A new hydrodynamic model using an exact Riemann solver

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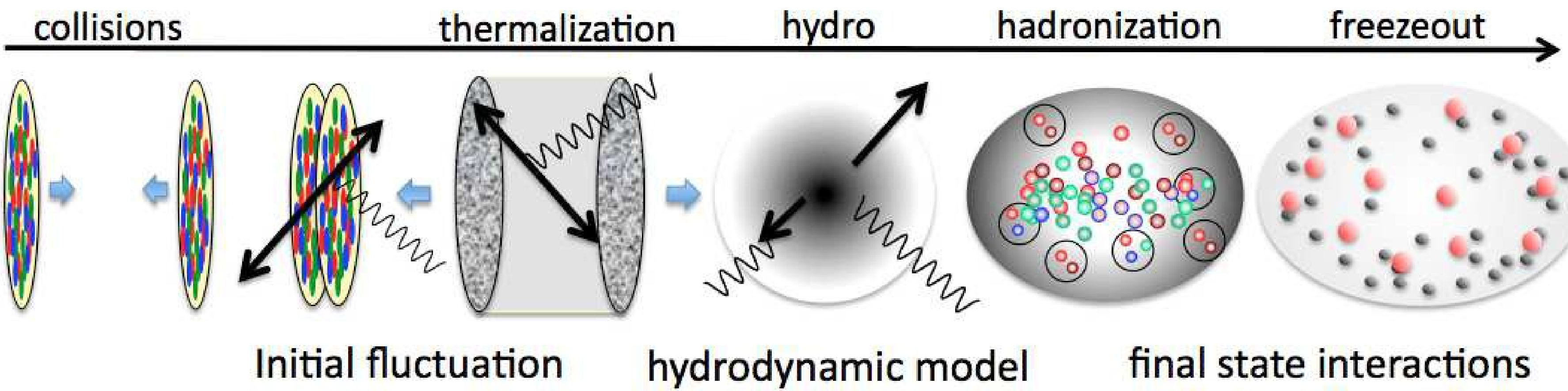
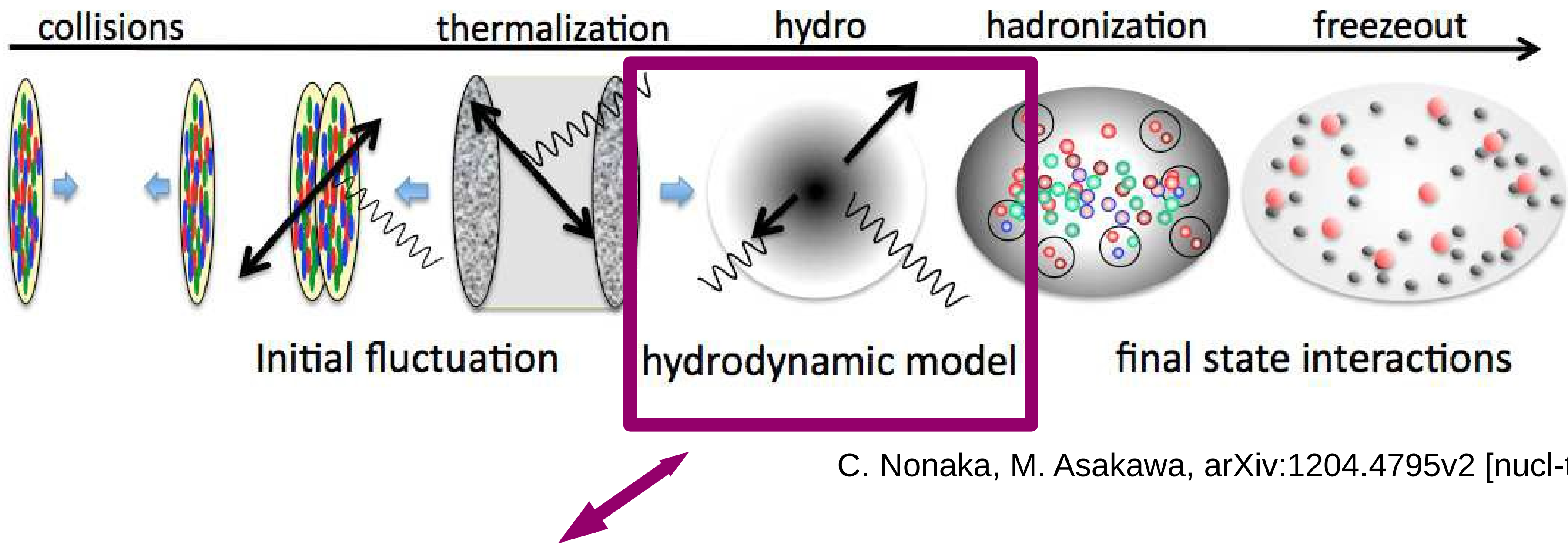
Excited QCD
8. - 14. 3. 2015
Tatranská Lomnica

Hydrodynamic expansion



C. Nonaka, M. Asakawa, arXiv:1204.4795v2 [nucl-th]

Hydrodynamic expansion



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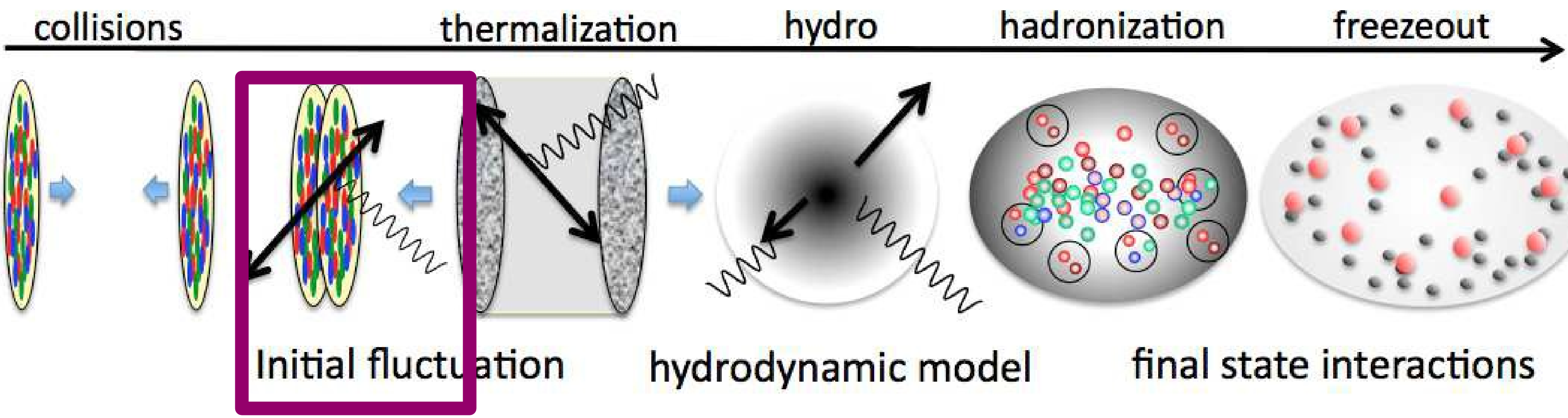
$$\partial_\mu n^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$p = p(\epsilon, n)$$

