# The role of the kinematical constraint and non-linear effects in the CCFM equation

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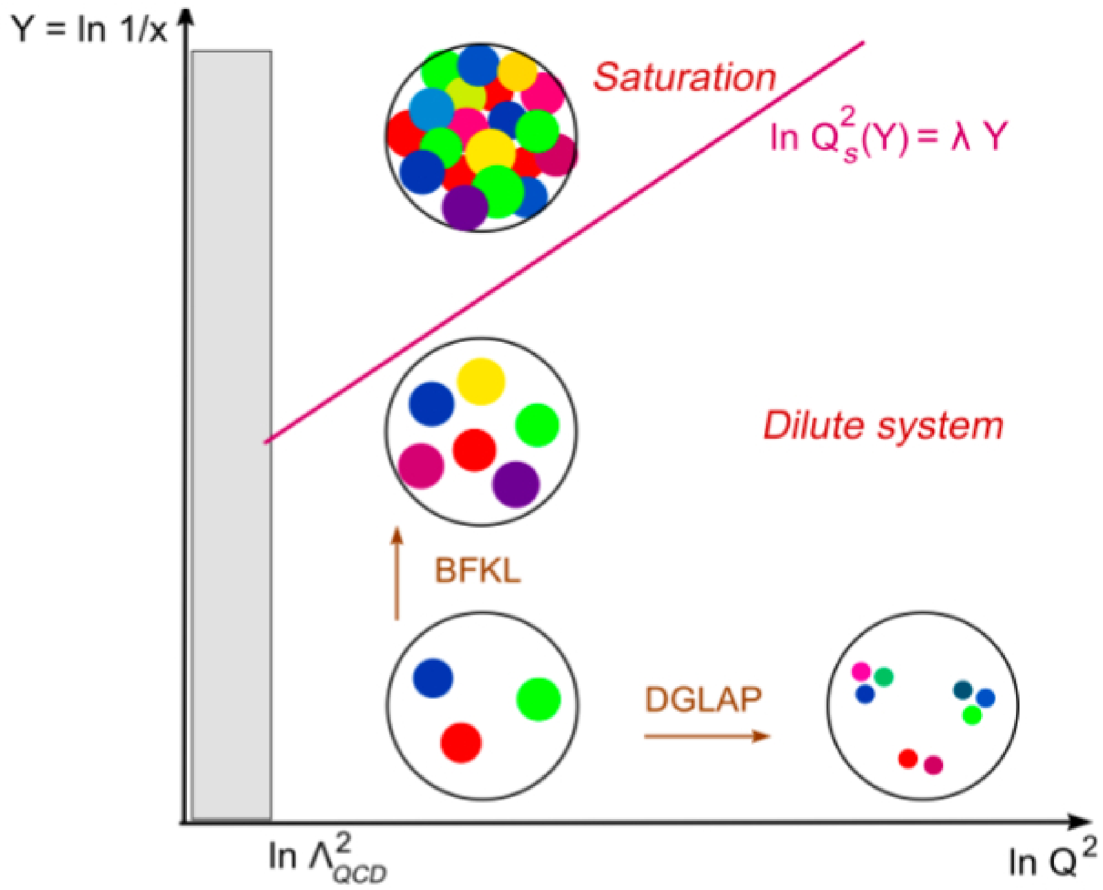
# Short introduction




- BFKL equation sums terms $(\alpha_s ln(1/x))^n$ – small-x physics

# Origin of the kinematical constraint

- The kinematical constraint is assumed to hold in derivation of the BFKL equation
  - *t*-channel gluon momentum $$k^2 \simeq -\mathbf{k}^2$$ (next slide)

Kwiecinski, Martin, Sutton Z.Phys. C71 (1996) 585-594

- In the final result it is omitted because:
  - should cause "subleading/minor" effects
  - makes things complicated for analytical studies

- How big is the effect of enforcement of the kinematical constraint?

