Is the spacetime Euclidean inside the hadrons?

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Excited OCD 2015, Tatranska Lomnica

Outline

- Introduction and Motivation
- SDEBSE models in M and E spaces
- LRA M Model I and Improved Model
- Oscilating quark propagator
- Conclusion

Introduction

Confinement is a conjecture

Colorless objects are the only visible excitations. It implies that only few among many QCD Green's functions describe asymptotic states. Quarks and gluons are the simplest colored guys that do not span physical Hilbert space. QCD is the field theory

- lattice (unavoidable Euclidean), Polyakov loops, Wilson loops
- Dyson-Schwinger equations, which includes Bethe-Salpeter E. for mesons and glue-balls and Fadeev E. for baryons , etc..

Introduction- Motivation

Confinement is related with string picture, Regge trajectory $M^2 \simeq n$

Can QCD DSEs reproduce Regge trajectories in Lorentz and gauge covariant manner?

99% papers done in Euclidean space this talk - Minkowski space QCD

Euclidean BSE needs complex argument for excited states, to get the results for excited states requires knowledge of GFs for complex argument. Problem with numerics.

How to determine confinement for light quarks?

Confinement test is not straightforward in Euclidean space:

Position space Schwinger function at rest frame

$$\Delta(\tau) = \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{ip_4\tau + i\vec{p}.x} G(p^2)$$

if there are symptotics states then they cane be read from. If Δ changes the sign the excitation asymptotic with G do not appear in the Hilbert space of observables.

The example is Gribov propagator, which has double complex conjugate pole. Generalization of the sort

$$G_g(p^2) = \frac{p^2 - c}{(p^2 - a)^2 + b^2}$$

fits a vast data on gluon and quark propagators.

Confinement is encoded in the analytical structure of GF of confined excitation! Confinement could be easily visible when one keeps full Minkowski space solution.

Motivation

DSEs+pion in E space, see C. Roberts, P. Maris, P.C. Tandy, A. Krassnigg, G. Eichmann, etc... based on an extension of the pionering work (25 y ago!) $K \simeq e^{-q^2/L^2}$

"S.J. Stainsby, R.T. Cahill, Is space-time euclidean âĂIJinsideâĂİ hadrons?, PLA (1990)"

Developing (numerical) method, which works in the Euclidean as well as in the Minkowski space, incorporates confinement, , incorporates DCSB, provides hadron spectroscopy for excited states (and form factors at not distinct future) First for charmonia V.S. PRD 86, (2012);V.S. PRD 90 (2015) arXiv:1207.2621 ,V.S. and P. Bicudo, QCD-TNTII (2011)

BSE + DSEs formalism

BSE for mesons

$$\Gamma(q,P) = -i\lambda(P)\int \frac{d^4k}{(2\pi)^4}\left[S(q-P/2)\Gamma(p,q)S(q+P/2)\right]^{i,j} V(k,q,P)_{i,k,l,j} \ ,$$

where Latin letters $i, j...$ represent Dirac indices. Method of Iterations + normalization (give up with matrix inverse method and angle expansion) is soluble in Minkowski space. Solution : $\lambda(P) = 1$. Explicitly for the pseudoscalar

$$\Gamma_P(q,P) = \gamma_5\left(A(q,P) + \not{P}C(q,P) + \not{q}B(q,P) + [\not{q},\not{P}]D(q,P)\right) \ ,$$

