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Non-ordinary light meson couplings and the 1/Nc expansion

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Outline

- Motivation and Introduction
- States with fixed number of constituents
- States with growing number of constituents: Polyquarks
- If wide, then heavy
- Summary

Light scalars relevant in many aspects of hadron physics but hard to accommodate as ordinary $\bar{q}q$ mesons

Actually, there is mounting evidence that these states may not be ordinary quark-antiquark states

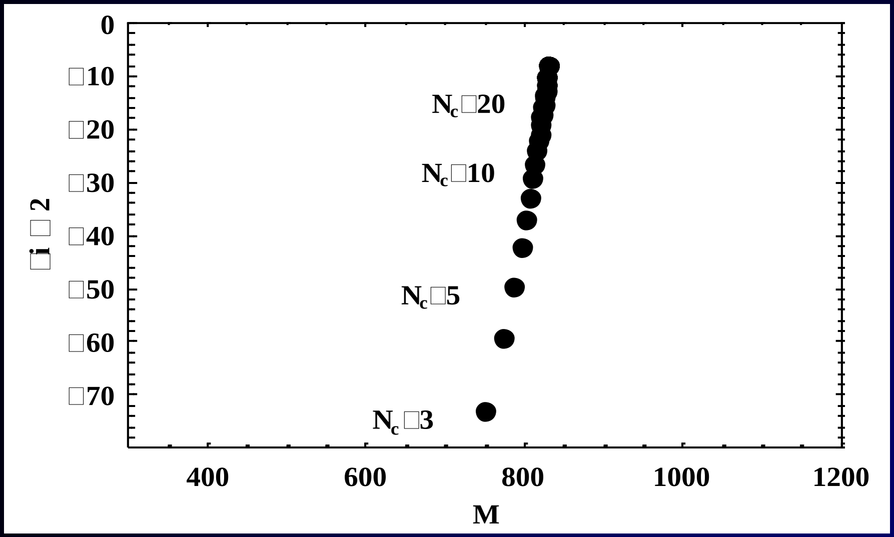
Jaffe, van Beveren,, Rupp, Tornqvist, Roos, Close, Schecter, Sannino, Fariborz, Black, Oset, Oller, JRP, Hanhart, Achasov, Kalashnikova, Maiani Polosa, Riquer, Giacosa, Parganlija, etc…

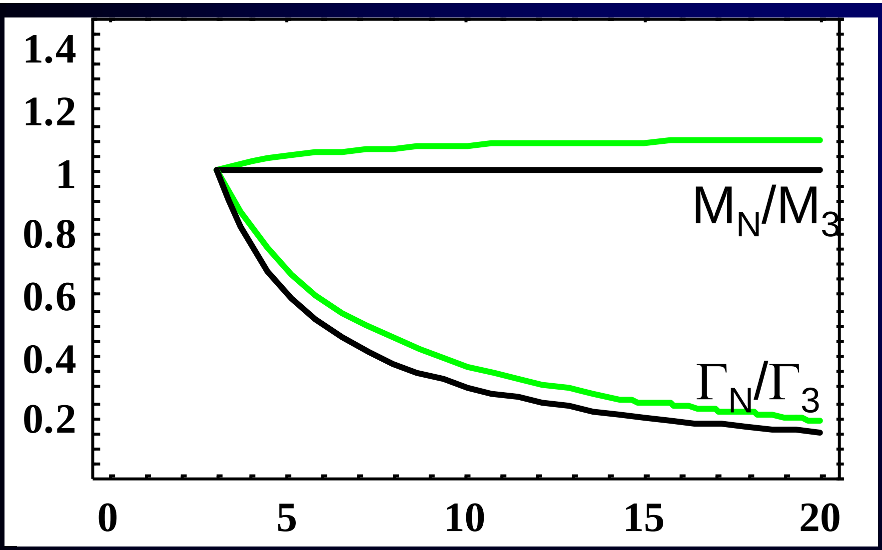
The scalar $\bar{q}q$ nonet may appear above 1 GeV

Support for this interpretation from observed different Nc behavior of vector and scalar states

