Hydrodynamics of QCD

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Excited QCD 2015

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Hydrodynamics of QCD

Hydrodynamics of. . . ?

Hydrodynamics of. . . ?

- Hydrodynamics is hydrodynamics, no matter the substance

Hydrodynamics of QCD

or

What
the use of
hydrodynamics
has taught us about
QCD matter

QCD matter

- matter in condensed matter physics sense

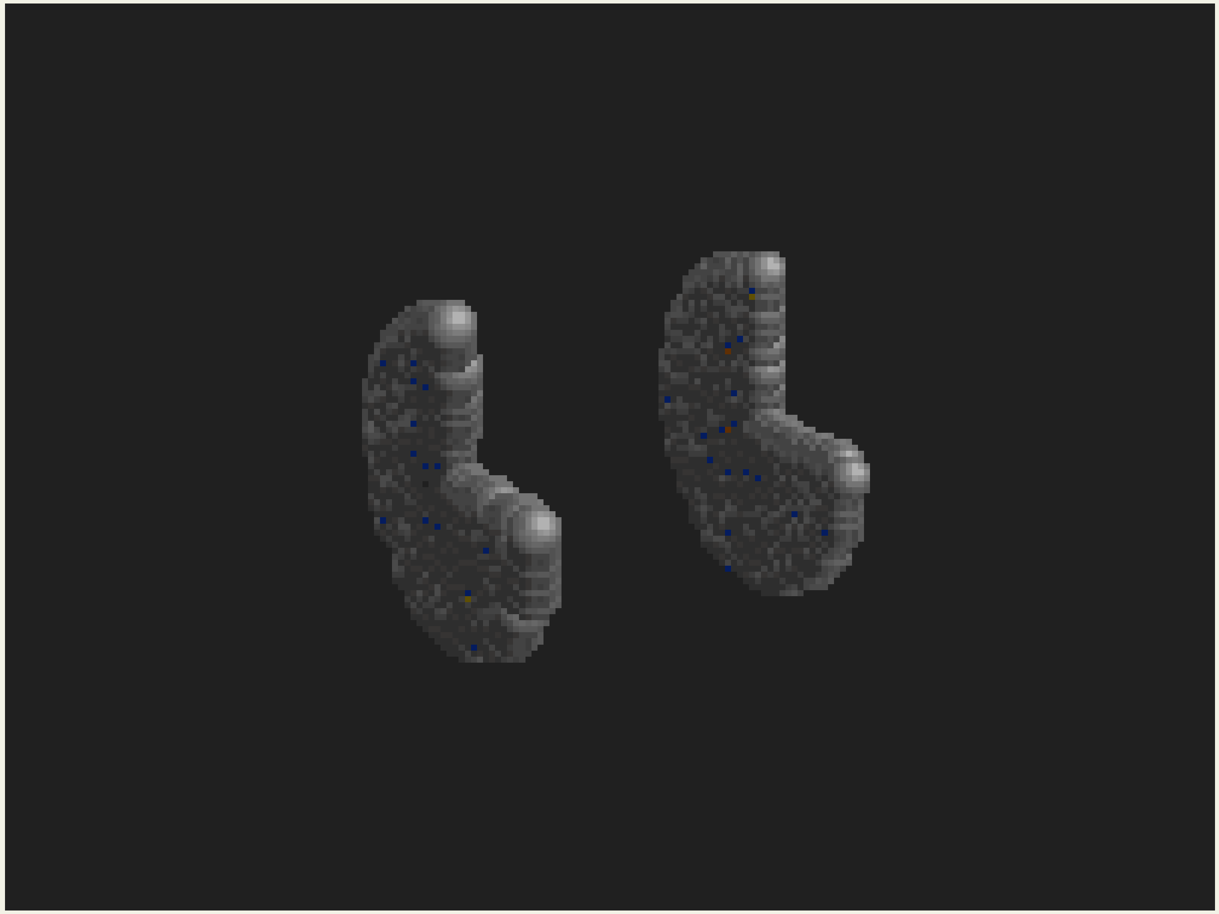
- so many particles that thermodynamical concepts
  - temperature
  - pressure
  - etc.

  apply

- interaction between constituents QCD, not QED

- in particular, partonic matter, i.e. QGP

Heavy-ion collision



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Hydrodynamics

local conservation of energy, momentum and baryon number:

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \textbf{and} \quad \partial_\mu N^\mu(x) = 0$$

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu} \\ N^\mu &= n u^\mu + \nu^\mu \end{aligned}$$

local, macroscopic variables: **energy density** $\epsilon(x)$
pressure $P(x)$
flow velocity $u^\mu(x)$

matter characterized by: **equation of state** $P = P(T, \{\mu_i\})$
transport coefficients $\eta = \eta(T, \{\mu_i\})$
$\zeta = \zeta(T, \{\mu_i\})$
$\kappa = \kappa(T, \{\mu_i\})$

Unknowns: **initial state, final state**

Dissipative hydrodynamics

$$T^{\mu\nu} = eu^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$\pi^{\mu\nu}$: shear-stress tensor, Π: bulk pressure

relativistic Navier-Stokes: $\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$

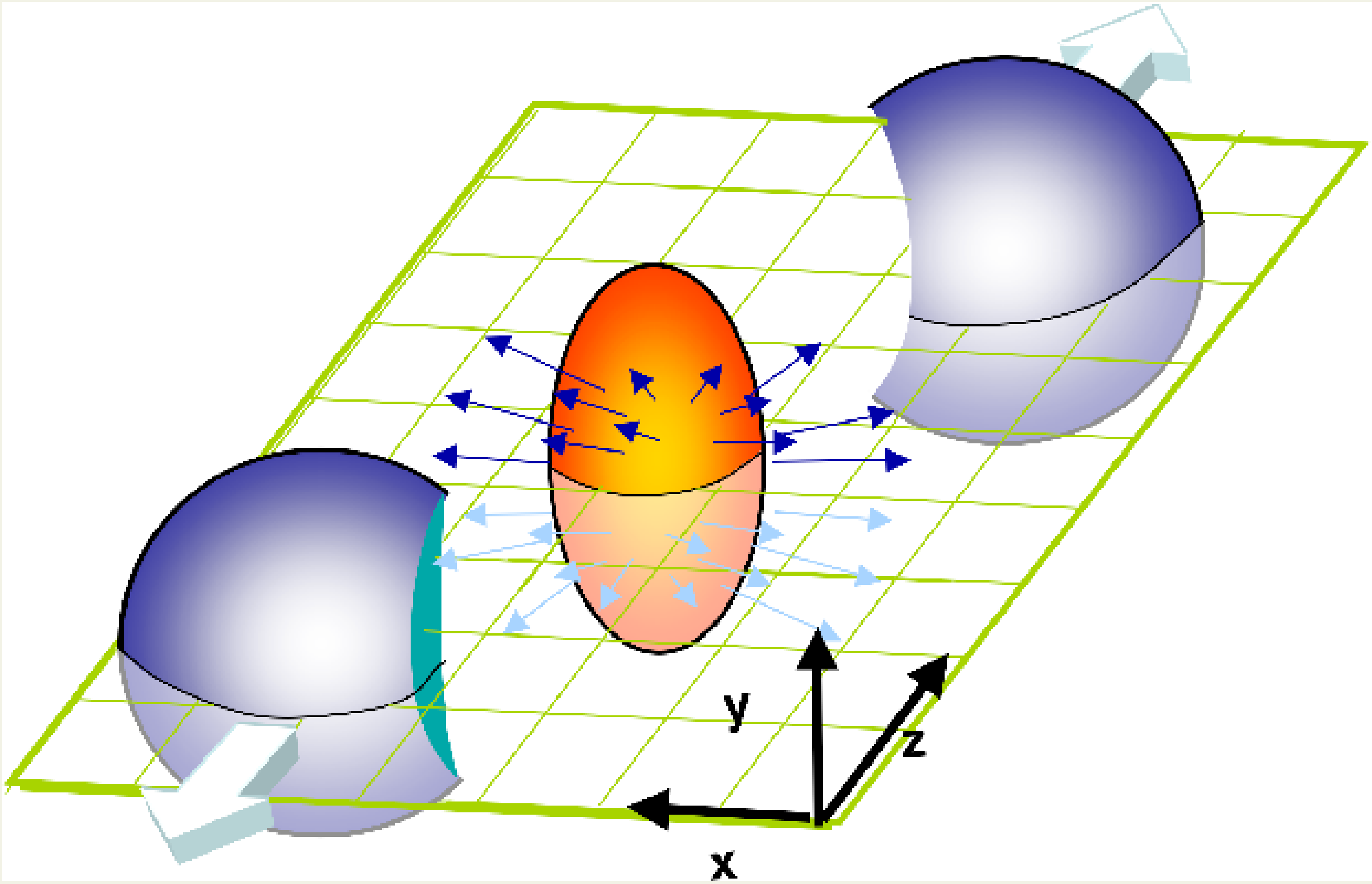
- unstable and acausal

Israel-Stewart: $\pi^{\mu\nu}$ independent dynamical variable

$$\left\langle u^\lambda \partial_\lambda \pi^{\mu\nu} \right\rangle = \frac{\pi^{\mu\nu}_{\mathrm{NS}} - \pi^{\mu\nu}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_\lambda u^\lambda + \cdots$$

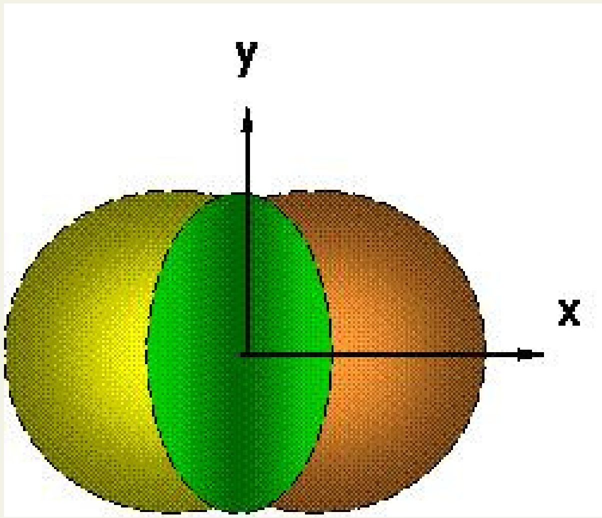
- η: shear viscosity coefficient
- τ_π: shear relaxation time, known for massless particles

