Tetraquarks from the Bethe-Salpeter equation

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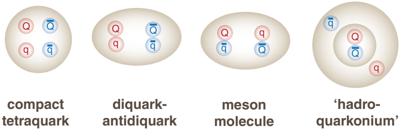
Outline

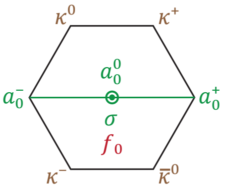
- **Introduction**
- Some background: **Bethe-Salpeter equations, rainbow-ladder,** applications to mesons and baryons
- **Tetraquarks** as meson-meson / diquark-antidiquark systems
  Heupel, GE, Fischer, PLB 718 (2012)
- **Tetraquarks** as **four-quark** systems
  Heupel, GE, Fischer, in preparation
- **Summary**

Introduction

- Increasing evidence for four-quark states in **charmonium & bottomonium** spectrum: X(3872), Y(4260), charged Z states, ...
  Godfrey 0910.3409, Brambilla et al., EPJ C71 (2011) & EPJ C74 (2014), Olsen, Front. Phys. 10 (2015)



- But already **light scalar** (0^{++}) **mesons** don't fit into the conventional meson spectrum:



a_0 (980 MeV) } $u\bar{u}, d\bar{d}, u\bar{d}$
σ (500 MeV) }
κ (680 MeV) $u\bar{s}, d\bar{s}$
f_0 (980 MeV) $s\bar{s}$

- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

  $\Gamma(\sigma, \kappa) \approx$ 550 MeV
  $\Gamma(a_0, f_0) \approx$ 50–100 MeV

- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have masses similar to axial-vectors: $a_1, f_1 \sim$ 1.3 GeV

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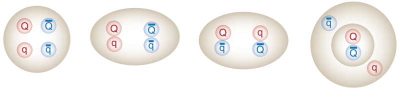
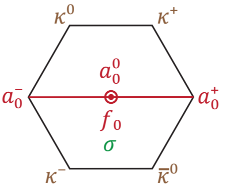
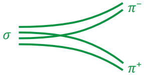


- But already **light scalar** (0^{++}) **mesons** don't fit into the conventional meson spectrum:
  What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004



a_0 (980 MeV) }
f_0 (980 MeV) } $us\overline{us}$, ...
κ (680 MeV) $us\overline{ud}$, ...
σ (500 MeV) $ud\overline{ud}$

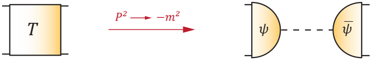
- Explains **mass ordering**:
  f_0 and a_0 have same strangeness content
- Explains **decay widths**:
  f_0 and a_0 couple to K$\overline{\text{K}}$,
  large widths for σ, κ



- Actual **scalar $q\bar{q}$ ground states** ~ 1.3–1.5 GeV?
- Large Nc, unitarized ChPT, quark models, ELSM, ...
  Pelaez 2004, Weinberg 2013, Knecht & Peris 2013, Cohen & Lebed 2014, Giacosa 2006, Bicudo, Cardoso 2010, Parganlija, Giacosa, Rischke 2010, . . .

Bethe-Salpeter equations

- Extract hadron properties from **poles** in $q\bar{q}, qqq, qq\bar{q}\bar{q}$ **scattering matrices:**



- defines onshell **Bethe-Salpeter amplitude.** Simplest example: **pion**

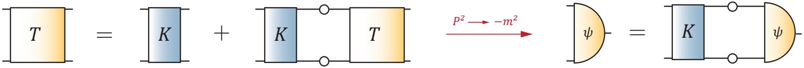
$$\psi(q,P) = \gamma_5 \; (f_1 + f_2 \, \not{P} + f_3 \, \not{q} + f_4 \, [\not{q}, \not{P}] \;) \; \otimes \; \text{Color} \; \otimes \; \text{Flavor}$$

most general Dirac-Lorentz structure,
Lorentz-invariant dressing functions:

$$f_i = f_i(q^2, q \cdot P, P^2 = -m^2)$$

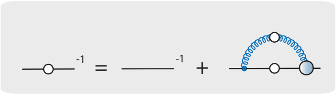
- Use **scattering equation** (inhomogeneous BSE) to obtain T in the first place: $T = K + K\,G_0\,T$