Ordinary $\bar{q}q$ states should behave as $M=O(1)$; $\Gamma=O(1/N_c)$

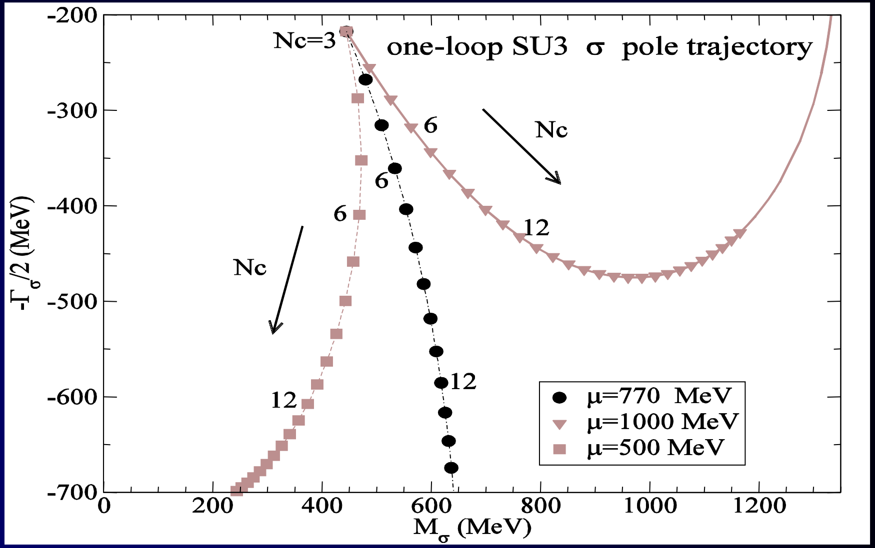
The ρ(770) behaves as a qq

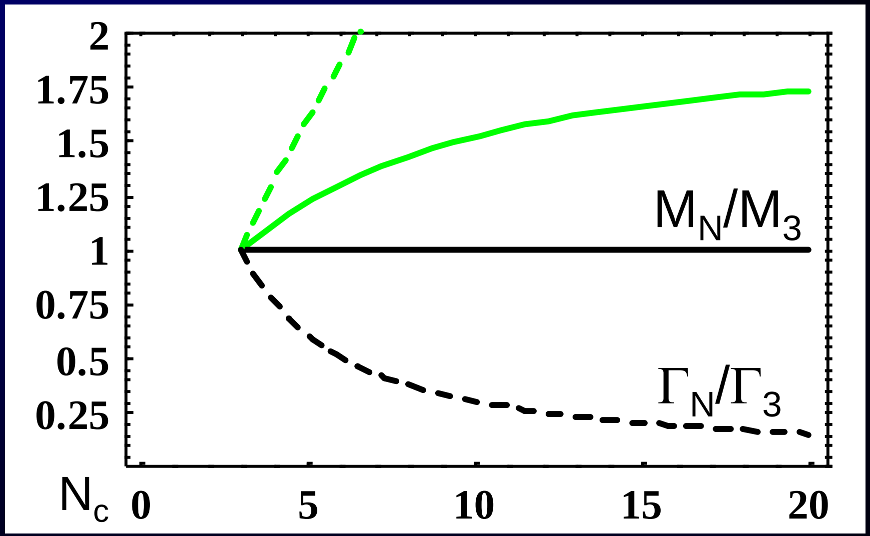




Similarly for the K*(892)

Whereas the f0(500) does NOT





Similar conclusions for the K(800), $f_0(980)$ and $a_0(980)$

● Such non-ordinary behavior does NOT seem to be present in pure states within large Nc QCD (We will see that)

● Some degree of mixing is very likely present between different states

$$|\sigma\rangle = \alpha \left| \bar{q}q - like \right\rangle + \beta \left| \left(\bar{q}q \right)^2 - like \right\rangle + \gamma \left| gg - like \right\rangle ...$$

● It is therefore interesting to find out how different kinds of states couple to each other

● Recent interest on the lattice also, where some groups have studied the Nc behavior up to Nc=17 (G. Bali et al.)