BS wave function χ

$$\Gamma(q,P) = S^{-1}(q-P/2)\chi(q,P)S^{-1}(q+P/2) \ ,$$

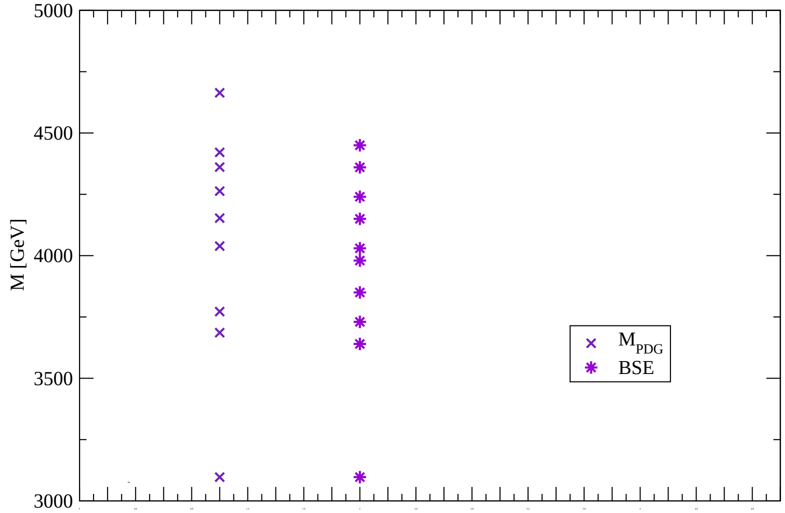

Figure 1: Excited vectors from Euclidean BSE V.S. PRD 86, 096004 (2012)

Highly excited states of meson have been gained by the method!

Excited pions and scalars in Euclidean and Minkowski space

Property of the solution in Minkowski space Digression: Gribov propagator arXiv:1412.7863

$$G_g(p^2) = \frac{1}{(p^2 - a^2) - b^2}\ ;\ \Pi(s = p^2) = iI(s) = i\int \frac{d^4l}{(2\pi)^4} G(l)G(l-q)$$

define

$$D = s(4a - s) + 4b^2\ ;\ D_1 = s(4a - s) + 4b^2 + 4ibs\ ;\ D_2 = 4a + 4ib - s$$

$$
\begin{aligned}
\Pi(s) &= \frac{i}{2b(4\pi)^2}\left\{\frac{1}{2s}\ln\frac{(a+s/4)^2+b^2}{a^2+b^2}-\frac{1}{4s}ln\frac{(a-s/4)^2}{a^2}\right.\\
&+ \left.\frac{1}{2b}tan^{-1}\frac{b}{a}-\frac{tan^{-1}\sqrt{\frac{s}{4a-s}}}{s\sqrt{\frac{4a-s}{s}}}\right\}\\
&- \frac{1}{4b^2(4\pi)^2}\left\{\frac{\sqrt{D}}{s}tan^{-1}\frac{-2ib+s}{\sqrt{D}}-\frac{\sqrt{D}}{s}tan^{-1}\frac{-2ib}{\sqrt{D}}-c.c.\right.\\
&+ \frac{\sqrt{D_1}}{s}tan^{-1}\frac{2ib}{\sqrt{D_1}}-\frac{\sqrt{D_1}}{s}tan^{-1}\frac{2ib-s}{\sqrt{D_1}}-c.c.\\
&- \left.\frac{\sqrt{D_2}}{s}tan^{-1}\frac{s}{\sqrt{D_2}}-c.c.\right\}
\end{aligned}
$$

= finite, pure imaginary expression at timelike and spacelike!!! branch points known

Ladder-Rainbow model I for pions

Light mesons require proper treatment! Rainbow-Ladder truncation of SDEs system V.S. IJTP 2015

The **rainbow-ladder approximation** enables us to write down the effective charge α:

$$\Gamma^{\mu}(k,p)G_{\mu\nu}(k-p) \rightarrow \gamma^{\mu}G_{\mu\nu}^{[free]}(k-p)\alpha(k-p)$$

defined as a simplified product of the quark-gluon vertex and the gluon propagator. The BSE and DSE kernel introduced above can be related with the charge α in the following way:

$$K(x) = \frac{\alpha(x)}{\pi x} = (8\pi^3)C\lambda^2 \frac{e^{-\frac{\sqrt{-x}}{\lambda}}\Theta(-x) + e^{i\frac{\sqrt{x}}{\lambda}}\Theta(x)}{(x-\lambda^2)^2+\lambda^4},$$

$$
\begin{aligned}
S^{(-1)} &= \not p - m_o - \Sigma(p) \\
\Sigma(p) &= i \int \frac{d^4k}{(2\pi)^4} G_{\mu\nu}(p-k) \frac{\lambda^a}{2} \gamma_\mu S(k) \Gamma^a_\nu(p,q) \, ,
\end{aligned}
$$

$$
\Gamma^j_{\pi_n}(p,P) = i \int \frac{d^4k}{(2\pi)^4} \chi(k,P)^j_{\pi_n} K(k,p,P) \tag{1}
$$

where $K(k,p,P)$ is quark-antiquark interaction kernel and is identical with $K(x)$ where the total momentum satisfies $P^2 = m^2_{\pi_n}$. RLA ensures AWT, emg WTI and pion is a Goldstone boson.

