Non-ordinary light meson couplings and the 1/Nc expansion

J.R. Peláez

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In collaboration with T. Cohen, F.J. Llanes Estrada & J. Ruiz de Elvira

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• Motivation and Introduction
• States with fixed number of constituents
• States with growing number of constituents: Polyquarks
• If wide, then heavy
• Summary
Light scalars relevant in many aspects of hadron physics but hard to accommodate as ordinary $\bar{q}q$ mesons.

Actually, there is mounting evidence that these states may not be ordinary quark-antiquark states.

Jaffe, van Beveren, Rupp, Tornqvist, Roos, Close, Schecter, Sannino, Fariborz, Black, Oset, Oller, JRP, Hanhart, Achasov, Kalashnikova, Maiani Polosa, Riquer, Giacosa, Parganlija, etc...

The scalar $\bar{q}q$ nonet may appear above 1 GeV.

Support for this interpretation from observed different $N_c$ behavior of vector and scalar states.

Ordinary $\bar{q}q$ states should behave as $M=O(1)$; $\Gamma=O(1/N_c)$.
Introduction and motivation

The $\rho(770)$ behaves as a qq

Whereas the $f_0(500)$ does NOT

Similarly for the $K^*(892)$

Similar conclusions for the $K(800)$, $f_0(980)$ and $a_0(980)$
• Such non-ordinary behavior does NOT seem to be present in pure states within large Nc QCD (We will see that)

• Some degree of mixing is very likely present between different states

\[ |\sigma\rangle = \alpha |q\bar{q} - \text{like}\rangle + \beta \left( |q\bar{q}\rangle^2 - \text{like}\right) + \gamma |gg - \text{like}\rangle \ldots \]

• It is therefore interesting to find out how different kinds of states couple to each other

• Recent interest on the lattice also, where some groups have studied the Nc behavior up to Nc=17 (G. Bali et al.)

Note that the Nc behavior cannot separate \( q\bar{q} \) from \( q\bar{q} + \text{glue} \), or even some kind of tetraquarks so we can only distinguish “classes” of states transforming the same under Nc. (Not exactly a Fock expansion)
● Motivation and Introduction

● States with fixed number of constituents
Naively we mean fixed number of “valence” constituents. The QCD mass gap is $O(1)$ so that constituent masses are $O(1)$ and therefore these states have $M=O(1)$.

These are: $\bar{q}q$ mesons, hybrids, glueballs and tetraquarks.

First one calculates the state normalization:

- **quark-antiquark**
  \[
  |q\bar{q}\rangle = \frac{\delta_{ij}}{\sqrt{N_c}} |q^i\bar{q}^j\rangle.
  \]

- **hybrid**
  \[
  |q\bar{q}g\rangle = \sqrt{\frac{2}{N_c^2 - 1}} T_{ij}^a |q_i\bar{q}_j g^a\rangle.
  \]

- **glueball**
  \[
  |gg\rangle = \frac{\delta^{ab}}{\sqrt{2(N_c^2 - 1)}} |g^a g^b\rangle.
  \]

**Two pions**

\[
|\pi\pi\rangle = \frac{\delta^{ik} \delta^{jl}}{\sqrt{2N_c(N_c - 1)}} |q^i q^j \bar{q}^k \bar{q}^l\rangle.
\]
• Tetraquarks are more subtle. For a while it was believed that they did not exist = had growing width with Nc. (Coleman and Jaffe)

• However, S. Weinberg has recently shown that, if they exist, they would be narrow, behaving generically as normal mesons. PRL110(2013)261601

• Knecht and Peris realized that for certain open flavor configurations they can be even narrower. PRD88 (2013)036016

• We are interested in the generic ones, plus the ππ state.

• Recall that from color only “tetraquarks” and “molecules” are not orthogonal (Fierz transformations)

Attention has to be paid to how their poles appear in different diagrams
Following Weinberg, tetraquarks appear in meson-meson scattering ONLY IN CONNECTED or CORRELATED DIAGRAMS,

Disconnected Free propagation

Correlated diagrams are suppressed !!

As a consequence, the normalization is NOT 1/Nc like $|\pi\pi\rangle$, but

$$|T_0\rangle = \frac{\delta_i^k \delta_j^l}{\sqrt{N_c}} |q^i q^j q^k q^l\rangle_{\text{correlated}},$$

We now have all normalizations
Next one calculates the $1/N_c$ leading term in the mixing, taking into account normalizations $g \sim N_c^{1/2}$ factors and color loops, for which is best the double line notation.