Ideal hydrodynamics:

$$T^{\mu\nu}_{(0)} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

Initial conditions



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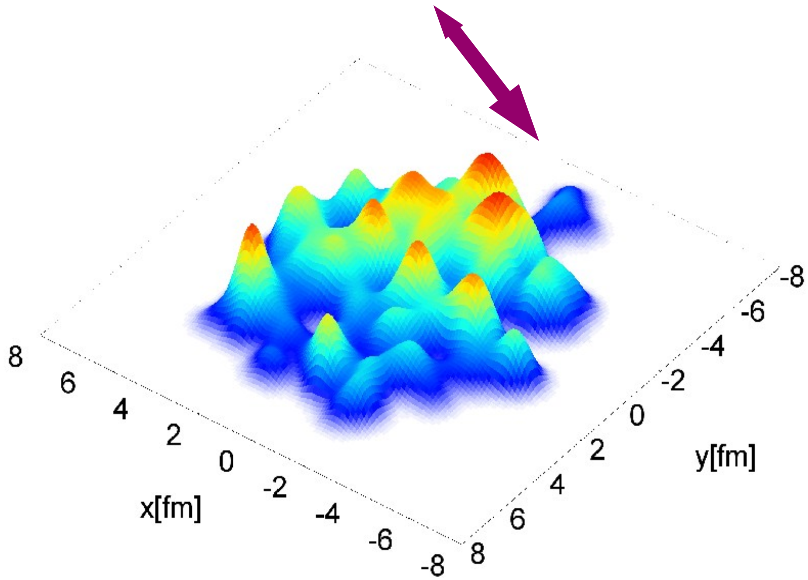
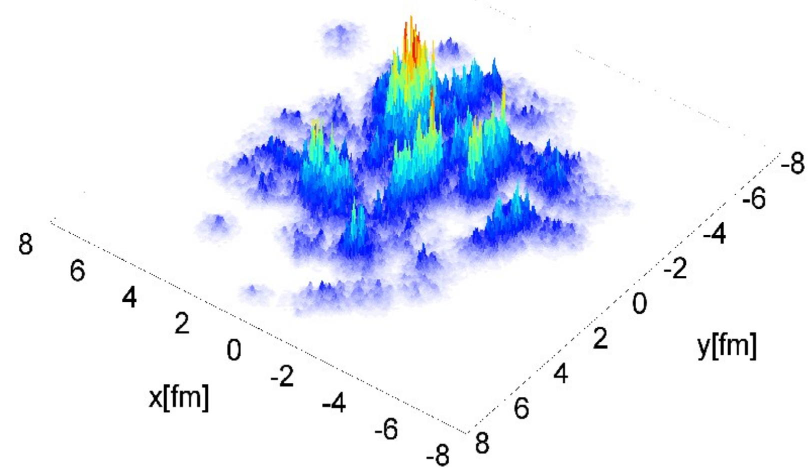


MC Glauber

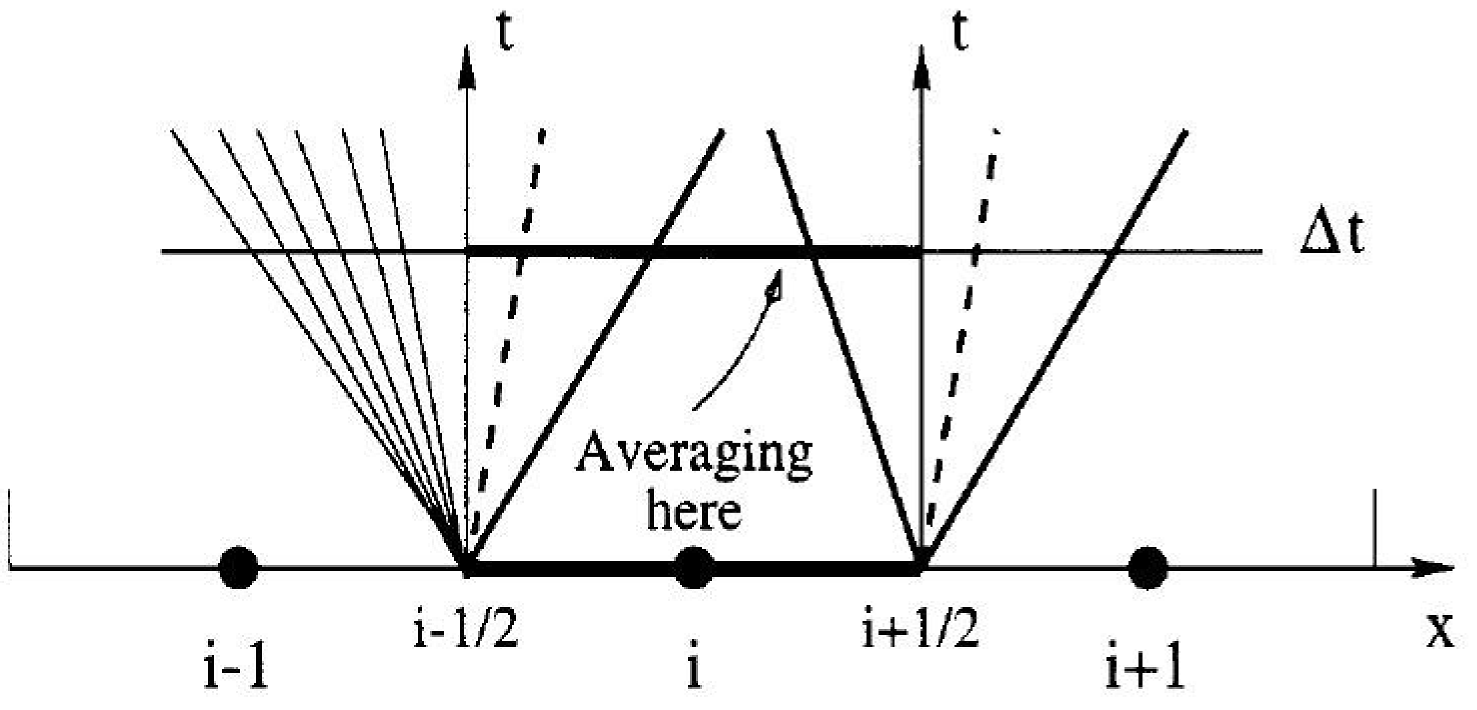


IP Glasma

B. Schenke et al., Phys. Rev. Lett. 108 (2012) 252301

Numerical method

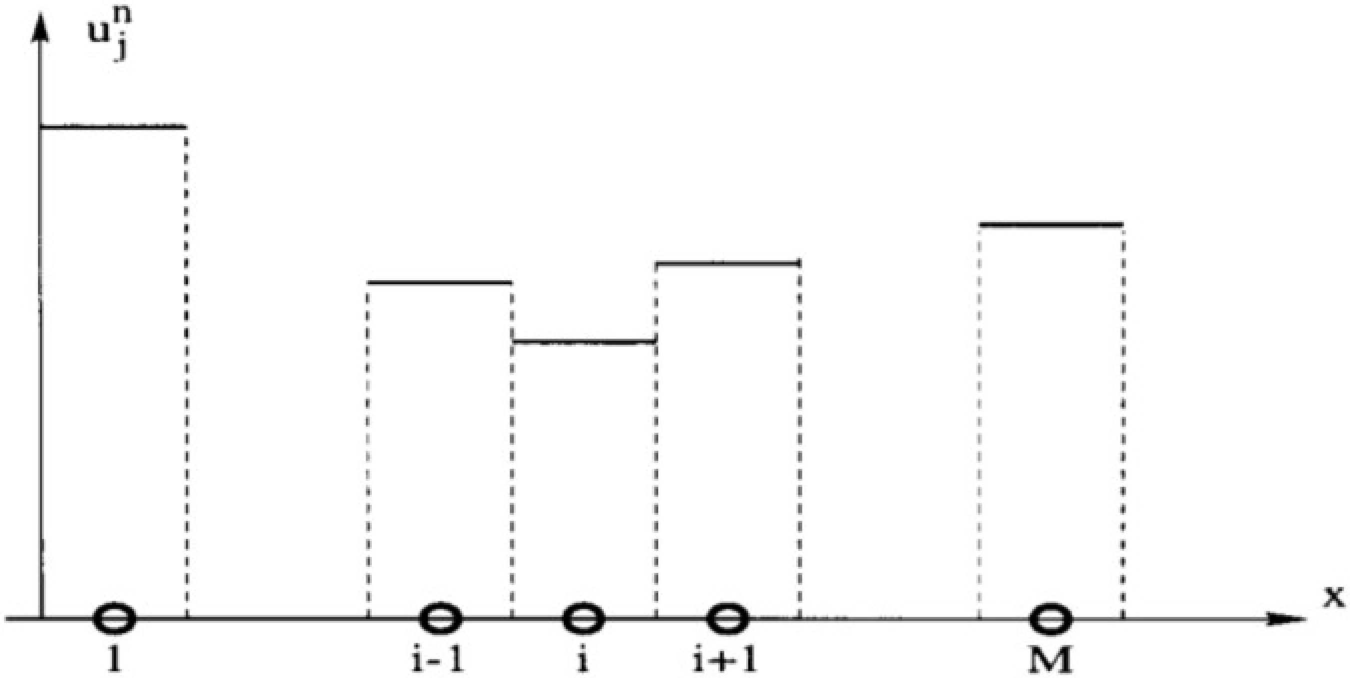
- Godunov method: computing the flow of conserved variables on cell boundaries