Note that the Nc behavior cannot separate qqbar from qqbar+glue, or even some kind of tetraquarks so we can only distinguish "classes" of states transforming the same under Nc. (Not exactly a Fock expansion)

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States with fixed number of constituents as Nc grows: Normalizations

- Naively we mean fixed number of "valence" constituents. The QCD mass gap is O(1) so that constituent masses are O(1) and therefore these states have M=O(1)

- These are: $\bar{q}q$ mesons, hybrids, glueballs and tetraquarks

- First one calculates the state normalization

-quark-antiquark

$$|q\bar{q}\rangle = \frac{\delta^{ij}}{\sqrt{N_c}}|q^i\bar{q}^j\rangle.$$

$$\Rightarrow \quad |\pi\pi\rangle = \frac{\delta^{ik}\delta^{jl}}{\sqrt{2N_c(N_c-1)}}|q^i q^j \bar{q}^k \bar{q}^l\rangle.$$

Two pions

-hybrid

$$|q\bar{q}g\rangle = \sqrt{\frac{2}{N_c^2-1}}T^a_{ij}|q_i\bar{q}_j g^a\rangle.$$

-glueball

$$|gg\rangle = \frac{\delta^{ab}}{\sqrt{2(N_c^2-1)}}|g^a g^b\rangle.$$

States with fixed number of constituents as Nc grows: Tetraquarks

- Tetraquarks are more subtle. For a while it was believed that they did not exist = had growing width with Nc. (Coleman and Jaffe)

- However, S.Weinberg has recently shown that, if they exist, they would be narrow, behaving generically as normal mesons. PRL110(2013)261601

- Knecht and Peris realized that for certain open flavor configurations they can be even narrower. PRD88 (2013)036016

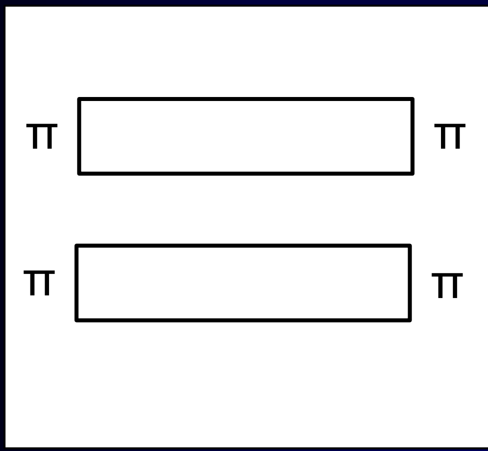
- We are interested in the generic ones, plus the $\pi\pi$ state.

- Recall that from color only "tetraquarks" and "molecules" are not orthogonal (Fierz transformations)

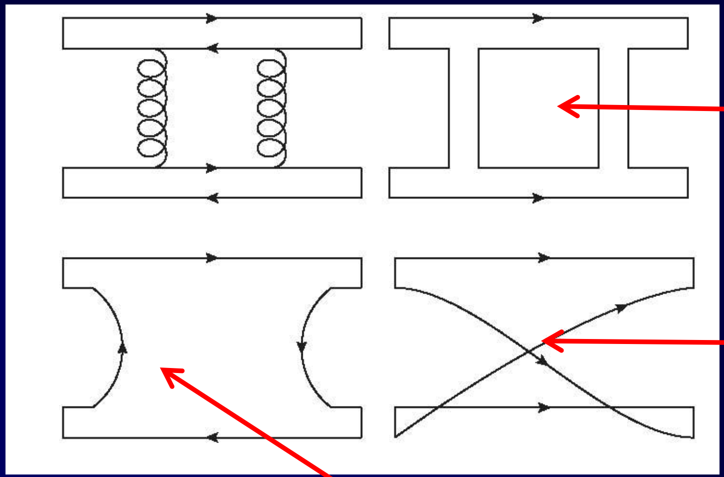
Attention has to be paid to how their poles appear in different diagrams

States with fixed number of constituents as Nc grows: Normaliztions

- Following Weinberg, tetraquarks appear in meson-meson scattering ONLY IN CONNECTED or CORRELATED DIAGRAMS,



Disconnected
Free propagation



$1/Nc^2$

1/Nc

1/Nc

Correlated diagrams are suppressed !!