Confinement of quarks - no on mass shell pole, no production thresholds. Absence of these singularities at real axis of momenta allows us to look for the solution at Minkowski space. First solve quark DSE and then BSE:

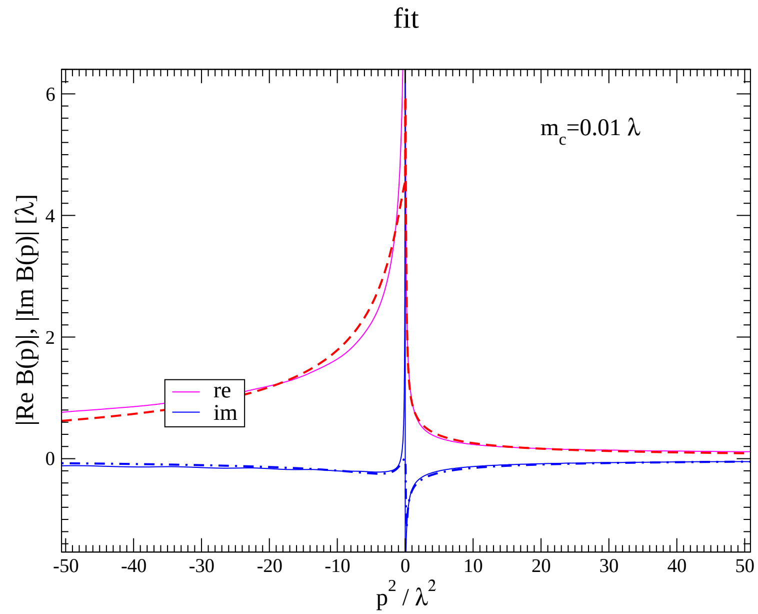

Figure 2: Solution of QCD gap equation: $S^{-1}(p) = 1/Z(p^2)\left[\not p - M(p^2)\right]$ Minkowski space complex selfenergy -the quark dynamical mass function $M = B$ - and the analytical fit are displayed.

S is required entries for BSE

$$\Phi(k,P)_{\pi_n} = \gamma_5 \Phi_{E_{\pi n}}(k,P)$$

$$\Gamma_i(p_0,p^2;P) = i\Sigma_j \int_{-\infty}^{\infty} dk_o \int_{-k^2}^{\infty} dk^2 \sqrt{k^2-k_o^2} \int_{-1}^{1} dz f_j(k,^2,p^2,k.p,k.P)$$

where z is cosine of the angle between internal and external spacelike momenta.

Numerical integrations 'over the hyperbolas' $k^2 = g^{\mu\nu}k_\mu k_\nu = k_0^2 - \vec{k}^2$ dictated by Minkowski metric, turns a numerical search to longstanding but succesfull game.

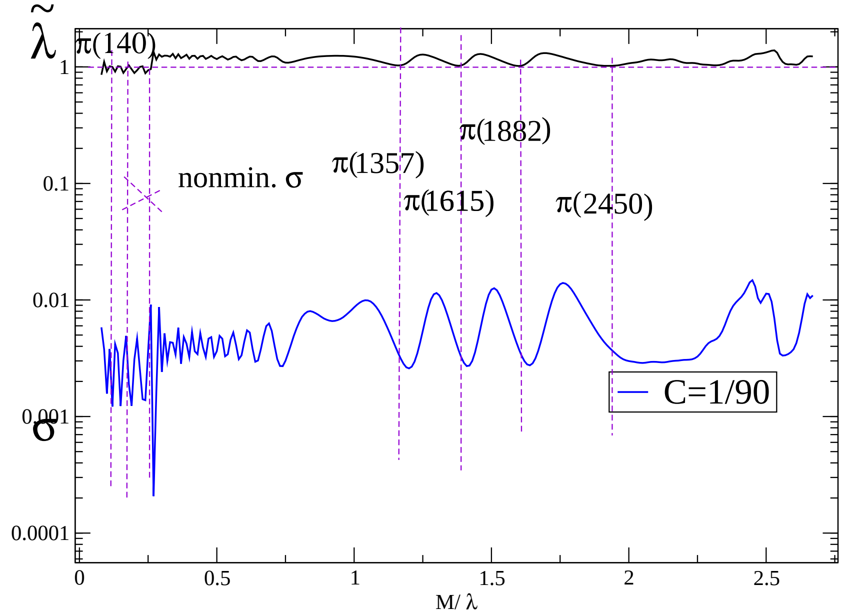

Figure 3: Example of numerical search for pseudoscalar, the eigenvalue $\tilde{\lambda}$ and the weighted error σ are shown. Vertical lines label where the solutions are, while crossed vertical lines represent quasisolution fakes.