# Origin of the kinematical constraint

- Kinematical constraint motivation and derivation

$$k^2 \simeq -\mathbf{k}^2$$ (BFKL derivation)

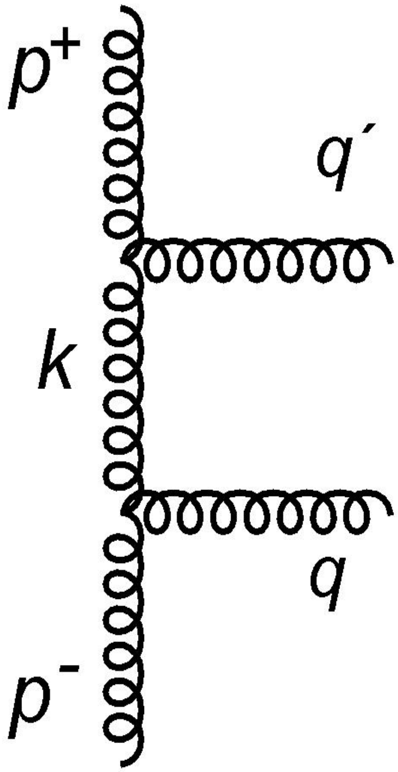
$$k = z\,p^+ + \bar{z}p^- + k_\perp$$ (Sudakov decomposition)

$$k^2 = -z\,\bar{z}\,\hat{s} - \mathbf{k}^2$$

$$q^2 = \bar{z}(1-z)\hat{s} - \mathbf{q}^2 = 0$$

$$\mathbf{k}^2 > \frac{z\,\mathbf{q}^2}{1-z}$$

$$\boxed{\mathbf{k}^2 > z\,\mathbf{q}^2}$$



# Origin of the kinematical constraint

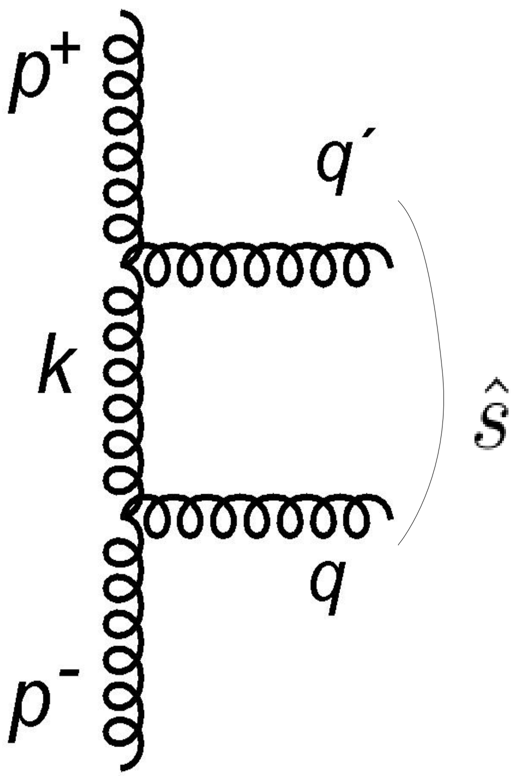
- Kinematical constraint motivation and derivation

$$\mathbf{k}^2 > z\,\mathbf{q}^2$$

$$z > x$$

$$\mathbf{q}^2 < \mathbf{k}^2/x \simeq \hat{s}$$

- Local “energy conservation” condition



# Kinematical constraint in the BFKL equation

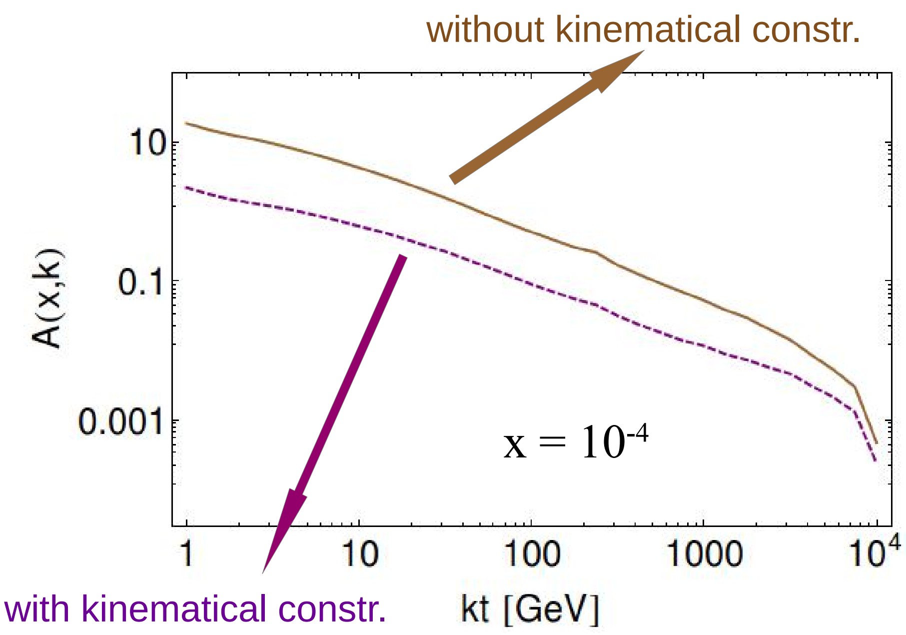
- The equation

$$\phi(x,\mathbf{k}^2) = \tilde{\phi}_0(\mathbf{k}^2) + \bar{\alpha}_S \int_x^1 \frac{dz}{z}\Delta_R(z,\mathbf{k}^2,\mu)\Big[\int \frac{d^2\mathbf{q}}{\pi\mathbf{q}^2}\theta(\mathbf{q}^2-\mu^2)\phi(x/z,|\mathbf{k}+\mathbf{q}|^2)\Big)$$