E. F. Toro: Riemann Solvers and Numerical Methods for Fluid Dynamics

Riemann problem

- Piece-wise constant distribution of data in the numerical grid



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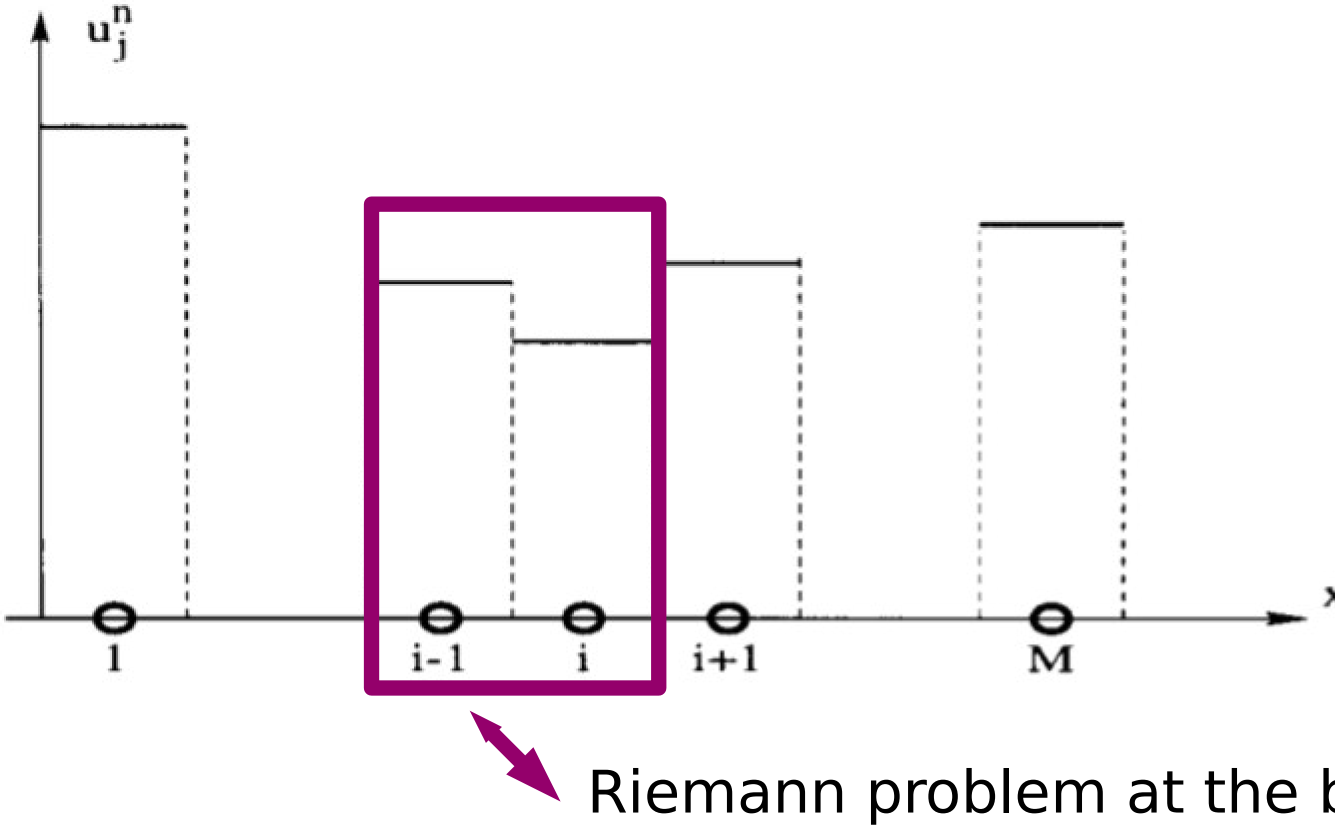
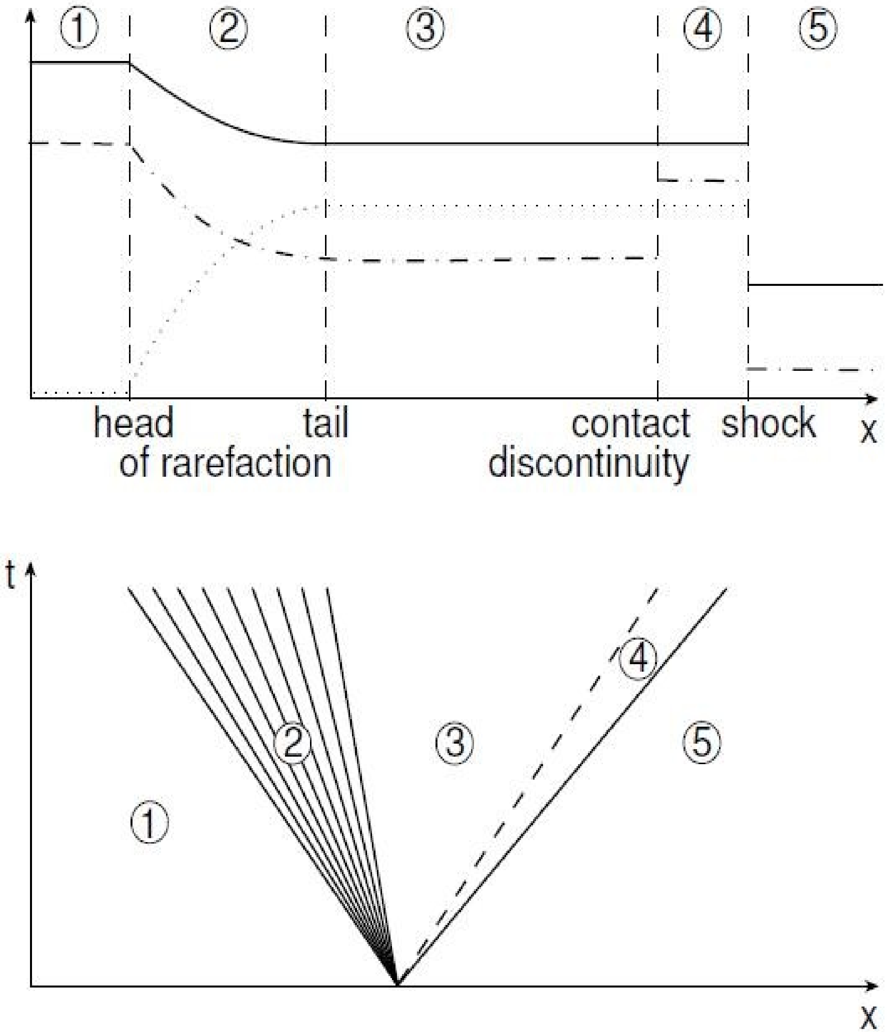


Riemann problem at the boundary[7]

