Center Vortices as a Model of the QCD vacuum

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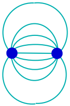
Why non-perturbative phenomena?

- Why confinement?
- Why chiral symmetry breaking?

QED
electric charges



dipole field

QCD
color charges

gluon string

QCD-vacuum is non-trivial

What is condensed in the vacuum?

Models of QCD vacuum

Vacuum is a condensate of

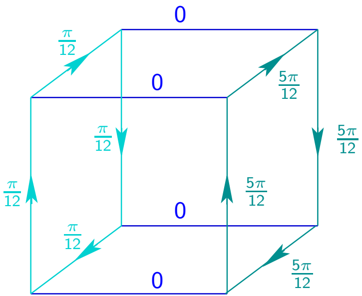
- instantons
  are field configurations of minimal action
  competition: action $\longleftrightarrow$ entropy
  $\Rightarrow$ action is not minimal
  $\Rightarrow$ (almost) no instantons in QCD vacuum
  $\Rightarrow$ in lattice QCD instantons produced by cooling
- color magnetic monopoles
- center vortices = closed quantised magnetic flux tubes

compare predictions of these models:
Wilson loops and string tensions,
abelian dominance versus center dominance

Color magnetic monopoles

identify by singular gauge fields

lattice: non-trivial cubes: $\mathrm{div}\vec{B} \neq 0$



$U_\Box = \frac{\pi}{3} = 60°$

$\sum_\Box U_\Box = 2\pi$

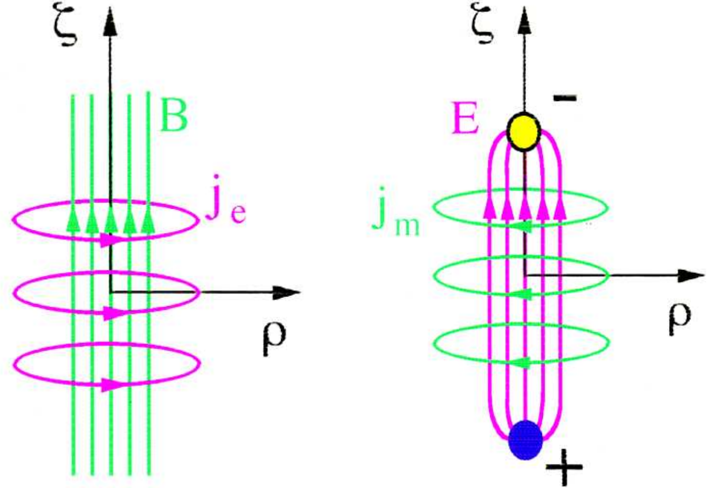
$\longrightarrow$ in a U(1) subgroup of SU(2) or SU(3)

$$U_\mu(x) = \underbrace{\begin{pmatrix} \sqrt{1-|c_\mu(x)|^2} & c_\mu(x) \\ -c_\mu^\star(x) & \sqrt{1-|c_\mu(x)|^2} \end{pmatrix}}_{\text{W-bosons}} \underbrace{\begin{pmatrix} \mathrm{e}^{\mathrm{i}\theta_\mu(x)} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta_\mu(x)} \end{pmatrix}}_{\in U(1)}$$

Maximal abelian gauge, abelian projection

Confinement due to Magnetic Monopoles

type II superconductor | dual superconductor



magnetic fluxoid quantisation | electric fluxoid quantisation

Center Vortices

➜ *'t Hooft 1979, Nielsen, Ambjorn, Olesen, Cornwall, 1979*
Mack, 1980; Feynman, 1981

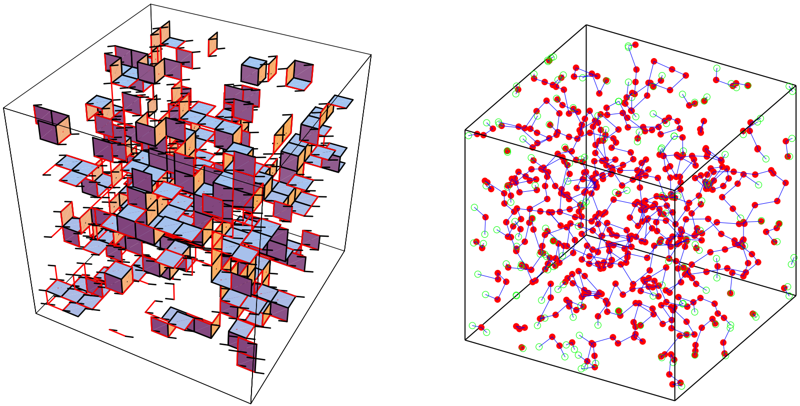
- QCD vacuum is a condensate of closed magnetic flux-lines, they have topology of tubes (3D) or surfaces (4D),
- magnetic flux corresponds to the center of the group,
- Vortex model may explain ...
  - **Confinement** $\rightarrow$ piercing of Wilson loop $\equiv$ crossing of static electric flux tube and moving closed magnetic flux
  - **Topological charge**: intersection points, writhing points and color structure

