of the gauge group.**

$$\sigma_R = \sigma_F \frac{x_R g(x_R)}{(p-1)g(p-1)}$$ **with** Z_p **the maximum subgroup of the center**

$$x_R = p_R(p - p_R)$$ **where** $p_R = n_R \operatorname{mod}(p)$

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Thus, for example all theories which preserve a maximum Z_5 subgroup of the center have three distinct string tensions depending on the n-ality of R:

$\sigma = 0$ for all n-ality divisible by 5

$\sigma = \sigma_1$ for all n-ality with remainder of 1 or 4 when dividing by 5 (n_R=1,4,6,9,11,14,16,19, 21, 24…)

$\sigma = \sigma_2$ for all n-ality with remainder of 2 or 3 when dividing by 5 (n_R=2,3,7,8,12,13,17,18, 22, 23…)

The fact that the pattern of which string tensions are equal depends on the maximum subgroup of the center of the theory helps make clear the connection of confinement and center symmety.

Note that if there is no nontrivial subgroup of the center left invariant all string tensions vanish.

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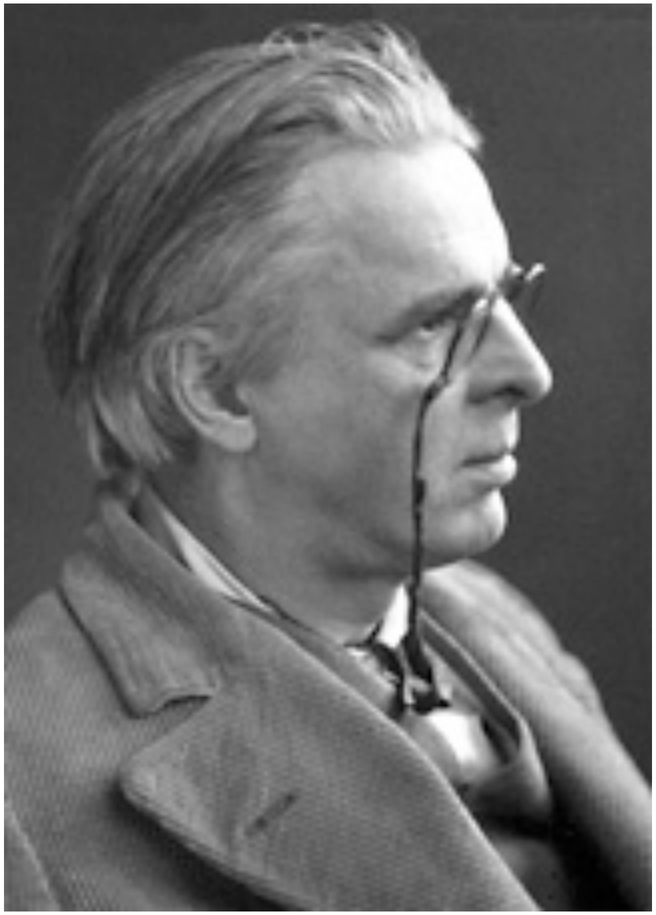


An update of *Second Coming*:

"Things fall apart; a nontrivial subgroup of the centre cannot hold"