Oscilating quark propagator

MODEL II Improved Rainbow-Ladder model for pions

Define the BSE/DSE kernels in a way the angular integration can be done analytically

UV part:

$$K_{UV}(q^2) = C_{UV}\frac{d}{dq^2}\ln\ln\left[\tau + (q^2/\Lambda^2_{QCD})^2\right]$$

$$K_{UV}(q^2) = \frac{q^2}{\tau\Lambda^2_{QCD} + {q^2}^2}\frac{1}{\frac{1}{2}ln\left[\tau + (q^2/\Lambda^2_{QCD})^2\right]}$$

with $C_{UV} = \frac{\gamma_m}{2\pi} = \frac{13}{(33-2N_f)2\pi} = 0.0714$ for $N_f = 2$

IF part

$$
\begin{aligned}
K_{IF}(q^2) &= C_{IF}\frac{d}{dq^2}\,e^{[-\sqrt{-q^2/L^2}/l(q^2)]}ln[\frac{(q^2-L)^2+L^4}{a^4+L^4}] \\
l(q^2) &= ln^{\gamma}(e+q^2/L_2^2) \\
K_{IF}(q^2) &= e^{[-\sqrt{-q^2/L^2}/l(q^2)]}\frac{2(q^2-L^2)}{(q^2-L^2)^2+L^4}+...
\end{aligned}
$$

BSE is then reduced to 2dim integral equation for dominant Φ_5,

Tuned model for excited pions $L =$ unit of the model $\simeq \Lambda_{QCD}$ $L_2 =$ few$L$ $C_{IF} \simeq 3/100$. $\gamma = 1/2$

Adding UV part is possible only if A is considered

Numericaly including A is disaster and often lead to worse num. stability, tuning the model is hard and t- consuming approximative guess based on Euclidean LR SDE approximation in Landau gauge.

$$A = (1 + K/ln^{1/2}(e + q^2/\Lambda^2_{QCD}) \tag{2}$$

with $K = 2$ at least! , so $Z(0) = 1/A(0) = 1/3$.

further approx. $\Phi_5 = \gamma_5 \Phi_A(q.P, q^2; P^2)$,

I allow different constant prefactor for BSE $\simeq 1$ to get rid of my wrong guess of A

in fact $C_{BSE} \simeq \frac{1}{3} C_{DSE}$ for $A(0) = 3$

to get a correct intercept one needs slightly softer coupling for the ground state

$C_{inf} = 1/100 - 1/160$ (or LR with single variable q^2 is not completely adequate)

Tunning the model

Solution:

