Unquenching the three-gluon vertex

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Der Wissenschaftsfonds.

Doktoratskolleg Graz "Hadrons in Vacuum, Nuclei and Stars"

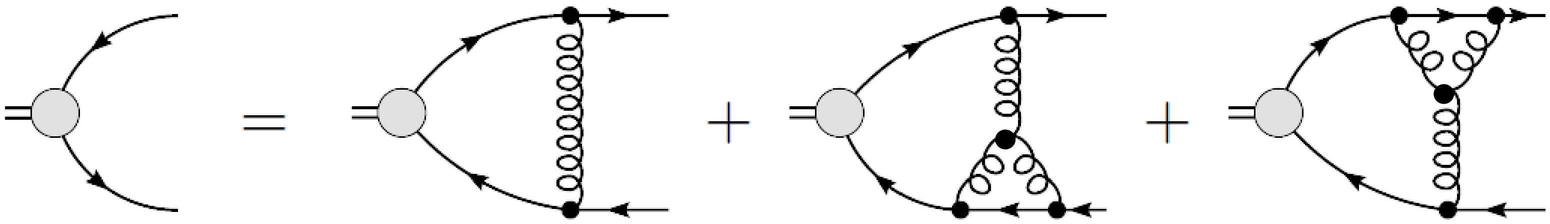
Hadron properties from first principles

- hadron properties (masses, decay constants …) depend on strong interaction between their constituents and can be described by DSE + BSE

- Rainbow-Ladder approximation uses effective interaction (only tree-level structure of quark-gluon vertex)

  ➡ Beyond Rainbow-Ladder

- for example:



Taken from C.S. Fischer and R. Williams, Phys. Rev. Lett. 103, 122001

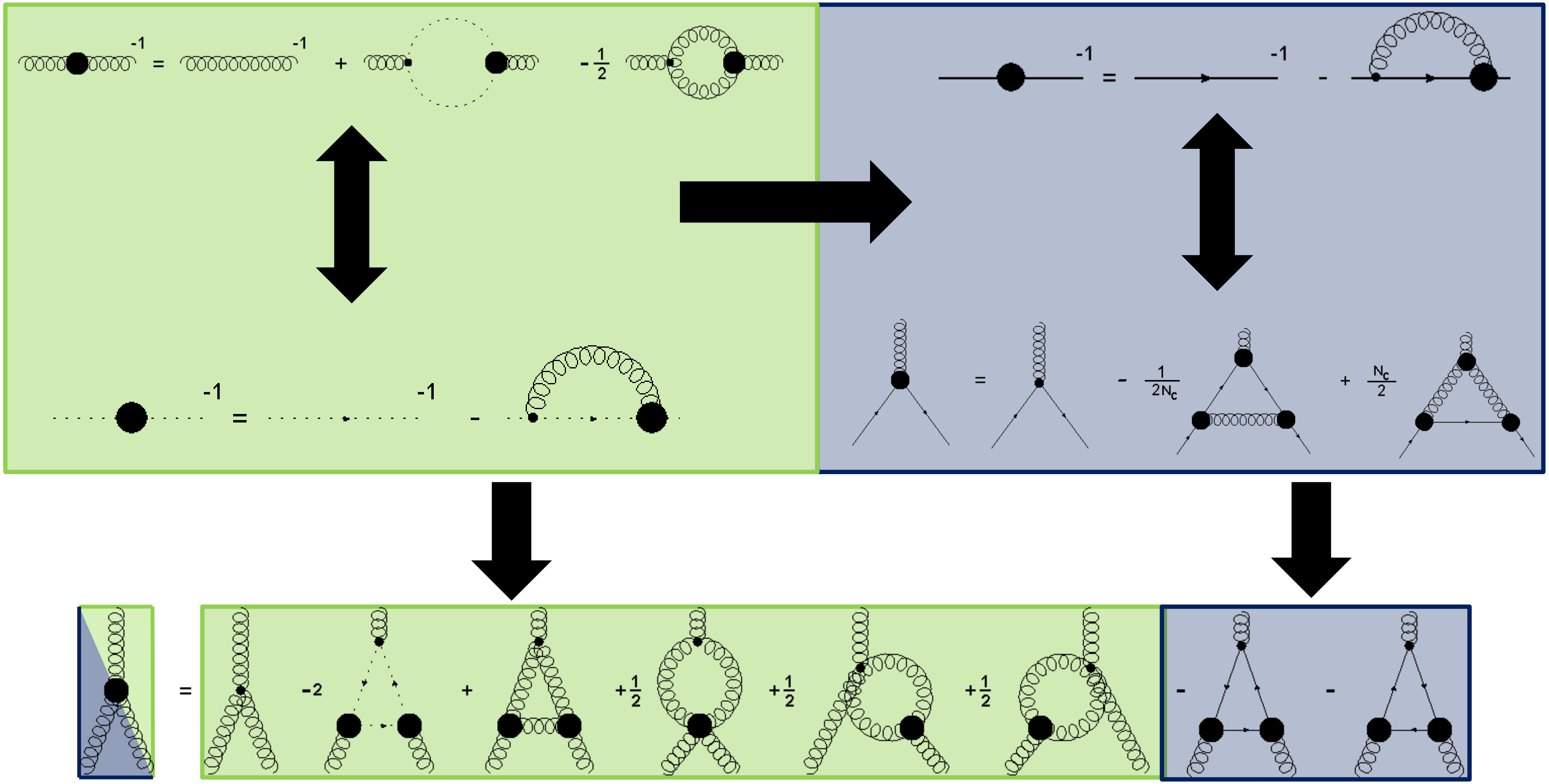
- three-gluon vertex required. YM sector results in **(Blum et al. 2014, Eichmann et al. 2014)**

  ➡ unquenching

The Dyson-Schwinger approach to correlation functions within a self-consistent calculation (in Landau gauge)

	Yang-Mills Sector	Unquenching	Quark Sector
Propagators	gluon propagator ghost propagator (1997, 2008)	gluon propagator (2003, 2012)	quark propagator
3 point functions	ghost-gluon vertex (2004, 2013) three-gluon vertex (2014)		quark-gluon vertex (2014)
4 point functions	four-gluon vertex (2014)		

Picture of Contents



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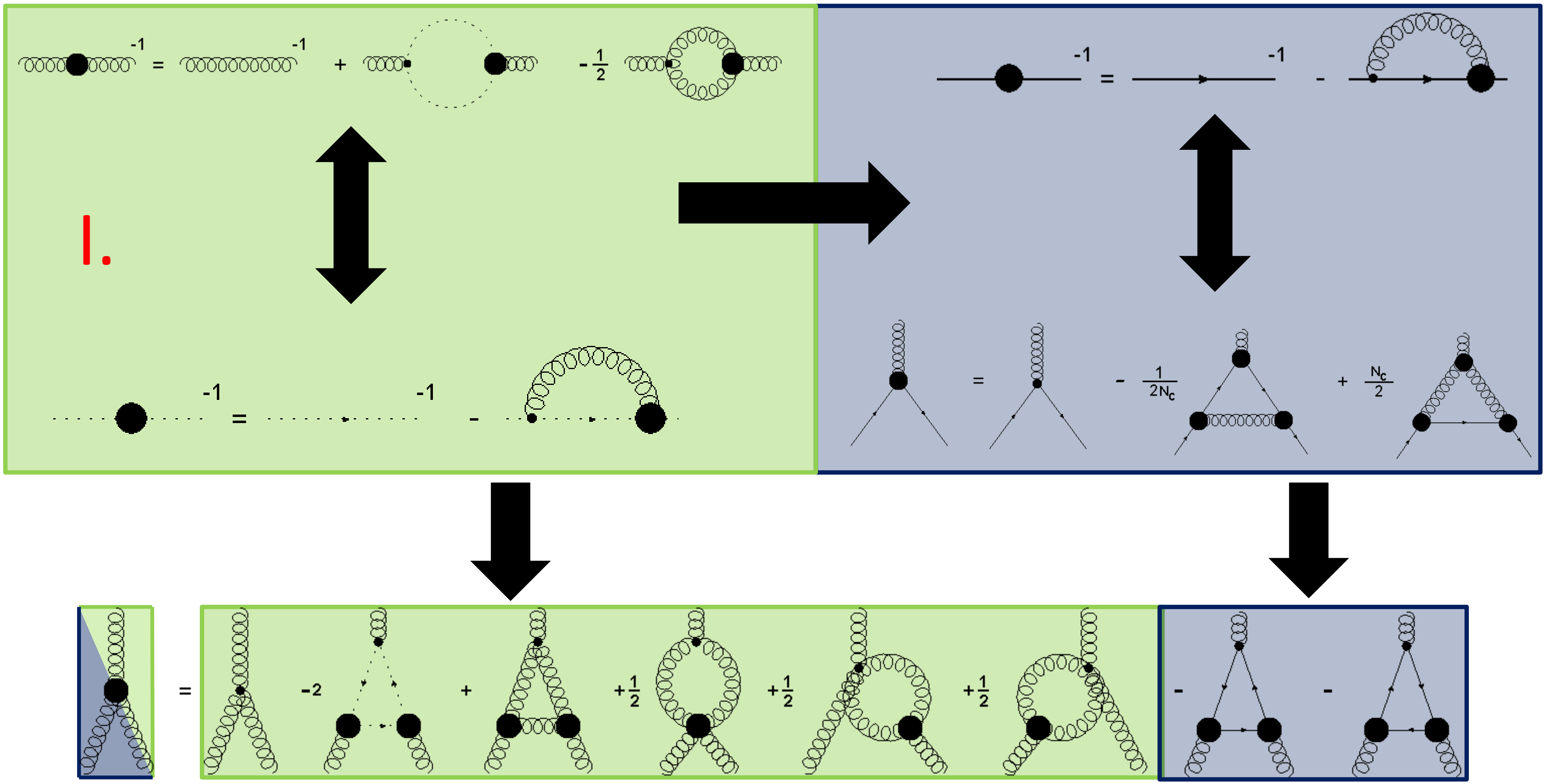

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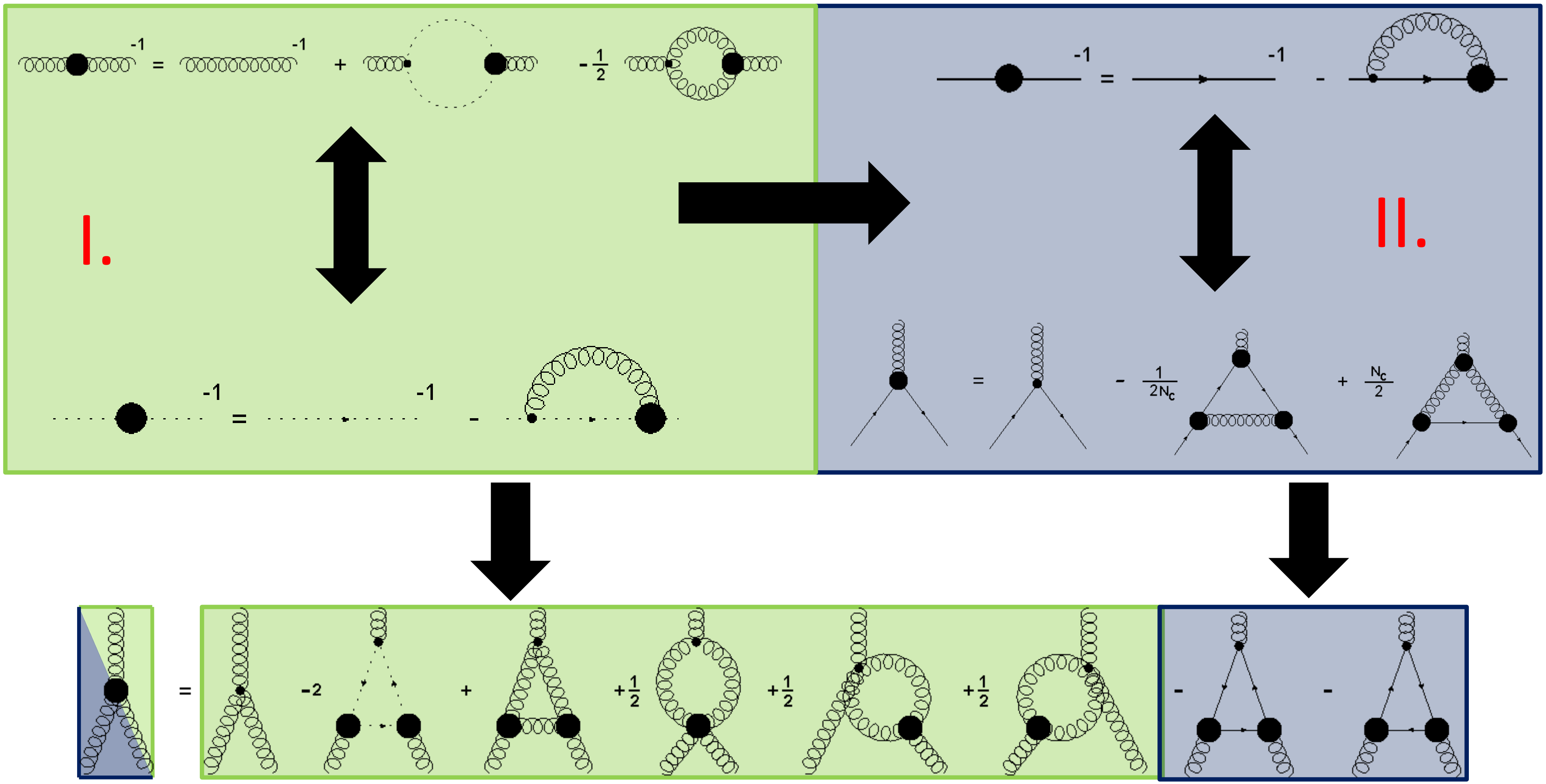

