Refined lattice/model investigation of $ud\bar{b}\bar{b}$ tetraquark candidates with heavy spin effects taken into account

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Outline

Goals, motivation

- study exotic mesons (tetraquarks/mesonic molecules) by combining lattice QCD and phenomenology
- Lattice part: compute the potential of two heavy valence quarks in the presence of two additional light valence quarks
- use a model calculation to decide whether the potentials are sufficiently attractive to generate a bound state

Heavy-heavy-light-light tetraquarks

- for this talk: investigate possibly existing $\bar{Q}\bar{Q}qq$ tetraquark states
  - the light quarks q are u/d-quarks (degenerate mass)
  - for the heavy quarks $\bar{Q}$ the static approximation is used
  - the static approximation saves computation time and allows for an easy extraction of the potential
  - approximation most appropriate for $\bar{Q}\bar{Q} = \bar{b}\bar{b}$
- compute potentials of the two heavy quarks in the background of two light quarks via lattice QCD

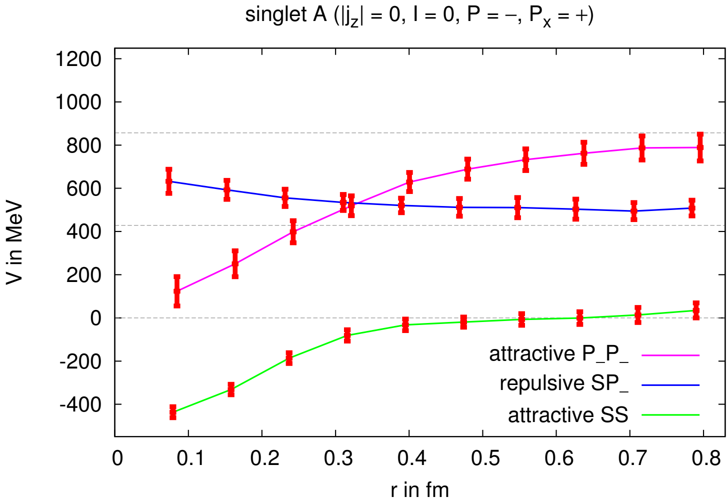
Extracting the potential

- To extract the potentials, compute the temporal correlation function of the trial state

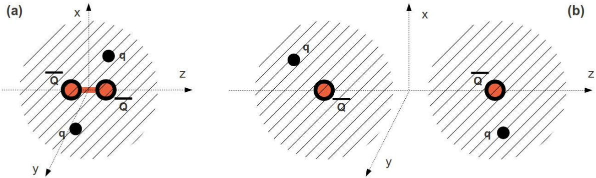
$$(L)_{\alpha\beta}\,(S)_{\gamma\delta}\left(\bar{Q}_{\gamma}\left(\vec{r}_1\right)q_{\alpha}^{(1)}\left(\vec{r}_1\right)\right)\left(\bar{Q}_{\delta}\left(\vec{r}_2\right)q_{\beta}^{(2)}\left(\vec{r}_2\right)\right)|\Omega\rangle,$$

  the exponential decay of the correlation function as $t \to \infty$ determines the energy of the potential $V(r)$ at $r = |\vec{r}_1 - \vec{r}_2|$
  - essential to be in static approximation, quarks automatically arranged according to QCD dynamics
  - matrix S couples static degrees of freedom
  - the spins of the static quarks are irrelevant in the static approximation, different choices of S lead to the same potential
  - therefore the matrix L that couples the light spin degrees of freedom determines the spin and parity of the state
  - it is *not* possible to couple static and light degrees of freedom, leads to mixing of different sectors
  - $q^{(1)}q^{(2)} \in \{ud - du, uu, dd, ud + du\}$, determine Isospin

Some potentials



Course of attractive potential



P. Bicudo and M. Wagner, "Lattice QCD signal for a bottom-bottom tetraquark," Phys. Rev. D 87, 114511 (2013) [arXiv:1209.6274 [hep-ph]]

- two different scenarios
  a) diquark interaction for small separation
  b) screening of color charge due to light quarks for big separation
- fit of continuous functions to lattice data was made with these cases in mind

Binding of B-Mesons?