Riemann problem

- Exact solution: reconstructing flow on both sides of the interface
- Shock/rarefaction wave
- Solving at the interface:

$$v_L^x(\epsilon_{new}) = v_R^x(\epsilon_{new})$$



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Testing the scheme

- Sound wave propagation: precision, numerical viscosity

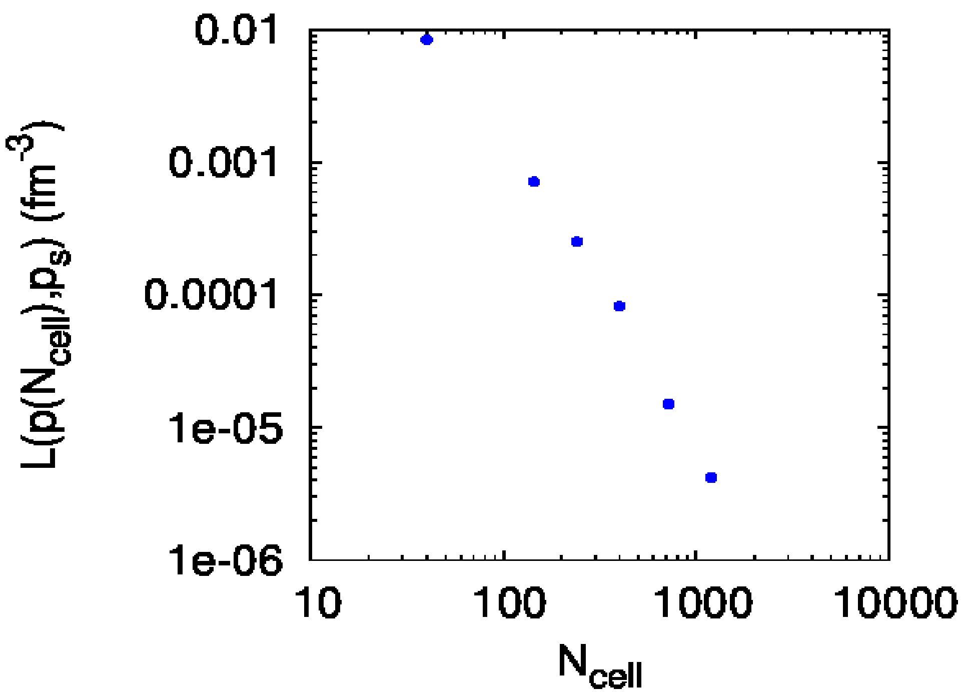
$$p_{init}(x) = p_0 + \delta p \sin(2\pi x/\lambda), v_{init}(x) = \frac{\delta p}{c_{s0}(e_0 + p_0)} \sin(2\pi x/\lambda)$$

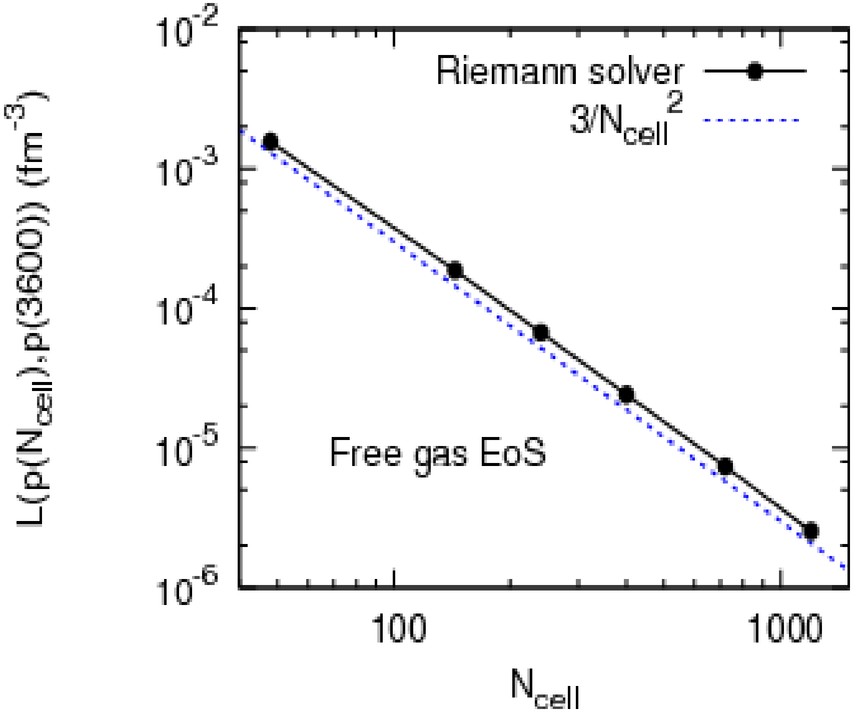
$$p_0 = 10^3 fm^{-4}, \delta p = 10^{-1} fm^{-4}, \lambda = 2 fm$$

Sound wave propagation

L1 norm:

$$L(p(N_{cell}),p_s)=\sum_{i=1}^{N_{cell}}|p(x_i,\lambda/c_s;N_{cell})-p_s(x_i,\lambda/c_s)|\frac{\lambda}{N_{cell}}$$



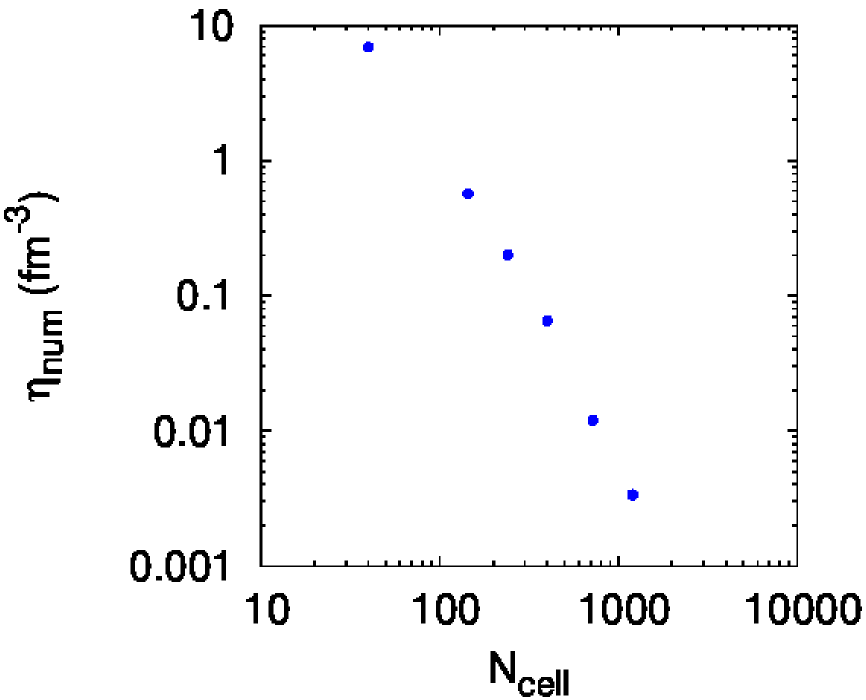


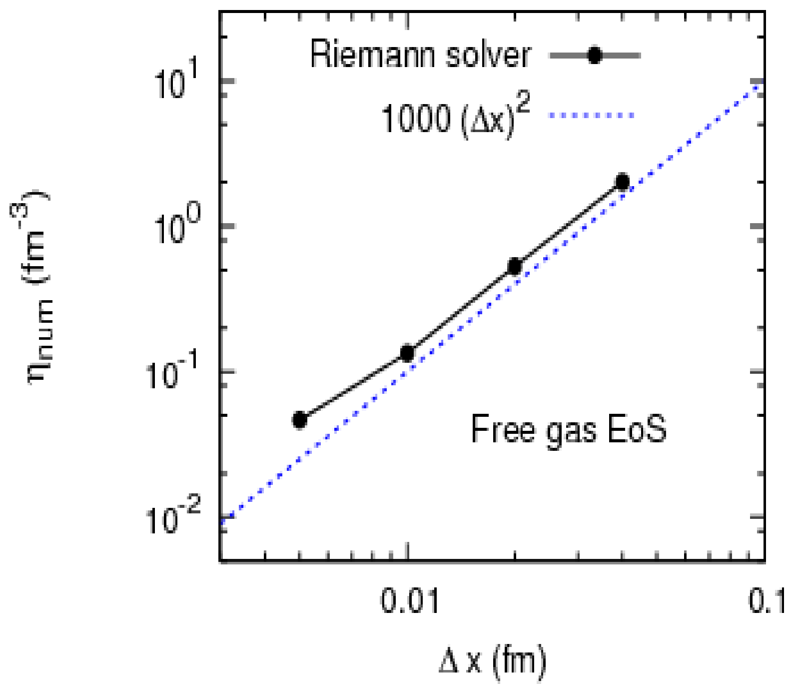
Y. Akamatsu et al., J. Comput. Phys. 256 (2014) 34