1. Guess parameters and solve SDE

2. Substitute the result to BSE , look the pionic spectrum

3. go back to 1. untill the pion is pion

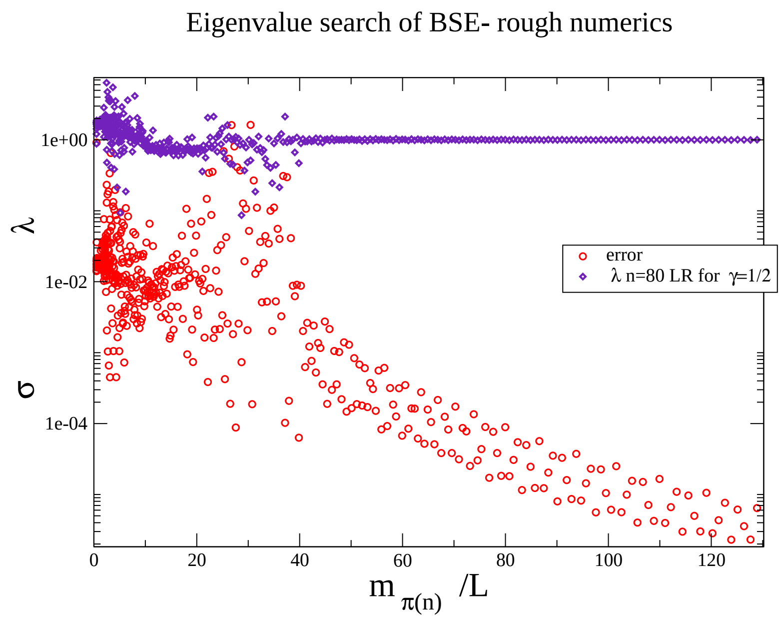

Figure 4: Numerical search

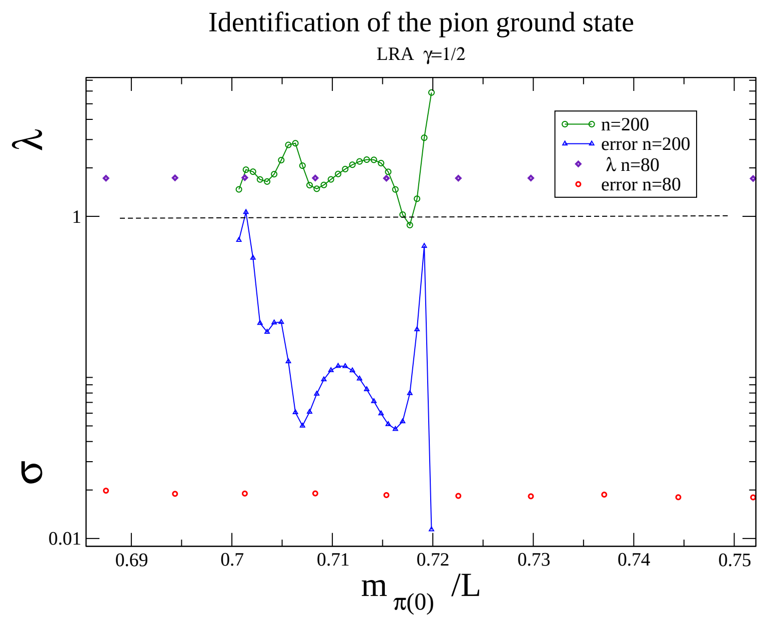

Figure 5: Numerical search

Tunning the model

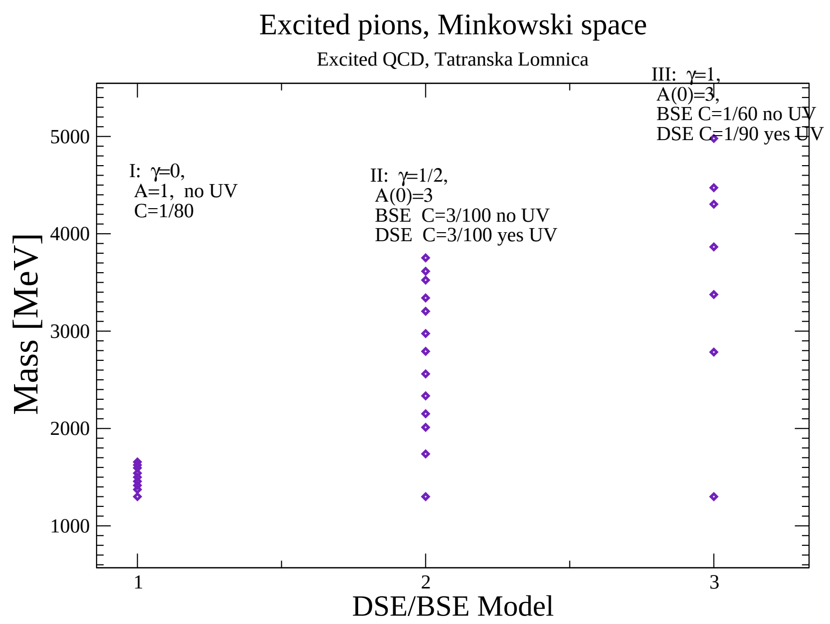