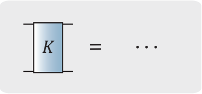
Homogeneous BSE
for **BS amplitude:**



Bethe-Salpeter equations

Kernel is closely related to **quark Dyson-Schwinger equation:**





Rainbow-ladder:
tree-level vertex +
effective coupling

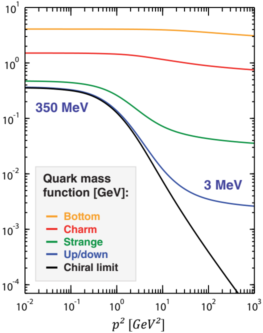
$$\alpha(k^2) = \alpha_{\text{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\text{UV}}(k^2)$$

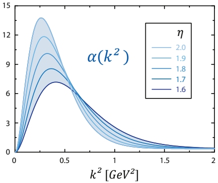
Maris, Roberts, Tandy,
PRC 56 (1997), PRC 60 (1999)

- **Dynamical breaking of chiral symmetry** generates "constituent- quark masses"

$$S_0(p) = \frac{-i\not{p}+m}{p^2+m^2} \longrightarrow S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

- **Vector & axial symmetries** automatically preserved:
  - ⇒ Goldstone theorem, massless pion in χL
  - ⇒ em. current conservation
  - ⇒ Goldberger-Treiman





Adjust scale Λ to observable, keep width η as parameter

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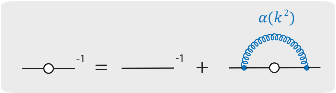

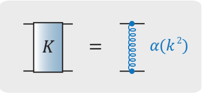

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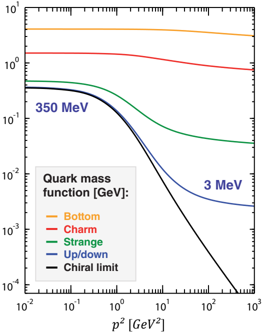

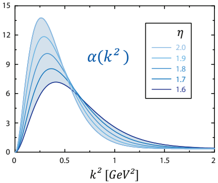

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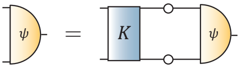
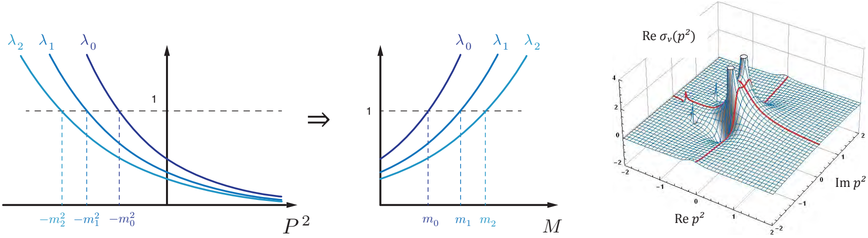
- BS amplitude makes only sense **onshell**, but homogeneous BSE = **eigenvalue equation**, can be solved for offshell momenta:



$$K\,\psi_i = \lambda_i(P^2)\,\psi_i\,, \qquad \lambda_i \xrightarrow{P^2 \to -m_i^2} 1$$

- Largest eigenvalue ⇔ ground state, smaller ones ⇔ excitations



- Restricted by singularity structure in **quark propagator** (but no **physical threshold!**): mesons: $M < 2m_p$, baryons: $M < 3m_p$, $m_p \sim 500 MeV$

⇒ include residues (numerically difficult) or **extrapolate eigenvalue**

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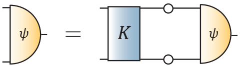