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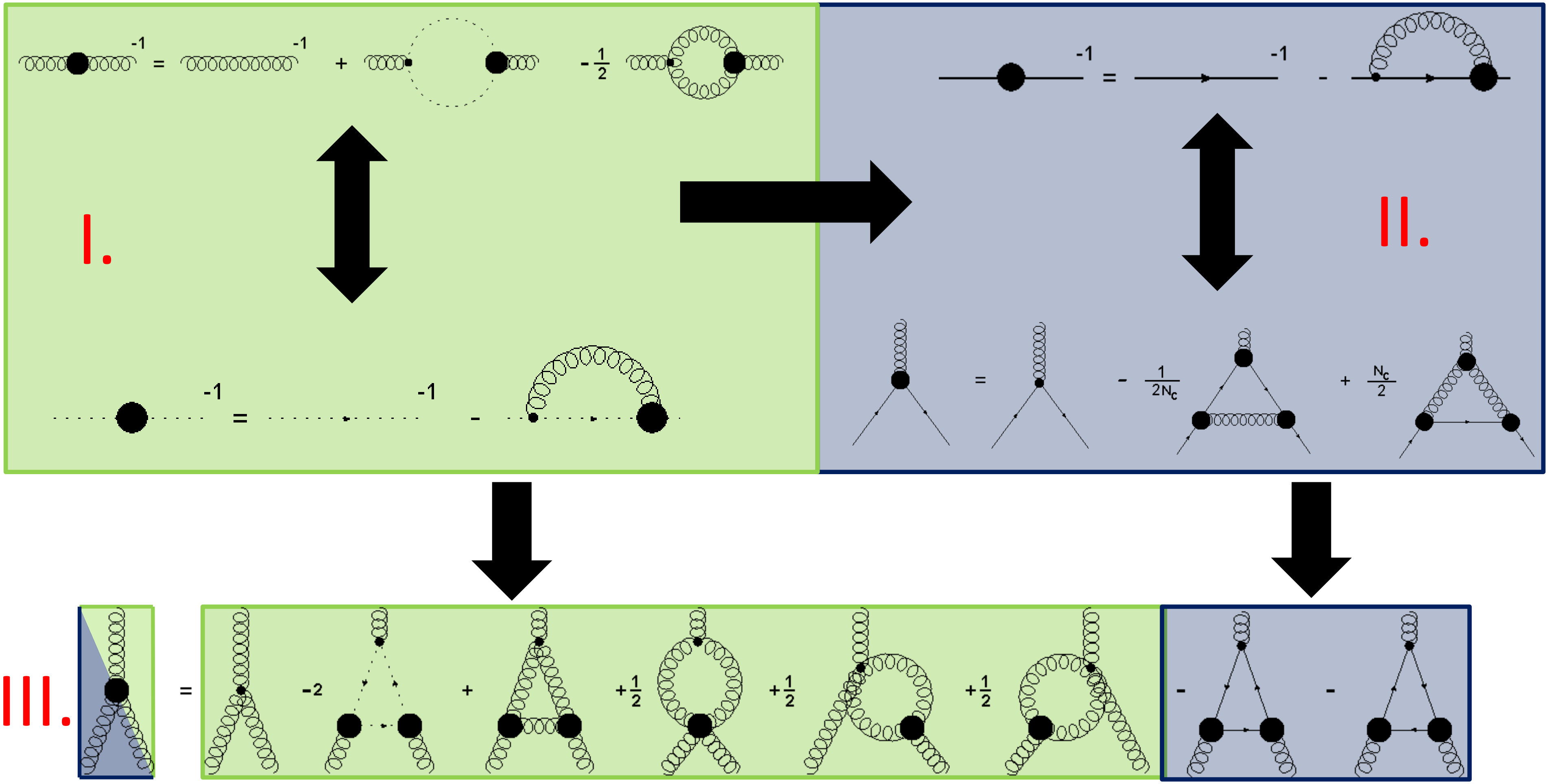
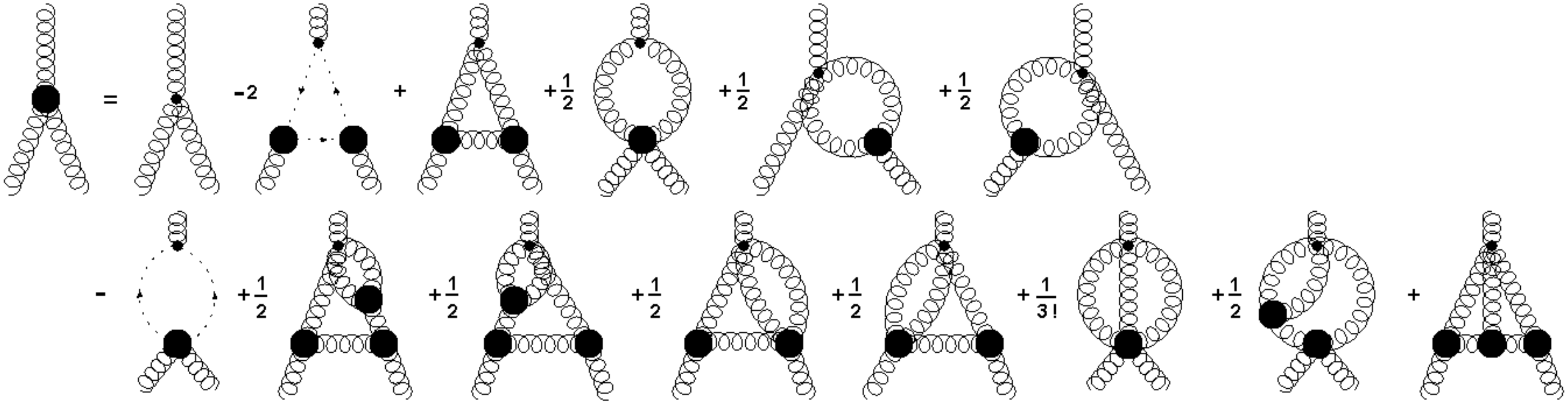


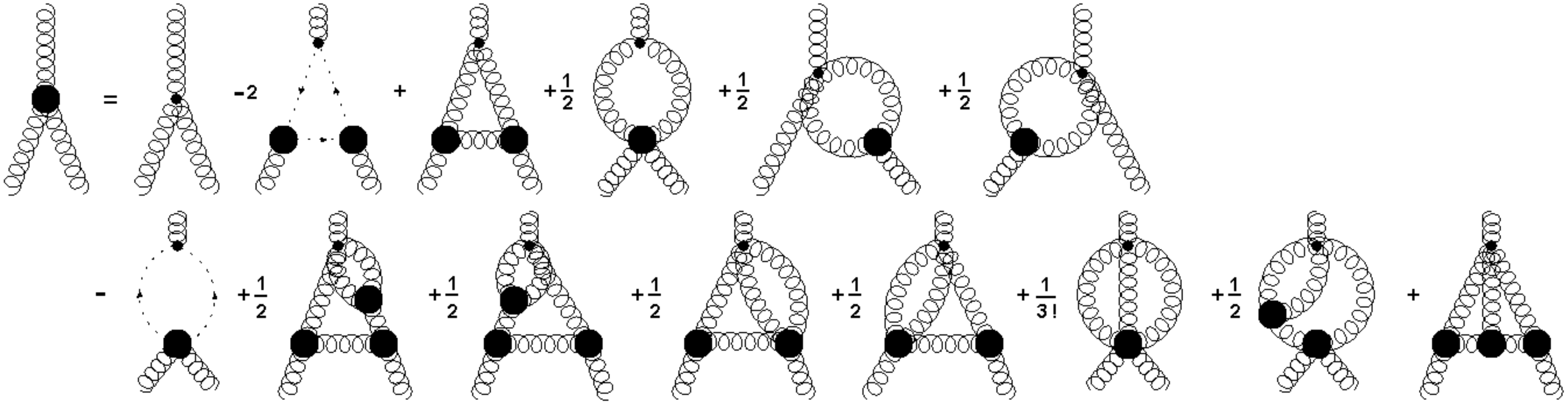
I. The three-gluon vertex DSE in Yang-Mills Theory

The three-gluon vertex DSE



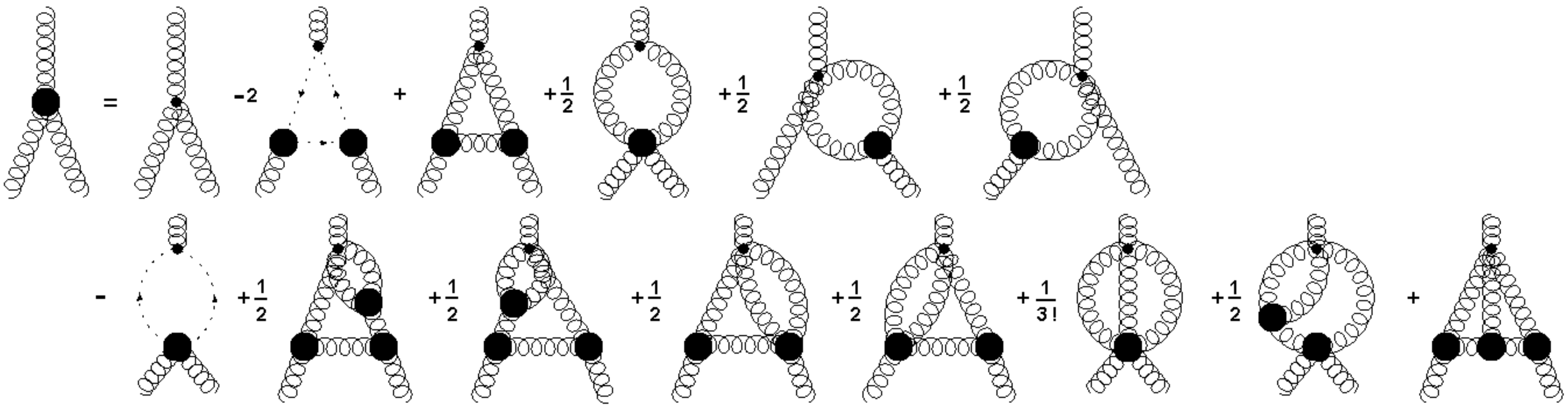
- thick blobs: dressed vertices
- thin blobs: bare vertices
- **all internal lines are dressed !**