Figure 6: Tunning the model, only few states are shown

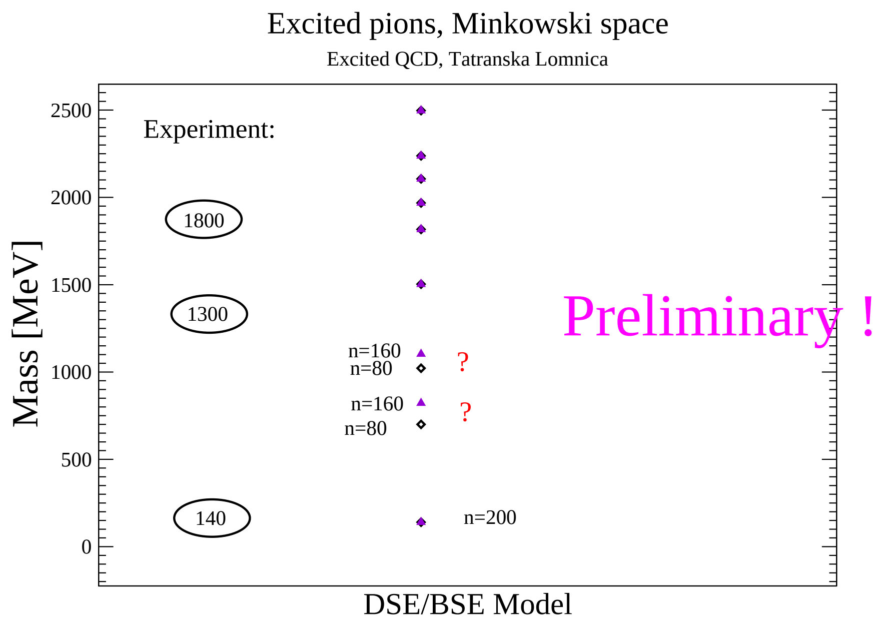

Figure 7: Preliminary results, only few states are shown

Regge trajectory

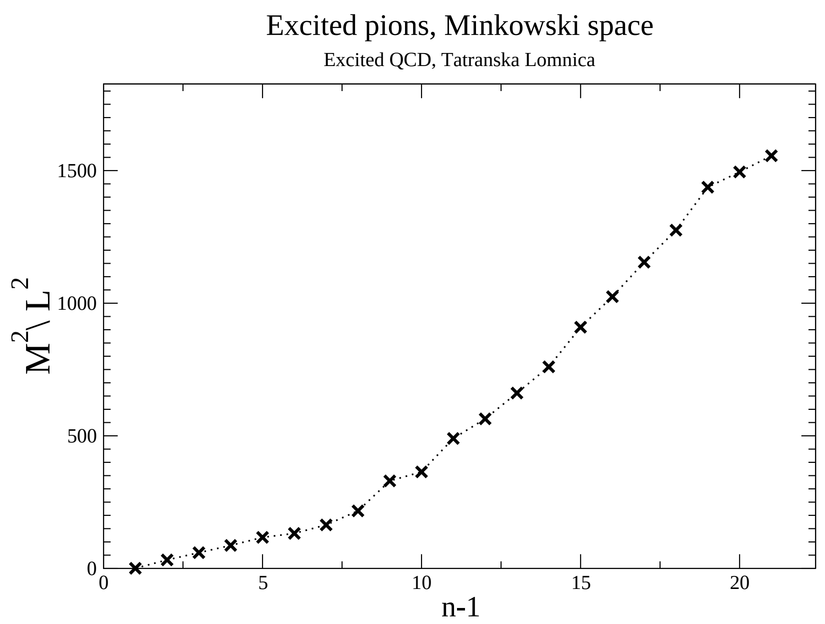

Figure 8: Regge traj. for pions $M^2 \simeq n$ is obtained ithout a presence of singular kernel. No linear string potential is needed.