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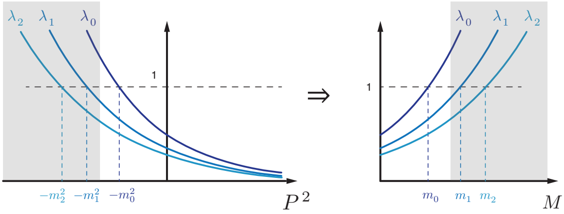

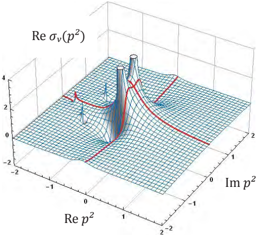

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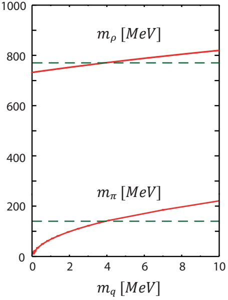
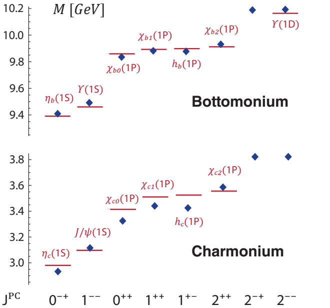
Mesons

- Rainbow-ladder works well for **pseudoscalar & vector mesons:** masses, form factors, decays, ...

  Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999);
  Bashir et al., Commun. Theor. Phys. 58 (2012)

Pion is Goldstone boson, satisfies GMOR: $m_\pi^2 \sim m_q$





- **Heavy mesons**

  Blank, Krassnigg, PRD 84 (2011),
  Fischer, Kubrak, Williams, EPJ A 51 (2015)

— exp
◆ calc

- Rainbow-ladder good for **'s-wave'** dominated states

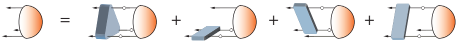
- Need to go **beyond rainbow-ladder** for scalar & axialvector mesons, excited states, η-η', ...

  Fischer, Williams & Chang, Roberts, PRL 103 (2009)
  Alkofer et al., EPJ A38 (2008), Bhagwat et al., PRC 76 (2007), ...

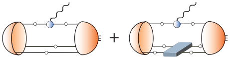
e.g. σ meson: 600-700 MeV in RL ⟶ $\gtrsim$ 1 GeV

Baryons

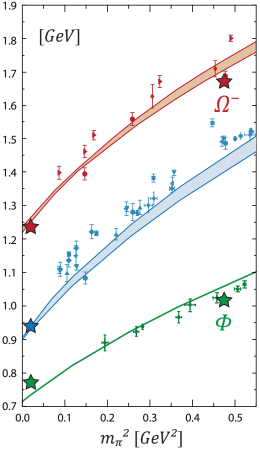
- Covariant Faddeev equation for **baryons:**
  keep 2-body interactions & rainbow-ladder,
  but no further approximations: $M_N = 0.94\,\text{GeV}$
  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011),
  Sanchis-Alepuz, Fischer, PRD 90 (2014)



- **Baryon form factors:**
  nucleon and Δ FFs, $N \to \Delta\gamma$ transition
  GE, PRD 84 (2011), Sanchis-Alepuz, Williams, Alkofer, PRD 87 (2013),
  Alkofer, GE, Sanchis-Alepuz, Williams, 1412.8413



- **Scattering amplitudes:**
  Compton scattering
  GE & Fischer, PRD 85 (2012) & PRD 87 (2013)
  hadronic light-by-light for muon g-2
  GE, Fischer, Heupel, Williams, 1411.7876



Delta:
Sanchis-Alepuz et al., PRD 84 (2011)

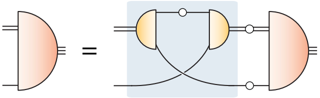
Nucleon:
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010);
GE, PRD 84 (2011)

ρ–meson:
Maris & Tandy, PRC 60 (1999)

Tetraquarks: two-body equation

Use **quark-diquark model** as template:

- Assumption: separable qq scattering matrix ⇒
  Faddeev equation simplifies to **quark-diquark BSE**