The three-gluon vertex DSE

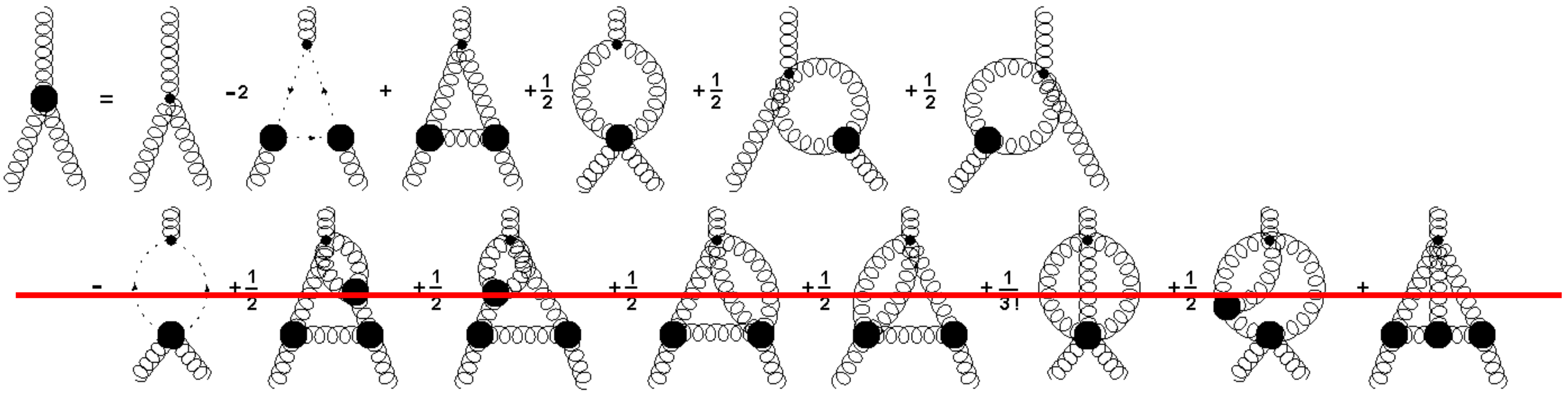


truncation

The three-gluon vertex DSE

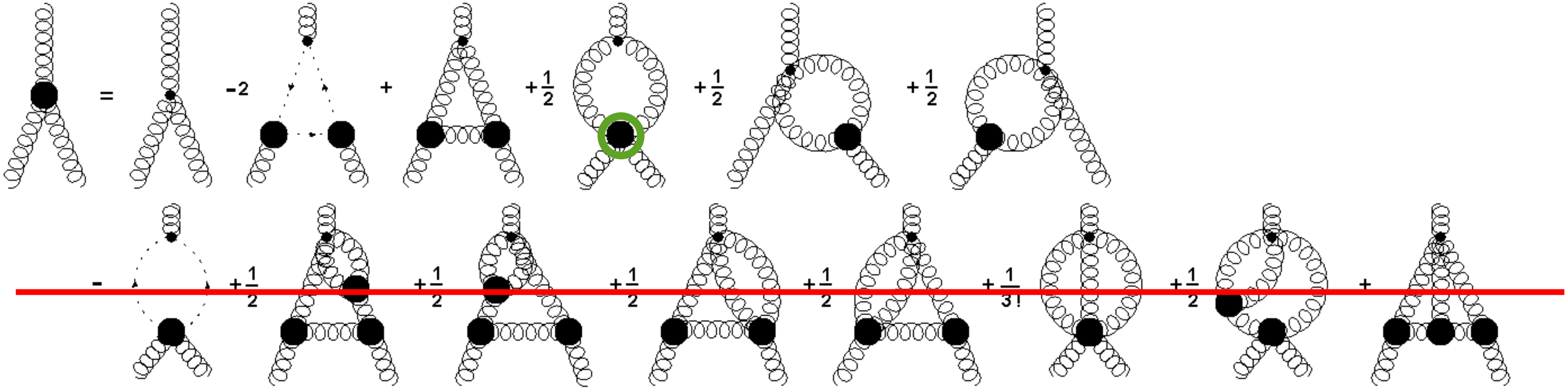


The three-gluon vertex DSE



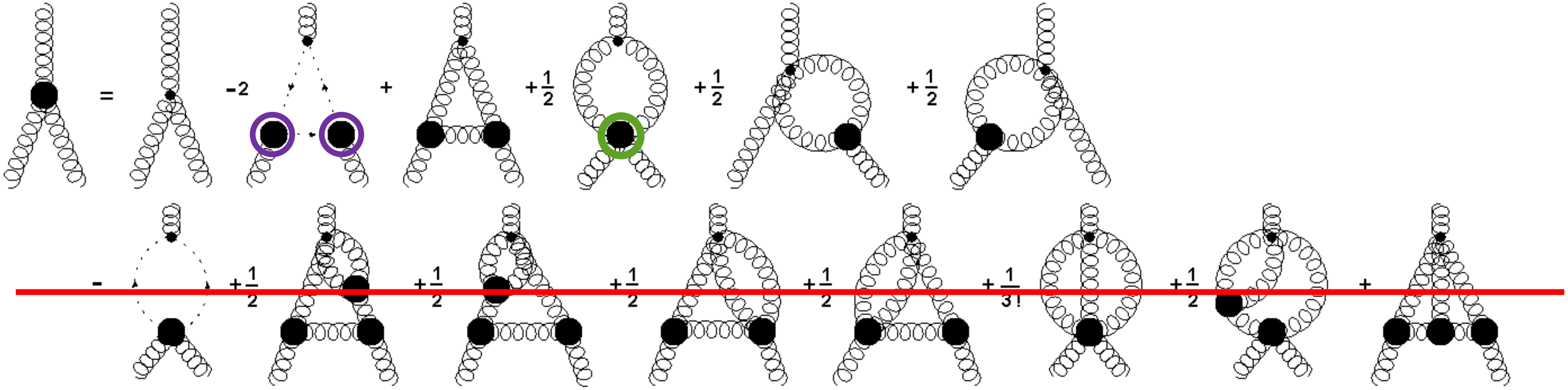
- neglect all 2-loop diagrams + vertices without tree-level counterpart

The three-gluon vertex DSE



- neglect all 2-loop diagrams + vertices without tree-level counterpart
- dressed 4-gluon vertex ⟶ model

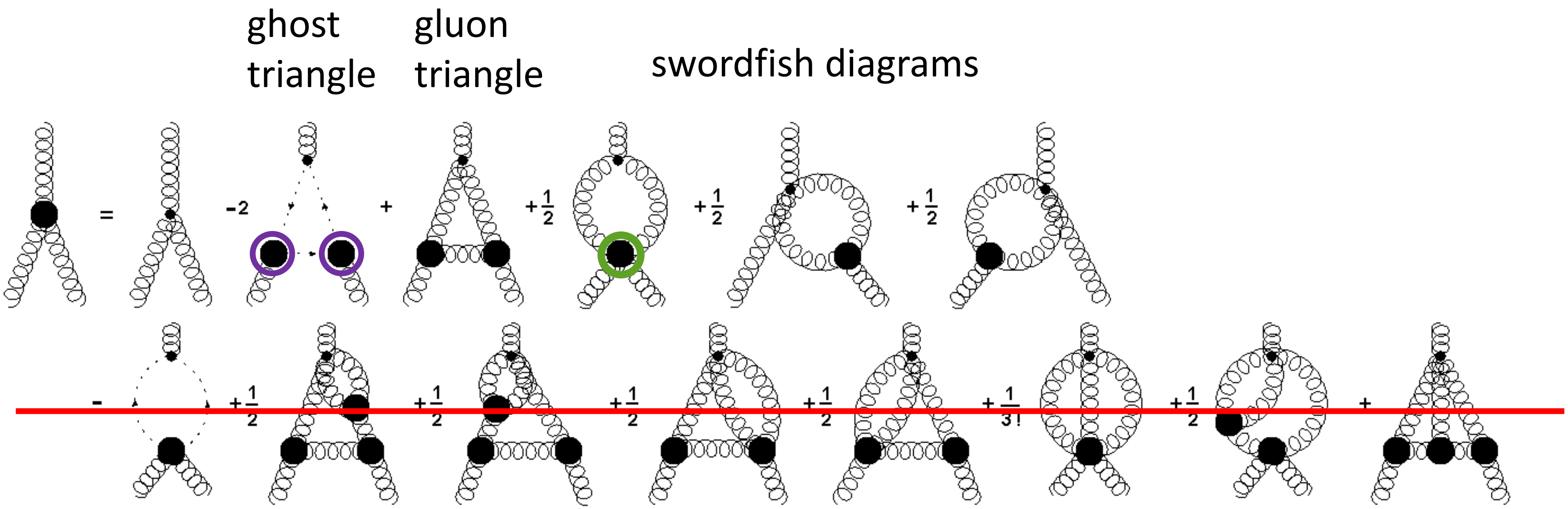
The three-gluon vertex DSE



- neglect all 2-loop diagrams + vertices without tree-level counterpart
- dressed 4-gluon vertex ⟶ model
- dressed ghost-gluon vertex ⟶ input from solved gh-gl vertex DSE

(M.Q. Huber and L. von Smekal JHEP 1304 (2013) 149)

The three-gluon vertex DSE



- neglect all 2-loop diagrams + vertices without tree-level counterpart
- dressed 4-gluon vertex → model
- dressed ghost-gluon vertex → input from solved gh-gl vertex DSE

(M.Q. Huber and L. von Smekal JHEP 1304 (2013) 149)

Dressed propagators and vertices in Landau gauge

dressed propagators:

- **ghost propagator:** $D^G(p^2) = -\frac{G(p^2)}{p^2}$
- **gluon propagator:** $D_{\mu\nu}(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$

$G(p^2)$, $Z(p^2)$ = dressing functions

Dressed propagators and vertices in Landau gauge

dressed vertices:

In Landau gauge the full dynamics of the theory are described by the transverse part[1]

ghost-gluon vertex: $\Gamma_{\mu}^{A\bar{c}\,c}(k;p,q) = ig\ A(k;p,q)p_{\mu} + \text{long. terms}$

three-gluon vertex: 4 transverse + 10 longitudinal tensors

$$\Gamma_{\mu\nu\varrho}^{A^3}(p,q,k) = \sum_{i=1}^{4} F_i\,(p,q,k)\tau_{\mu\nu\rho}^{(i)} \ \text{+ long. terms}$$

four-gluon vertex: we employ a model

1: C.S. Fischer, A. Maas and J. M. Pawlowski, *Annals Phys.* 324 (2009) 2408

Dressed propagators and vertices in Landau gauge

the four-gluon vertex model:

- cancellations between gluon-triangle and swordfish diagrams
- model must take into account the balance between these diagrams

➡ especially strength in **midmomentum** regime important

- we make the following ansatz:

$$\Gamma^{A^4,abcd}_{\mu\nu\varrho\sigma}(p,q,k,r) = (a\tanh(b/\overline{p}^2_{A^4}) + 1)D^{A^4,UV}(p,q,k,r)\Gamma^{(0)A^4,abcd}_{\mu\nu\varrho\sigma}(p,q,k,r)$$