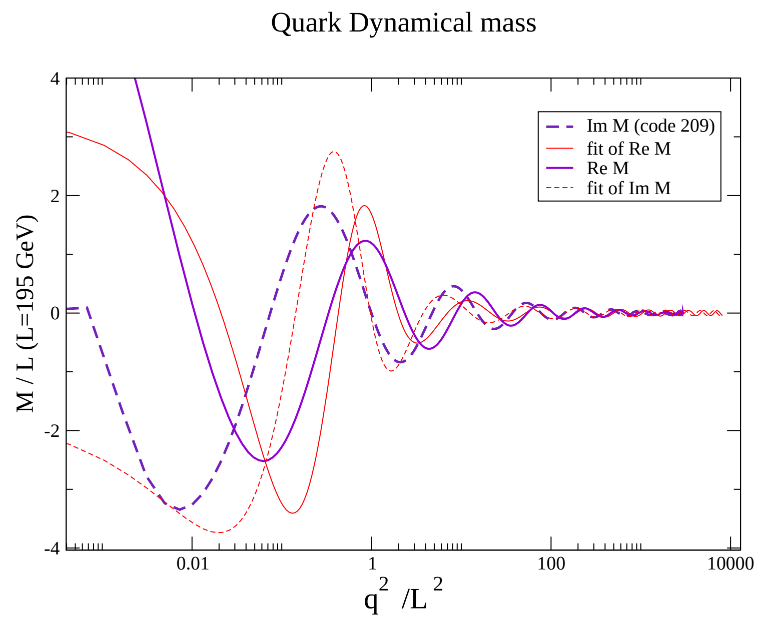

Figure 9: behaviour of dynamical quark mass at timelike region

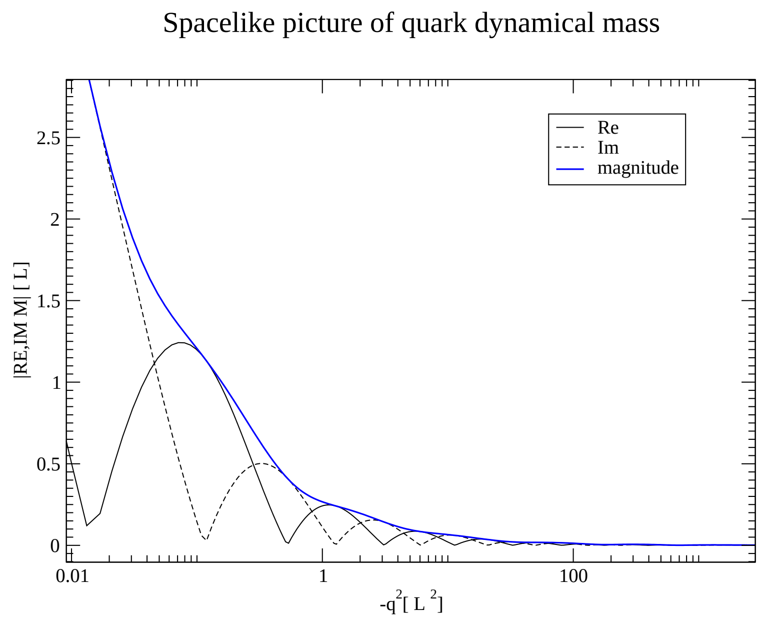

Figure 10: Dynamical quark mass at timelike region for $\gamma = 1/2$

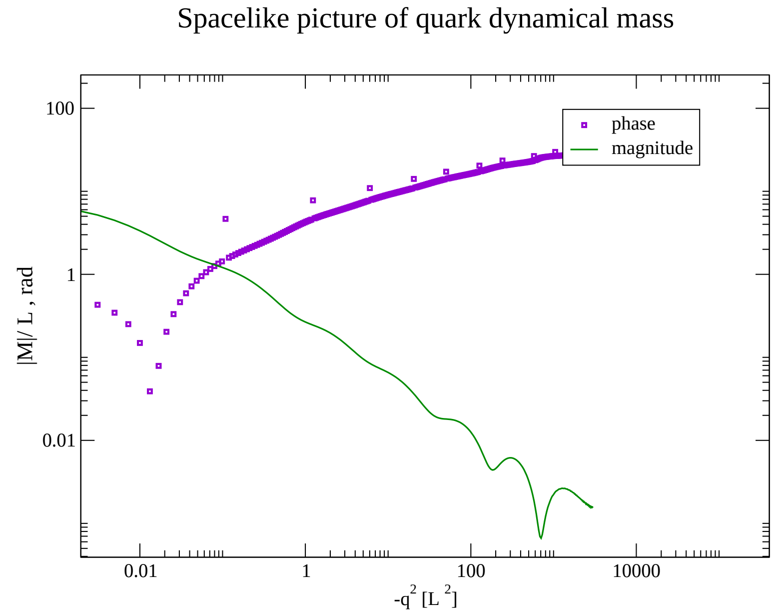

Figure 11: Dynamical quark mass at spacelike region for $\gamma = 1/2$

Back to the kernel

Where the oscilating behaviour comes from?