- **Quark exchange** between quark & diquark binds nucleon
- All quark and diquark properties calculated from quark level,
  same rainbow-ladder interaction:

  scalar diquark $\sim$ **800 MeV, axialvector diquark** $\sim$ **1 GeV**

- N and Δ masses & form factors very similar:
  quark-diquark model is good approximation for three-body equation

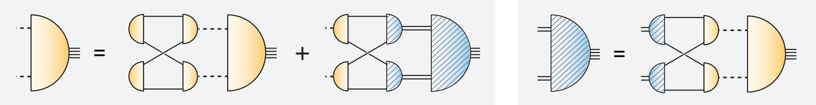
  Nucleon and Δ electromagnetic FFs, $N \to \Delta\pi$ decay, $N \to \Delta\gamma$ transition

  GE, PhD Thesis, 0909.0703, GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009),
  Nicmorus, GE, Alkofer, PRD 82 (2010), Mader, GE, Blank, Krassnigg, PRD 84 (2011), GE, Nicmorus, PRD 85 (2012)

Tetraquarks: two-body equation

Use **quark-diquark model** as template:

- Assumption: separable qq, $q\bar{q}$ scattering matrices ⇒ coupled **diquark-antidiquark / meson-meson** equations: Heupel, GE, Fischer, PLB 718 (2012)



- **Quark exchange** between mesons and diquarks binds tetraquark
- Coupled equations can be contracted into single **meson-meson equation**, where diquarks appear only internally (not vice versa!)

  ⇒ **meson molecule** with **diquark-antidiquark admixture!**

So far:

- 0^{++}, isoscalar, 4 identical quarks: $nn\bar{n}\bar{n}$, $ss\bar{s}\bar{s}$, $cc\bar{c}\bar{c}$, ...
- keep only **pseudoscalar meson** and **scalar diquark**, calculated in rainbow-ladder

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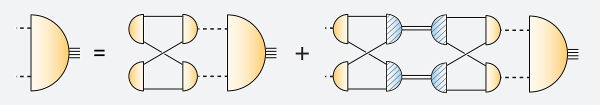

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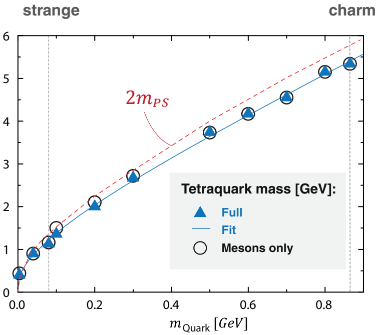
Tetraquarks: two-body equation

Tetraquark masses:

Heupel, GE, Fischer, PLB 718 (2012)

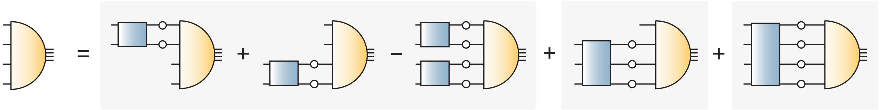
- up/down: $m \sim$ 400 MeV $\Leftrightarrow$ σ / f_0 (500)?
- The σ is so light because it 'feels' **Goldstone nature of the pion** - diquarks completely irrelevant!
- Resolves problem with diquark-antidiquark interpretation: ' 2 x 800 MeV - binding energy ' ~ 500 MeV?!
- **All-strange** tetraquark: $m \sim$ 1.2 GeV
  all-charm tetraquark: $m \sim$ 5.3 GeV
  (below $2\eta_c$ threshold)



$\Rightarrow$ Artifact of 2-body approximation or genuine result?
What about κ, a_0 / f_0 ?