Sound wave propagation

Numerical viscosity:

$$\eta_{num} = \frac{-3\lambda}{8\pi^2} c_{s\,0}(e_0 + p_0)\ln\left[1 - \frac{\pi}{2\lambda\,\delta p} L(p(N_{cell}, p_s))\right]$$

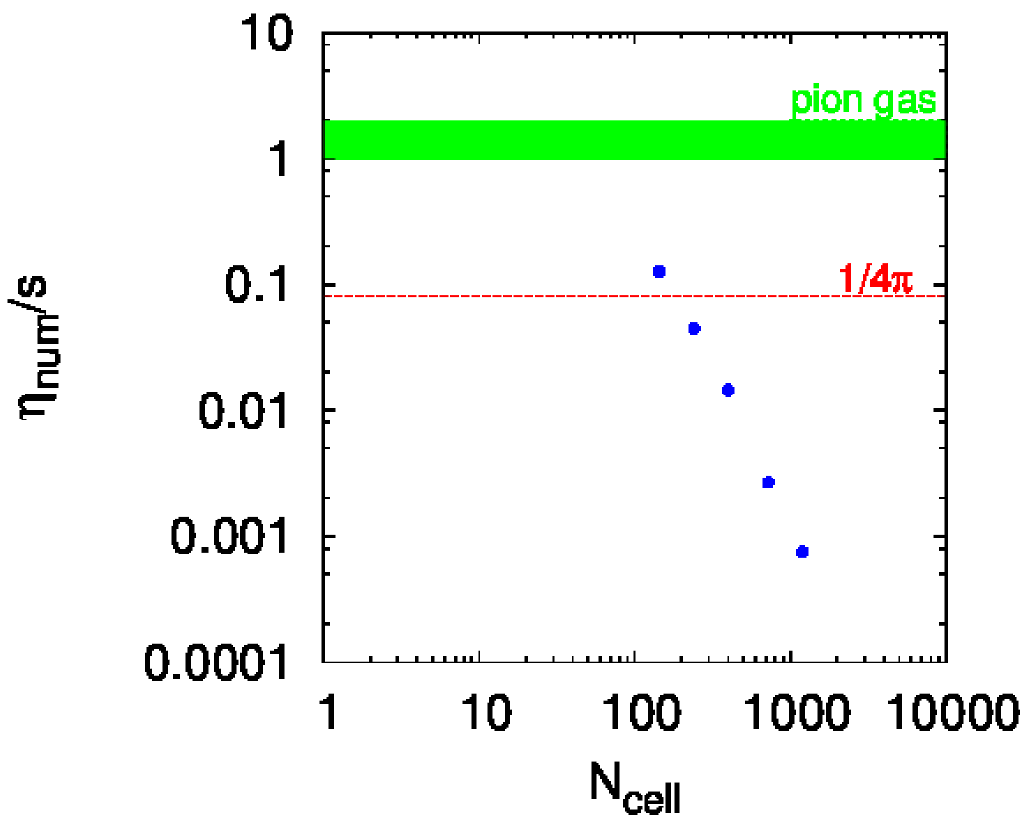




Y. Akamatsu et al., J. Comput. Phys. 256 (2014) 34

Sound wave propagation

Numerical viscosity to entropy ratio:



Testing the scheme

- Sound wave propagation: precison, numerical viscosity

$$p_{init}(x) = p_0 + \delta p \sin(2\pi x/\lambda), v_{init}(x) = \frac{\delta p}{c_{s0}(e_0 + p_0)} \sin(2\pi x/\lambda)$$

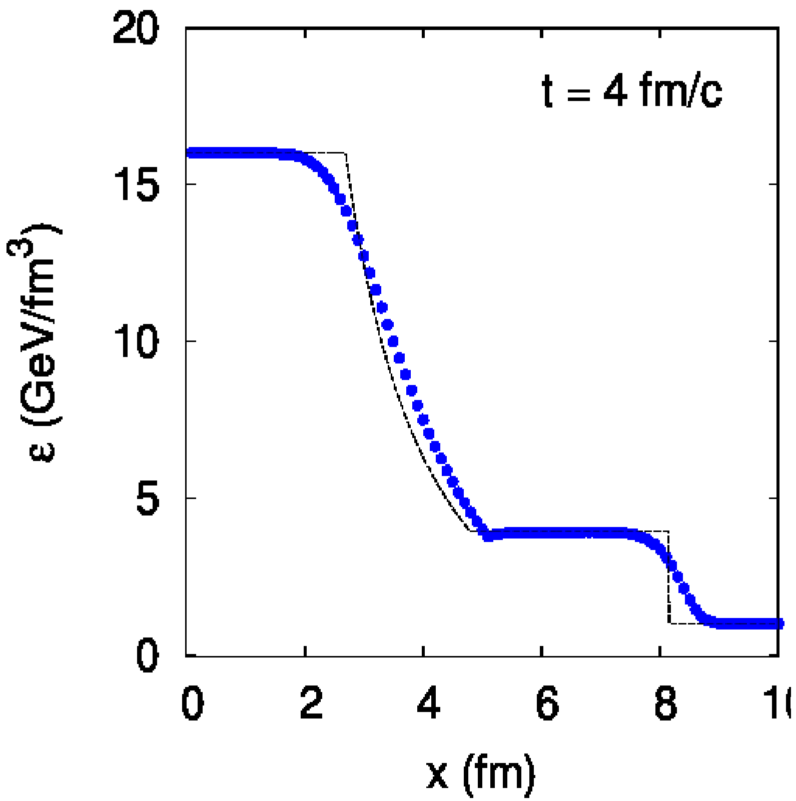
$$p_0 = 10^3 fm^{-4}, \delta p = 10^{-1} fm^{-4}, \lambda = 2 fm$$