$$\overline{p}^2_{A^4} = (p^2+q^2+k^2+r^2)/2$$

- parameters a,b can be varied ➡ produces a band of solutions

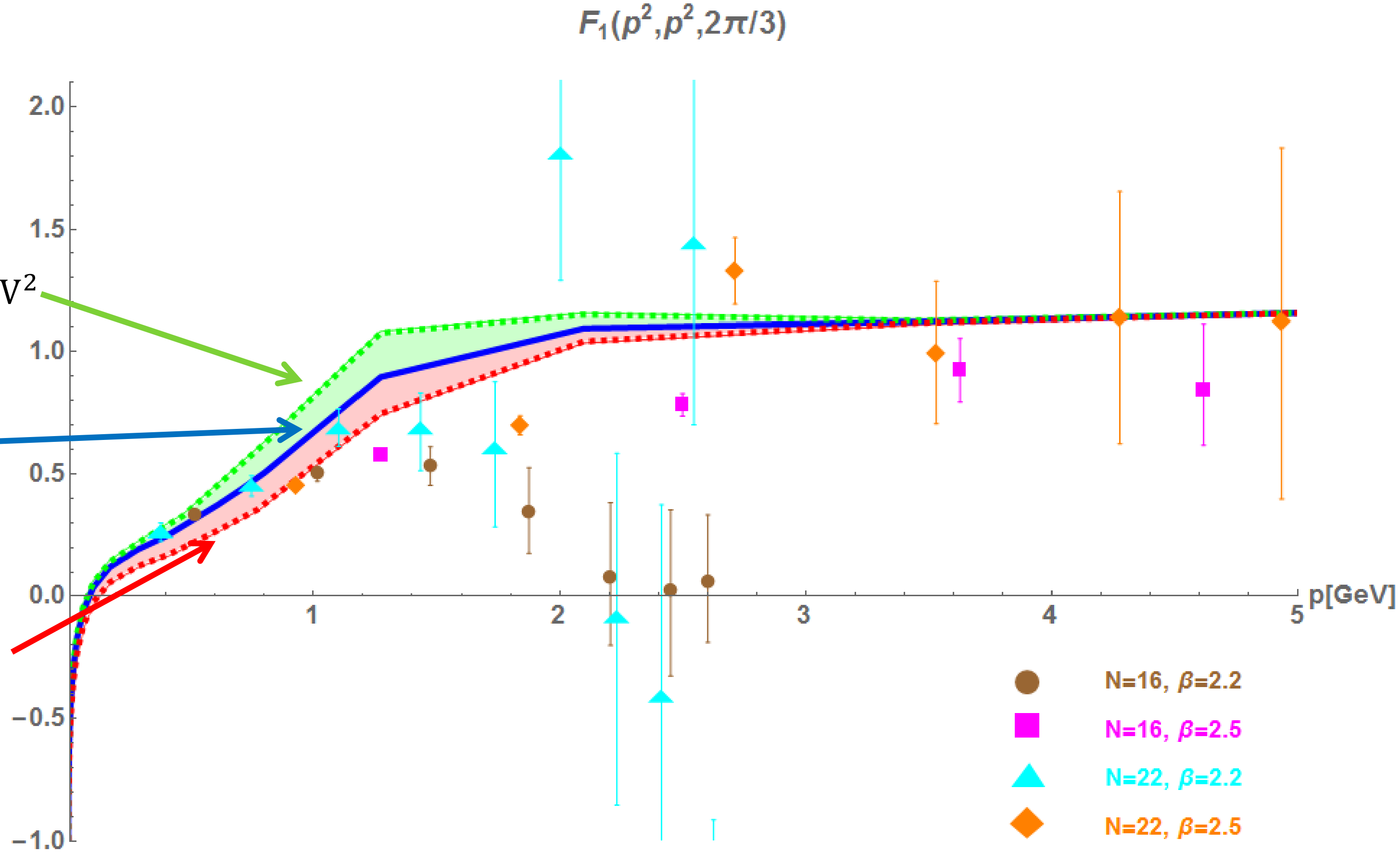
A. Blum, M. Q. Huber, M. Mitter and L. von Smekal, Phys. Rev. D 89, 061703 (2014)

comparison with lattice data[1]

upper band: a=1.5, b=(1.48-1.97) GeV^2

a=1.5, b=1.97 GeV^2

lower band: a=1.5-2 b=1.97 GeV^2



1: A. Cucchieri, A. Maas and T. Mendes, Phys. Rev. D77 (2008) 094510.

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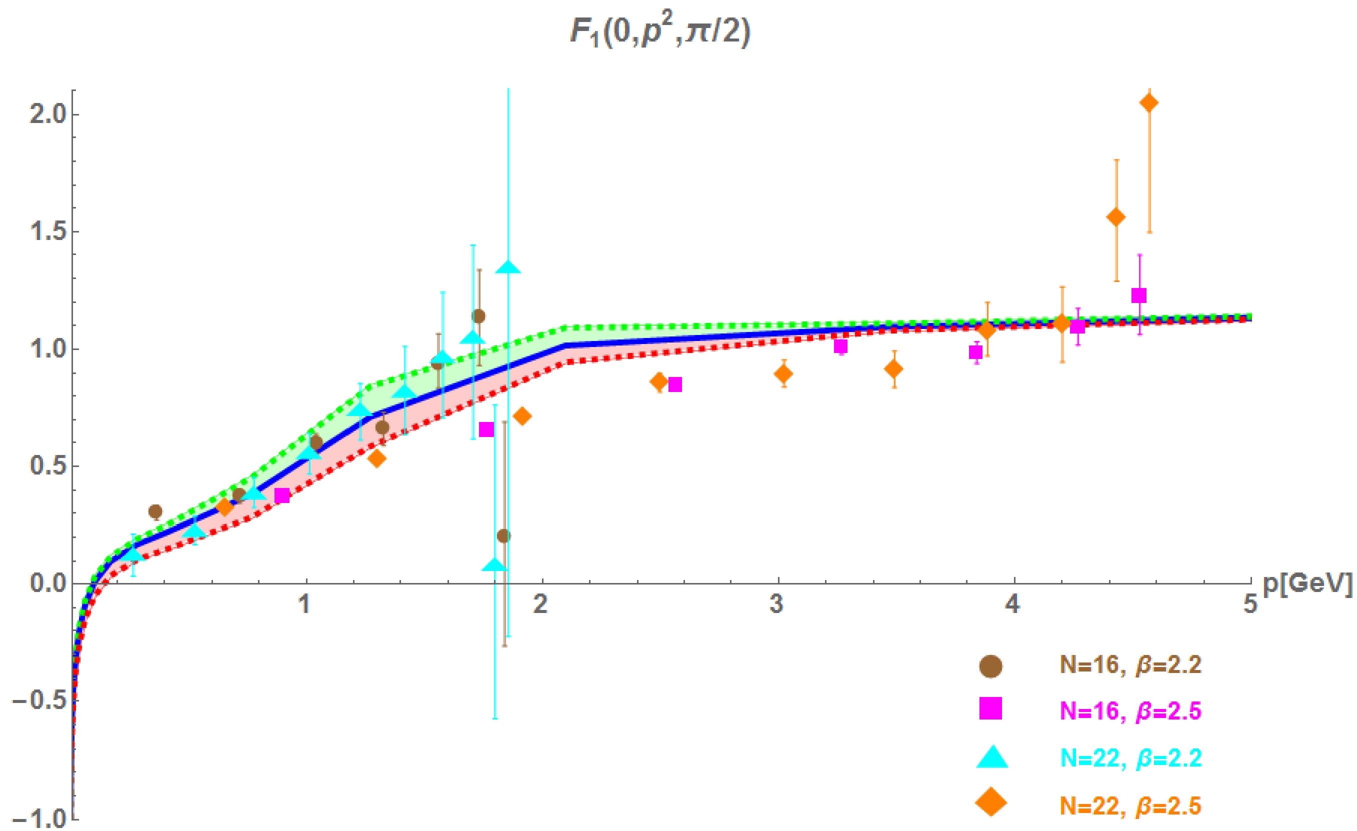

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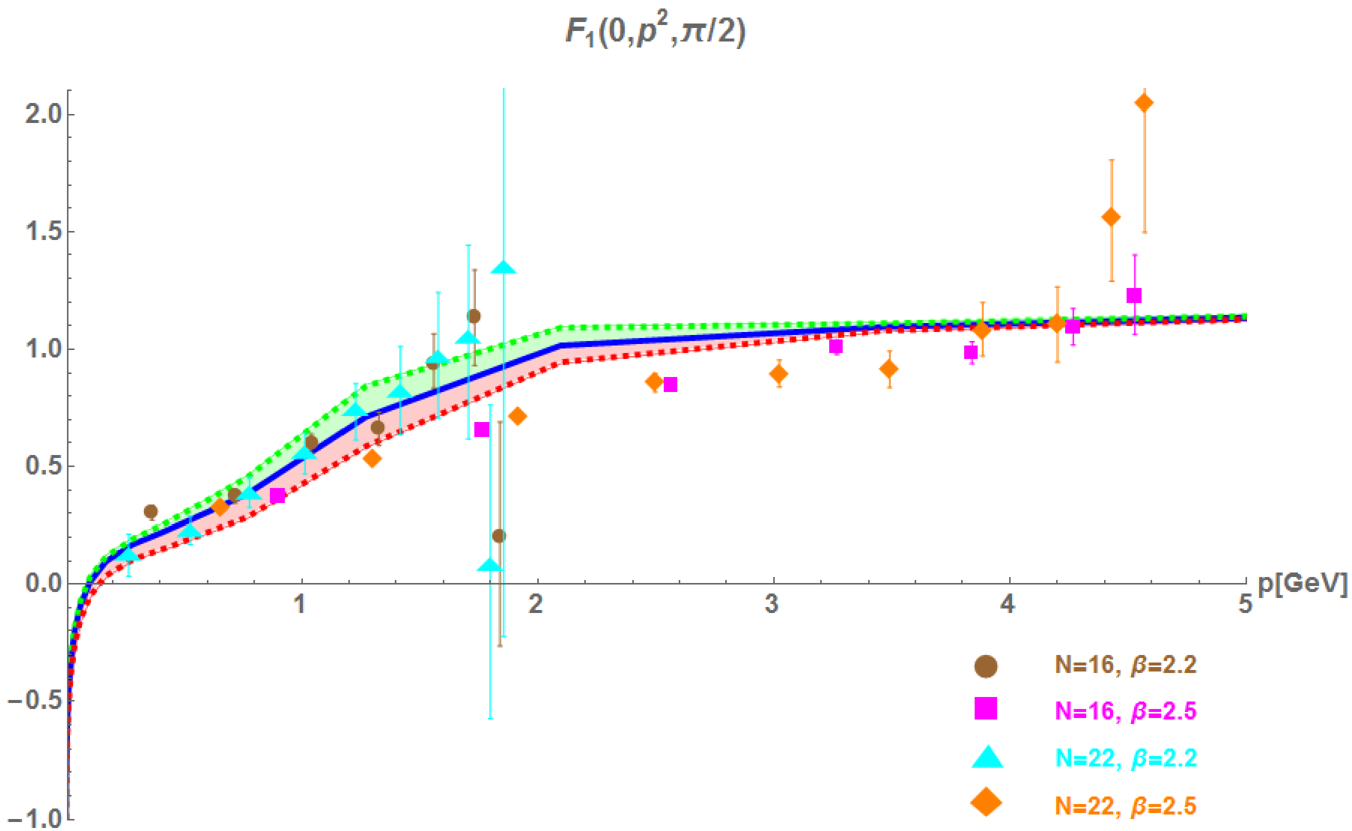
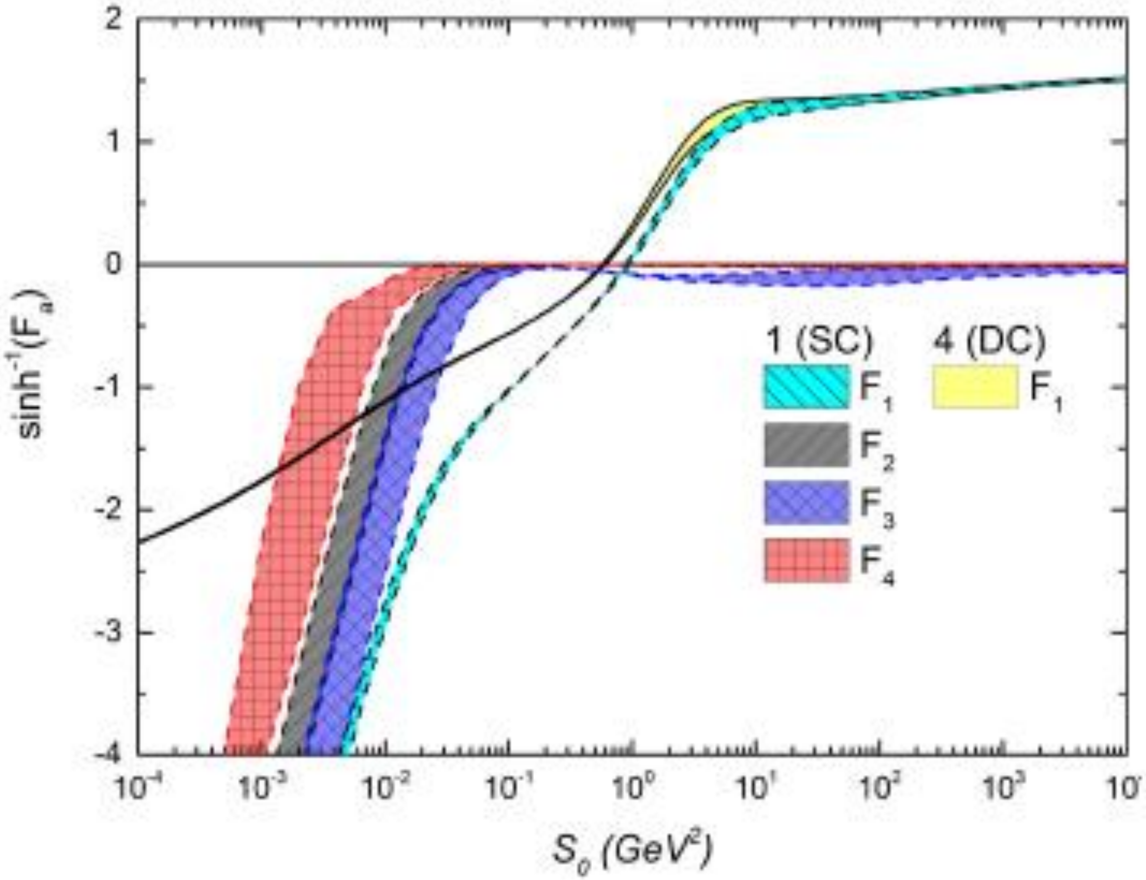
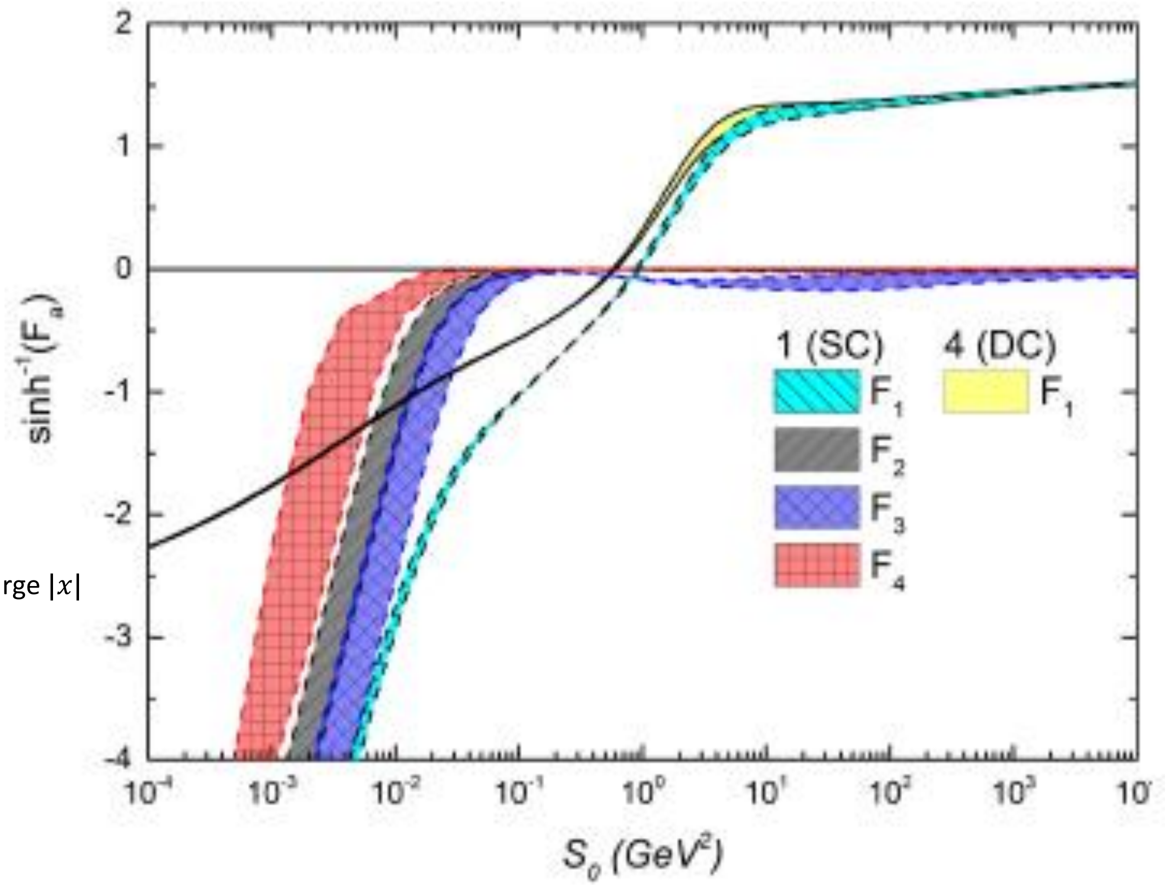