    ➜ *Engelhardt, Reinhardt (2000), Jordan, Höllwieser, M.F., Heller (2007), Höllwieser, Engelhardt (2014)*
  - **Spontaneous chiral symmetry breaking**: also center-projected configurations show χSB

➜ *Forcrand, Elia (1999), Höllwieser, M.F., Greensite, Heller, Olejnik (2008), Schweigler, Höllwieser, M.F., Heller (2012,2013)*

Vortex Vacuum in SU(2)

Random Structure, Percolation Transition



3-dimensional cut through the dual of a 12^4-lattice.

Maximal center gauge, center projection

Wilson loops

- closed loops around rectangular $(R \times T)$, planar contour C

$$\text{Area law:} \qquad W(R,T) = \langle \prod_{x \in C} U_\mu(x) \rangle \rightarrow e^{-\sigma R T}$$

$$\text{Perimeter law:} \qquad W(R,T) = \langle \prod_{x \in C} U_\mu(x) \rangle \rightarrow e^{-\alpha (R+T)}$$

- quark-antiquark "test-pair"
- heavy quark potential in limit $T \rightarrow \infty$

$$\text{Area law:} \quad V(R) = -\lim_{T \rightarrow \infty} \frac{1}{T} \ln \langle W(R,T) \rangle \rightarrow -\sigma R$$

- Area law $\rightarrow$ Confinement
- $\sigma \ldots$ string tension $\rightarrow$ creutz ratio χ

$$\chi = \frac{W(R+1,T+1) W(R,T)}{W(R+1,T) W(R,T+1)} \rightarrow e^{-\sigma} \Rightarrow \sigma = -\ln \chi$$

Abelian Models in SU(2)

- monopole plasma in D=3 or 4 Euclidean Dimensions,

$$f_{ij} = \epsilon_{ijk} \frac{1}{2} \int d^3 r' \frac{(r - r')_k}{|r - r'|^3} \rho(r')$$

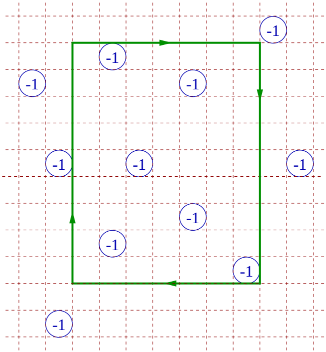
- dual superconductivity
- dyon ensembles, calorons, far field essentially abelian

Abelian Dominance Approximation:

long range fluctuations are mainly abelian

$$\begin{aligned} W(C) &= \frac{1}{2} \langle \mathrm{Tr} P \exp[i \oint_C dx^\mu A_\mu^a \frac{\sigma^a}{2}] \rangle \approx \frac{1}{2} \langle \mathrm{Tr} \exp[i \oint_C dx^\mu A_\mu^3 \frac{\sigma^3}{2}] \rangle \\ &= \langle \exp[i \frac{1}{2} \oint_{C=\partial S} dx^\mu A_\mu^3] \rangle = \langle \exp[i \frac{1}{2} \int_S d\sigma^{\mu\nu} f_{\mu\nu}^3] \rangle \end{aligned}$$

Area law for center projected loops in SU(2)

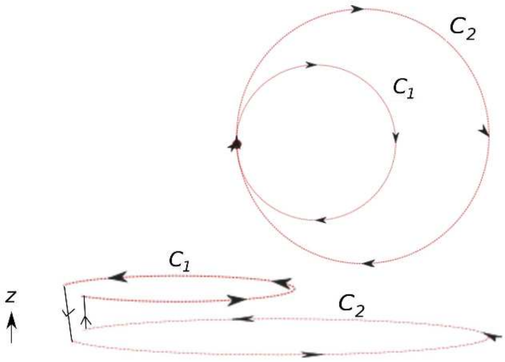


denote *f the probability that a plaquette has the value -1*

$$\langle W(A)\rangle = [f(-1) + (1-f)\cdot 1]^A = \exp[\underbrace{\ln(1-2f)}_{-\sigma} A], =$$

$$= \exp[-\sigma R \times T], \qquad \sigma \equiv -\ln(1-2f) \approx 2f$$

Double-winding Wilson loops



➜ *Greensite, Höllwieser, 2015*

check monopole and vortex picture in SU(2)