Tetraquarks: four-body equation

Start from **four-quark bound-state equation:**



Two-body interactions:

- $K \otimes I + I \otimes K - K \otimes K$ structure necessary to prevent overcounting in T-matrix $T = K + K\, G_0\, T$
  Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
- plus permutations:
  $(qq)(\bar{q}\bar{q}),\ (q\bar{q})(q\bar{q}),\ (q\bar{q})(q\bar{q})$
  (12) (34)   (23) (14)   (13) (24)

Three-body interactions (+ permutations)

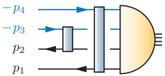
Four-body interactions

Keep **two-body interactions** with **rainbow-ladder kernel:**
well motivated by many other studies, tetraquark is **s-wave**

Structure of the amplitude

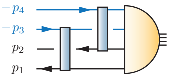
Bethe-Salpeter amplitude $\Gamma(p,q,k,P)$
depends on **four independent momenta:**

$$P = p_1 + p_2 + p_3 + p_4$$



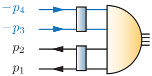
$p = \frac{1}{2}(p_2 + p_3 - p_1 - p_4)$

's channel'



$q = \frac{1}{2}(p_3 + p_1 - p_2 - p_4)$

'u channel'



$k = \frac{1}{2}(p_1 + p_2 - p_3 - p_4)$

't channel'

General structure quite complicated:

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2$$

$$\omega_1 = q \cdot k \qquad \eta_1 = p \cdot P$$
$$\omega_2 = p \cdot k \qquad \eta_2 = q \cdot P$$
$$\omega_3 = p \cdot q \qquad \eta_3 = k \cdot P$$

$$P^2 = -M^2$$

256 Dirac-Lorentz tensors

2 Color tensors:

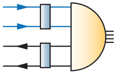
$3 \otimes \bar{3}, \quad 6 \otimes \bar{6}$ or
$1 \otimes 1, \quad 8 \otimes 8$
(Fierz-equivalent)

Structure of the amplitude

Keep **s waves** only:
Fierz-complete, **16** Dirac-Lorentz tensors

#	Structure
1	$(\mathcal{C}^T\gamma_5)_{2,1} \otimes (\gamma_5\mathcal{C})_{3,4}$
2	$\mathcal{C}^T\gamma_5\not{P} \otimes \gamma_5\mathcal{C} + \mathcal{C}^T\gamma_5 \otimes \gamma_5\not{P}\mathcal{C}$
3	$\mathcal{C}^T\gamma_5\not{P} \otimes \gamma_5\mathcal{C}$ - $\mathcal{C}^T\gamma_5 \otimes \gamma_5\not{P}\mathcal{C}$
4	$\mathcal{C}^T\gamma_5\not{P} \otimes \gamma_5\not{P}\mathcal{C}$
5	$\mathcal{C}^T\gamma_T^\mu \otimes \gamma_T^\mu\mathcal{C}$
6	$\mathcal{C}^T\gamma_T^\mu\not{P} \otimes \gamma_T^\mu\mathcal{C} + \mathcal{C}^T\gamma_T^\mu \otimes \gamma_T^\mu\not{P}\mathcal{C}$
7	$\mathcal{C}^T\gamma_T^\mu\not{P} \otimes \gamma_T^\mu\mathcal{C}$ - $\mathcal{C}^T\gamma_T^\mu \otimes \gamma_T^\mu\not{P}\mathcal{C}$
8	$\mathcal{C}^T\gamma_T^\mu\not{P} \otimes \gamma_T^\mu\not{P}\mathcal{C}$
9	$\mathcal{C}^T\mathbb{1} \otimes \mathbb{1}\mathcal{C}$
10	$\mathcal{C}^T\gamma_T^\mu\not{P} \otimes \gamma_T^\mu\mathcal{C} + \mathcal{C}^T\gamma_T^\mu \otimes \gamma_T^\mu\not{P}\mathcal{C}$
11	$\mathcal{C}^T\gamma_T^\mu\not{P} \otimes \gamma_T^\mu\mathcal{C}$ - $\mathcal{C}^T\gamma_T^\mu \otimes \gamma_T^\mu\not{P}\mathcal{C}$
12	$\mathcal{C}^T\gamma_T^\mu\not{P} \otimes \gamma_T^\mu\not{P}\mathcal{C}$
13	$\mathcal{C}^T\gamma_T^\mu\gamma_5 \otimes \gamma_T^\mu\gamma_5\mathcal{C}$
14	$\mathcal{C}^T\gamma_T^\mu\gamma_5\not{P} \otimes \gamma_T^\mu\gamma_5\mathcal{C} + \mathcal{C}^T\gamma_T^\mu\gamma_5 \otimes \gamma_T^\mu\gamma_5\not{P}\mathcal{C}$
15	$\mathcal{C}^T\gamma_T^\mu\gamma_5\not{P} \otimes \gamma_T^\mu\gamma_5\mathcal{C}$ - $\mathcal{C}^T\gamma_T^\mu\gamma_5 \otimes \gamma_T^\mu\gamma_5\not{P}\mathcal{C}$
16	$\mathcal{C}^T\gamma_T^\mu\gamma_5\not{P} \otimes \gamma_T^\mu\gamma_5\not{P}\mathcal{C}$