Regge form factor:

$$\Delta_R(z,\mathbf{k}^2) = \exp\left(-\bar{\alpha}_s \ln 1/z \ln \mathbf{k}^2/\mu^2\right)$$

enforcing $\mathbf{k}^2 > z\,\mathbf{q}^2$



# The CCFM equation - linear

- Angular ordering
- Kinematical constraint


Ciafaloni, Nucl. Phys. B296 (1988) 49
Catani, Fioranni, MarchesiniPhys. Lett. B234 (1990) 339
Marchesini, Nucl. Phys. B336 (1990) 18

H. Jung, G. P. Salam, Eur. Phys. J. C19 (2001) 351–360
H. Jung, Comput. Phys. Commun. 143 (2002) 100–111


$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta\left(\frac{\mathbf{k}^2}{(1-z)\bar{\mathbf{q}}^2} - z\right) \mathcal{E}(x/z,\mathbf{k}',|\bar{\mathbf{q}}|)$$
$$\times\ \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)\ ,$$

With the splitting function:

$$\mathcal{P}(z,\mathbf{k},\mathbf{q}) = \frac{1}{1-z} + \Delta_{NS}(z,\mathbf{k}^2,|\mathbf{q}|)\, \frac{1}{z}$$

Sudakov:

$$\Delta_S(p, z|\bar{\mathbf{q}}|, Q_0) = \exp\left( - \int_{(z\bar{\mathbf{q}})^2}^{p^2} \frac{d^2\mathbf{q}'}{\pi \mathbf{q}'^2} \int_0^{1-\frac{Q_0}{|\mathbf{q}'|}} dz' \frac{\bar{\alpha}_S}{1-z'} \right)$$

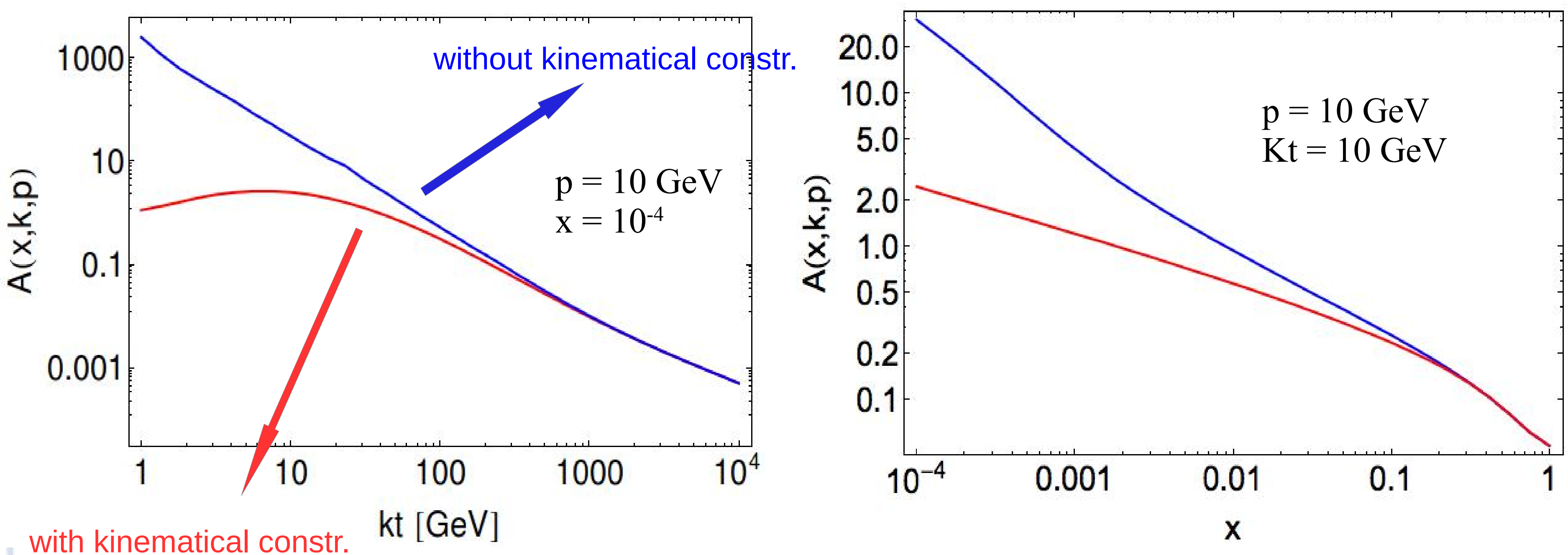
Non-Sudakov:

$$\Delta_{NS}(z,k_T,\bar{q}) = \exp\left\{ -\bar{\alpha}_S \int_z^1 \frac{dz'}{z'}\, \Theta\left(\frac{(1-z')k_T^2}{(1-z)^2\bar{q}^2} - z'\right) \times \int \frac{dq'^2}{q'^2}\, \Theta(k_T^2 - q'^2)\, \Theta(q' - z'\bar{q}) \right\}$$

# The CCFM equation - linear

$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta\left(\frac{\mathbf{k}^2}{(1-z)\bar{\mathbf{q}}^2} - z\right) \mathcal{E}(x/z,\mathbf{k}',|\bar{\mathbf{q}}|)$$

$$\times\ \theta(p - z|\bar{\mathbf{q}}|)\ \mathcal{P}(z,\mathbf{k},\mathbf{q})\ \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)\ ,$$



# Implementations of the kinematical constraint

- Different implementations exist
- Most applications – kinematical constraint in the kernel neglected

$$
\begin{aligned}
\Delta_{NS}(z,k_T,\bar{q}) &= \exp\left(-\overline{\alpha}_S \int_z^{z_0} \frac{dz'}{z'} \int \frac{dq'^2}{q'^2} \Theta(k^2 - q'^2)\Theta(q' - z'\bar{q})\right) \\
&= \exp\left(-\overline{\alpha}_S \log\left(\frac{z_0}{z}\right) \log\left(\frac{k^2}{z_0 z \bar{q}^2}\right)\right),
\end{aligned}
$$

$$
z_0 = \begin{cases} 1\ , & \text{if} \quad (k_T/\bar{q}) \geq 1 \\ k_T/\bar{q}, & \text{if} \quad z < (k_T/\bar{q}) < 1 \\ z\ , & \text{if} \quad (k_T/\bar{q}) \leq z \end{cases}
$$