Elliptic flow v_2

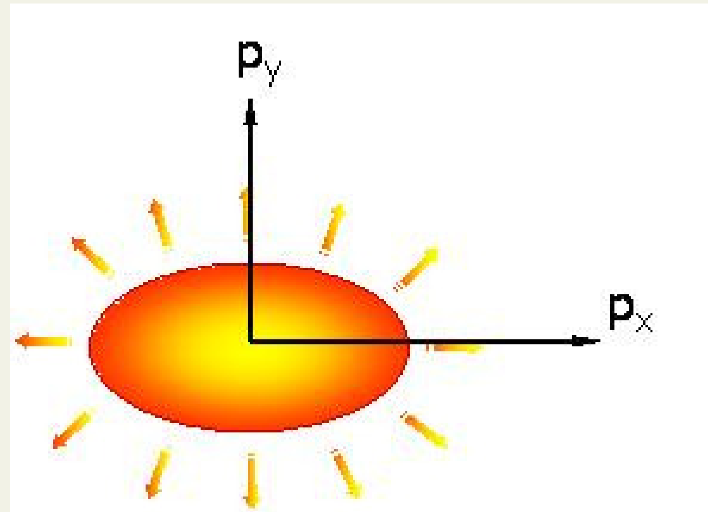


Elliptic flow v_2

spatial anisotropy → final azimuthal momentum anisotropy



⇒



- **Anisotropy in coordinate space + rescattering**

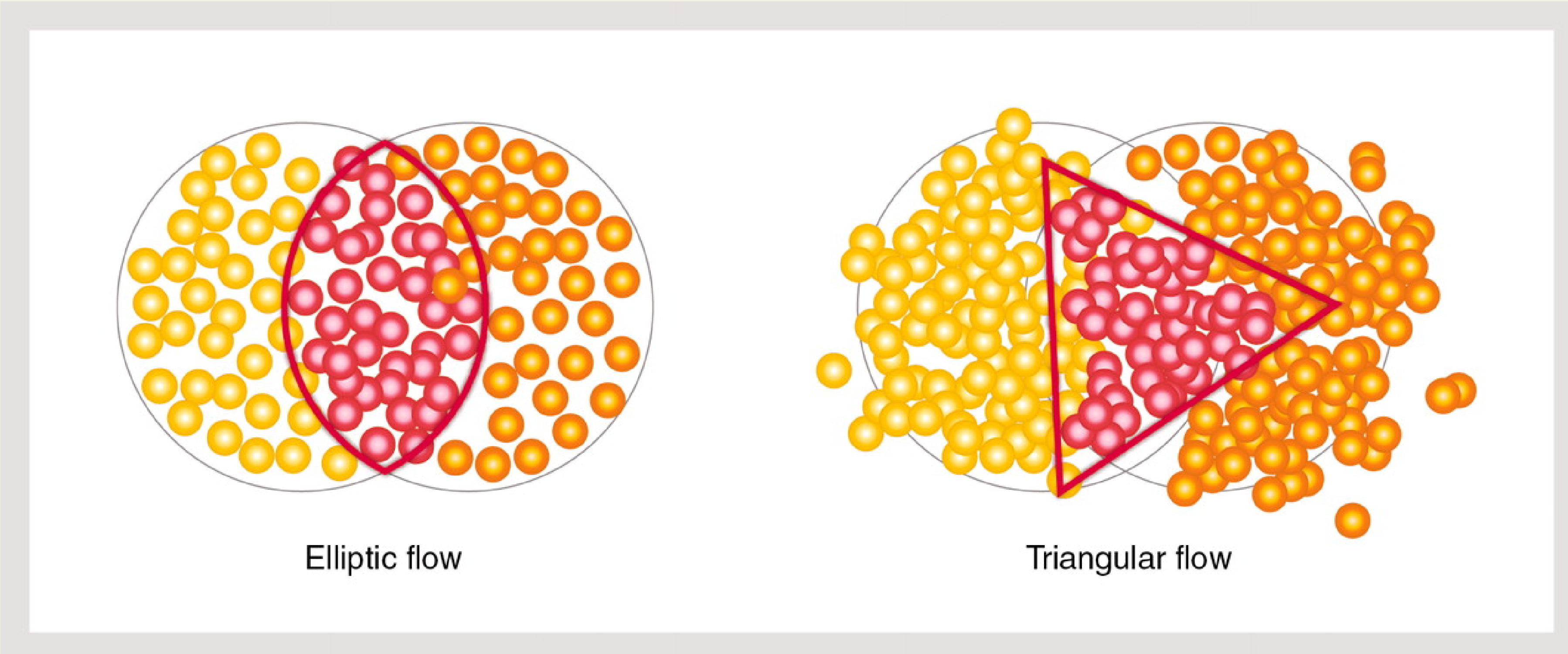
⇒ **Anisotropy in momentum space**

sensitive to speed of sound $c_s^2 = \partial p/\partial e$ **and shear viscosity** η

- **Quantified with Fourier expansion of momentum distribution:**

$$\frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T\, \mathrm{d}\phi} = \frac{1}{2\pi}\frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T}(1 + 2v_1(y,p_T)\cos\phi + 2v_2(y,p_T)\cos 2\phi + \cdots)$$

And other anisotropies v_n



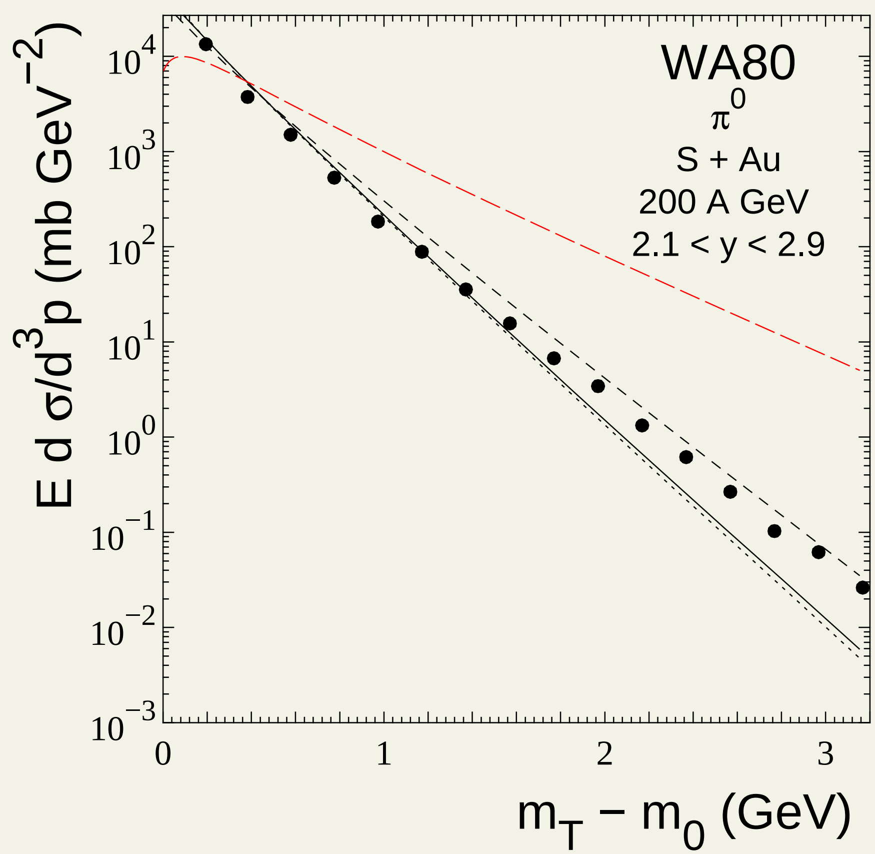
- some events have large v_2 — elliptic flow
- some events have large v_3 — triangular flow
- or something mixed, or. . .
- all coefficients v_n finite

$$\frac{dN}{dy d\phi} = \frac{dN}{dy}\left[1 + \sum_n 2 v_n \cos(2(\phi - \Psi_n))\right]$$

Equation of State

Pion gas EoS

Sollfrank *et al*, PRC55, 392 (1997)