Double-winding Wilson loops $C = C_1 + C_2$

- Sum of areas behavior in Abelian models:

$$\begin{aligned} W(C) &= \frac{1}{2}\langle \mathrm{Tr} P \exp[i \oint_C dx^\mu A^a_\mu \frac{\sigma^a}{2}]\rangle \approx \langle \mathrm{Tr}\ \exp[\frac{i}{2}\oint_C dx^\mu A^3_\mu]\rangle \\ &= \langle \exp[\frac{i}{2}\oint_{C_1} dx^\mu A^3_\mu]\ \exp[\frac{i}{2}\oint_{C_2} dx^\mu A^3_\mu]\rangle \\ &\approx \langle \exp[\frac{i}{2}\oint_{C_1} dx^\mu A^3_\mu]\rangle\langle \exp[\frac{i}{2}\oint_{C_2} dx^\mu A^3_\mu]\rangle \\ &\approx \exp[-\sigma(A_1 + A_2) - \mu P] \end{aligned}$$

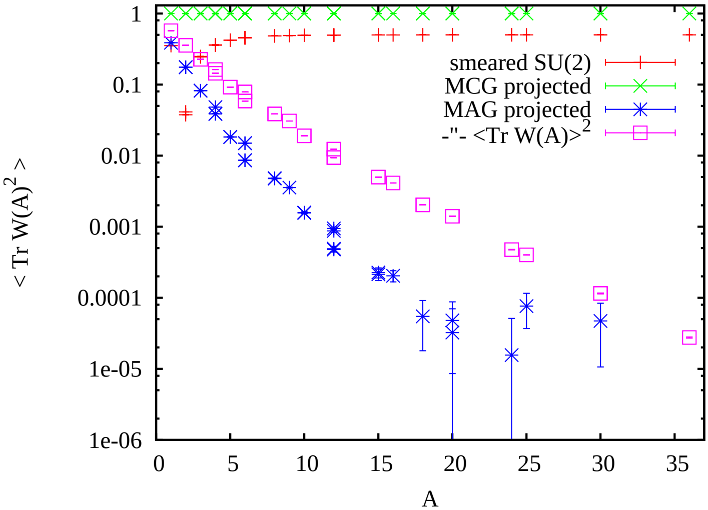
- vs. Difference of areas behavior in center vortex picture:

$$W(C) = \alpha \exp[-\sigma |A_1 - A_2|]$$

Winding around a vortex twice gives no contribution to $W(C)$:

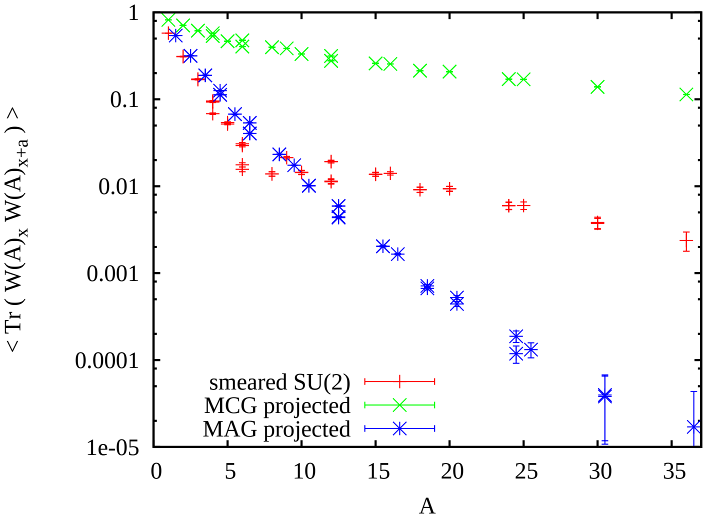
$$(-1)^2 = +1$$

Double-winding loops $C = C_1 = C_2$

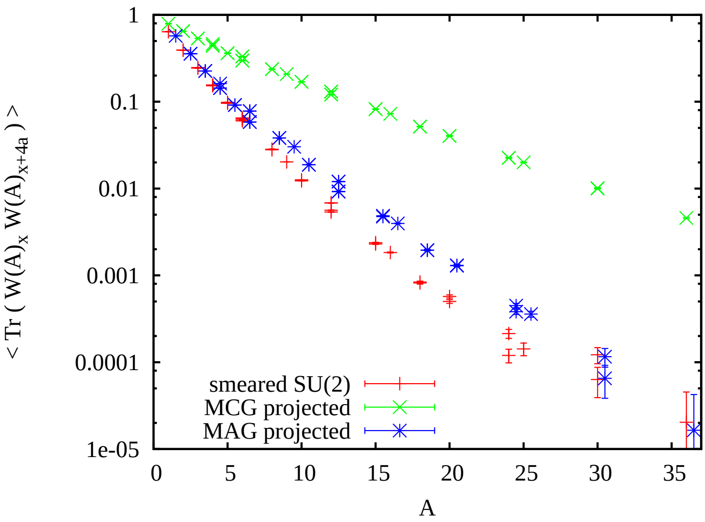


SU(2) group identity: $\mathrm{Tr}[U(C)U(C)] = 1 + \mathrm{Tr}_A U(C)$,
$\langle \mathrm{Tr}_A U(C) \rangle \ll 1 \Rightarrow W(C) \approx 1/2$

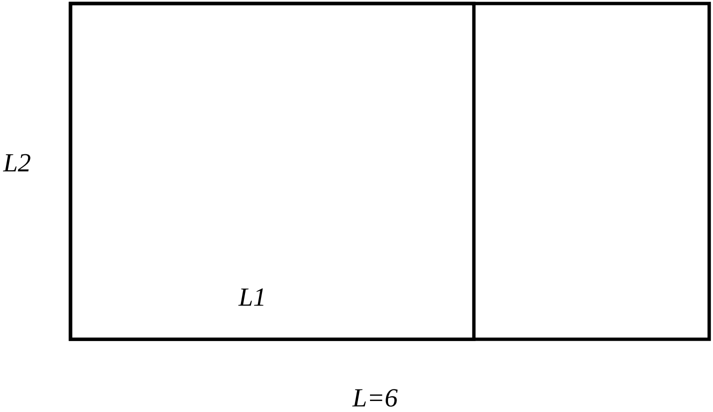
Double-winding loops $C = C_1 = C_2$



Double-winding loops $C = C_1 = C_2$

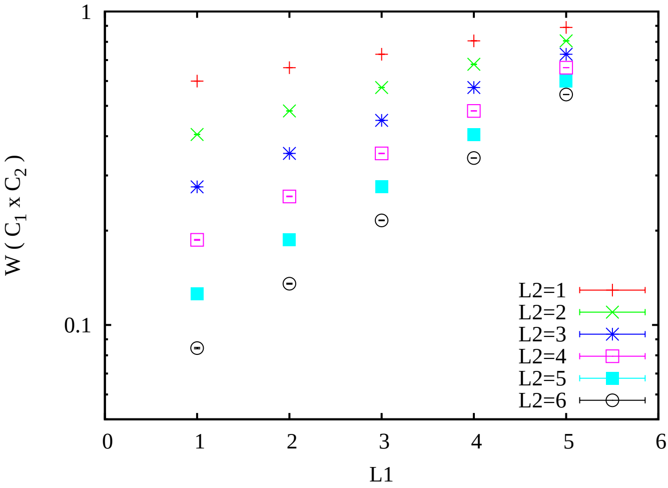


Double-winding loops



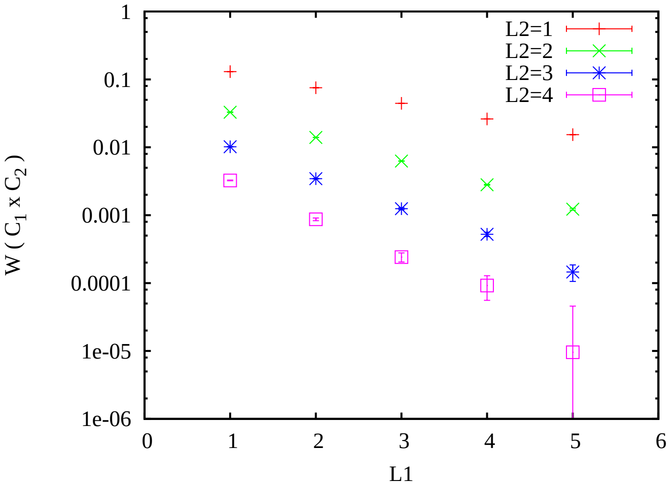
$$A_1 = 6L_2, \quad A_2 = L_1 L_2$$

Double-winding loops: Z(2)