- Full implementation + inclusion of the $\theta$-function in the kernel

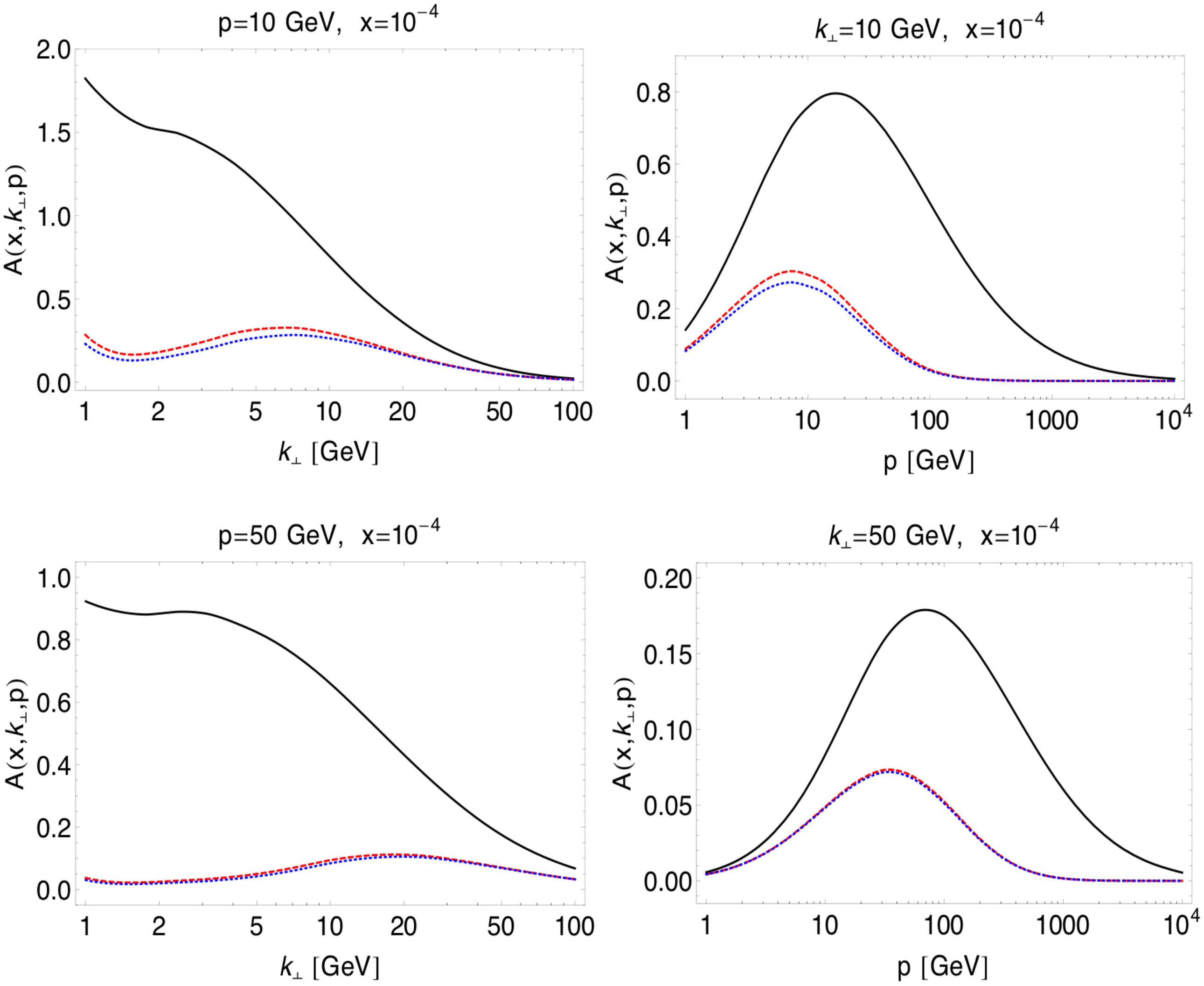
$$
\Delta_{NS}(z,k_T,\bar{q}) = \exp\Bigg\{ -\overline{\alpha}_S \int_z^1 \frac{dz'}{z'}\, \Theta\left(\frac{(1-z')k_T^2}{(1-z)^2\bar{q}^2} - z'\right) \times \int \frac{dq'^2}{q'^2}\, \Theta(k_T^2 - q'^2)\, \Theta(q' - z'\bar{q})\Bigg\}
$$

$$
\theta\left(\frac{k_T^2}{(1-z)\bar{q}^2} - z\right)
$$

# The CCFM equation - linear

- black line – implementation without the $\theta$-function
- red line – full implementation
- blue line – with non-linear term

- note the peak at $p \approx k_\perp$
- big difference between the 2 implementations