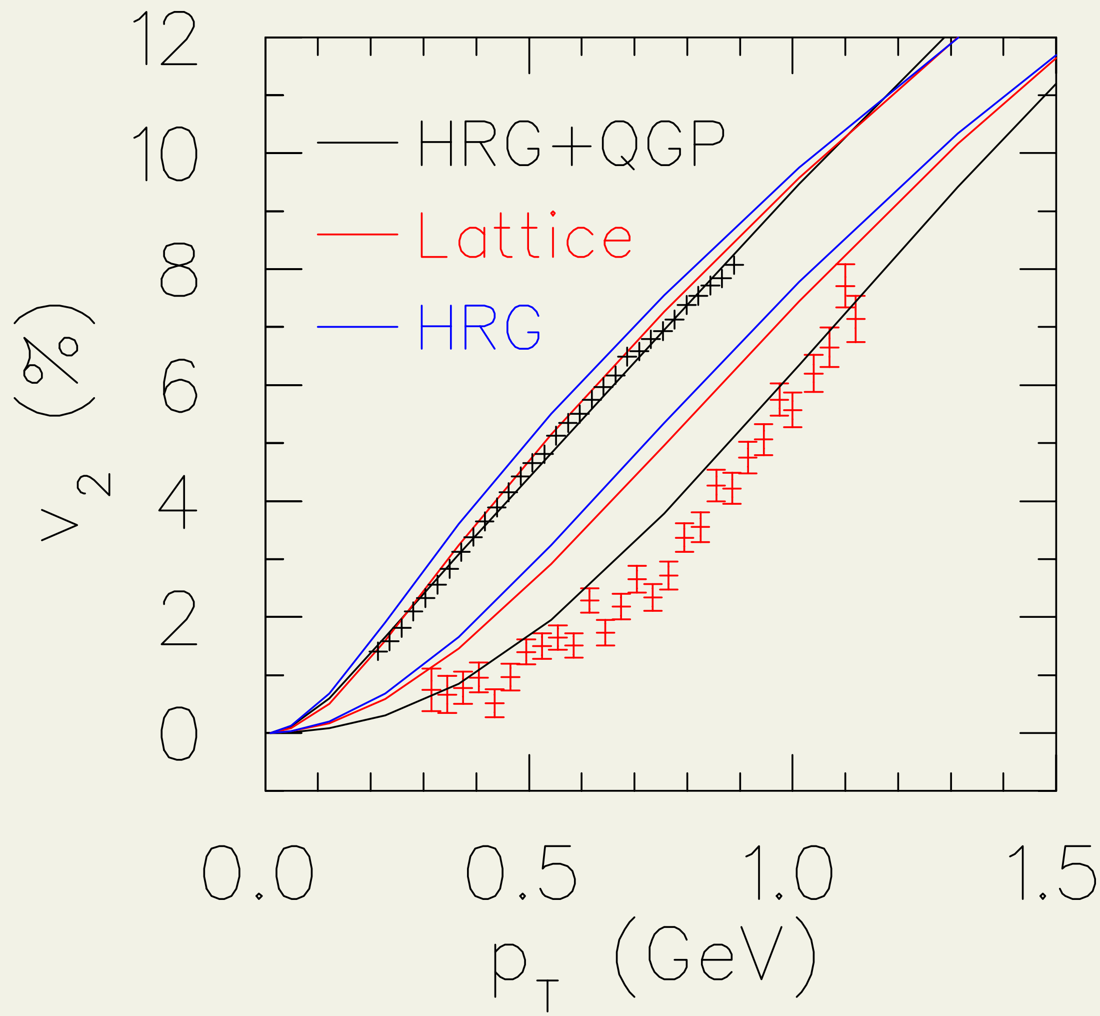
- **ideal pion gas EoS**

⇒ too stiff

- **must have "many" d.o.f's in the EoS**

difficult to say anything about EoS

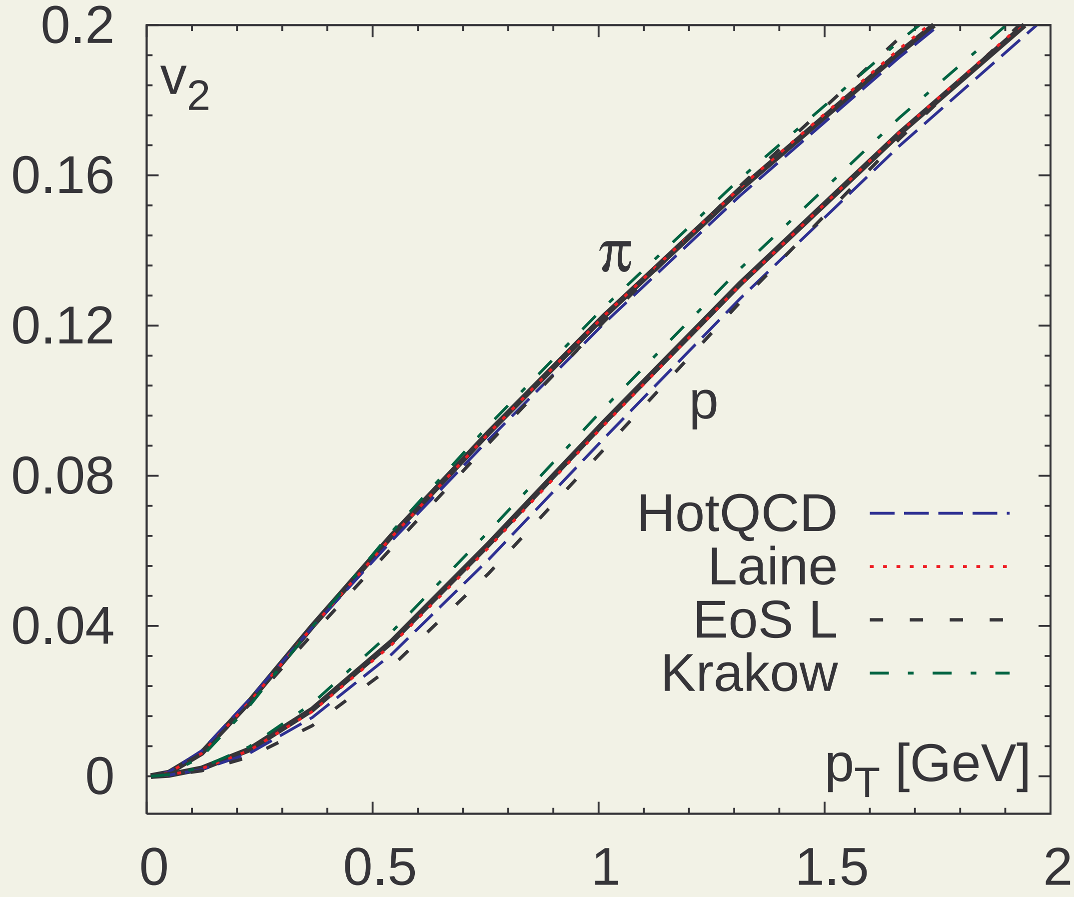
π **and** p $v_2(p_T)$ **in min.bias Au+Au collisions at RHIC**



- **HRG: no phase transition as close to data as lattice EoS**
- **HRG+QGP: Maxwell construction, closest to data**

difficult to say anything about EoS

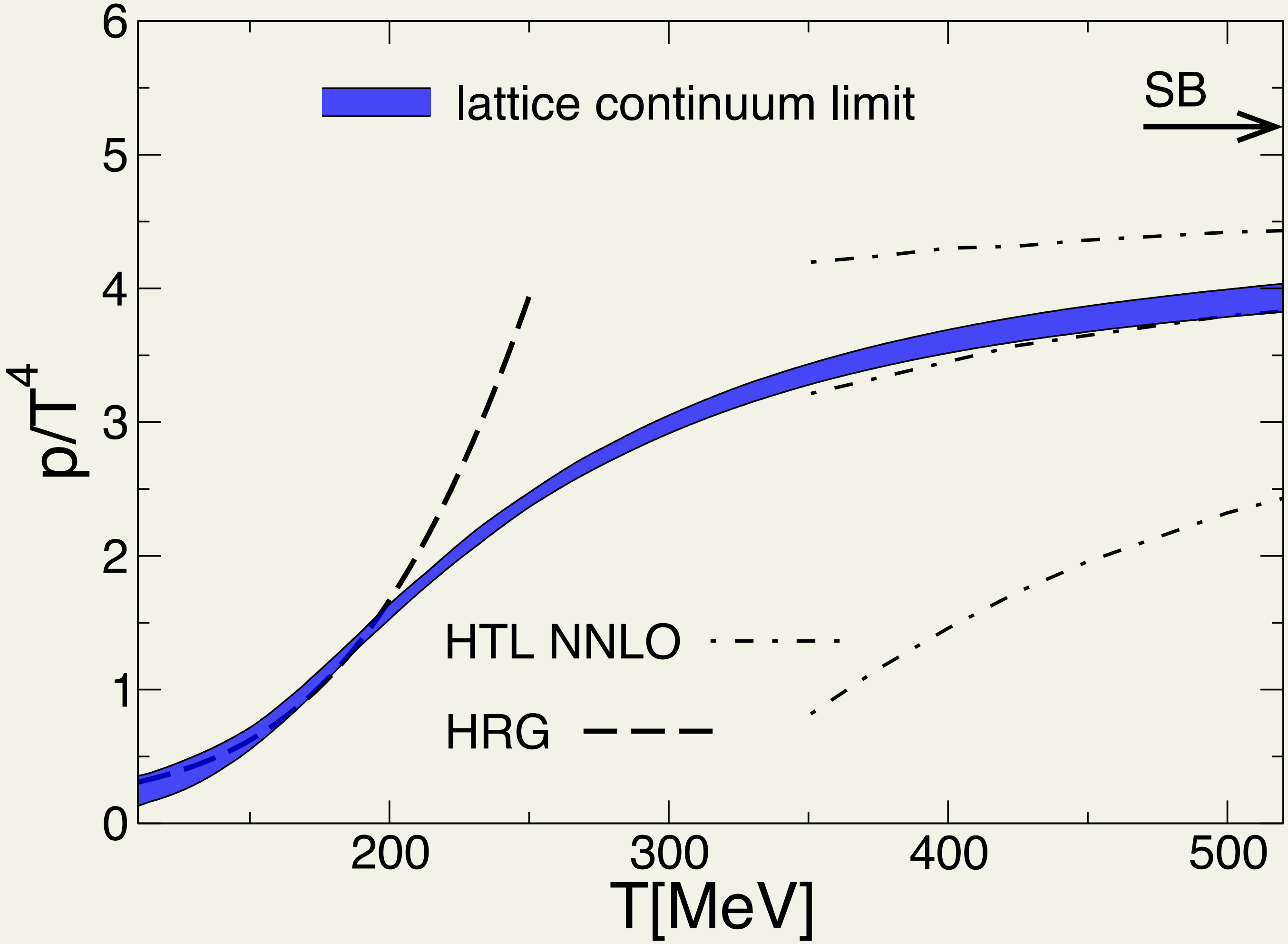
π and p $v_2(p_T)$ in b=7 fm Au+Au collisions at RHIC



- **comparison of various lattice parametrizations**
- **uncertainties of the model. . .**

Lattice EoS

Borsanyi *et al* [Budapest-Wuppertal Collaboration] Phys.Lett.B730:99,2014



- **compatible with data (when dissipation is included)**

η/s

why η/s?

Ideal:

$$(\epsilon + P)Du^\mu = \nabla^\mu P$$

where $D = u^\mu \partial_\mu$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$, $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

why η/s?

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Viscous:

$$\begin{aligned}(\epsilon + P)Du^\mu &= \nabla^\mu P - \Delta^\mu{}_\alpha \partial_\beta \pi^{\alpha\beta} \\ Du^\mu &= \frac{1}{\epsilon + P}\nabla^\mu P - \frac{2\eta}{\epsilon + P}\Delta^\mu{}_\alpha \partial_\beta \left[\nabla^{\langle\alpha} u^{\beta\rangle} + \cdots\right] + \cdots\end{aligned}$$

why η/s?

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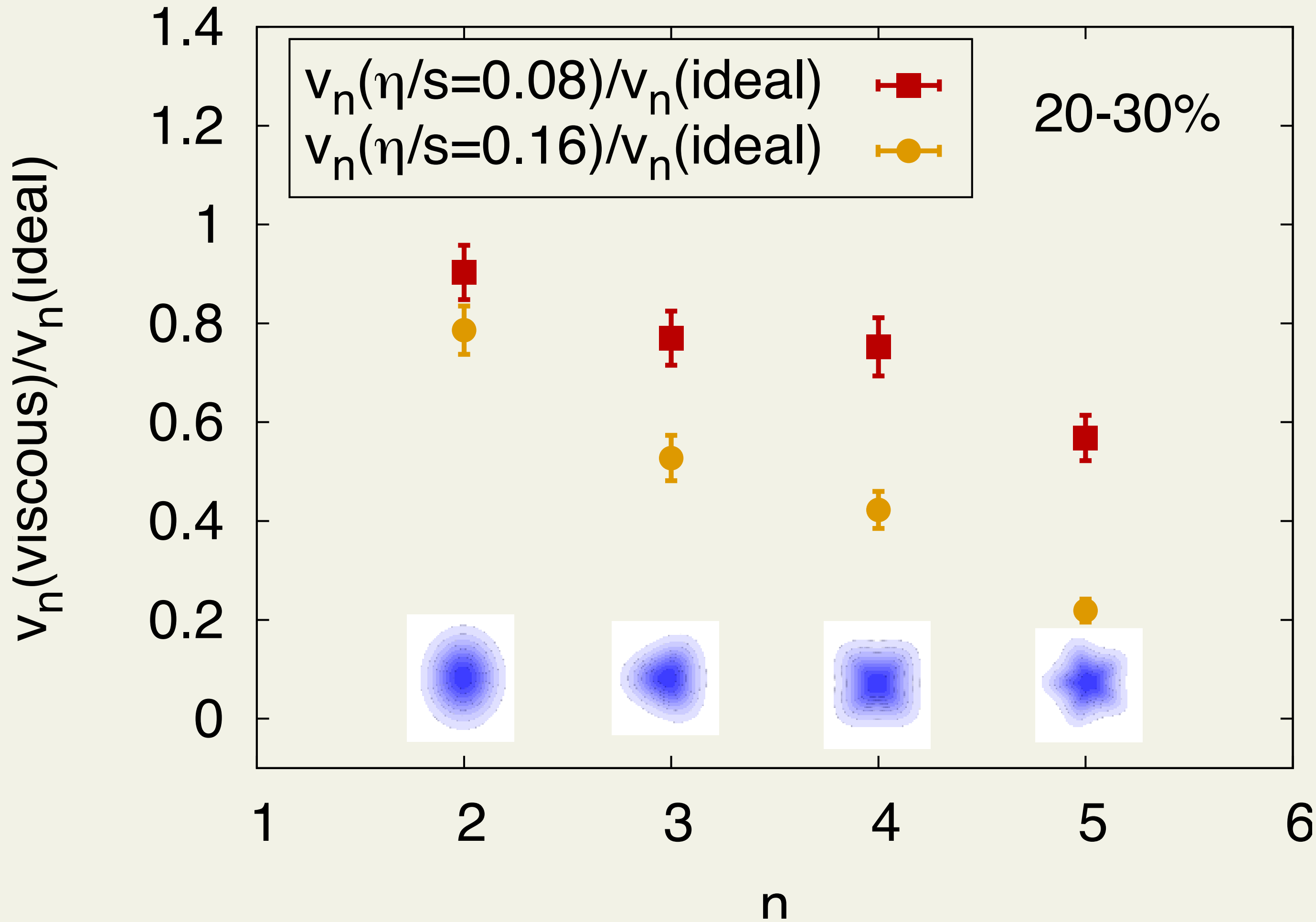
$\mu = 0 \Longrightarrow Ts = \epsilon + P :$