Results are in very good agreement with lattice data
Hints at dominance of tree-level dressing

G. Eichmann, R. Williams, R. Alkofer and M. Vujinovic, Phys. Rev. D 89, 105014 (2014): All Tensors



G. Eichmann, R. Williams, R. Alkofer and M. Vujinovic, Phys. Rev. D 89, 105014 (2014): All Tensors



$\sinh^{-1}(x) \approx x$ for x≤ 1

$\sinh^{-1}(x) \approx \log(x)$ for large $|x|$

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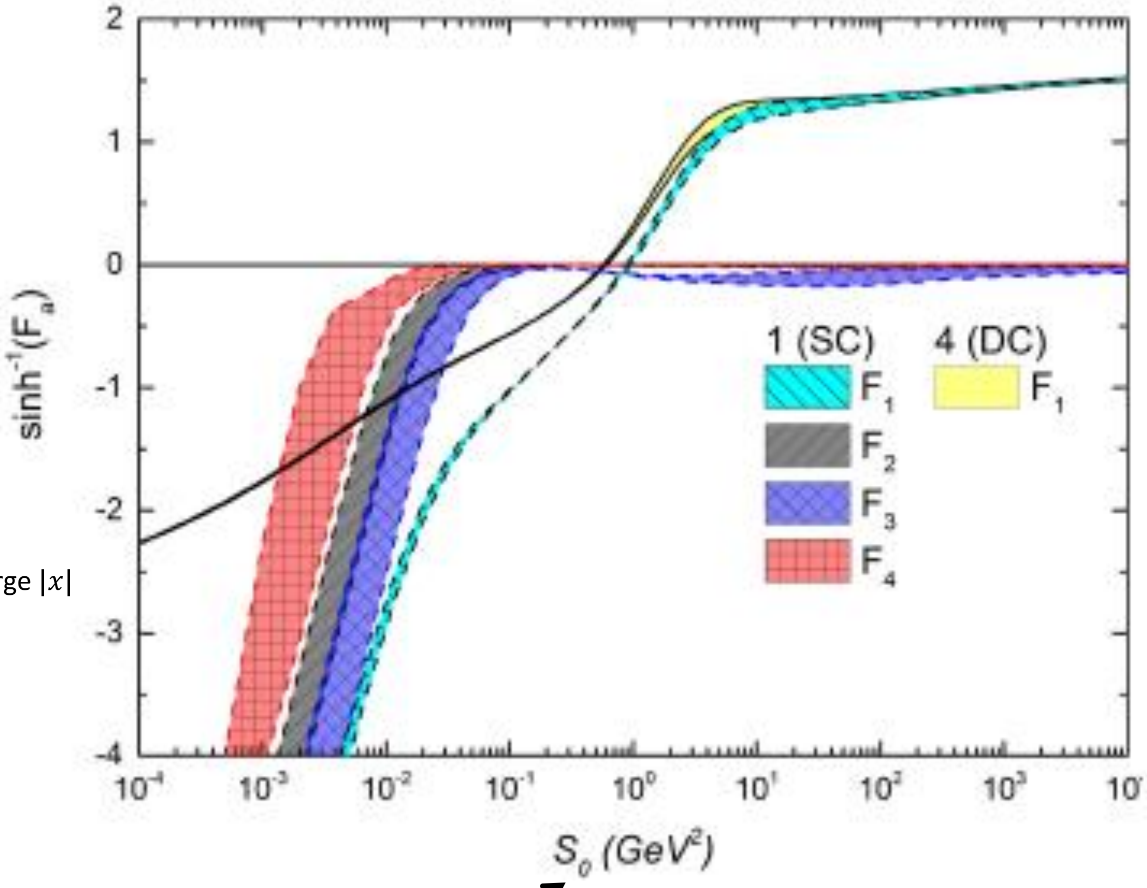
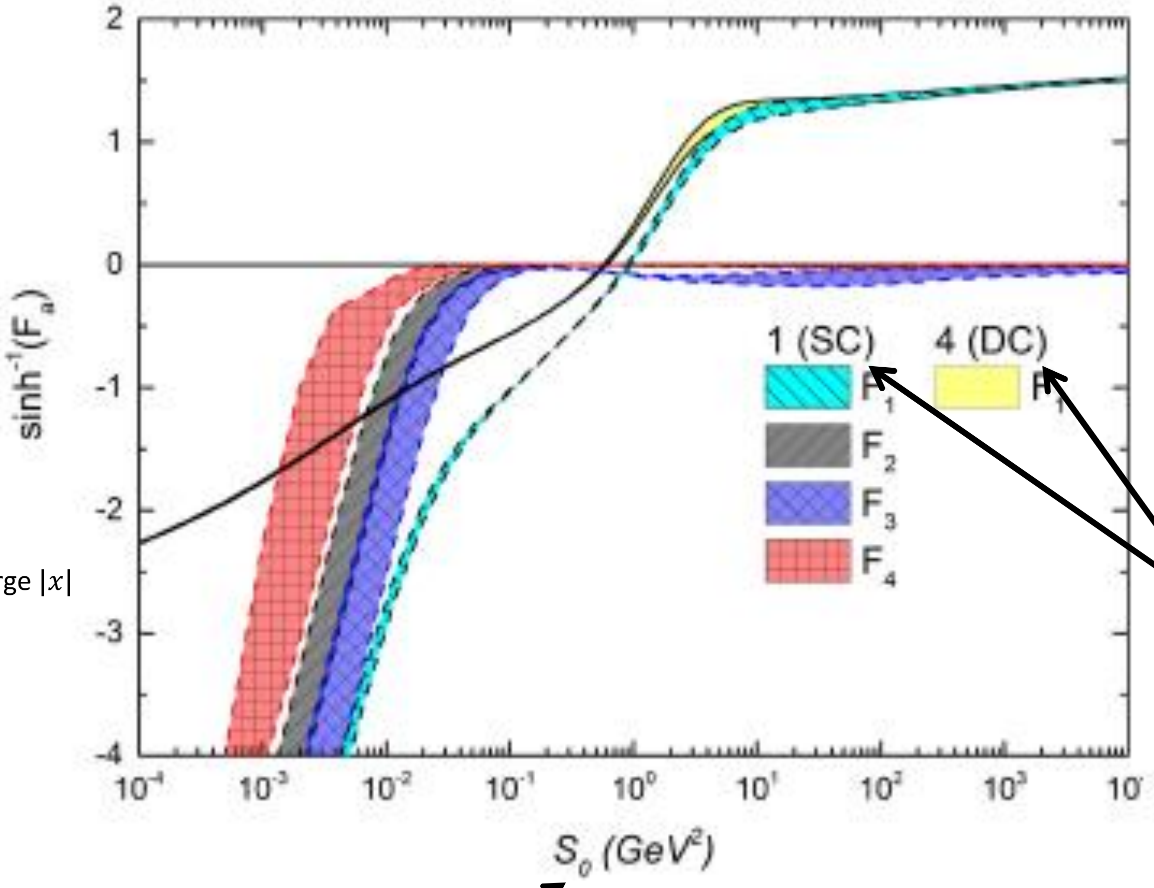