Quark-Gluon vertex is a source, not a Yang Mills part. Oscilation is generated by Γ_μ^T, e.g. γ_μ^T and F_5. Mechanism is visible only beyond LRA. It is enhanced by non-abelian vertices.

Appearance of Redundant phases

If one is willing to accept the scenario proposed an unusual conclusion is evident: the Green's functions are complex objects in Minkowski space, but not only above certain timelike scale but everywhere. This is in agreement with absence of quark pole masses, quark production threshold and Lehman representation for the quark propagator. The solution of quark gap equation is manifestly in agreement with confinement. This is obvious without intruding an area laws, Schwinger correlators and other indirect lattice/Euclidean space usual indicators.

Consequnces for observables:

Effective models uses physical degrees freedom (chpt) The corret or usual analyticity is ensured by the unitarity of S-matrix, formfactors are complex only above the thresholds

Minkowski space QCD? "Unusual" consequences for (not yet calculated) observables

- Charge conservation ,"$F(0) = 1$", is ensured by proper normalization of BSE

-F can be complex at other values of q, but it can differ from conventional results by the phase only, momentum dependence is allowed

This aditional phase would not be due to the threshold and absorption of physical particles but due to confinement.

Conclusion

The main purpose was to present the first Minkowski space nonperturbative calculation of bound state equations for pions. This was possible due to to the special property of Green's functions describing confined quarks and gluons. Although the approximations of DSBSEs in Minkowski space are limited by numerical convergence the presented models are an example, where working directly in Minkowski momentum space gives reliable results. It does not require (a numerically sometimes impossible) analytical continuation of the data coming from an auxiliary Euclidean space.

Model I $\lambda \simeq 1GeV$, Improved Model II $L = 195GeV$

Advantages of Minkowski space DSEBSE treatment

-it explains the light quark confinement -no poles, no production thresholds

- it gives the excited states without some extrapolations

- Minkowski QCD is our world QCD, no speculation about validity of Euclidean approximation is needed.

Weaknesses of Minkowski space DSEBSE treatment:

-numerical acces is limited, there are windows in par. space which can not be checked recently. It is not working for pure YM.

- swapping solutions of the quark gap equation- problem with convergence

- to get the low states , including $m_\pi = 140MeV$ is a hardest job

-going beyond single component in BSE approx. is time consuming again

Scalars

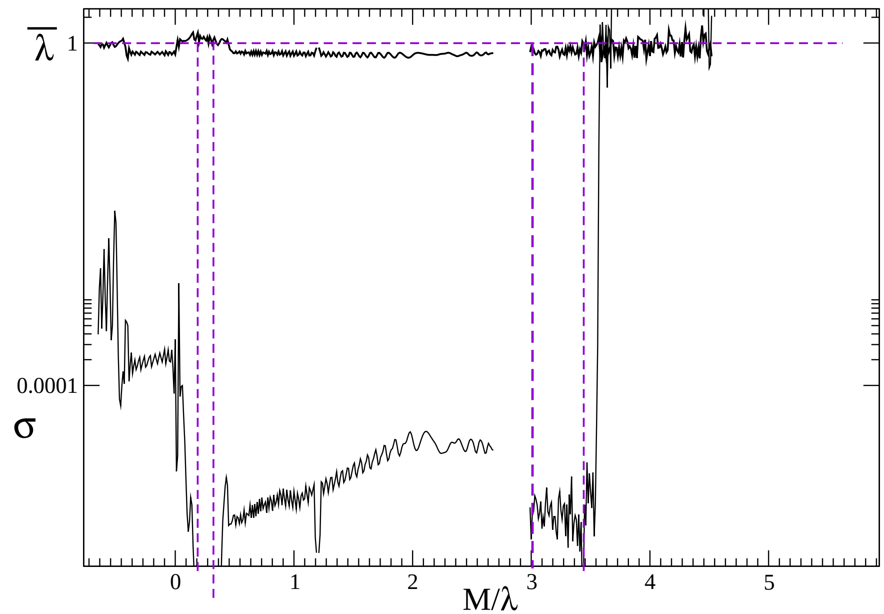

Figure 12: Example of eight weeks (on a single (3.5GHZ) processor machine) numerical search for scalars for $C = 1/90$, the eigenvalue $\tilde{\lambda}$ and the weighted error σ are shown. Vertical lines label where the physical solutions are. Recall PD : $a_0(980)$, noting that σ is isoscalar