$$Du^\mu = \frac{1}{\epsilon + P}\nabla^\mu P - \frac{2}{T}\frac{\eta}{s}\Delta^\mu{}_\alpha \partial_\beta \left[\nabla^{\langle\alpha} u^{\beta\rangle} + \cdots\right] + \cdots$$

Sensitivity to η/s

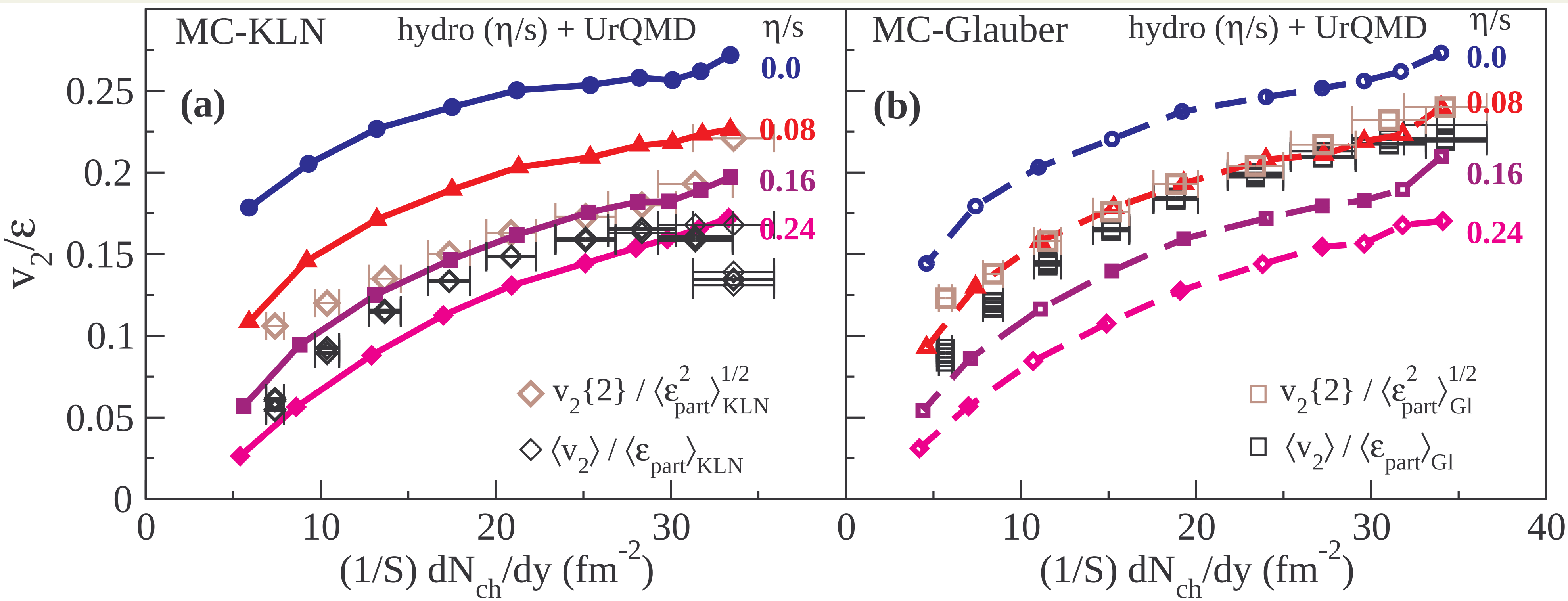
Schenke *et al.* Phys.Rev.C85:024901,2012



- **higher coefficients are suppressed more by dissipation**

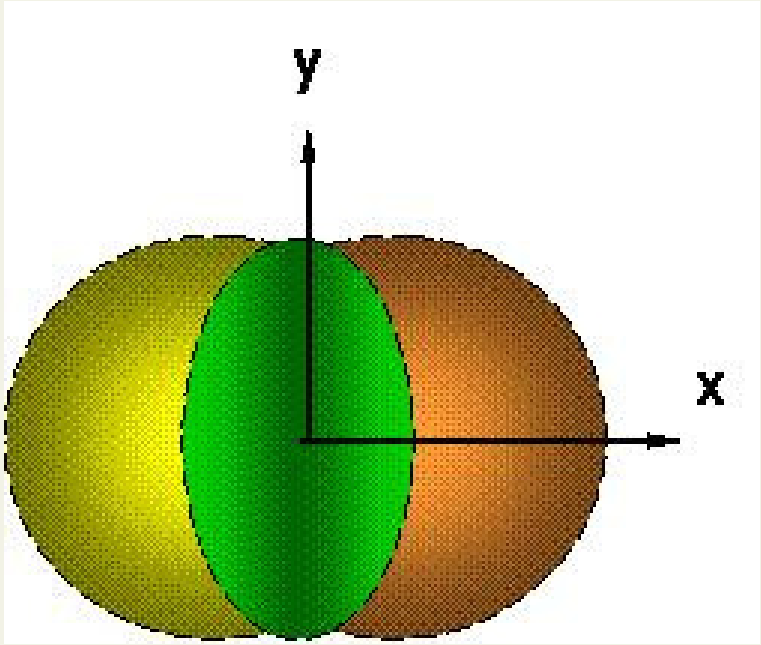
η/s from v_2

Shen *et al.* J.Phys.G38:124045,2011



- **MC-Glauber initialization:** $\eta/s = 0.08$
- **MC-KLN initialization:** $\eta/s = 0.2$

Initial shape



the larger the spatial anisotropy
the larger the momentum anisotropy

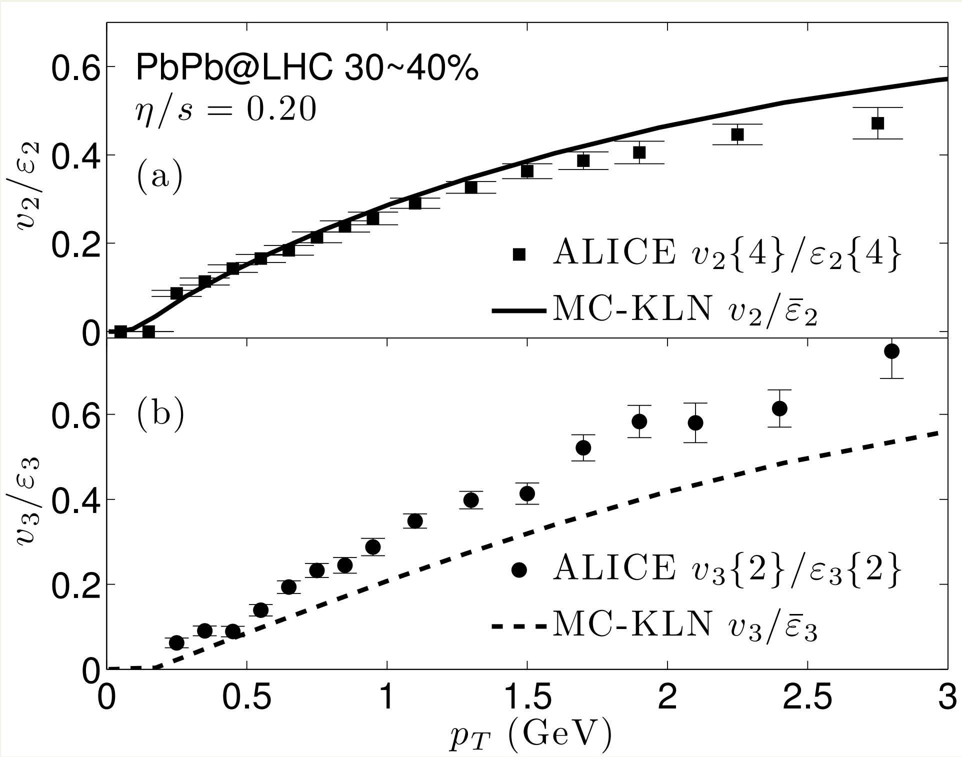
Initialization models:

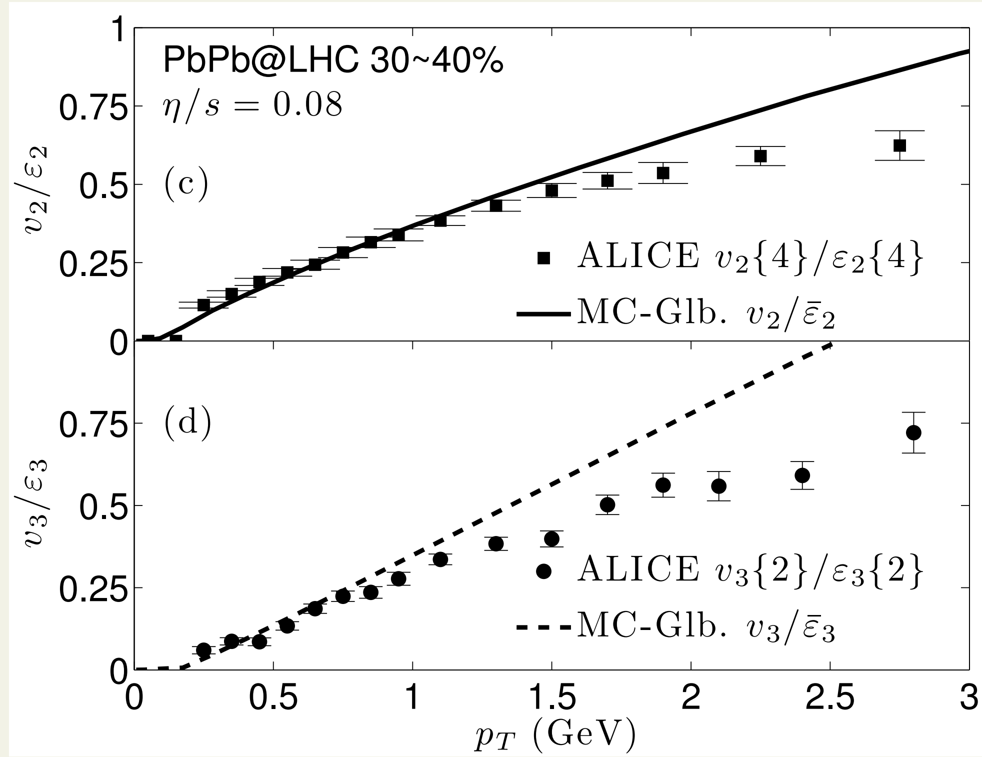
- **Glauber:** **Geometric model to calculate the number of participants.**
  Density/shape scales as a number of participants.