- Shock tube problem: initial discontinuity in energy density

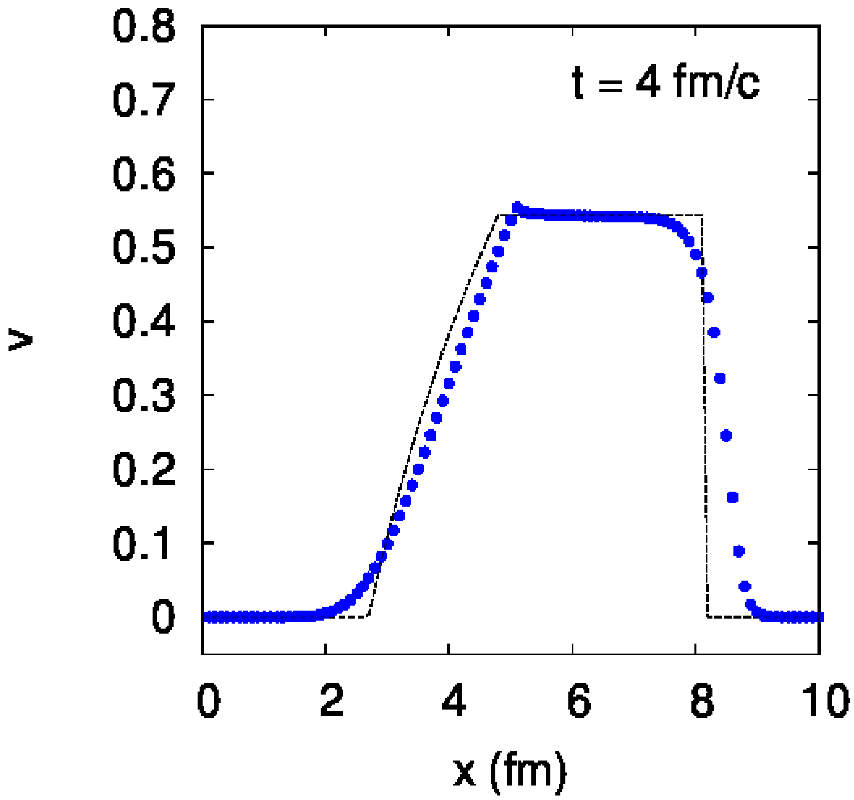
$$\varepsilon_L = 16\, GeV \cdot fm^{-3}, \varepsilon_R = 1\, GeV \cdot fm^{-3}$$

$$\lambda = 10\, fm, N_{cell} = 100, \Delta t = 0.04\, fm.c^{-1}$$

Shock tube problem

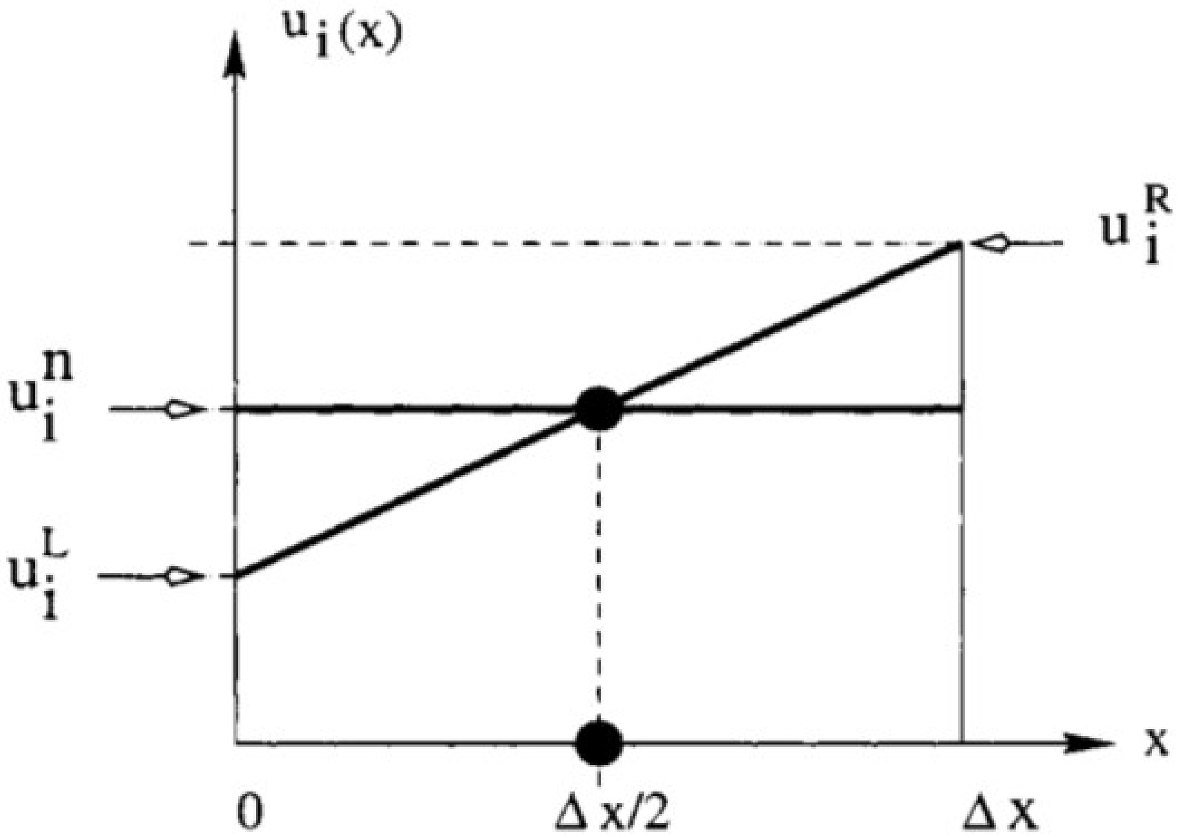


Energy profile after 100 steps



Velocity profile after 100 steps

Linear reconstruction



Piece-wise linear reconstruction of data in one cell

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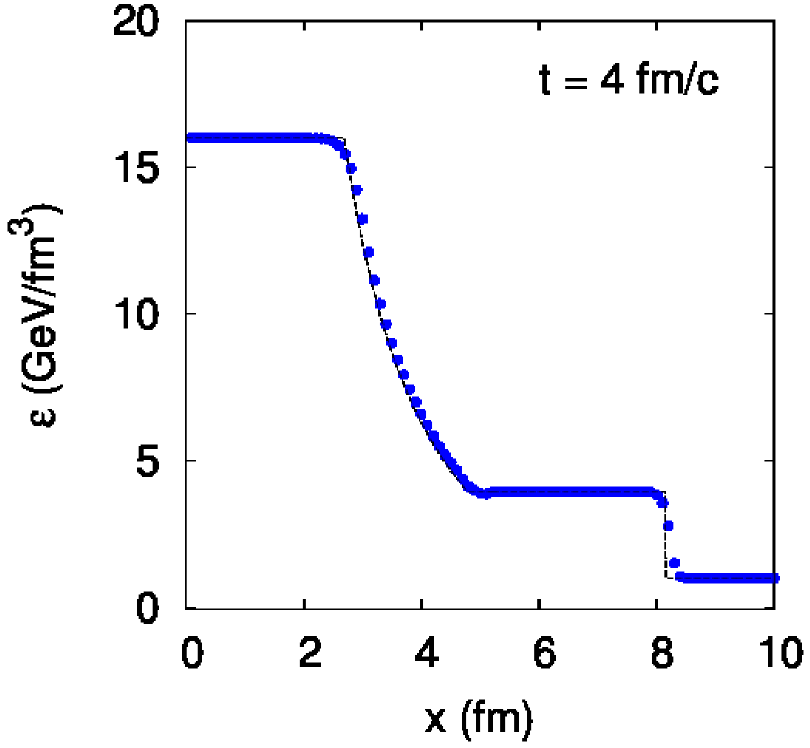
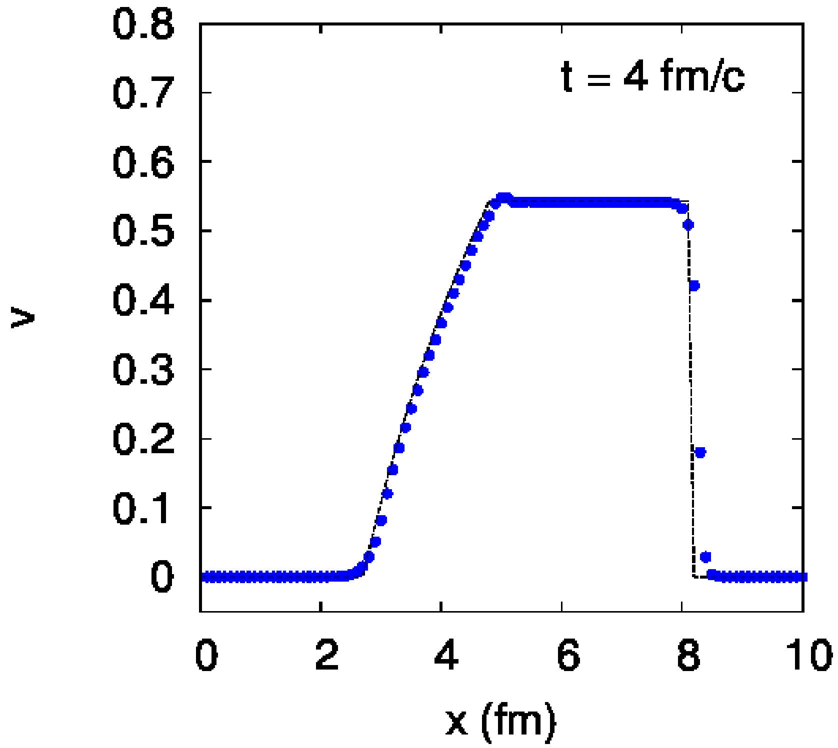
Shock tube problem