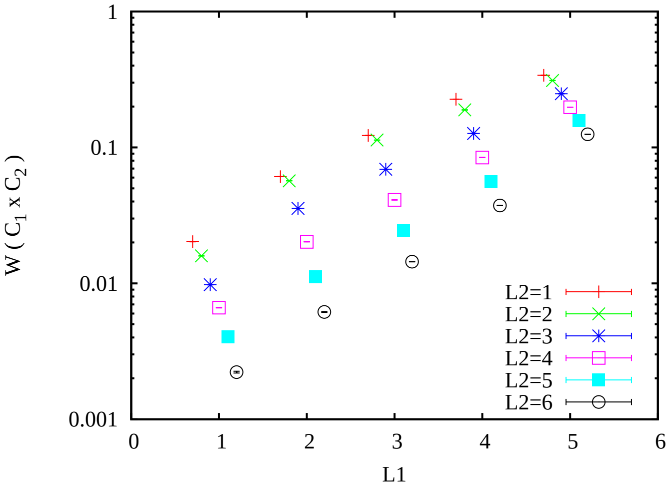
$$A_1 - A_2 = (6 - L_1)L_2$$

Double-winding Wilson loops: MAG



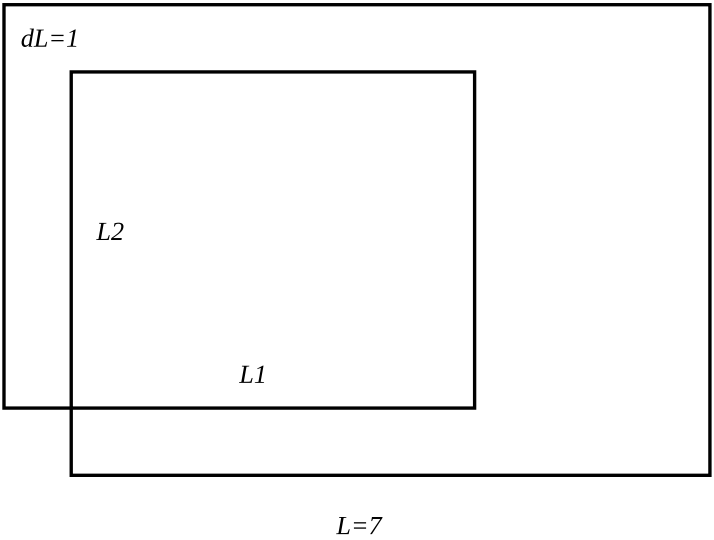
$$A_1 + A_2 = (6 + L_1)L_2$$

Double-winding Wilson loops: SU(2)



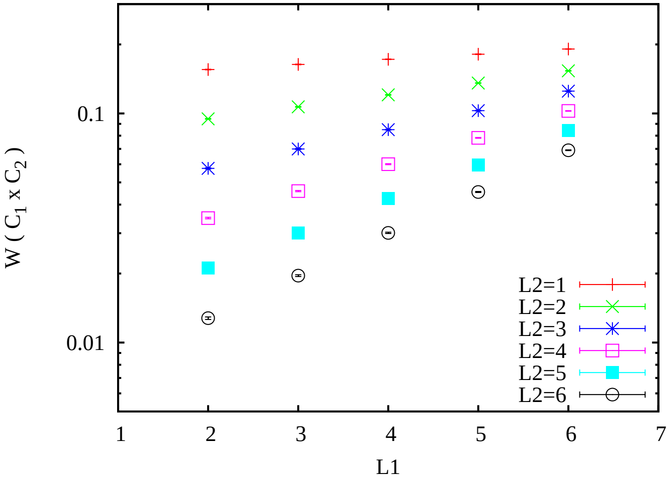
$$A_1 - A_2 = (6 - L_1)L_2 \quad \text{versus} \quad A_1 + A_2 = (6 + L_1)L_2$$