- **KLN:** **Color-Glass-Condensate based model — classical color field approximation**

v_3

Qiu *et al.* Phys.Lett.B707:151,2012

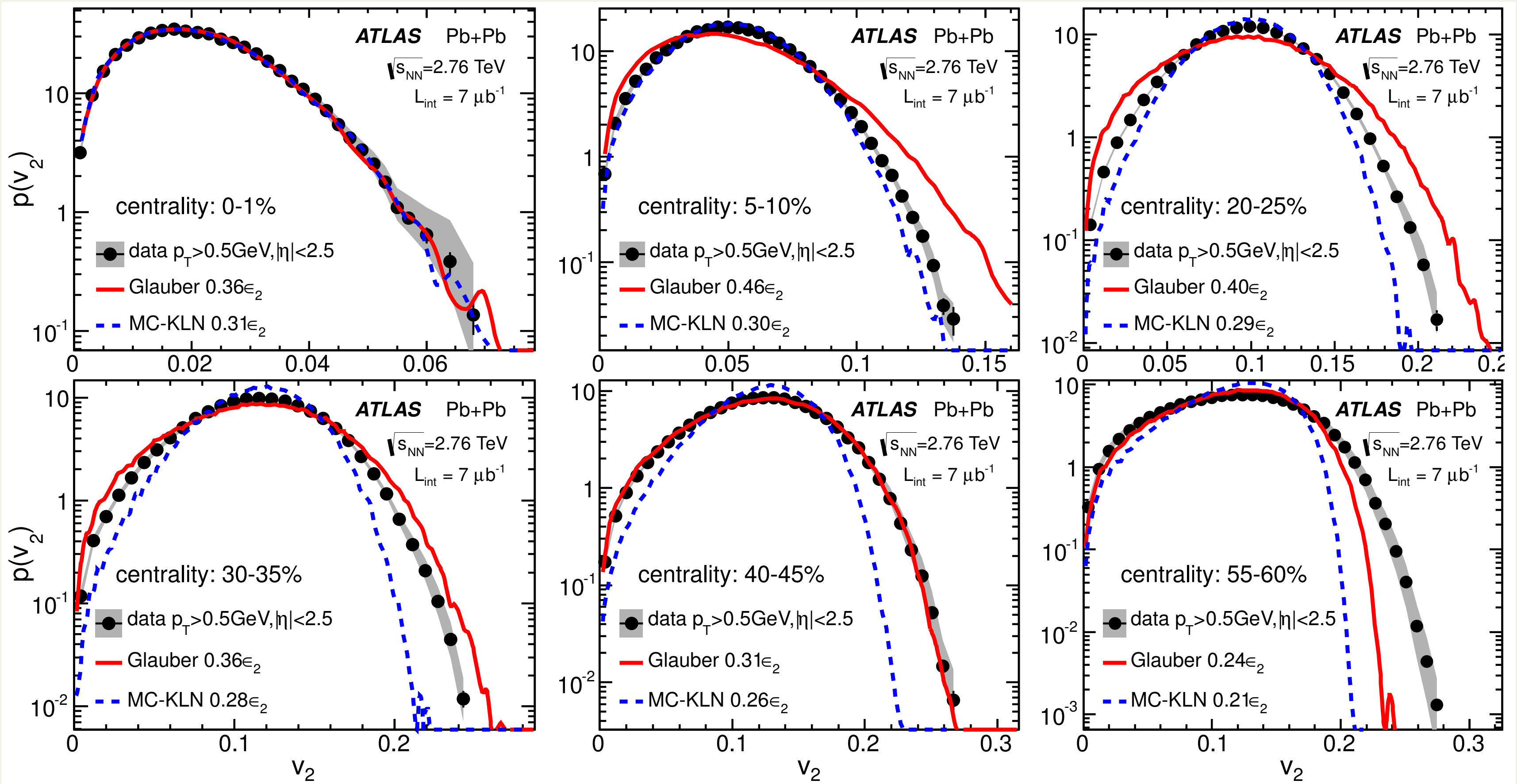




- **models can be distinguished**
- **MC-Glauber slightly favoured**

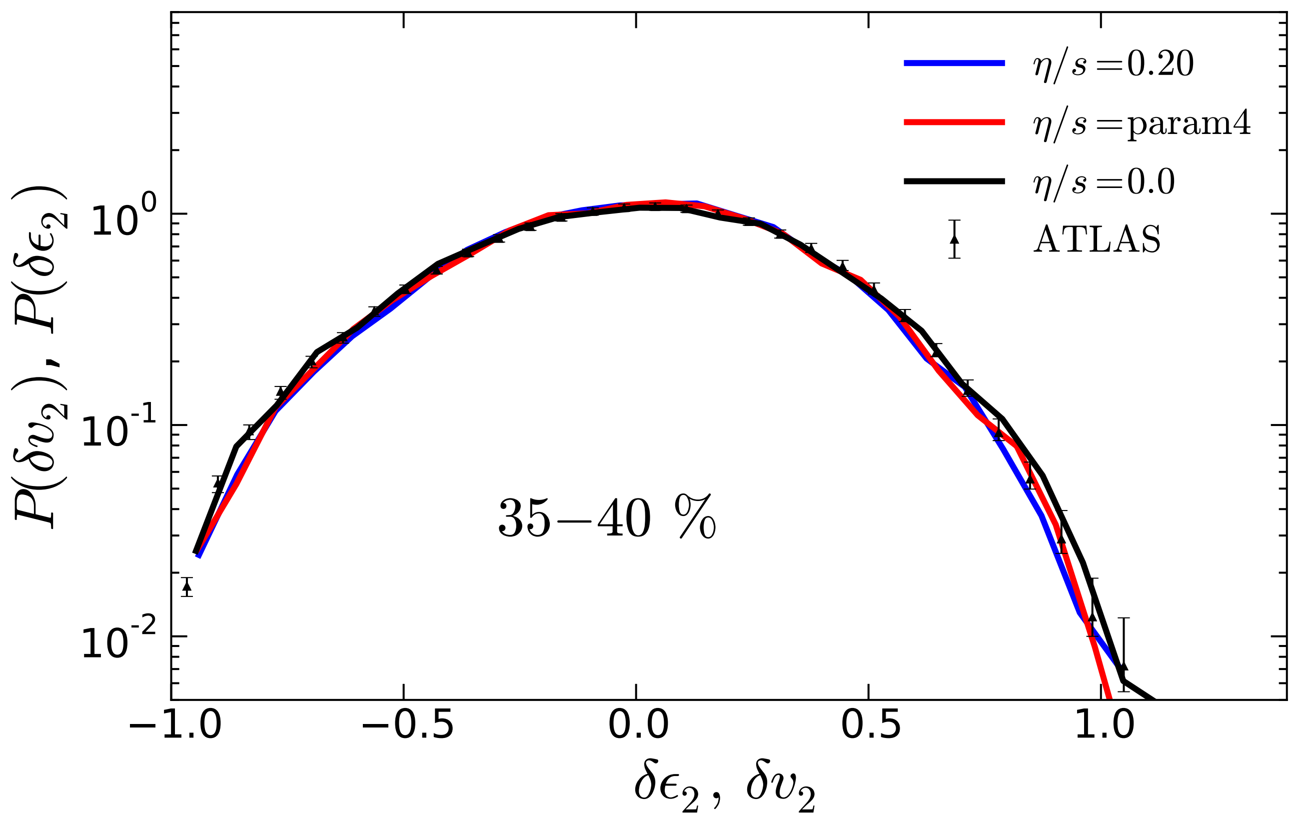
Flow fluctuations

Aad *et al.* [ATLAS Collaboration] JHEP 1311:183,2013



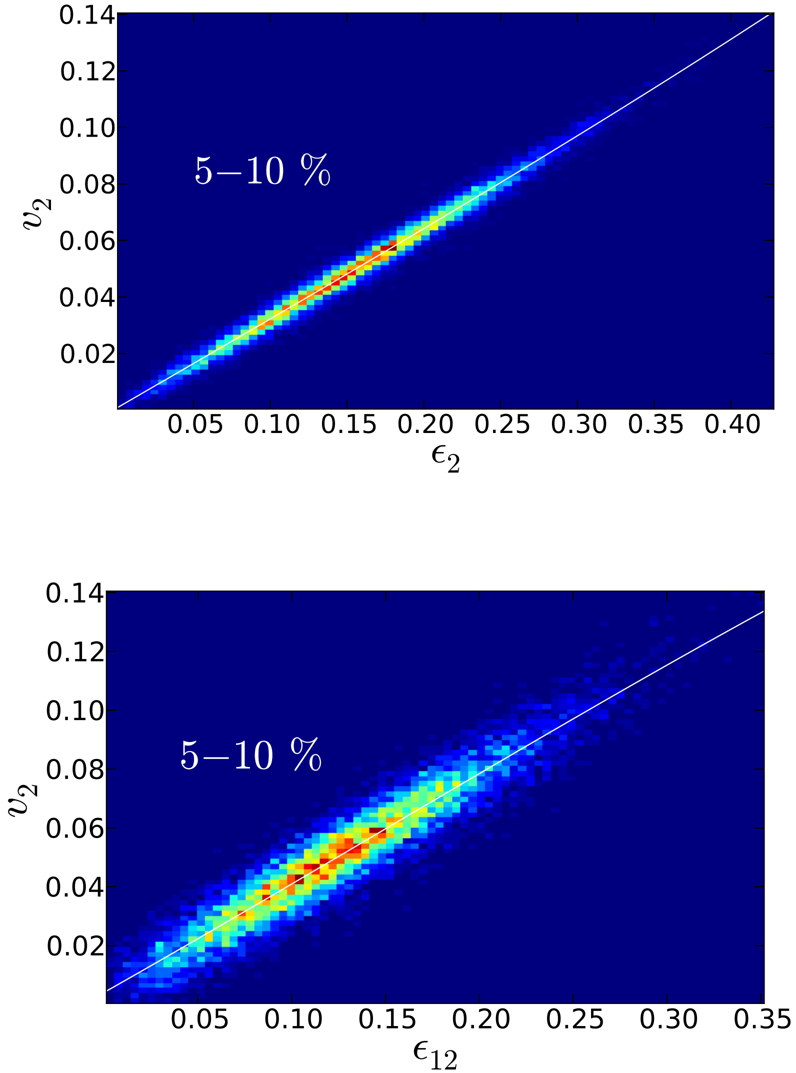
- $P(v_2)$ **compared to MC-Glauber and MC-KLN** $P(\varepsilon_2)$
- **MC-Glauber initialization: too wide**
- **MC-KLN initialization: too narrow**