Energy profile after 100 steps



Velocity profile after 100 steps

Testing the scheme

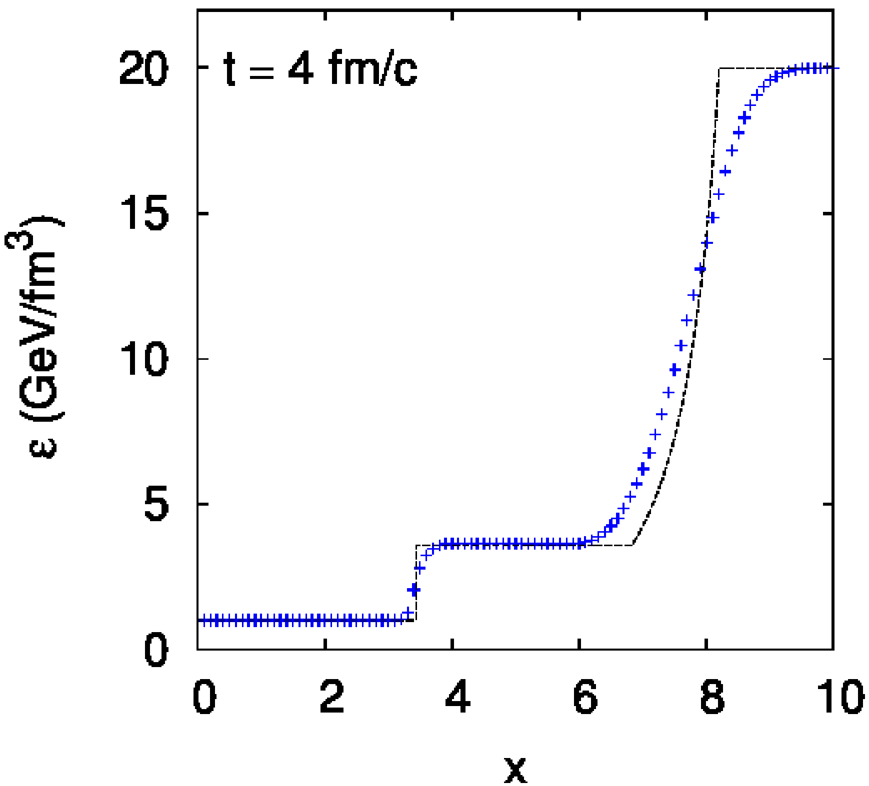
- Shock tube problem: initial discontinuity in energy density and tangential velocity

$$\varepsilon_L = 1\, GeV \cdot fm^{-3}, \varepsilon_R = 20\, GeV \cdot fm^{-3}$$