$\sinh^{-1}(x) \approx x$ for x≤ 1

$\sinh^{-1}(x) \approx \log(x)$ for large $|x|$

$S_0 = \frac{1}{3}(p^2 + q^2 + \sqrt{p^2}\sqrt{q^2}\cos\alpha)$

G. Eichmann, R. Williams, R. Alkofer and M. Vujinovic, Phys. Rev. D 89, 105014 (2014): All Tensors



$\sinh^{-1}(x) \approx x$ for x≤ 1

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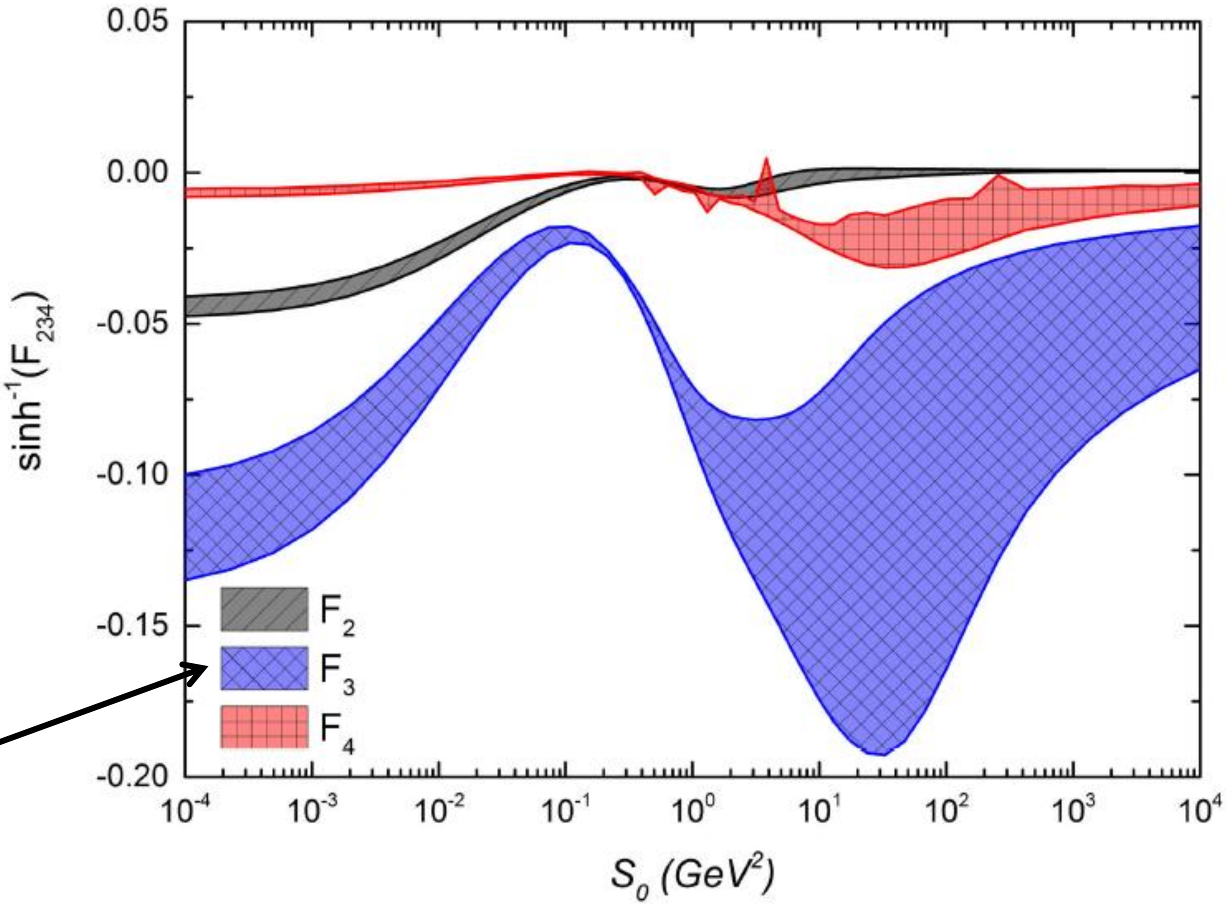
two families of solutions: decoupling and scaling

$S_0 = \frac{1}{3}(p^2 + q^2 + \sqrt{p^2}\,\sqrt{q^2}\,\cos\alpha)$

G. Eichmann, R. Williams, R. Alkofer and M. Vujinovic, Phys. Rev. D 89, 105014 (2014) : All Tensors



decoupling

Features of the three-gluon vertex

- three-gluon vertex features a zero crossing
- dominant contribution stems from dressing function F_1
  F_1 corresponds to tree-level tensor structure
- calculation of F_1 in good agreement with lattice data
- truncation scheme reliable

II. The quark-gluon vertex

The quark-gluon vertex DSE

based on the work by Andreas Windisch and Markus Hopfer

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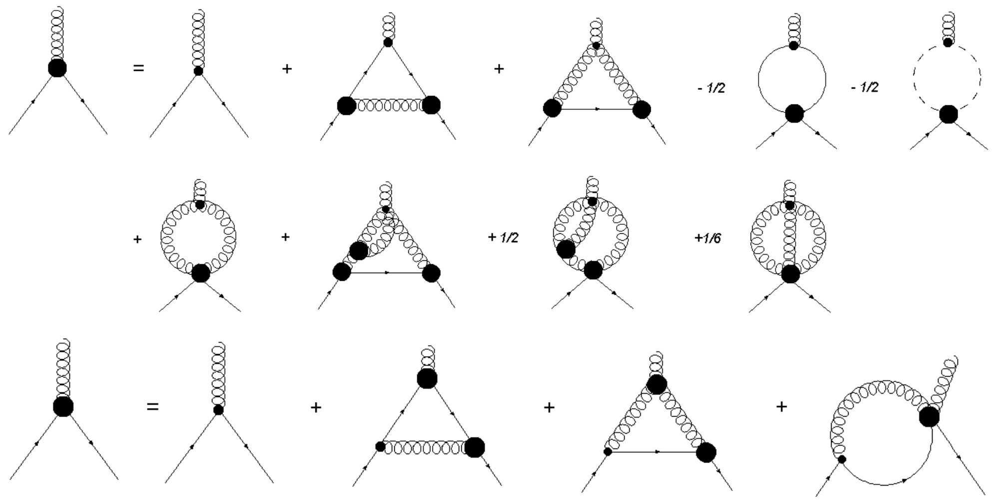
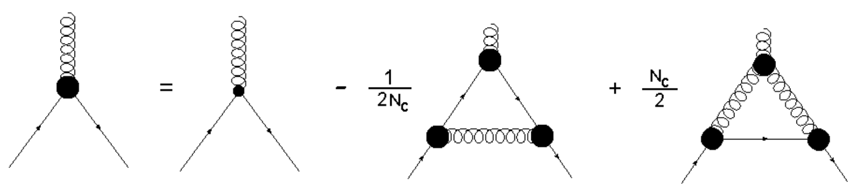


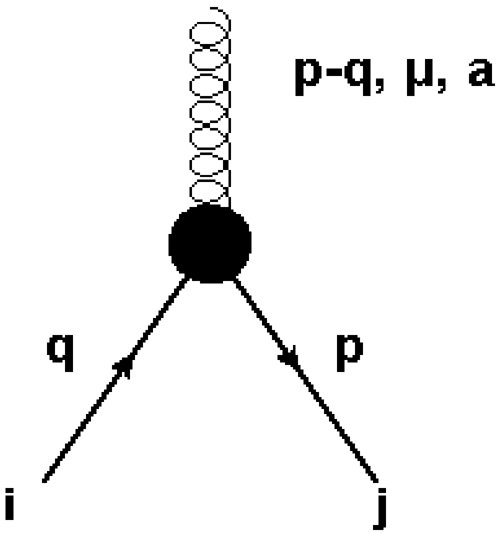
DSE



3PI

The contribution of two-quark-two-gluon scattering kernel is sizeable

The quark-gluon vertex DSE



- the quark-gluon vertex can be decomposed into 12 basis tensors

$$\bar{\Gamma}_{\mu}^{qgv}(p,q;p-q\ ) = \sum_{i=1}^{12} g_i\,(p,q;p-q)\lambda_{\mu}^{(i)}$$

- Naive Basis:
  - simple
  - all 12 tensors have to be calculated

$$\begin{pmatrix} \mathbb{1} \\ \not{p} \\ \not{q} \\ \not{p}\not{q} \end{pmatrix} \otimes \begin{pmatrix} \gamma_\nu \\ p_\nu \\ q_\nu \end{pmatrix}$$

- Ball-Chiu Basis:
  - free of kinematic singularities
  - too complex

Finding a basis for the quark-gluon vertex

Instead:

- use relative momentum $\Delta = p - q$ and total momentum $\Sigma = \frac{1}{2}(p+q)$
- in Landau gauge only transversely projected vertex relevant

$$\Gamma_\mu^{qgv}(q,p;\Delta) = P_{\mu\nu}(\Delta)\ \bar{\Gamma}_\nu^{qgv}(q,p;\Delta)$$

Transversely Projected Basis:

$$\Rightarrow \quad \mathcal{G} = P_{\mu\nu}(\Delta) \begin{pmatrix} \mathbb{1} \\ \not{\Sigma} \\ \not{\Delta} \\ \not{\Sigma}\not{\Delta} \end{pmatrix} \otimes \begin{pmatrix} \gamma_\nu \\ \Sigma_\nu \\ \Delta_\nu \end{pmatrix}$$

Finding a basis for the quark-gluon vertex

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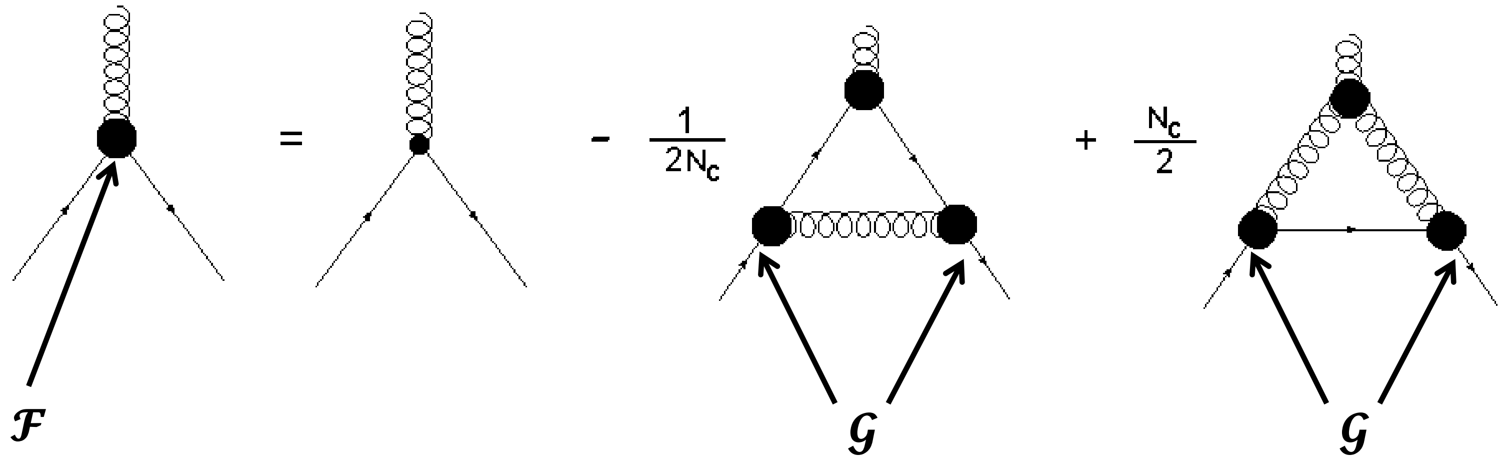
$$\Gamma_\mu^{qgv}(q,p;\Delta) = P_{\mu\nu}(\Delta)\ \bar{\Gamma}_\nu^{qgv}(q,p;\Delta)$$

Transversely Projected Basis:

$$\mathcal{G} = \{\gamma_\mu^T, i\,\hat{\Sigma}_\mu^T, i\,\hat{\slashed{\Delta}}\gamma_\mu^T, \hat{\slashed{\Delta}}\hat{\Sigma}_\mu^T, i\hat{\slashed{\Sigma}}^T\gamma_\mu^T, \hat{\slashed{\Sigma}}^T\hat{\Sigma}_\mu^T, \hat{\slashed{\Sigma}}^T\hat{\slashed{\Delta}}\gamma_\mu^T, \hat{\slashed{\Sigma}}^T\hat{\slashed{\Delta}}\hat{\Sigma}_\mu^T\}$$

Finding a basis for the quark-gluon vertex

- second projection with $P_{\mu\nu}\left(\hat{\Sigma}^T\right)$ onto γ_ν^T allows for construction of **orthonormal basis $\mathcal{F}$**



- Externally: use **orthonormal basis $\mathcal{F}$**
- Internally: use **transversal basis $\mathcal{G}$**
- convert from one basis set to the other in each iteration step