Double-winding Wilson loops

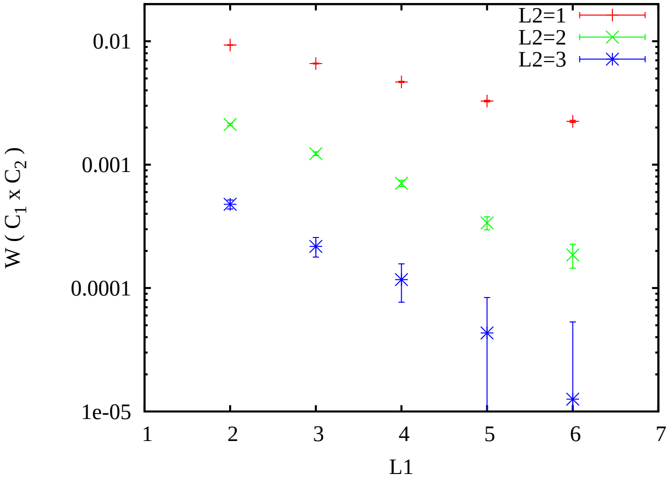


$$A_1 = 8(L_2 + 1) - 1, \quad A_2 = L_1 L_2$$

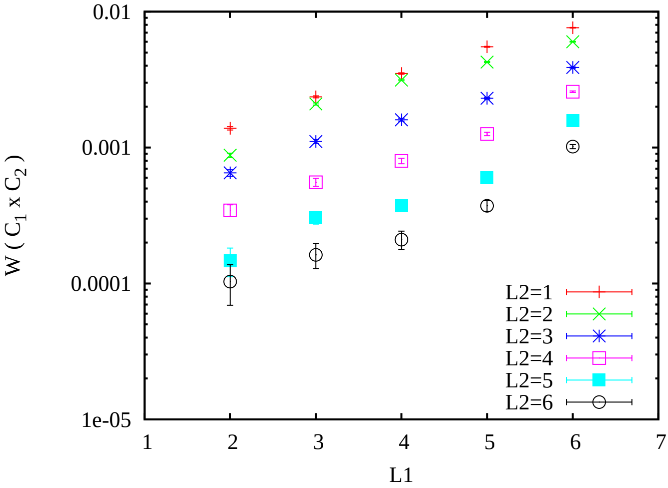
Double-winding Wilson loops: Z(2)



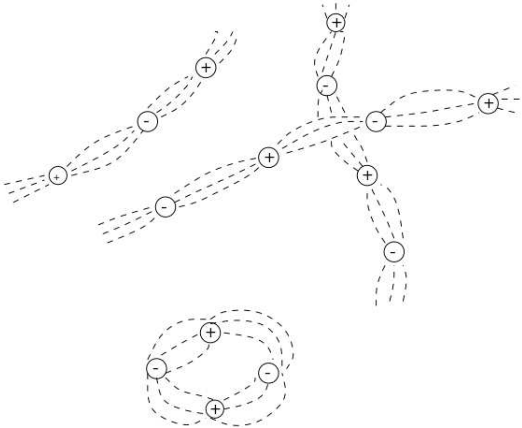
Double-winding Wilson loops: MAG



Double-winding Wilson loops: SU(2)



W-bosons change the field distribution

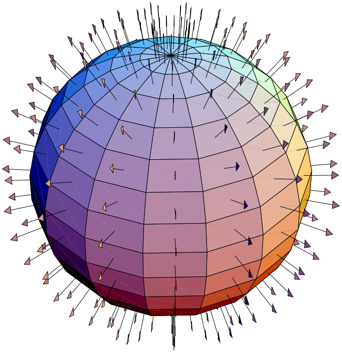


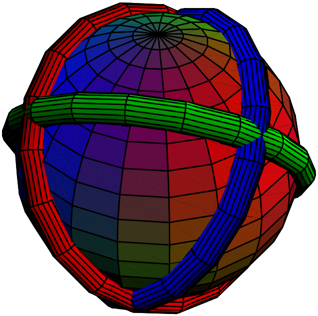
Monopoles arranged in monopole–antimonopole chains = Vortices

➜ *Ambjorm, Giedt, Greensite, 2000*

Monopoles as hint of color structure of vortices

The corresponding P-vortex is a sphere at radius R. The color structure of the spherical vortex, a hedgehog configuration, is illustrated in the left plot. The right plot illustrates the monopole lines after Abelian projection in the maximal Abelian gauge.





Conclusion

- monopol plasma, dyon gas, dual superconductor models predict sum of areas falloff of double-winding Wilson loops
- this sum of areas falloff contradicts results of lattice Monte-Carlo
- abelian models do not give the right spatial distribution of magnetic field
- abelian models neglect color structure of magnetic flux
- center vortex model predicts difference of areas fall off

Remind from ExQCD14:

- center vortices contribute to topological charge via intersections, writhing points and color structure
- all objects with topological charge contribute to near-zero modes via interaction
- all topological objects contribute to $\bar{\psi}\psi$ (Banks-Casher)

Thank You &

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