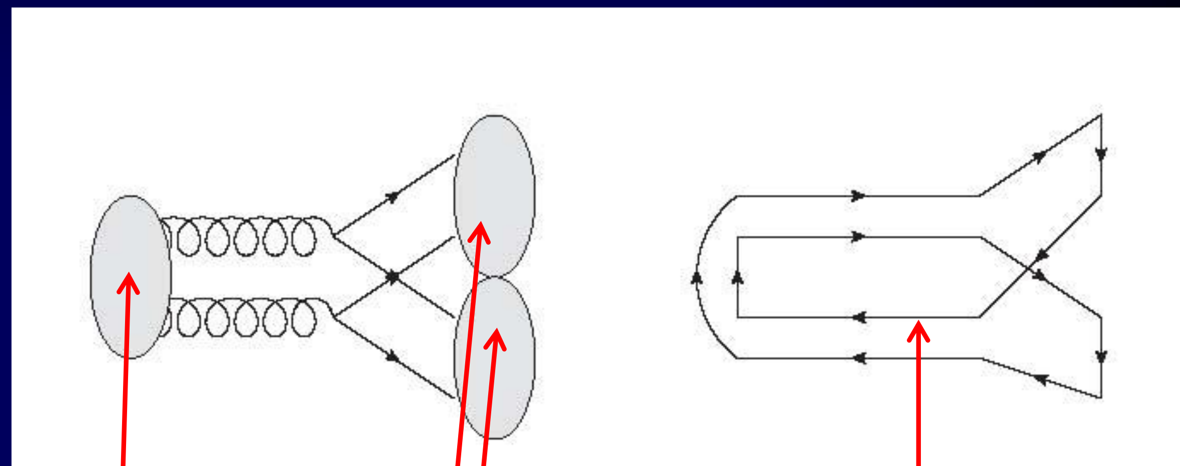
As a consequence, the normalization is NOT 1/Nc like |ππ>, but

$$|T_0\rangle = \frac{\delta_{ik}\delta_{jl}}{\sqrt{N_c}} |q^i q^j q^k q^l\rangle_{\text{correlated}},$$

We now have all normalizations

Next one calculates the 1/Nc leading term in the mixing, taking into account normalizations g~Nc$^{1/2}$ factors and color loops, for which is best the double line notation

- Example:
Glueball to two pions:



1/Nc

1/Nc

Color flow and g factors
1 closed line = Nc

$$\langle 0|T((gg)(\pi\pi))|0\rangle \propto \frac{1}{N_c}.$$

States with fixed number of constituents as Nc grows: Couplings

- And the same can be done with the other states, leading to the following results

TABLE III. We collect the couplings between configurations with fixed constituent number in leading order in the large N_c expansion. Note that the diagonal counts, of course, as the propagator (mass) and is of order 1.

	$q\bar{q}$	$\pi\pi$	gg	$T_0(qq\bar{q}\bar{q})$
$q\bar{q}$	$O(1)$	$O(\frac{1}{\sqrt{N_c}})$	$O(\frac{1}{\sqrt{N_c}})$	$O(1)$
$\pi\pi$		$O(1)$	$O(\frac{1}{N_c})$	$O(\frac{1}{\sqrt{N_c}})$
gg			$O(1)$	$O(\frac{1}{\sqrt{N_c}})$
$T_0(qq\bar{q}\bar{q})$				$O(1)$

Most of them well known, and the tetraquark ones recalculated.
Tetraquark lies in same Nc-equivalence class of ordinary quark-antiquark mesons

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States with growing number of constituents: The polyquark

- For Nc=3 we build tetraquarks from bilinears

$$\bar{\mathbb{B}}^i = \epsilon^{ijk} q^j q^k,$$
$$\mathbb{B}^i = \epsilon^{ijk} \bar{q}^j \bar{q}^k.$$