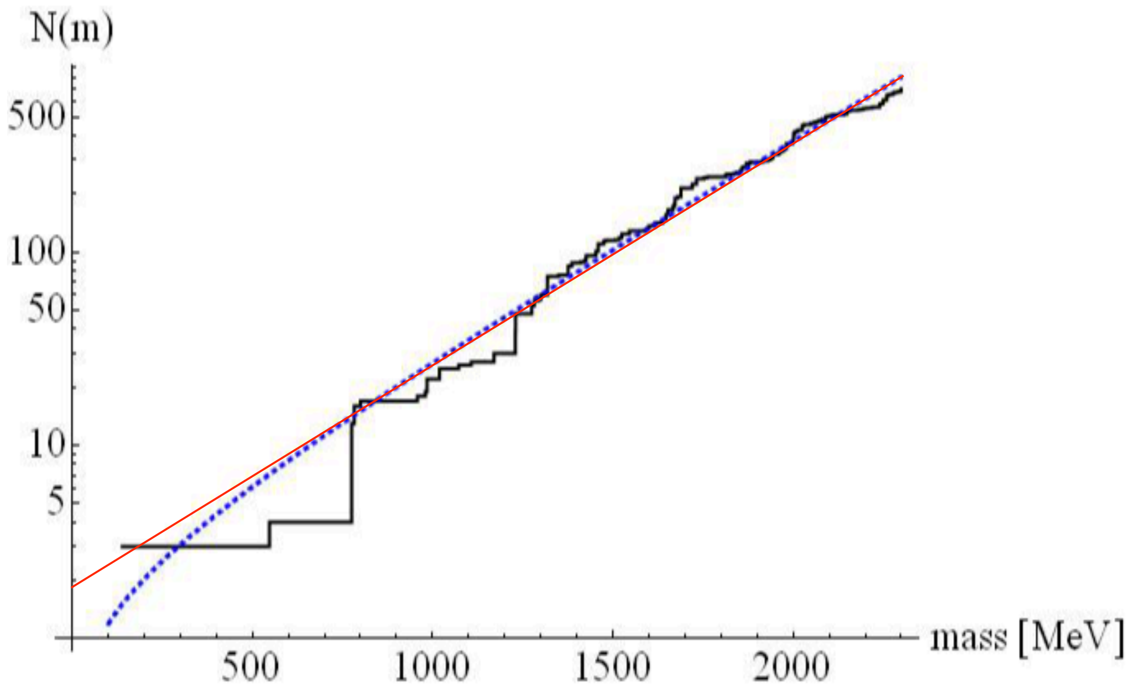
PART II—HAGGEDORN SPECTRA AND CENTER SYMMETRY

A Hagedorn spectrum is one in which the number of hadrons with mass less than *m* , *N(m)*, grows exponentially with *m*

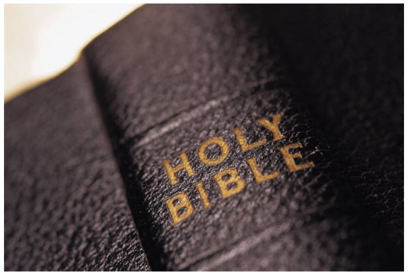
A. Armoni,TDC, S. Sen ArXive 1502.0136

Phenomenological support from the hadronic spectrum

***N*(*m*), the Integrated number of mesons with mass less then m**



Get data from the Particle Data Book



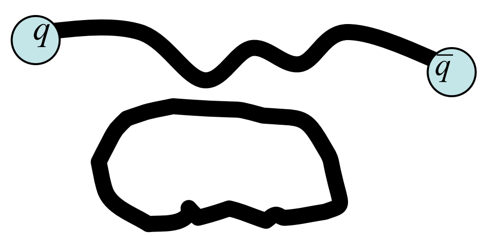
From TDC & Vojtech Krejcirik (2011)

- Concept of a Hagedorn spectrum is intellectually mudldled since hadrons are resonances not bound states and masses are not well-defined— except in large Nc limit.

- For large Nc gauge theories there is a set of criteria which if satified imply a Hagedorn spectrum (TDC JHEP 1006 (2010) 098, TDC &V Krejcirik JHEP 1108 (2011) 138). These include the theories being asymptotically free and confining in the sense of all states being color singlet, but do **not** explictly require center symmetry.

Confinement, QCD Strings and Hagedorn Spectra at Large Nc

- Hadronic strings are thought to emerge due to confinenemt in QCD as flux tubes.
- Picture becomes valid when excitations are high enough so that the length of the flux tube is much bigger than its width and string motion dominates
  - At large Nc, strings do not break and string picture should hold
  - Dynamics of flux tubes might then be expected to give excitation spectrum of hadrons.
    - Mesons are open strings.
    - Glueballs are closed strings.