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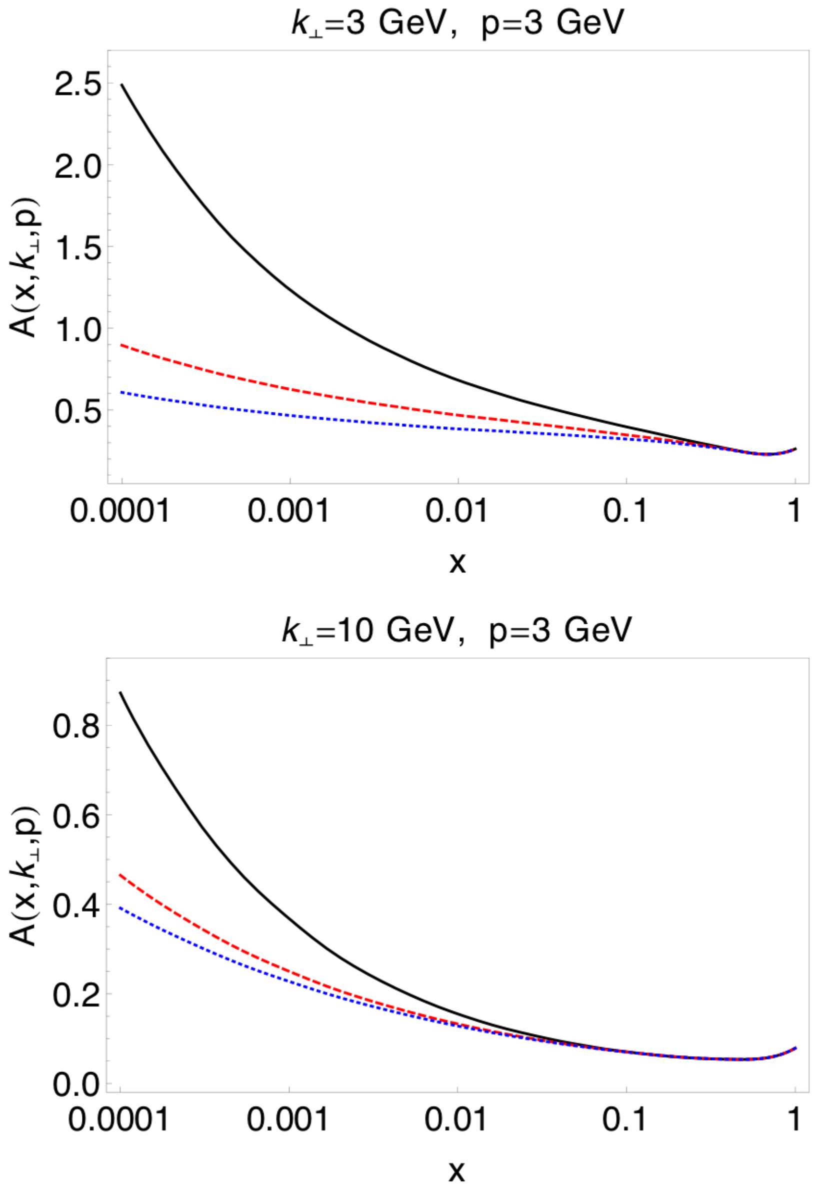
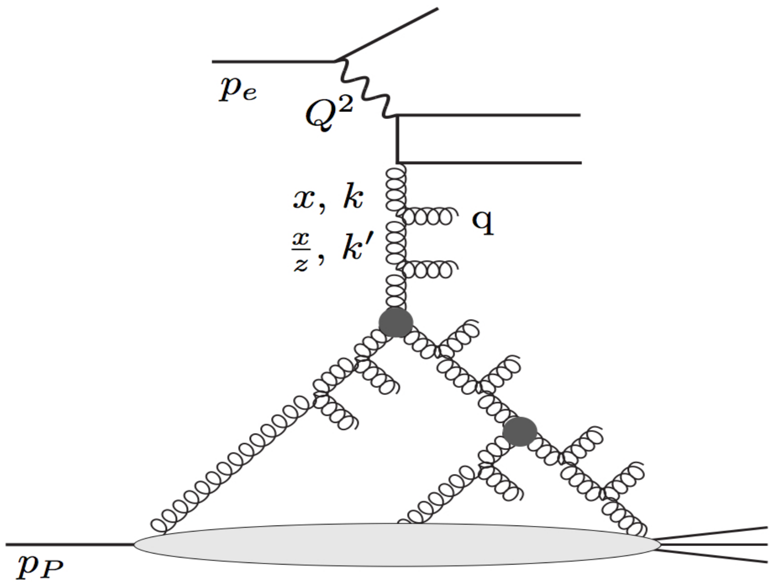


# The non-linear CCFM equation – KGBJS equation

- The equation
- Angular ordering
- Kinematical constraint



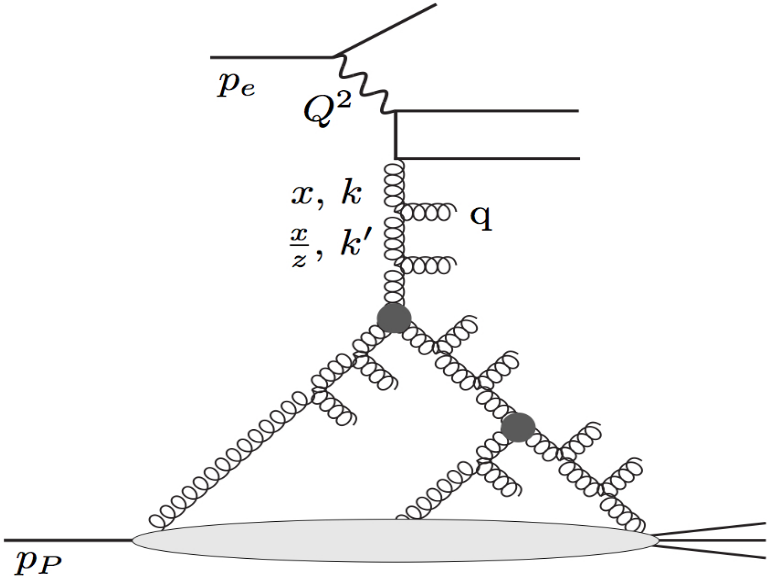

$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)$$

$$\times \left( \mathcal{E}(x/z, \mathbf{k}', |\bar{\mathbf{q}}|) - \frac{1}{\pi R^2} \delta\left( \bar{\mathbf{q}}^2 - \frac{\mathbf{k}^2}{(1-z)^2} \right) \bar{\mathbf{q}}^2\, \mathcal{E}^2(x/z, \bar{\mathbf{q}}, |\bar{\mathbf{q}}|) \right)$$