$$\Rightarrow \quad \mathbb{Q} = \overline{\mathbb{B}}\mathbb{B}.$$

- Generalizing this construction to arbitrary Nc

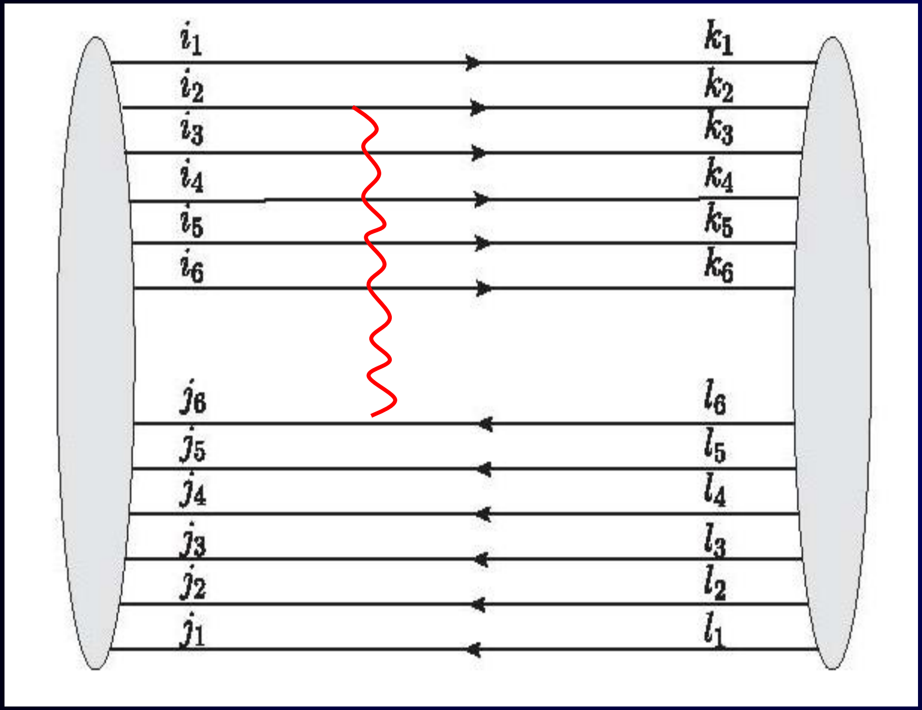
$$\mathbb{Q} \equiv \bar{\mathbb{B}}^a \mathbb{B}^a = \epsilon^{a j_i \cdots j_{N_c-1}} \epsilon^{a i_1 \cdots i_{N_c-1}} q^{i_1} \cdots q^{i_{N_c-1}} \bar{q}^{j_1} \cdots \bar{q}^{j_{N_c-1}},$$

- The “polyquark” has Nc-1 quark.antiquark pairs and its mass:

$$M_{(N_c-1)q\bar{q}} \propto N_c.$$

The polyquark

● With ~ Nc components, the number of diagrams at a given order Goes with Nc! And that changes the naive Nc



Example:

1 gluon exchange is suppressed

But there are ~ Nc!
Possible ways of exchanging it

Mean field approach

(as in Witten NPB (1979) for Baryons)

Still read Nc behavior from one graph

The polyquark: sequential decay

- Actually the polyquark can be understood from the nucleon-antinucleon state by removing a quark-antiquark pair

$$|\mathbb{Q}_{N_c-1}\rangle \equiv |\bar{\mathbb{B}}^a \mathbb{B}^a\rangle = \frac{a^a_{i_{N_c}}\, b^a_{j_{N_c}}}{\sqrt{N_c}} |\bar{N}N\rangle;$$

- The dominant decay is the sequential emission of a pion to become an Nc-2 polyquark

$$|\mathbb{Q}_{N_c-2}\rangle = \frac{a^a_{i_{N_c}}\, a^b_{i_{N_c}-1}\, b^a_{j_{N_c}}\, b^b_{j_{N_c}-1}}{\sqrt{2N_c(N_c-1)}} |\bar{N}N\rangle.$$

- We have explicitly calculated it…

$$\langle 0|\mathcal{T}\left(\left(\pi \mathbb{Q}_{N_c-2}\right)\mathbb{Q}_{N_c-1}\right)|0\rangle \sim \sqrt{2}.$$

O(1)
As expected
(Witten)

The polyquark: sequential decay

We also calculated all intermediate sequential steps from Nc-n polyquark to π plus an Nc-n-1 polyquark

$$|\mathbb{Q}_{N_c-n}\rangle = \frac{a^{a_1}_{i_{N_c}} \cdots a^{a_n}_{i_{N_c-n}} b^{a_1}_{j_{N_c}} \cdots b^{a_n}_{j_{N_c-n}}}{\mathcal{N}} |\bar{N}N\rangle,$$

Normalized as….

$$\begin{aligned}
\mathcal{N}^2 &= \langle \bar{N}N | (a^{\dagger b_1}_{k_{N_c}} \cdots a^{\dagger b_n}_{k_{N_c-n}} b^{\dagger b_1}_{l_{N_c}} \cdots b^{\dagger b_n}_{l_{N_c-n}}) | (a^{a_1}_{i_{N_c}} \cdots a^{a_n}_{i_{N_c-n}} b^{a_1}_{j_{N_c}} \cdots b^{a_n}_{j_{N_c-n}}) |\bar{N}N\rangle \\
&= \epsilon^{\alpha i^{a_1}_n \cdots i^{a_n}_{N_c-n}} \epsilon^{\alpha k^{b_1}_n \cdots k^{b_n}_{N_c-n}} \epsilon^{\beta j^{a_1}_n \cdots j^{a_n}_{N_c-n}} \epsilon^{\beta l^{b_1}_n \cdots l^{b_n}_{N_c-n}} \\
&= \delta^{\alpha\beta} \delta^{\alpha\beta} N_c (N_c-1)^2 \cdots (N_c-n+1)^2 \\
&= n \frac{N_c!^2}{(N_c-n)!^2},
\end{aligned}$$