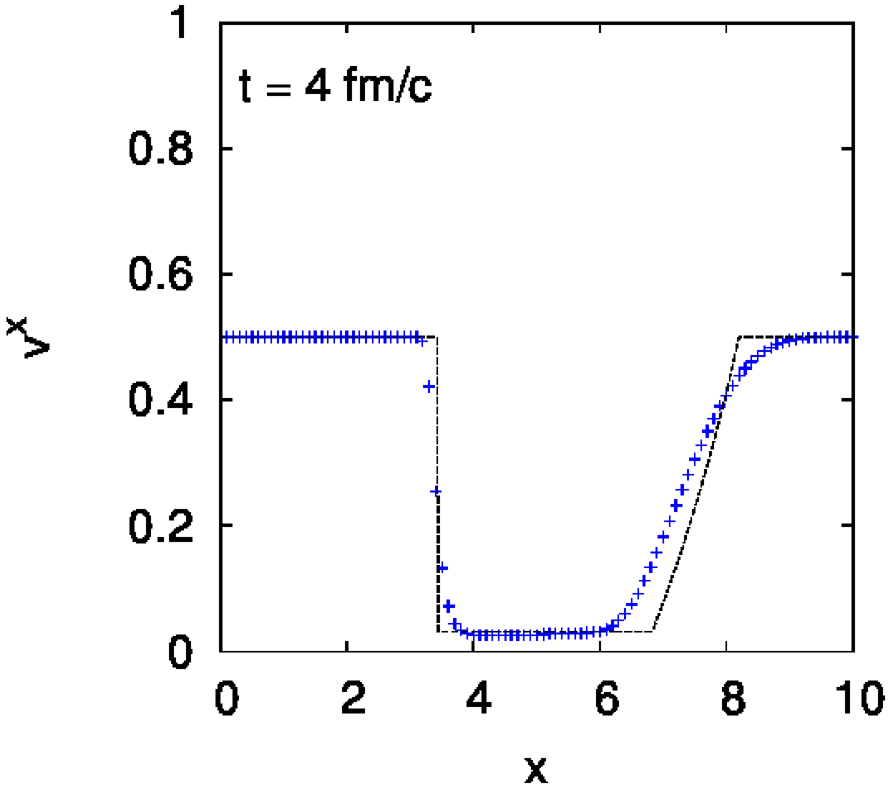
$$v_L^t = 1/3\, c\, , v_R^t = 1/2\, c$$

$$\lambda = 10\, fm\, , N_{cell} = 100, \Delta t = 0.04\, fm.c^{-1}$$

Shock tube problem

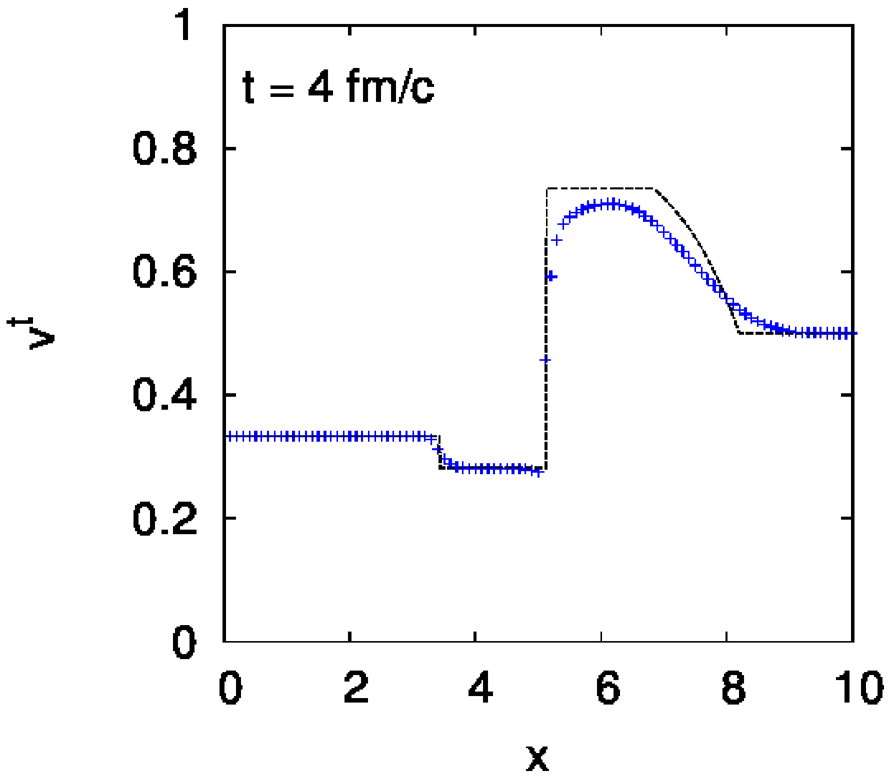


Energy profile after 100 steps



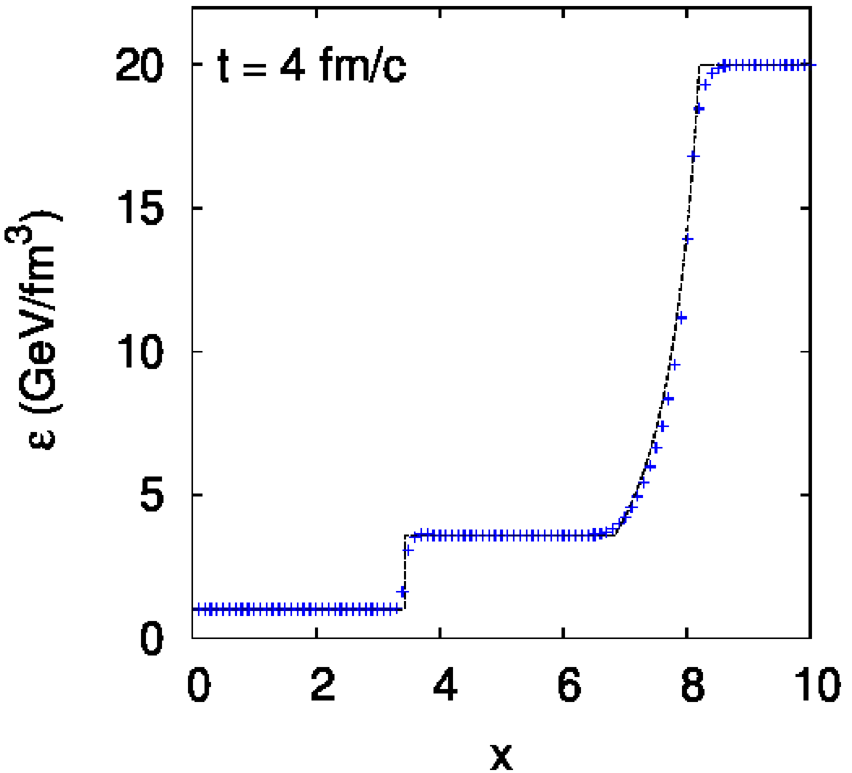
Normal velocity profile after 100 steps

Shock tube problem

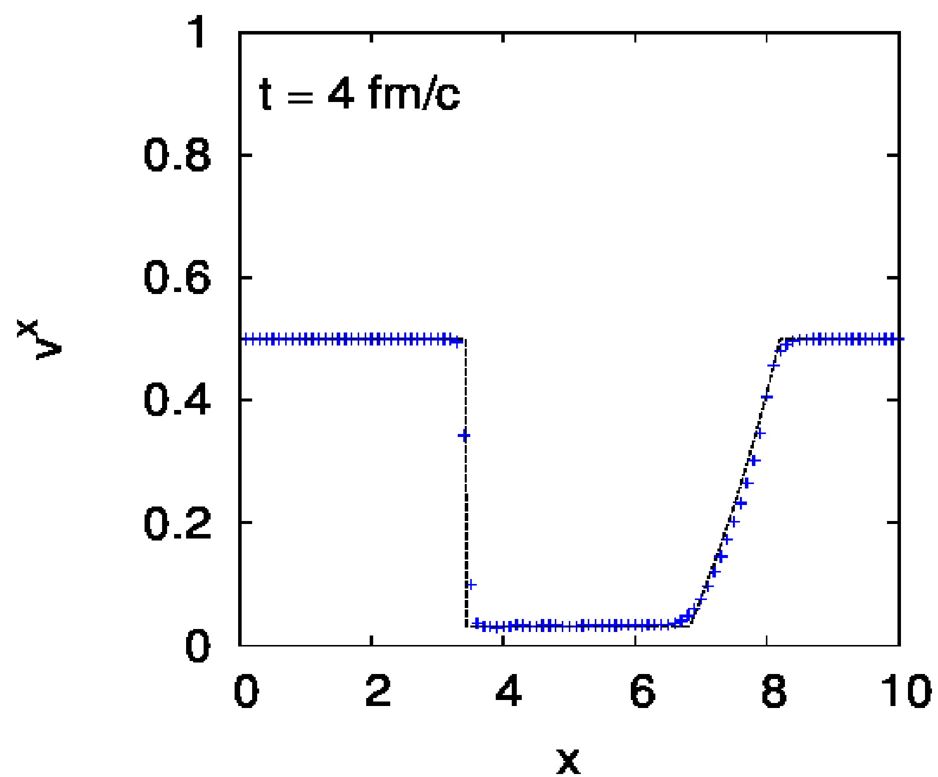


Tangential velocity profile after 100 steps

Shock tube problem

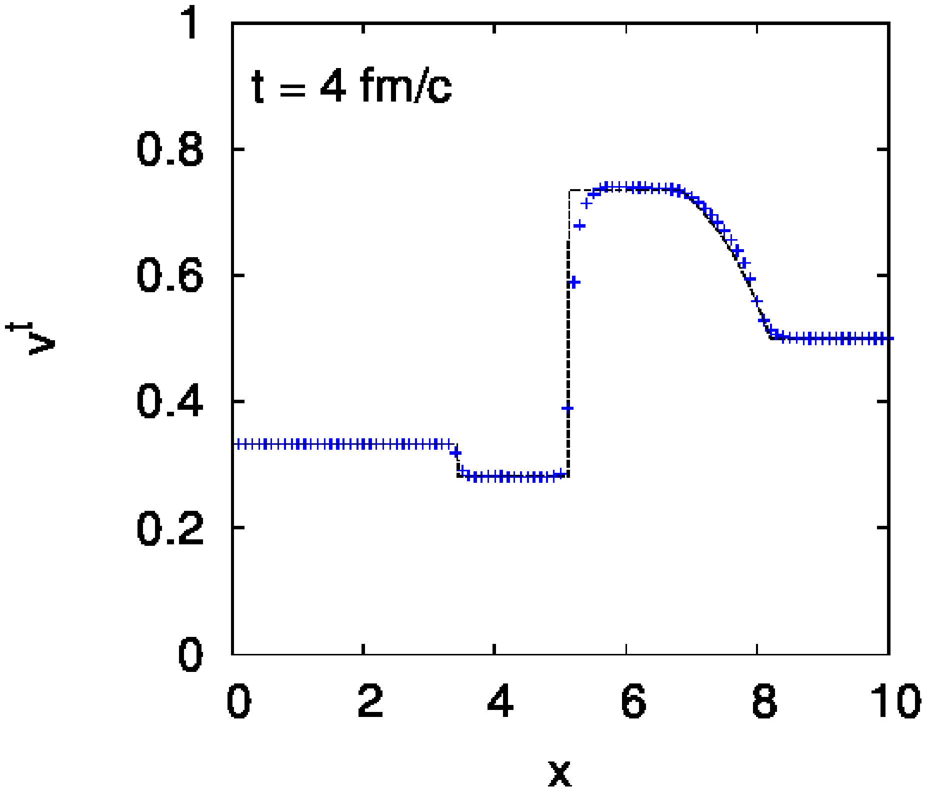


Energy profile after 100 steps



Normal velocity profile after 100 steps

Shock tube problem



Tangential velocity profile after 100 steps

Summary

- Ideal hydrodynamics code for quark-gluon plasma modeling
- Successfull testing in 1D
- Firsts tests in 3D
- Simulating jets penetrating the medium and the response of the medium to the energy deposited