- **Example:**
  Glueball to two pions:

\[
\langle 0 | T((gg)(\pi\pi)) | 0 \rangle \propto \frac{1}{N_c}.
\]
And the same can be done with the other states, leading to the following results

<table>
<thead>
<tr>
<th></th>
<th>$q\bar{q}$</th>
<th>$\pi\pi$</th>
<th>$gg$</th>
<th>$T_0(qq\bar{q}\bar{q})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q\bar{q}$</td>
<td>$O(1)$</td>
<td>$O\left(\frac{1}{\sqrt{N_c}}\right)$</td>
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<td>$T_0(qq\bar{q}\bar{q})$</td>
<td></td>
<td></td>
<td></td>
<td>$O(1)$</td>
</tr>
</tbody>
</table>

Most of them well known, and the tetraquark ones recalculated. Tetraquark lies in same Nc-equivalence class of ordinary quark-antiquark mesons.
● Motivation and Introduction

● States with fixed number of constituents

● States with growing number of constituents: Polyquarks
States with growing number of constituents: The polyquark

- For $N_c=3$ we build tetraquarks from bilinears

$$\bar{B}^i = e^{ijk} q^j q^k,$$
$$B^i = e^{ijk} \bar{q}^j \bar{q}^k.$$ 

- Generalizing this construction to arbitrary $N_c$

$$Q = \bar{B}^a B^a = \epsilon^{aj_1 \cdots j_{N_c-1}} \epsilon^{ai_1 \cdots i_{N_c-1}} q^{i_1} \cdots q^{i_{N_c-1}} \bar{q}^{j_1} \cdots \bar{q}^{j_{N_c-1}},$$

- The “polyquark” has $N_c-1$ quark.anti-quark pairs and its mass:

$$M_{(N_c-1)\bar{q}q} \propto N_c.$$
With ~ $N_c$ components, the number of diagrams at a given order goes with $N_c!$ And that changes the naive $N_c$.

Example:

1 gluon exchange is suppressed.

But there are ~ $N_c!$ Possible ways of exchanging it.

Mean field approach
(as in Witten NPB (1979) for Baryons)

Still read $N_c$ behavior from one graph.
The polyquark: sequential decay

- Actually the polyquark can be understood from the nucleon-antinucleon state by removing a quark-antiquark pair

\[
|Q_{Nc-1}\rangle = |B^aB^a\rangle = \frac{a_{iNc}^a b_{jNc}^a}{\sqrt{N_c}} |\bar{NN}\rangle;
\]

- The dominant decay is the sequential emission of a pion to become an Nc-2 polyquark

\[
|Q_{Nc-2}\rangle = \frac{a_{iNc}^a a_{iNc-1}^b b_{jNc}^a b_{jNc-1}^b}{\sqrt{2N_c(N_c-1)}} |\bar{NN}\rangle.
\]

- We have explicitly calculated it…

\[
\langle 0|T((\pi|Q_{Nc-2}\rangle|Q_{Nc-1}\rangle)|0\rangle \sim \sqrt{2}.
\]

O(1)  As expected (Witten)
The polyquark: sequential decay

We also calculated all intermediate sequential steps from $N_c-n$ polyquark to $\pi$ plus an $N_c-n-1$ polyquark

$$|Q_{N_c-n}\rangle = \frac{a_{i_{N_c}}^{a_1} \ldots a_{i_{N_c-n}}^{a_n} b_{j_{N_c}}^{b_1} \ldots b_{j_{N_c-n}}^{b_n}}{\mathcal{N}} |\bar{NN}\rangle,$$

Normalized as….

$$N^2 = \langle \bar{NN}| (a_{k_{N_c}}^{\dagger b_1} \ldots a_{k_{N_c-n}}^{\dagger b_n} b_{l_{N_c}}^{\dagger b_1} \ldots b_{l_{N_c-n}}^{\dagger b_n}) (a_{i_{N_c}}^{a_1} \ldots a_{i_{N_c-n}}^{a_n} b_{j_{N_c}}^{b_1} \ldots b_{j_{N_c-n}}^{b_n}) |\bar{NN}\rangle$$
$$= e^{i a_{i_{N_c}}^{a_1} \ldots a_{i_{N_c-n}}^{a_n} b_{j_{N_c}}^{b_1} \ldots b_{j_{N_c-n}}^{b_n} - i^{a_1} \ldots l_{N_c-n}^{b_n} + \delta_{N_c-n}^{a_1} \ldots \delta_{N_c-n}^{b_n}}$$

$$= \delta^{a_1 b_1} \ldots \delta^{a_n b_n} N_c (N_c - 1)^2 \ldots (N_c - n + 1)^2$$

$$= n \frac{N_c!^2}{(N_c - n)!^2},$$

Whose sequential decay goes as….