Table 2.8: Symmetrized Momentum independent s-wave tensor structures.



$$k = \tfrac{1}{2}\,(p_1 + p_2 - p_3 - p_4)$$

't channel'

$\tau_i(p,q,k,P) \quad \otimes \quad$ Color $\quad \otimes \quad$ Flavor

256 Dirac-Lorentz tensors

2 Color tensors:

$3 \otimes \overline{3}, \quad 6 \otimes \overline{6}$ or
$1 \otimes 1, \quad 8 \otimes 8$
(Fierz-equivalent)

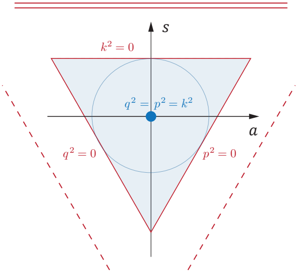
Structure of the amplitude

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \tfrac{1}{4}(p^2+q^2+k^2)$$

- **Doublet:** $\mathcal{D}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}\sqrt{3}(q^2-p^2)\\ p^2+q^2-2k^2\end{bmatrix}$

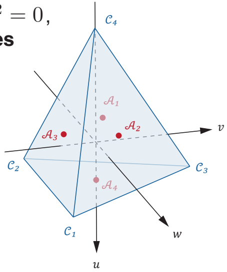
Mandelstam triangle,
outside: **meson and diquark poles!**



Lorentz invariants can be grouped into **multiplets of the permutation group S4:**
GE, Fischer, Heupel, Williams, 1411.7876

- **Triplet:** $\mathcal{T}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}2(\omega_1+\omega_2+\omega_3)\\ \sqrt{2}(\omega_1+\omega_2-2\omega_3)\\ \sqrt{6}(\omega_2-\omega_1)\end{bmatrix}$

tetrahedron bounded by $p_i^2=0$,
outside: **quark singularities**



- Second **triplet:**
3dim. sphere

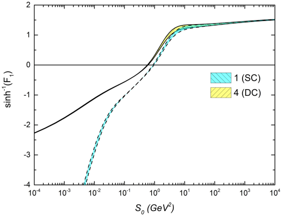
$$\mathcal{T}_1 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}2(\eta_1+\eta_2+\eta_3)\\ \sqrt{2}(\eta_1+\eta_2-2\eta_3)\\ \sqrt{6}(\eta_2-\eta_1)\end{bmatrix}$$

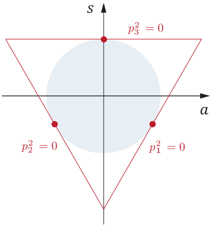
Structure of the amplitude

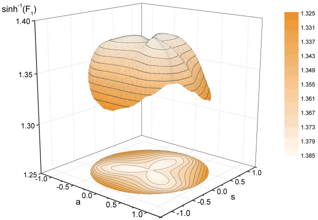
Idea: use symmetries to figure out **relevant** momentum dependence:

$$f_i\,(\,\mathcal{S}_0\,,\triangledown\,,\triangle\,,\bigcirc\,)$$

- cf. **photon four-point function** ⇔ hadronic LbL scattering contribution to **muon g-2**
  GE, Fischer, Heupel, Williams, 1411.7876
- cf. **three-gluon vertex:** angular variation in Mandelstam plane is negligible, only $\mathcal{S}_0$ relevant GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)