- Flux tubes in QCD are only a sensible description for highly excited hadronic states.
- Restriction to highly excited states critical for stringy description as well: string theory in four dimension is diseased for low-lying states (tachyons, massless spin 2 state…)

- Flux tubes in QCD are only a sensible description for highly excited hadronic states.
- Restriction to highly excited states critical for stringy description as well: string theory in four dimension is diseased for low-lying states (tachyons, massless spin 2 state…)
  - **One might hope that a theory which approaches a string theory in four dimensions might work for highly excited states**
  - **If so, highly excited states in QCD should be stringy and the emergent string theory for highly excited states would be expected to have a Hagedorn spectrum since transverse vibrations of a string theory with unbreakable strings is know to have a Hagedorn spectrum in greater than 1+1 dimension .**

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- **A recent survey of a wide class of large Nc gauge theories showed that all theories considered for which a hagedorn spectrum could be deduced from the general argument either had an explicit center symmetry or an emergent one in the large Nc limit and conversely confining theories with an explicit or emergent center had a hagedorn spectrum**

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- **A recent survey of a wide class of large Nc gauge theories showed that all theories considered for which a hagedorn spectrum could be deduced from the general argument either had an explicit center symmetry or an emergent one in the large Nc limit and conversely confining theories with an explicit or emergent center had a hagedorn spectrum**
  - **In this context an emergent symmetry means that as Nc becomes large, a nontrivial class of correlation functions become identical to correlators in a theory with an explicit center up to corrections which vanish as Nc $\rightarrow \infty$**

– **Note that these emergent centers can be highly nontrivial. For example, SU(Nc) gauge theories with quarks in 2-index representations have correlators which become equivalent to those of a center symmetric theory with quarks in the adjoint.**

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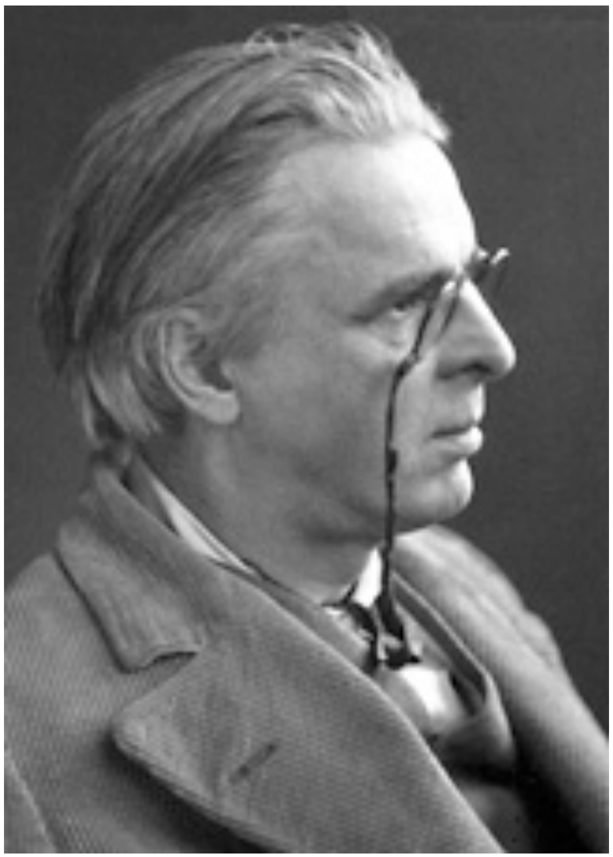
- Remarkably, the existence of a center symmetry appears to be the key rather stringy dynamics
  - For example, the general argument shows that QCD in 1+1 dimension and quarks in the adjoint has a Hagedorn spectrum.
    - This theory IS center symmetric
    - This theory does NOT have a Hagedorn spectrum due to stringy vibrations of a flux tube in physial space since in 1+1 dimensions the flux tube cannot undergo transverse vibrations.

Conclusion



Center symmetry plays an interesting and subtle role in how confinement is manifest.

Insight into this can be gleaned by studying the behavior of a wide variety of gauge theories with different matter content and by looking at Hagedorn spectra at large Nc.