- it was argued, that some of these potentials can be interpreted as potentials between pseudoscalar bottom mesons (PDG: B-Mesons), which are the lightest bottom mesons
- the two most attractive potentials were taken as potentials for the non relativistic Schrödinger equation for the relative coordinate of the heavy quarks

$$\left(-\frac{1}{2\mu}\Delta + V(r)\right)\psi(\vec{r}) = E\psi(\vec{r}), \qquad \mu = m_B/2$$

- for the most attractive potential a bound state with binding energy $E \approx -50\,\mathrm{MeV}$ was found
- **however**, due to the spin coupling in the operators, which is static to static and light to light, the Meson content is not clear

Outline

Fierz identity

- rearrange static-static-light-light operators to identify meson-meson structure
- use the following Fierz identity

$$(L)_{\alpha\beta}\,(S)_{\gamma\delta}\left(\bar{Q}_{\gamma}\,(\vec{r}_1)\,q^{(1)}_{\alpha}\,(\vec{r}_1)\right)\left(\bar{Q}_{\delta}\,(\vec{r}_2)\,q^{(2)}_{\beta}\,(\vec{r}_2)\right)$$
$$= \frac{1}{16}\,\mathrm{Tr}\left(\Gamma_b(S^T)(\Gamma_a^T)L\right)\left(\bar{Q}\,(\vec{r}_1)\,(\Gamma^a)q^{(1)}\,(\vec{r}_1)\right)\left(\bar{Q}\,(r_2)\,(\Gamma^b)q^{(2)}\,(\vec{r}_2)\right),$$

where the 16 linear independent Γ^i are the natural basis of the Clifford algebra that is generated by the (euclidean) $\gamma-$matrices

- on the rhs of the equation one now has a superposition of Meson-Meson operators

Choosing L and S

- matrix L determines to which static potential the superposition corresponds to
- S is to be chosen so that the Meson operators correspond to pseudoscalar/vector mesons B or $B^{\star}$ (lightest mesons containing $\bar{b}$)
- one can show that there are:
  - four linear independent choices for L
  - four independent choices for S for each potential
- S and L can *not* be chosen so that only B Mesons come up
- three of the four choices for L lead to the same potential, so we will be working (after isospin is fixed) with two potentials: V_j and V_5

New effects

- to conclude: one potential corresponds to a linear combination of BB, $BB^\star$ and $B^\star B^\star$
- $m_{B\star} - m_B \approx 50\,\mathrm{MeV}$, which is the same order of magnitude of the binding energy in the old model
- furthermore, one has now attractive *and* repulsive potentials
- these concurrent effects must be taken into account

Wavefunction

- non relativistic Schrödinger equation $H\Psi = E\Psi$
- the Hamiltonian will act on a sixteen component wavefunction
- the components correspond to the sixteen possibilities to combine $\left(B, B_x^\star, B_y^\star, B_z^\star\right) = \left(B, \vec{B}^\star\right) = \underline{B}$ with $\underline{B}$:

$$\Psi = \begin{pmatrix} B\underline{B} \\ B_x^\star\underline{B} \\ B_y^\star\underline{B} \\ B_z^\star\underline{B} \end{pmatrix}$$

Hamiltonian: free part

- Hamiltonian can be split in free and interacting part $H = H_0 + H_{int}$
- free part

$$H_0 = M \otimes \mathbb{1} + \mathbb{1} \otimes M + \frac{\vec{p}_1^{\,2}}{2} \left(M \otimes \mathbb{1}\right)^{-1} + \frac{\vec{p}_2^{\,2}}{2} \left(\mathbb{1} \otimes M\right)^{-1} ,$$

where

$$M = \mathrm{diag}\left(m_B, m_{B^\star}, m_{B^\star}, m_{B^\star}\right)$$

Hamiltonian: interaction part

- the interaction part has the following form

$$H_{int} = T^{-1} V T$$

- the matrix V contains the two previously identified potentials

$$V = \mathrm{diag}\left(V_5, V_j, V_j, V_j\right) \otimes \mathbb{1}$$

- the matrix T is defined so that when acting on Ψ the superpositions of $B^{(\star)}B^{(\star)}$ according to the Fierz identity are obtained

Outline

1. Introduction
2. Incorporating heavy spin
   - Rearranging operators
   - Setting up the coupled Schrödinger equation
3. Solving the coupled Schrödinger equation
   - Partial decoupling
   - Reduction to an ODE
   - Numerical solution
4. Outlook

Direct solution?