K. Kutak et al., JHEP 1202 (2012) 117, arXiv:1111.6928
M. D., JHEP 1307 (2013) 087, arXiv:1209.6092
K. Kutak, D. Toton: JHEP 1311 (2013) 082, arxiv:1306.3369

# The non-linear CCFM equation – KGBJS equation

- Angular ordering
- Kinematical constraint
- Non-linear term
- Saturation effects




$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)$$

$$\times \left(\mathcal{E}(x/z,\mathbf{k}',|\bar{\mathbf{q}}|) - \frac{1}{\pi R^2}\delta\left(\bar{\mathbf{q}}^2 - \frac{\mathbf{k}^2}{(1-z)^2}\right)\bar{\mathbf{q}}^2\, \mathcal{E}^2(x/z,\bar{\mathbf{q}},|\bar{\mathbf{q}}|)\right)$$

related to the hadronic size
connection to soft MPI

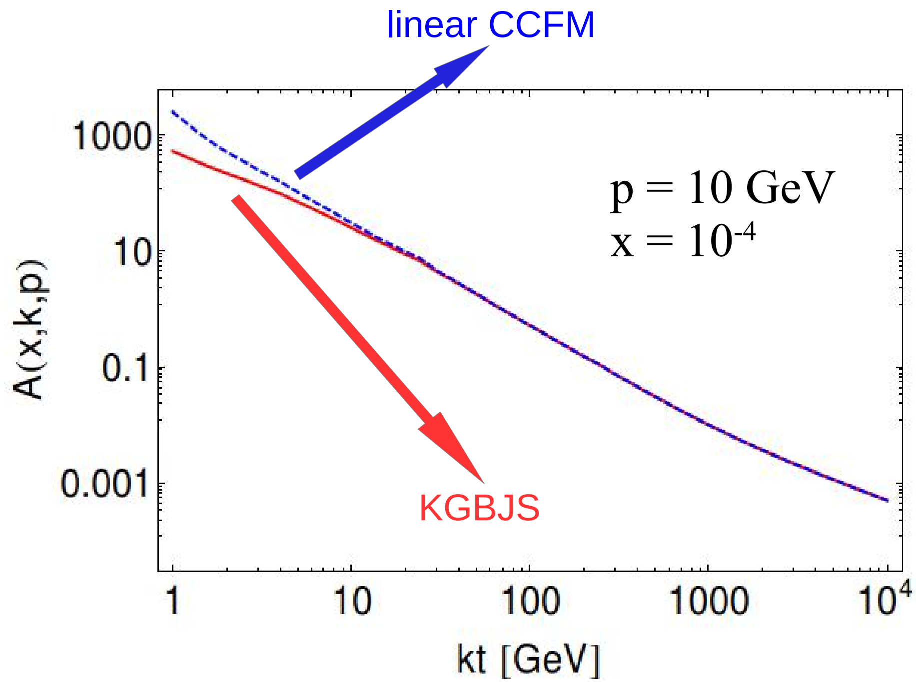
K. Kutak et al., JHEP 1202 (2012) 117, arXiv:1111.6928
M. D., JHEP 1307 (2013) 087, arXiv:1209.6092
K. Kutak, D. Toton: JHEP 1311 (2013) 082, arxiv:1306.3369

# The CCFM equation – non-linear KGBJS

$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)$$

$$\times \left( \mathcal{E}(x/z, \mathbf{k}', |\bar{\mathbf{q}}|) - \frac{1}{\pi R^2} \delta\left( \bar{\mathbf{q}}^2 - \frac{\mathbf{k}^2}{(1-z)^2} \right) \bar{\mathbf{q}}^2\, \mathcal{E}^2(x/z, \bar{\mathbf{q}}, |\bar{\mathbf{q}}|) \right)$$

Suppression for small transverse
Momentum – large amplitudes



# The CCFM equation – non-linear KGBJS

- note the change of the slope at $p \approx k_\perp$ !