Sensitivity of $P(\delta v_2)$ to viscosity

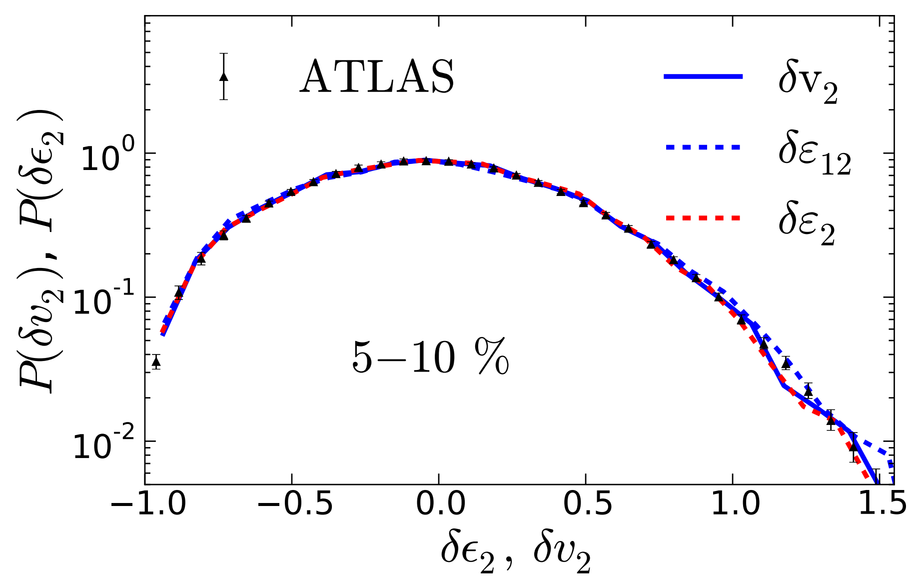


- Hydro response shows no sensitivity to η/s. (Note: we scale out the average v_2)

(non)linear-response?



5-10 %

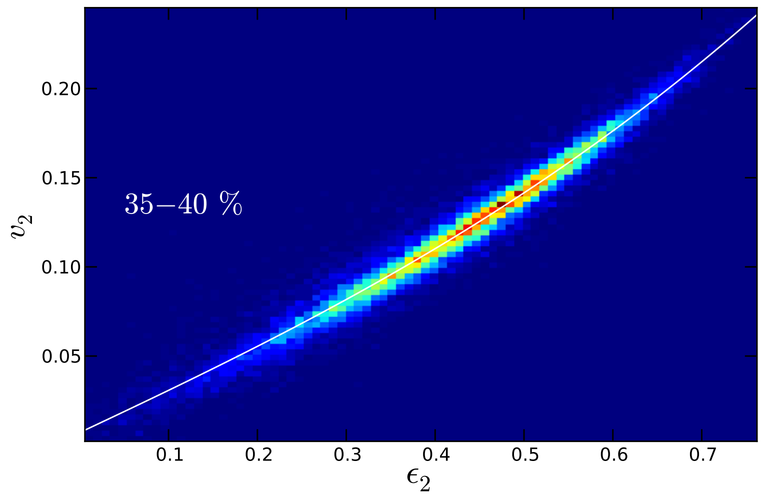


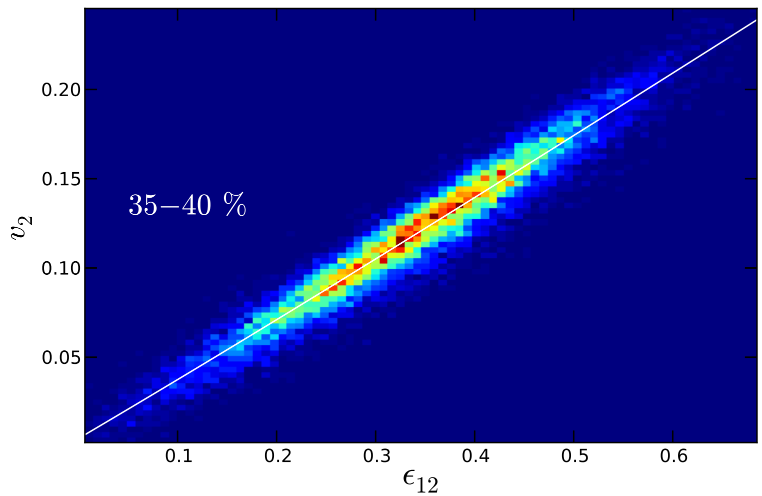
$$\varepsilon_{m,n} e^{in\Psi_{m,n}} = -\{r^m e^{in\phi}\}/\{r^m\},$$

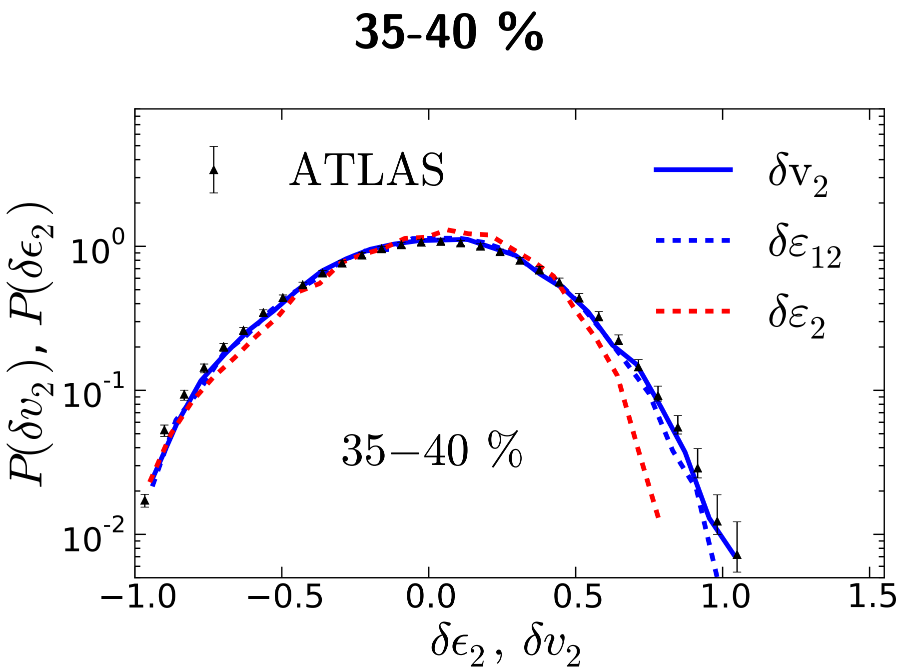
$$\varepsilon_2 \equiv \varepsilon_{2,2} \text{ vs } \varepsilon_{1,2}$$

Full azimuthal structure: $m = 0, \ldots, \infty$

(non)linear-response?







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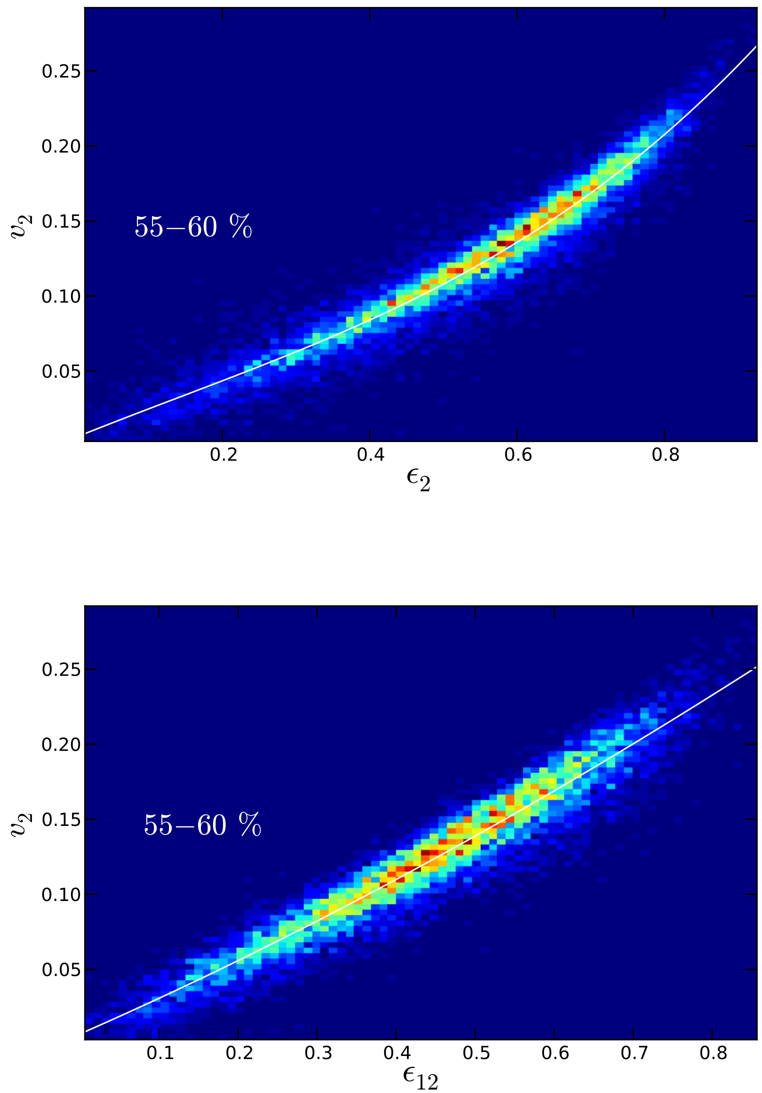