- a direct solution of the coupled equation is numerically not feasible
- however, it is possible to decouple the equations partially
- to this end introduce a linear transformation that splits up H_{int} into blocks
- these blocks correspond to the total angular momentum of the system
- $\rightarrow$ problem is reduced to one 2x2, one 3x3 and one uncoupled equation

$J = 0$ case

- for the rest of the talk, 2x2 case
- corresponds to total angular momentum of 0
- isosinglet channel, contains most attractive potential

2x2 equation

- Hamiltonian of the 2x2 problem:

$$H_{0,2x2} = \begin{pmatrix} 2m_B & 0 \\ 0 & 2m_{B^\star} \end{pmatrix} + \left(\frac{\vec{p}_1^2}{2} + \frac{\vec{p}_2^2}{2} \right) \begin{pmatrix} \frac{1}{m_B} & 0 \\ 0 & \frac{1}{m_{B^\star}} \end{pmatrix}$$

$$H_{int,2x2} = \begin{pmatrix} \frac{1}{4}\left(V_5 + 3V_j\right) & \frac{\sqrt{3}}{4}\left(V_5 - V_j\right) \\ \frac{\sqrt{3}}{4}\left(V_5 - V_j\right) & \frac{1}{4}\left(3V_5 + V_j\right) \end{pmatrix}$$

- V_5 is an attractive potential, V_j is (weakly) repulsive
- in an approximation it is possible to go to center of mass and the problem of determining the ground state energy can be reduced so solving an *ordinary* differential equation

Ordinary differential equation

- the ordinary differential equation to solve is

$$D\chi(r) = E\chi(r),$$

where

$$D = \begin{pmatrix} -\frac{1}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + 2m_B & 0 \\ 0 & -\frac{1}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + 2m_{B^\star} \end{pmatrix} + H_{int,2x2}(r),$$

$$\chi(r) = \begin{pmatrix} \chi_1(r) \\ \chi_2(r) \end{pmatrix}$$

and $r \in (0, \infty)$

Boundary conditions

- demanding D to be self-adjoint and the wavefunction to be normalizable leads to the boundary conditions

$$\lim_{r\to 0} u(r) = u(\infty) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

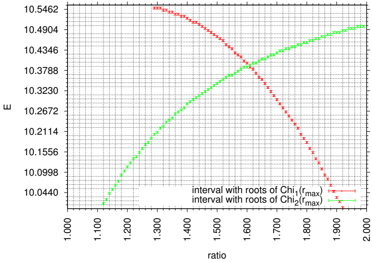
- furthermore, one can show that the asymptotic behaviour of χ as $r \to 0$ is

$$\chi \sim \begin{pmatrix} Ar \\ Br \end{pmatrix},$$

where $A, B \in \mathbb{R}$

Unphysical data

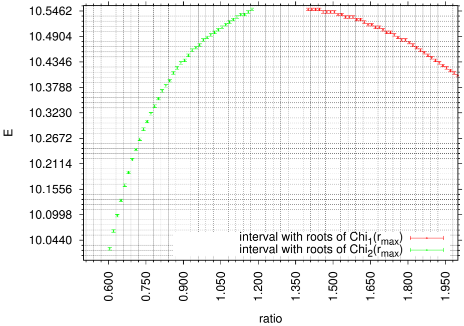
- to find a solution one integrates the equation numerically and varies E and A/B until the boundary conditions are met
- to get an idea of finding a bound state we illustrate the situation with the attractive potential V_5 chosen unphysically strong



- these intervals parametrise 2 continuous functions, the crossing point of these function gives E and A/B of the bound state

Physical data

- using a rough fit to the lattice data for the potential one gets the following plot



- therefore, we do *not* expect binding in this case
- "tweak" one parameter of the fit from 0.293 to about 0.33 to get bound state

Outline

What still needs to be done

- lattice data has statistical and systematic errors, they need to be taken into account
- the other cases (3x3 and uncoupled, isotriplet) have to be solved
- instead of using $B^{(\star)}B^{(\star)}$, experimentally more interesting to study $B^{(\star)}\bar{B}^{(\star)}$ systems

Outline

Lattice details

- two flavors of dynamical Wilson twisted mass quarks
- lattice spacing $0.08\,\mathrm{fm}$
- u/d quark masses unphysically heavy, corresponding to $m_\pi \approx 340\,\mathrm{MeV}$
- spatial extension of the lattice $L = 1.9\,\mathrm{fm}$, leading to $m_\pi L = 3.3$

Some potentials II