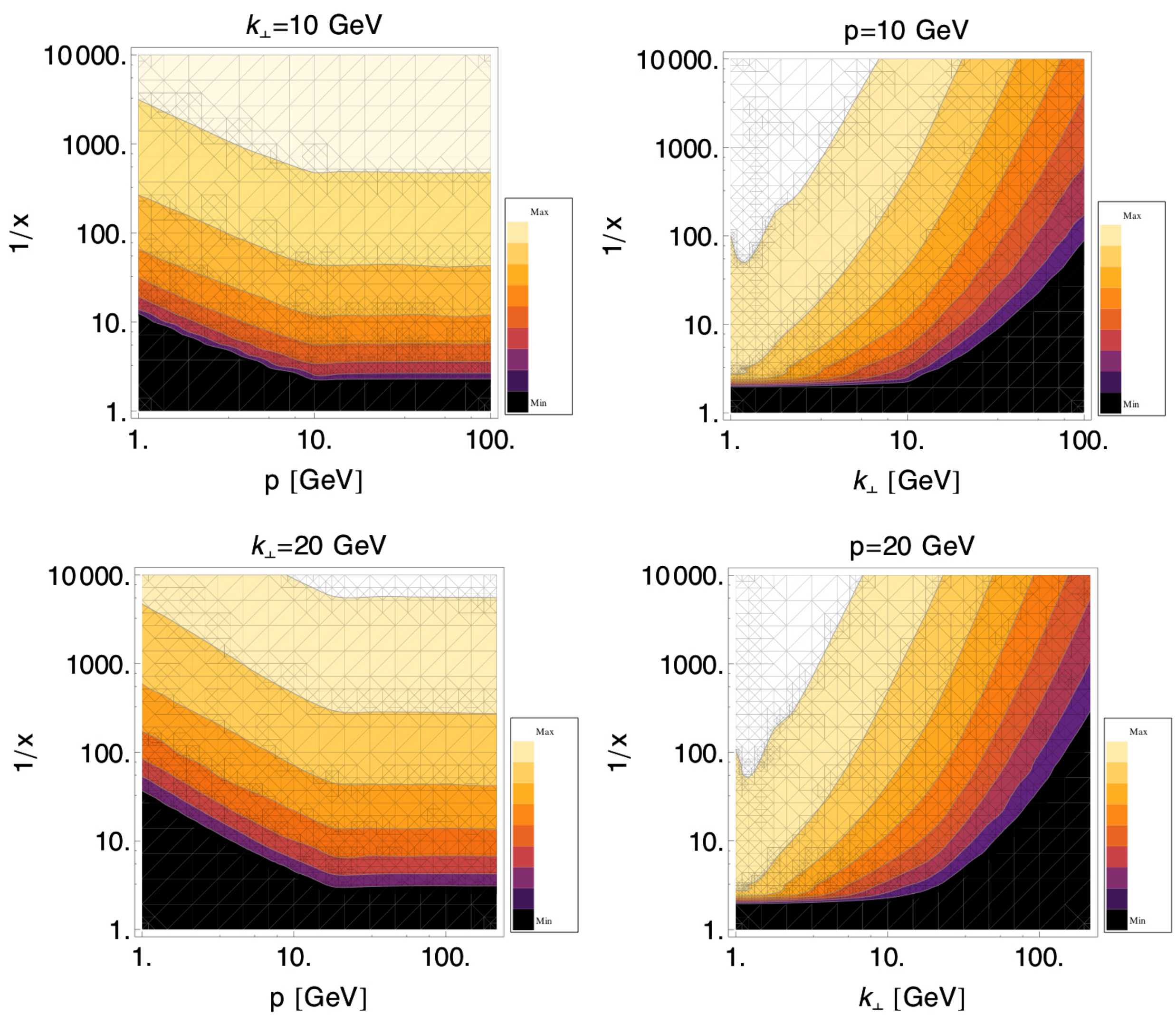
$$\beta(x, k_T, p) = \frac{|\mathcal{A}_{CCFM}(x, k_T, p) - \mathcal{A}_{KGBJS}(x, k_T, p)|}{\mathcal{A}_{CCFM}(x, k_T, p)}$$

- saturation condition for $k_\perp$

$$\beta(x, Q_s(x, p), p) = const$$

- saturation condition for p

$$\beta(x, k_T, P_s) = const$$



# Summary

- The kinematical constraint
  - is required by consistency of derivation of the BFKL equation
  - locally induces energy conservation condition
- Numerical results show that
  - represents a big correction – has a big effect on the solution of given evolution equation
- Suppression of the amplitude at low transversal momentum by the non-linear term in the CCFM equation

## Prospects:

- Fit to data ($F_2$ ?)