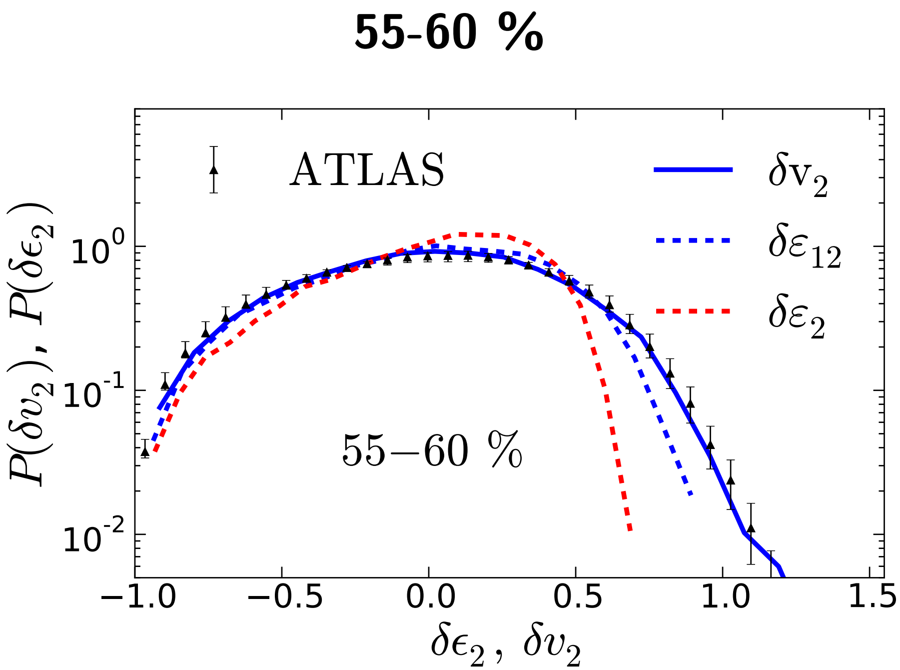

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Flow fluctuations

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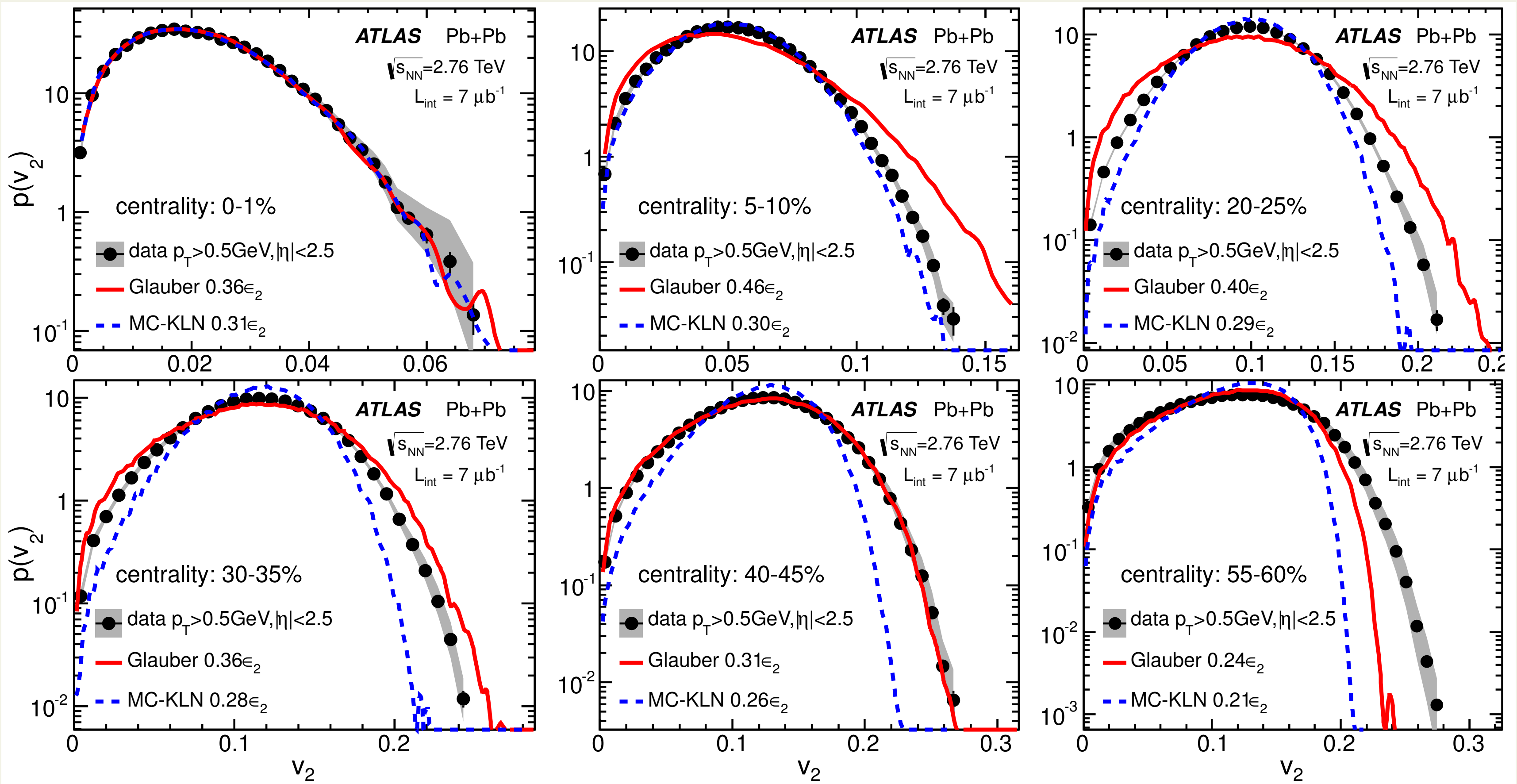

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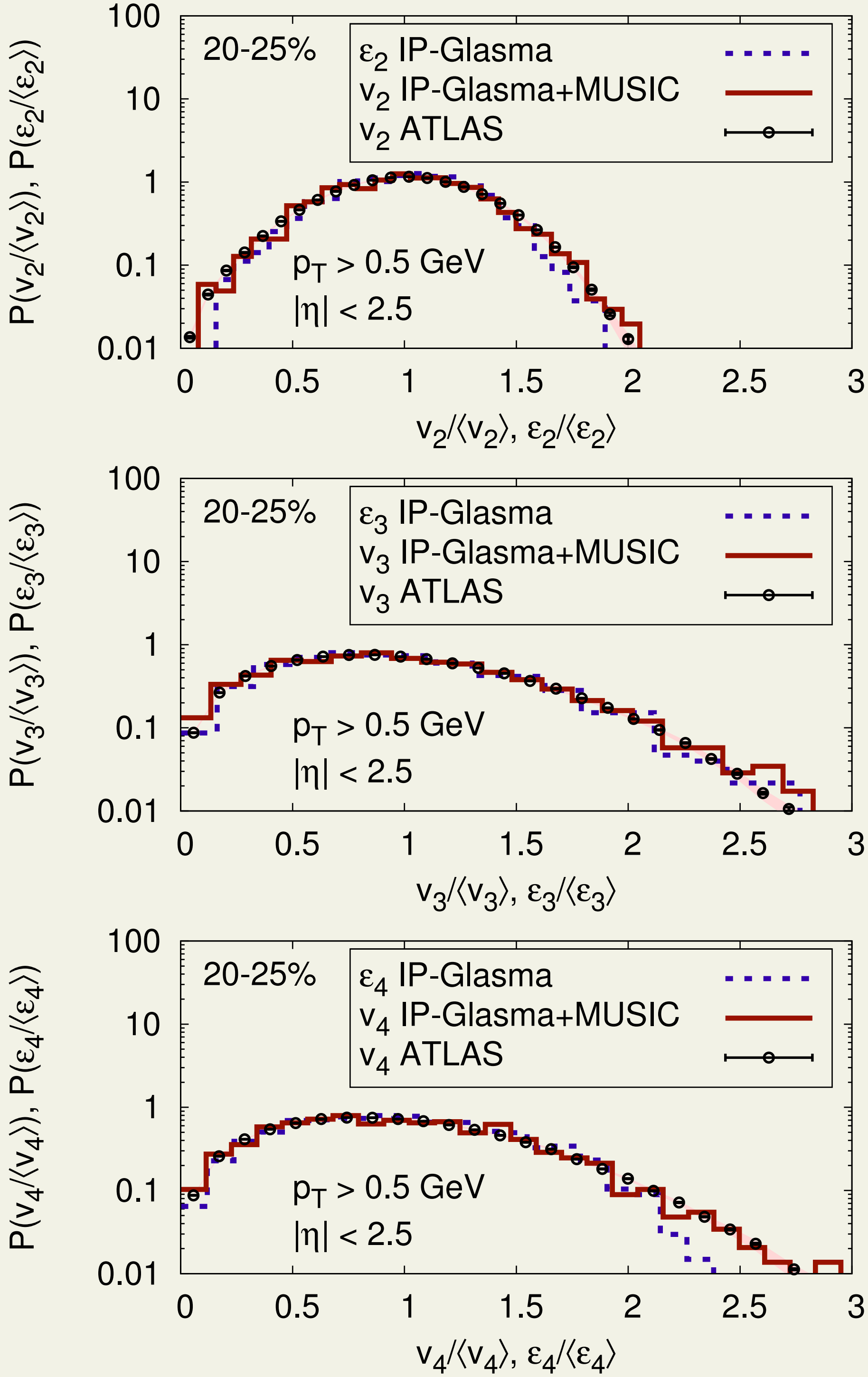

IP-Glasma initial state

Gale *et al.* Phys.Rev.Lett.110:012302,2013

- **reproduces $P(v_n)$**
- **reproduces v_n**

Other recent initializations reproducing the fluctuations:

- **pQCD + saturation (EKRT)**

Paatelainen *et al.* Phys.Lett.B731:126,2014

- **Trento**

Moreland *et al.* arXiv:1412.4708

$$\eta/s = \text{const.} \longrightarrow (\eta/s)(T)$$

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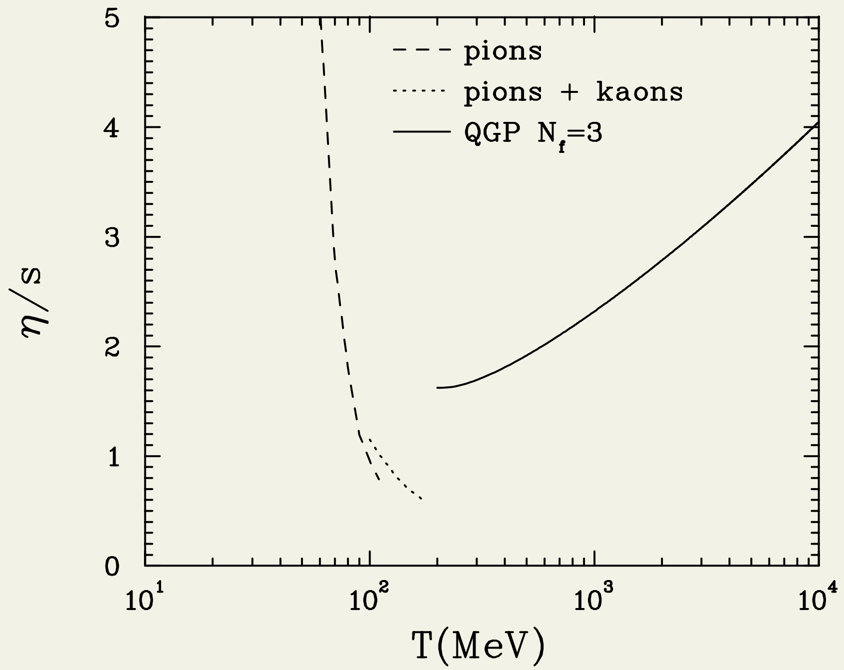
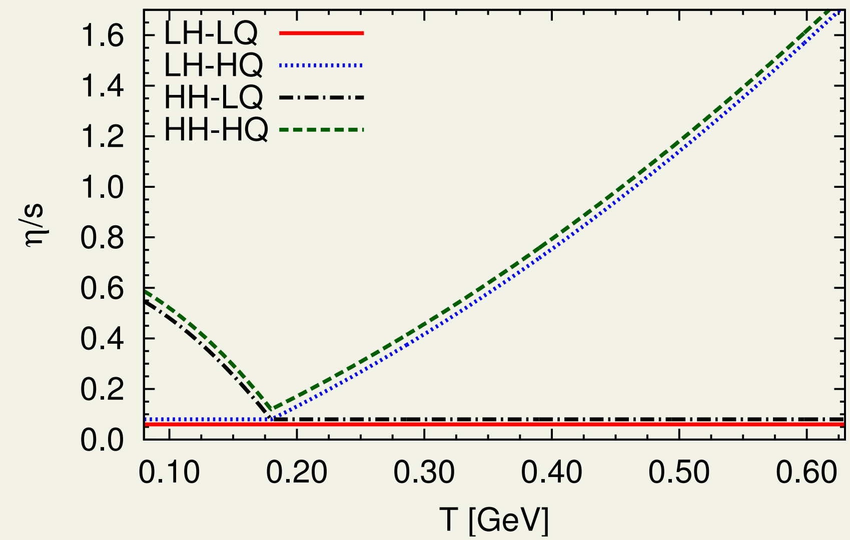
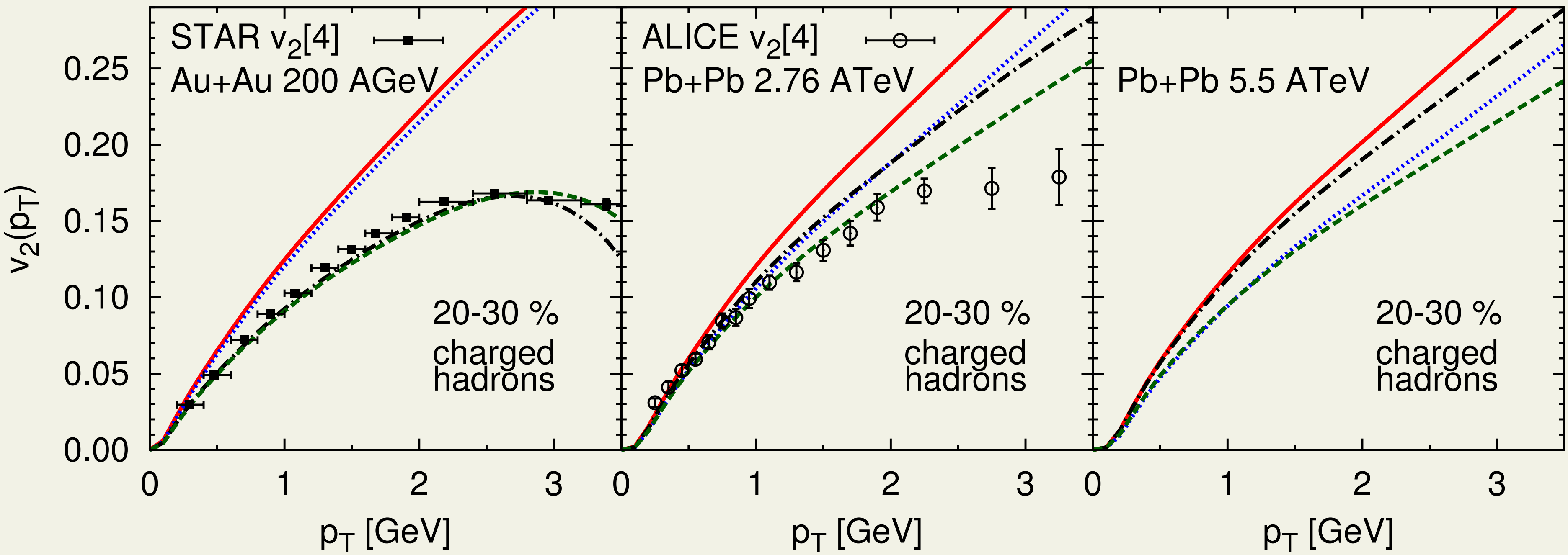


theory expectation

sensitivity to $(\eta/s)(T)$

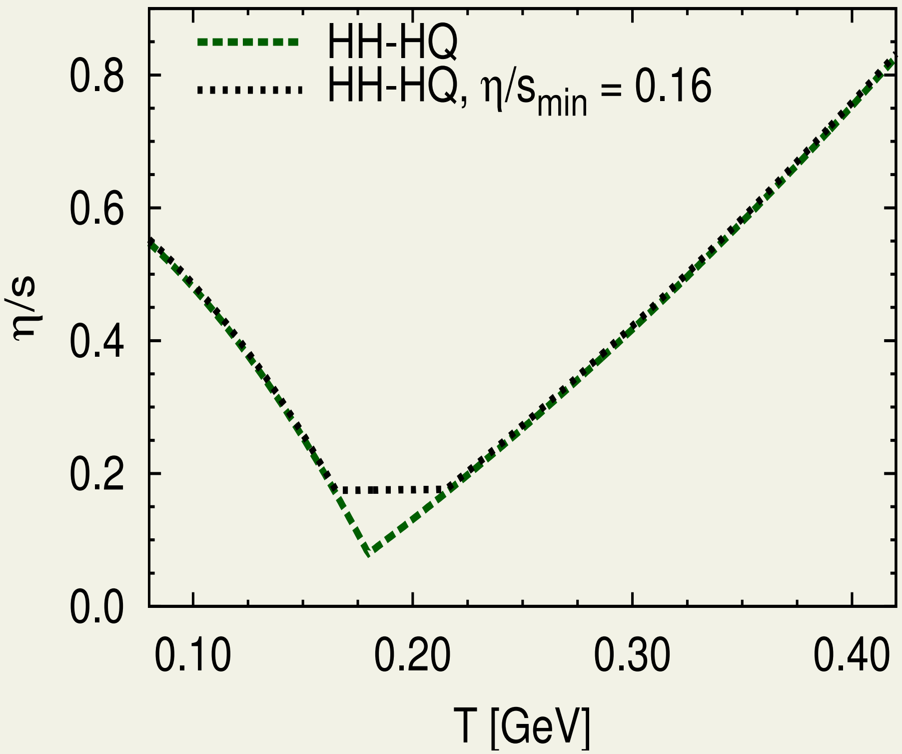


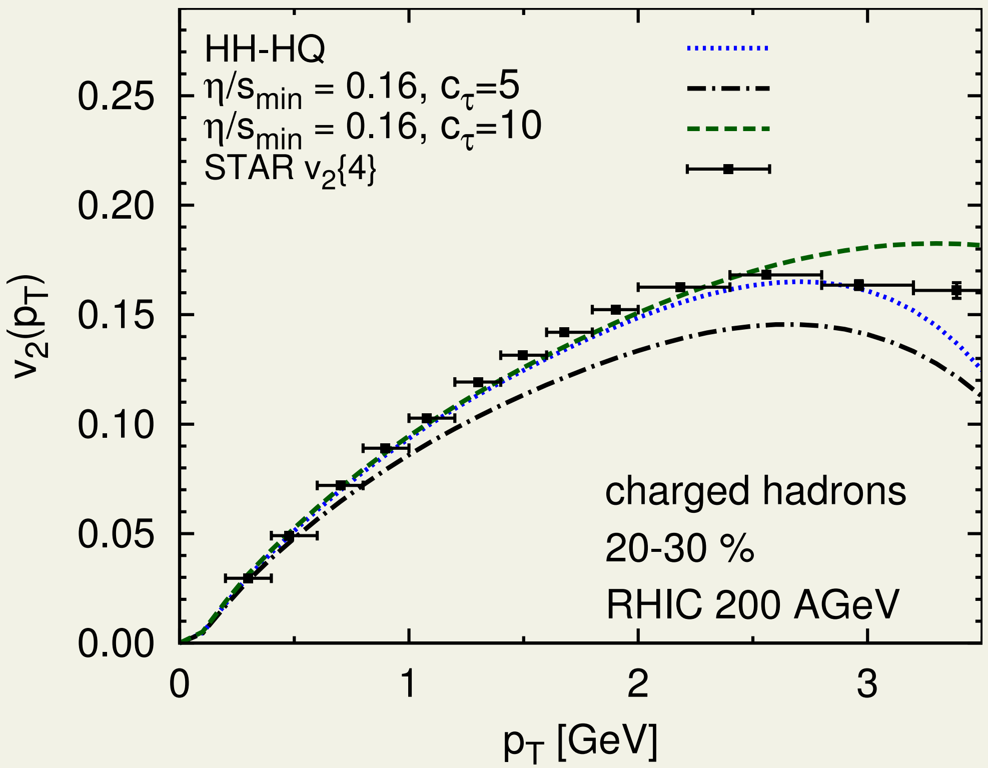
- parametrizations of $(\eta/s)(T)$
  L="low", H="high"
  H="hadronic", Q="qgp"



- weak dependence on QGP η/s at RHIC
- increasing sensitivity to QGP η/s with collision energy at LHC
- sensitive to minimum of η/s

Niemi *et al.* Phys.Rev.C86:014909,2012





- $\tau_\pi = c_\tau \eta/(e+P)$
- v_2 **sensitive to** **both** η **and** τ_π
- **how to disentangle?**

pA?

- **Fluid dynamical behaviour: all data explained by functions**

$$P = P(T, \{\mu_i\}), \qquad \eta/s = (\eta/s)(T, \{\mu_i\})$$

- **properties of matter should not change with $\sqrt{s}$ or colliding nuclei**

- **does this work at $p + A$?**

Summary

- **EoS has large # of d.o.f.**
- $(\eta/s)(T)$ **has low minimum**

Summary

- **EoS has large # of d.o.f.**
- $(\eta/s)(T)$ **has low minimum**
- **extracting** $(\eta/s)(T)$ **difficult**
  - τ_π**?**
  - **initial state? (there is progress)**
- **fluctuations provide more constraints**
- **fluid dynamical behaviour: all systems described by same** $(\eta/s)(T)$