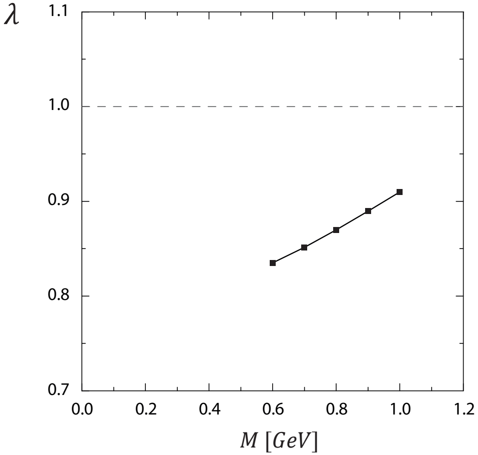




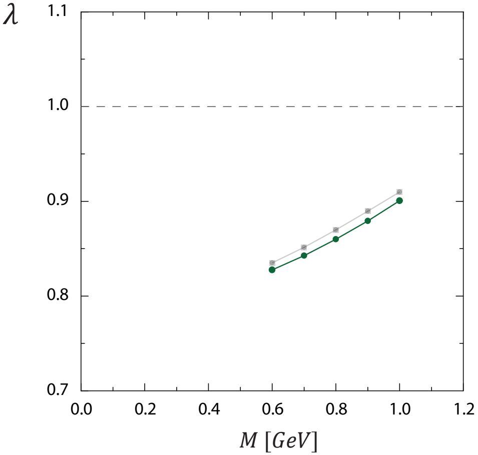
→ see also talks by **Markus Huber** and **Adrian Blum**

BSE eigenvalue

$f_i\,(\,\mathcal{S}_0\,, \bigtriangledown\,, \triangle\,, \bigcirc\,)$

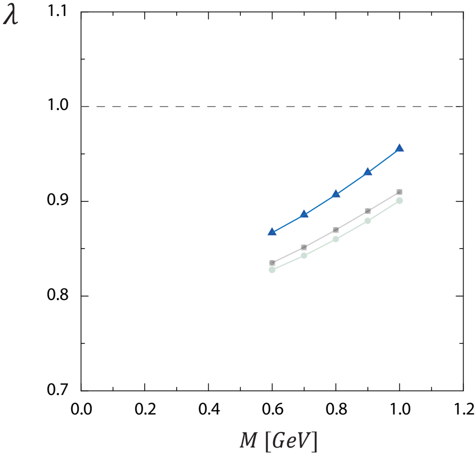


BSE eigenvalue



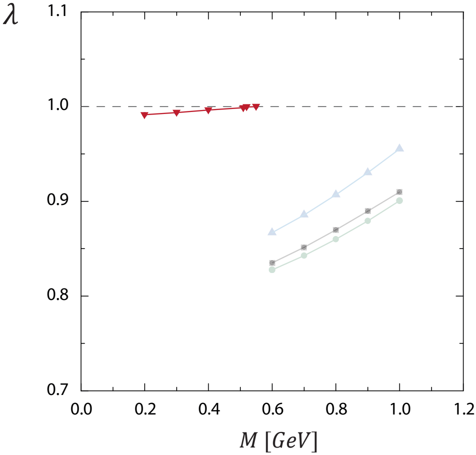
BSE eigenvalue

$f_i\,(\,\mathcal{S}_0\,,$  $)$

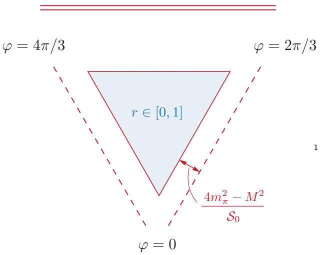


BSE eigenvalue

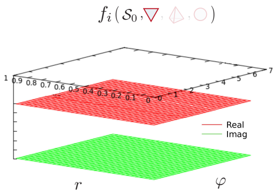
$f_i\,(\,\mathcal{S}_0\,,\nabla,\triangle,\bigcirc\,)$