Whose sequential decay goes as….

$$\begin{aligned}
\langle 0 | \mathcal{T}((\pi \mathbb{Q}_{N_c-n-1}) \mathbb{Q}_{N_c-n}) | 0 \rangle &= \frac{(N_c-n)!(N_c-n-1)!}{\sqrt{N_c}\sqrt{n(n+1)} N_c!^2} \\
&\times \langle \bar{N}N | \pi (a^{\dagger b_1}_{k_{N_c}} \cdots a^{\dagger b_{n+1}}_{k_{N_c-n-1}} b^{\dagger b_1}_{l_{N_c}} \cdots b^{\dagger b_{n+1}}_{l_{N_c-n-1}}) (a^{a_1}_{i_{N_c}} \cdots a^{a_n}_{i_{N_c-n}} b^{a_1}_{j_{N_c}} \cdots b^{a_n}_{j_{N_c-n}}) |\bar{N}N\rangle \\
&\simeq \sqrt{1+\frac{1}{n}}.
\end{aligned}$$

O(1) As expected (Witten)

Thus the width of the polyquark is O(1), it is not necessarily narrow, although its mass grows with Nc

The polyquark: other decays

We have also calculated the decays to all other configurations, both for Nf=1 and 2

	$q\bar{q}$	gg	$\pi\pi$	$T(qq\bar{q}\bar{q})$	$(N_c-1)\pi$
$N_f=1$	$(N_c-1)!!(\frac{c}{N_c})^{(N_c-4)/2}$	$N_c!!(\frac{c}{N_c})^{(N_c-2)/2}$	$N_c!!(\frac{c}{N_c})^{(N_c-4)/2}$	$N_c!!(\frac{c}{N_c})^{(N_c-3)/2}$	c^{N_c-1}
$N_f=2$	$(N_c-1)!!(\frac{c}{N_c})^{(N_c-4)/2}$	$N_c!!(\frac{c}{N_c})^{(N_c-2)/2}$	$N_c!!(\frac{c}{N_c})^{(N_c-4)/2}$	$N_c!!(\frac{c}{N_c})^{(N_c-3)/2}$	$\frac{(N_c/2)^2!}{N_c^{N_c/2}}$

They are all exponentially suppressed, including that to 2 pions

Only exception: the O(1) sequential decay, which dominates the width: $M_P=O(Nc)$; $\Gamma_P=O(1)$

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Growing widths with Nc ?

ππ scattering is 1/Nc ("weak"), so strong coupling may look forbidden, but… if $\Gamma \sim Nc^{2\gamma}$

$$\mathcal{M}_{\pi\pi} = \frac{g^2_{R\pi\pi}}{s - m_R^2 + \Gamma_R^2/4 + im\Gamma_R} \propto \frac{N_c^{2\gamma}}{N_c^{4\gamma}}$$

Is still consistent if $1/2 \leq \gamma$, even if M=O(1)

However, since the couling to two pions grows as $g \sim Nc^{\gamma}$, the resonance propagator would be "dressed" with g^2 corrections. Keeping the real part as O(1) would require fine tuning

$$S_\Phi^{-1} = S_\Phi^{(0)-1} - (ig)^2 \int \frac{d^4 q}{(2\pi)^4} S_\phi(q^2) S_\phi((p-q)^2) V(q, p-q),$$

We have shown this fine tuning with a unitarized ChPT example

If wide, then naturally heavy

SUMMARY

Explicitl calculation of 1/Nc leading behavior of the coupling between different kinds of meson states:

-Quark-antiquark,hybrids, glueballs, conventional tetraquarks, and “polyquarks” explicit expressions can be found on the published paper

-Hybrids and conventional tetraquarks fall in the same Nc-equivalence class of ordinary quark-antiquark mesons and become narrow at large Nc

-Only the polyquark does not necessarily become narrow as Nc grows

-We have also argued why if wide at large Nc, then also heavy at large Nc

“Pure” configurations do not explain the sigma Nc behavior,

-Mixing with heavier q´-like states?

-Subdominant Nc effects (rescattering, meson-meson states?)