$$\langle 0| T([\pi Q_{N_c-n-1} Q_{N_c-n}]|0\rangle = \frac{(N_c - n)! (N_c - n - 1)!}{\sqrt{N_c} \sqrt{n(n+1)N_c!^2}}$$
$$\times \langle \bar{NN}| \pi (a_{k_{N_c}}^{\dagger b_1} \ldots b_{l_{N_c-n-1}}^{\dagger b_n} \ldots b_{l_{N_c-n-1}}^{\dagger b_n}) (a_{i_{N_c}}^{a_1} \ldots a_{i_{N_c-n}}^{a_n} b_{j_{N_c}}^{b_1} \ldots b_{j_{N_c-n}}^{b_n}) |\bar{NN}\rangle$$

$$= \sqrt{1 + \frac{1}{n}},$$

$O(1)$ As expected (Witten)

Thus the width of the polyquark is $O(1)$, it is not necessarily narrow, although its mass grows with $N_c$
The polyquark: other decays

We have also calculated the decays to all other configurations, both for \( N_f=1 \) and 2

<table>
<thead>
<tr>
<th></th>
<th>( q\bar{q} )</th>
<th>( gg )</th>
<th>( \pi\pi )</th>
<th>( T(qq\bar{q}q) )</th>
<th>( (N_c - 1)\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_f = 1 )</td>
<td>( (N_c - 1)!! \left(\frac{c}{N_c}\right)^{(N_c-4)/2} )</td>
<td>( N_c!! \left(\frac{c}{N_c}\right)^{(N_c-2)/2} )</td>
<td>( N_c!! \left(\frac{c}{N_c}\right)^{(N_c-4)/2} )</td>
<td>( N_c!! \left(\frac{c}{N_c}\right)^{(N_c-3)/2} )</td>
<td>( c^{N_c-1} )</td>
</tr>
<tr>
<td>( N_f = 2 )</td>
<td>( (N_c - 1)!! \left(\frac{c}{N_c}\right)^{(N_c-4)/2} )</td>
<td>( N_c!! \left(\frac{c}{N_c}\right)^{(N_c-2)/2} )</td>
<td>( N_c!! \left(\frac{c}{N_c}\right)^{(N_c-4)/2} )</td>
<td>( N_c!! \left(\frac{c}{N_c}\right)^{(N_c-3)/2} )</td>
<td>( \frac{(N_c/2)^2!}{N_c^{N_c/2}} )</td>
</tr>
</tbody>
</table>

They are all exponentially suppressed, including that to 2 pions

Only exception: the \( O(1) \) sequential decay, which dominates the width:

\[ M_P = O(N_c) \; ; \; \Gamma_P = O(1) \]
Motivation and Introduction

States with fixed number of constituents

States with growing number of constituents: Polyquark

If wide, then heavy
Growing widths with $N_c$?

$\pi\pi\pi$ scattering is $1/N_c$ (“weak”), so strong coupling may look forbidden, but... if $\Gamma \sim N_c^{2\gamma}$

$$M_{\pi\pi} = \frac{g_{R\pi\pi}^2}{s - m_R^2 + \Gamma_R^2/4 + i m \Gamma_R} \propto \frac{N_c^{2\gamma}}{N_c^4}.$$  

Is still consistent if $1/2 \leq \gamma$, even if $M=O(1)$

However, since the coupling to two pions grows as $g \sim N_c^\gamma$, the resonance propagator would be “dressed” with $g^2$ corrections. Keeping the real part as $O(1)$ would require fine tuning

$$S^{-1}_\Phi = S^{(0)-1}_\Phi - (ig)^2 \int \frac{d^4 q}{(2\pi)^4} S_\phi(q^2) S_\phi((p-q)^2) V(q,p-q).$$

We have shown this fine tuning with a unitarized ChPT example

If wide, then naturally heavy
Explicit calculation of $1/N_c$ leading behavior of the coupling between different kinds of meson states:

- Quark-antiquark, hybrids, glueballs, conventional tetraquarks, and “polyquarks” explicit expressions can be found on the published paper

- Hybrids and conventional tetraquarks fall in the same $N_c$-equivalence class of ordinary quark-antiquark mesons and become narrow at large $N_c$

- Only the polyquark does not necessarily become narrow as $N_c$ grows

- We have also argued why if wide at large $N_c$, then also heavy at large $N_c$

“Pure” configurations do not explain the sigma $N_c$ behavior,

- Mixing with heavier $q'$-like states?

- Subdominant $N_c$ effects (rescattering, meson-meson states?)