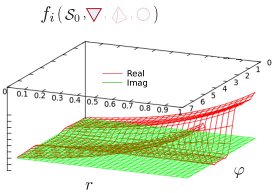
Pion poles



Small $\mathcal{S}_0$: no angular dependence

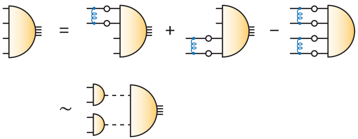


Larger $\mathcal{S}_0$: gap in Mandelstam triangle due to **pion poles!**



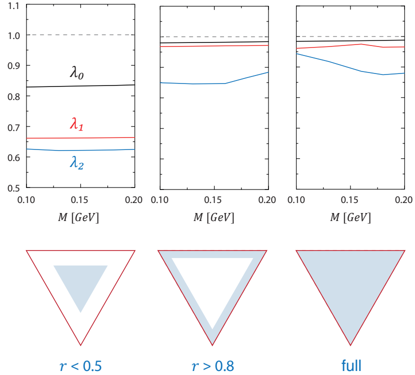
- Bethe-Salpeter amplitude sensitive to **pion poles outside the integration domain,** although equation knows nothing about pions
- drive tetraquark mass from 2 GeV to ~500 MeV
- **Poles enter integration domain** above threshold $M > 2m_\pi$: the tetraquark becomes a **resonance**

- Four-quark equation generates **bound state** together with its **decay channels!**



Tetraquark mass

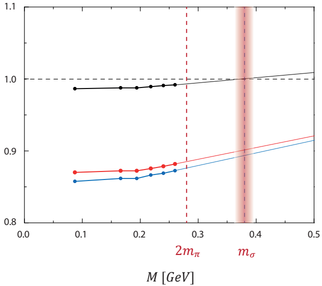
Tetraquark mass driven by momentum dependence close to **r = 1:** visible from phase space cuts (larger eigenvalue ⇔ smaller mass)



But dense eigenvalue spectrum: **spurious states?**

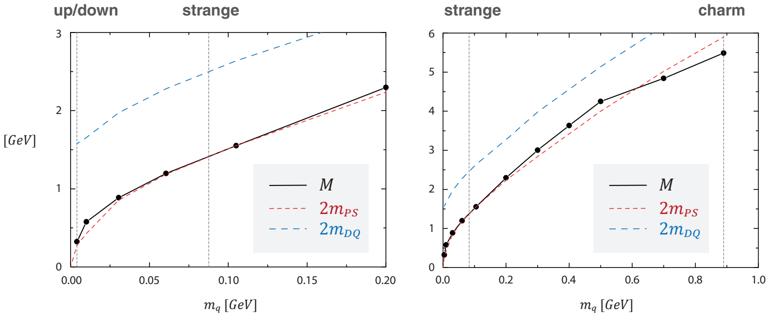
No, just numerical artifact:
pion poles at large $\mathcal{S}_0$ (UV!)
not properly resolved

⇒ Implement pion (and diquark) poles analytically: ground state unchanged, but low-lying excitations disappear



Tetraquark mass

Evolution with **current-quark mass:**



- Resonance just above $\pi\pi$ threshold,
  becomes bound state in charm-quark region
- $\sigma \sim$ **380 MeV**, $\kappa \sim$ **700 MeV**, $a_0 / f_0 \sim$ **920 MeV**

Summary

- Two-body and four-body equations give consistent results, suggest **light scalar mesons are tetraquarks**

- $\sigma \sim$ **380 MeV**, $\kappa \sim$ **700 MeV**, $a_0 / f_0 \sim$ **920 MeV**

- Dominated by pseudoscalar Goldstone bosons, diquarks irrelevant: **'meson molecule'** (but resonance)

- Extract **widths?**
  Maybe, not sure yet (look for poles in complex plane)

- Tetraquarks in **heavy-quark regime?**
  Maybe, but rainbow-ladder problematic for heavy-light systems

- First solution of **genuine four-quark BSE** (which is also a **resonance!**)