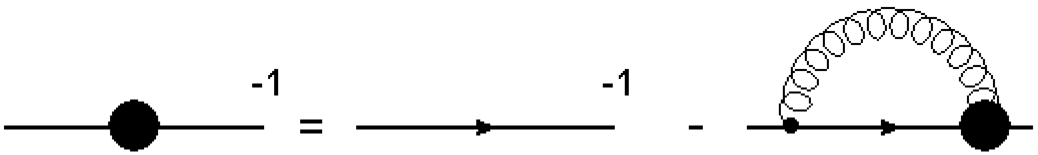
The quark propagator DSE

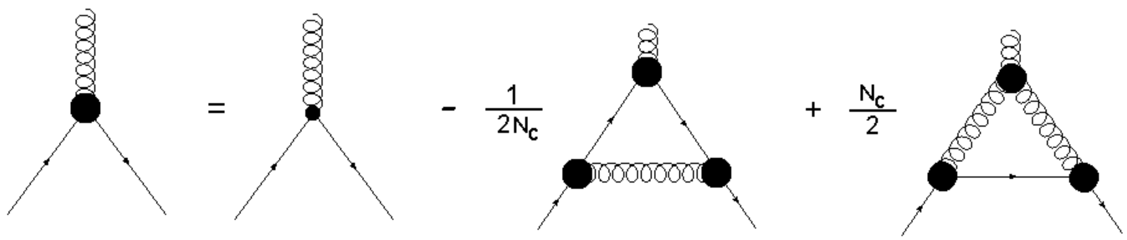
dressed quark propagator: $S(p) = \frac{1}{-i\not{p}\,\mathrm{A}(p^2)+\mathrm{B}(p^2)} = \mathrm{Z_f}(p^2)\,\frac{i\not{p}+\mathrm{M}(p^2)}{p^2+\mathrm{M}^2(p^2)}$

quark wave function renormalization: $\mathrm{Z_f}(p^2) = 1/\mathrm{A}(p^2)$

quark mass function: $\mathrm{M}(p^2) = \mathrm{B}(p^2)/\mathrm{A}(p^2)$

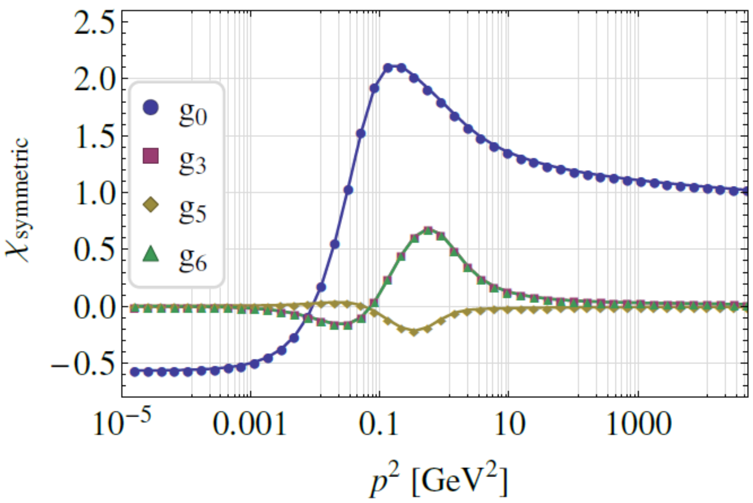
Solve coupled system of quark propagator + quark-gluon vertex DSE

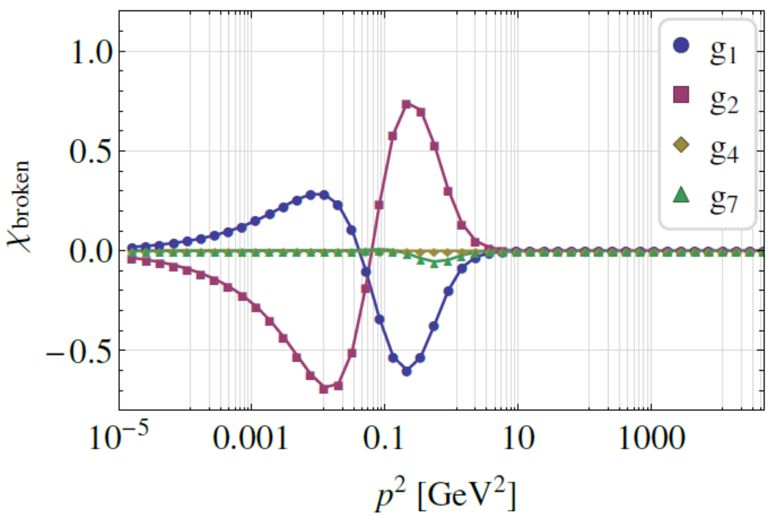




Quark-gluon vertex: Results

- all calculations are performed in the chiral limit
- important contribution from chirally broken dressing function g_2
- in contrast to the three-gluon vertex tensor structures beyond tree-level contribute significantly

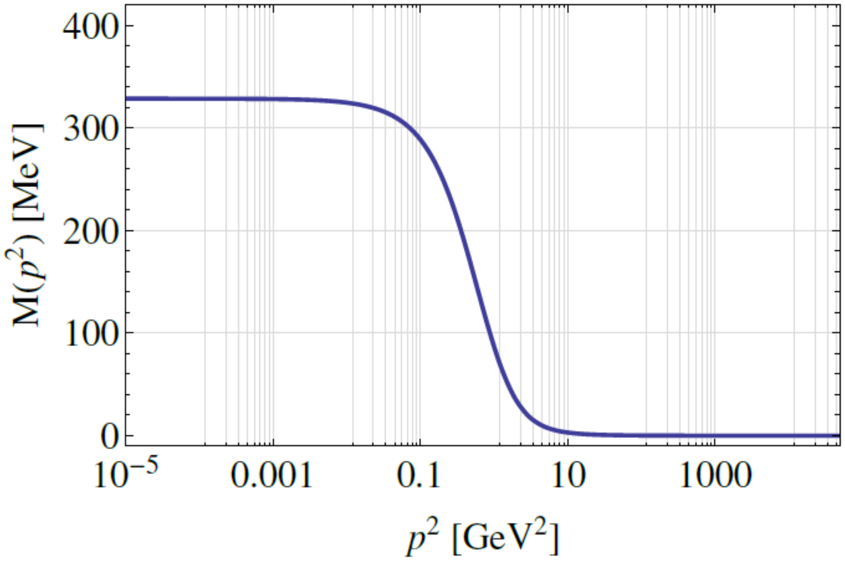


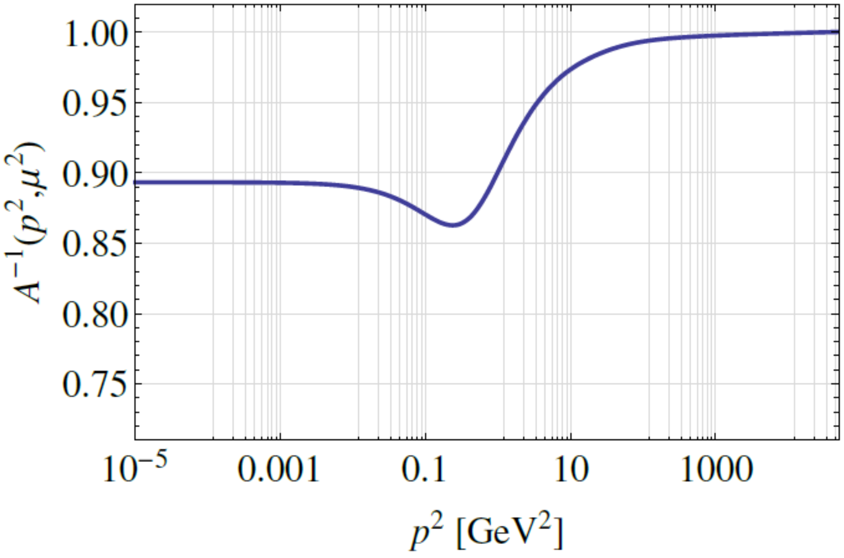


Taken from M. Hopfer, PhD thesis, Karl-Franzens-Universität 2014

Quark-gluon vertex: Impact on quark propagator

- mass generation starts at p $\approx 1\ GeV$
- behaviour of vertex dressing functions in IR (below p $\approx 0.1\ GeV$) has almost no effect on M(0) (or the chiral condensate)



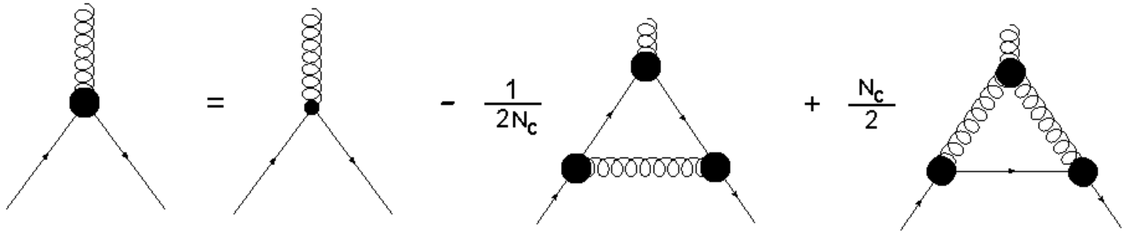


Taken from M. Hopfer, PhD thesis, Karl-Franzens-Universität 2014

Quark-gluon vertex: Results

Importance of different contributions to the quark-gluon vertex

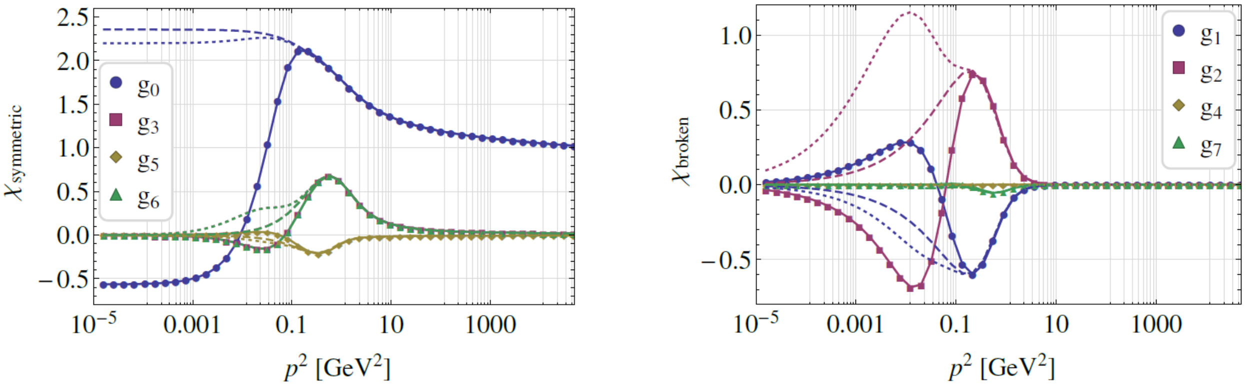
- the importance of different dressing functions can be extracted from their impact on M(0) and the chiral condensate
- only 5 out of 8 dressing functions are necessary for good results (3 dressing functions for a minimal setup)
- the Abelian diagram is supressed by the color factor N_c^2 as well as dynamically
- dynamical supression can be investigated in the adjoint representation



Impact of three-gluon vertex on quark-gluon vertex

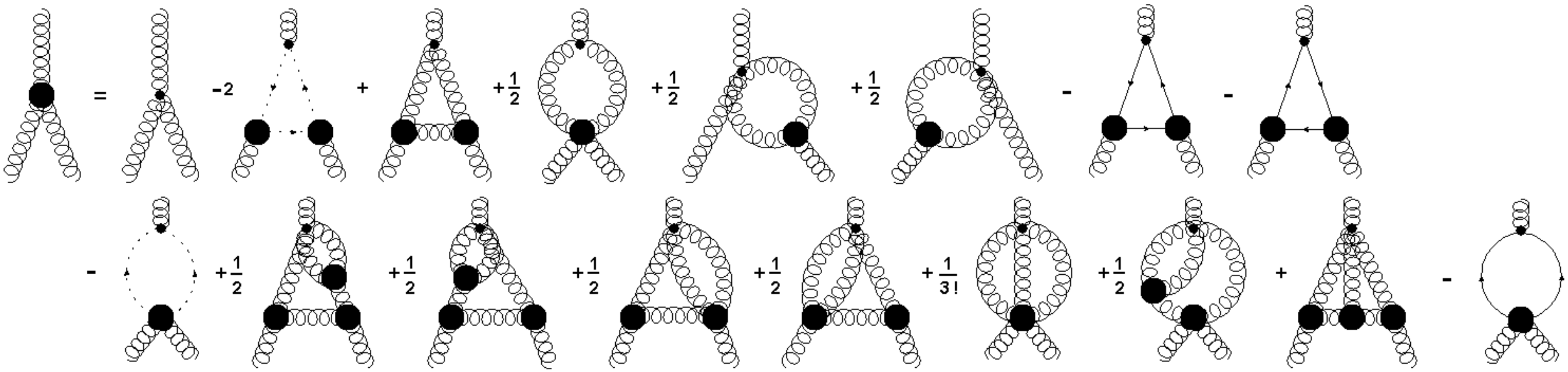
- for the dressed three-gluon vertex a model was employed
- the zero-crossing of the three-gluon vertex induces a zero crossing in (most of) the quark-gluon vertex dressing functions
- the IR-behaviour of the three-gluon vertex has only small impact on M(0) and chiral condensate, but behaviour in midmomentum + UV crucial
- to achieve self-consistency:

➡ include three-gluon vertex dynamically



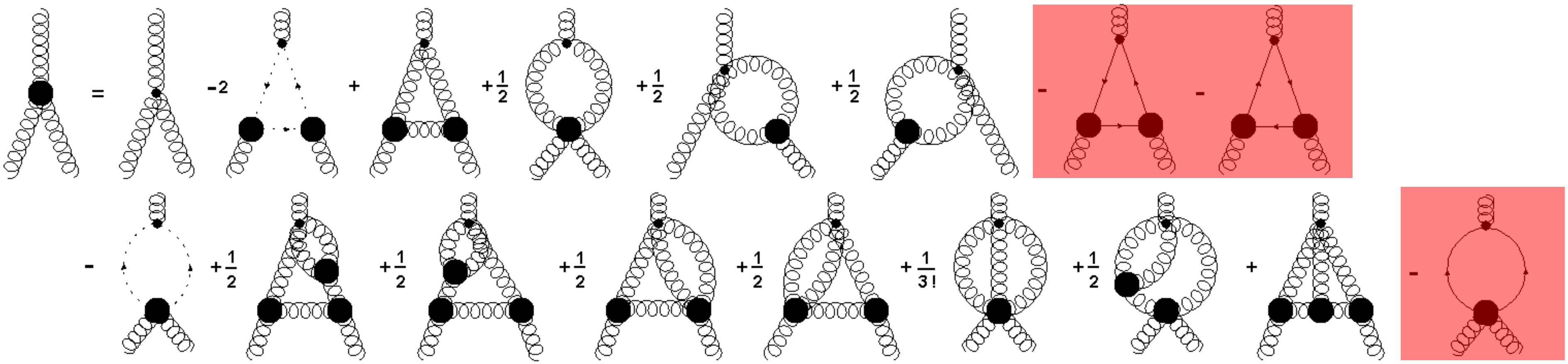
III. Unquenching the three-gluon vertex

The unquenched three-gluon vertex DSE



- employ the same truncation as for the YM three-gluon vertex
- contribution from quark-swordfish diagram may be included in future investigations by modelling the two-quark-two-gluon scattering kernel

The unquenched three-gluon vertex DSE



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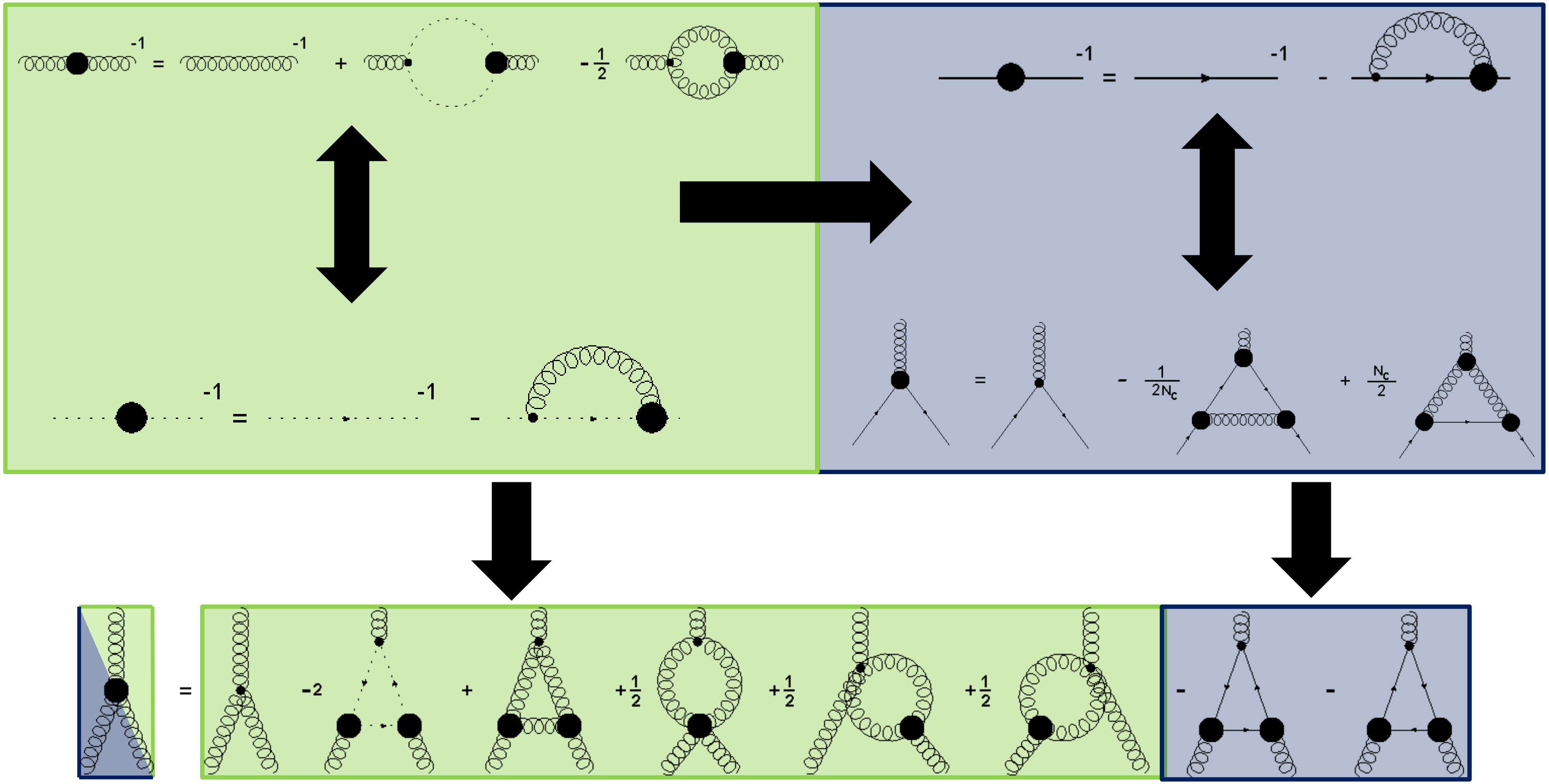
-1
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-1
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- 1/2
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-1
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- 1/(2N_c)
+ N_c/2
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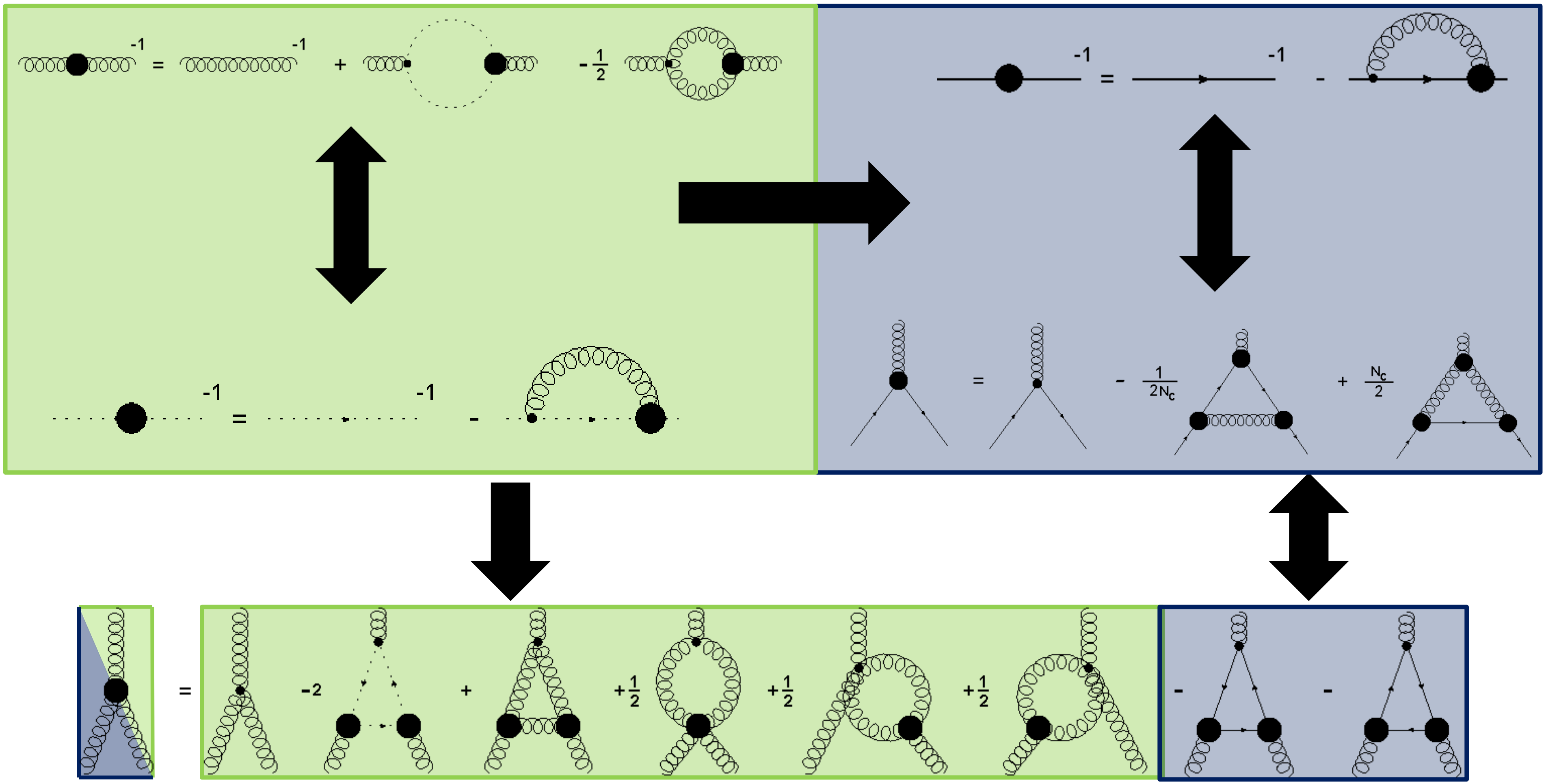
-2
+
+ 1/2
+ 1/2
+ 1/2
-
-


-1
-1
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- 1/2
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=
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-
1/2N_c
+
N_c/2
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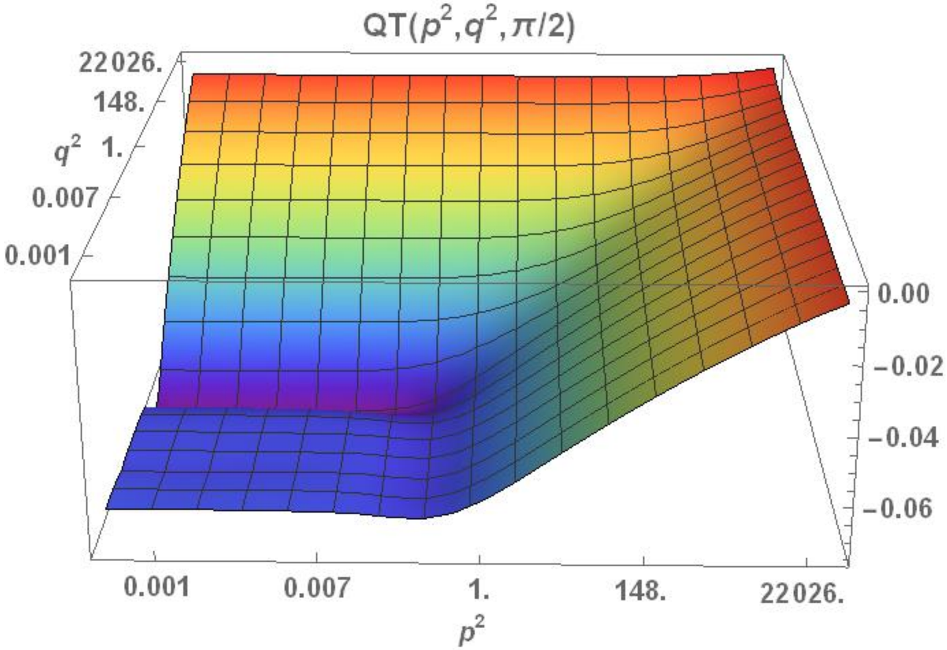
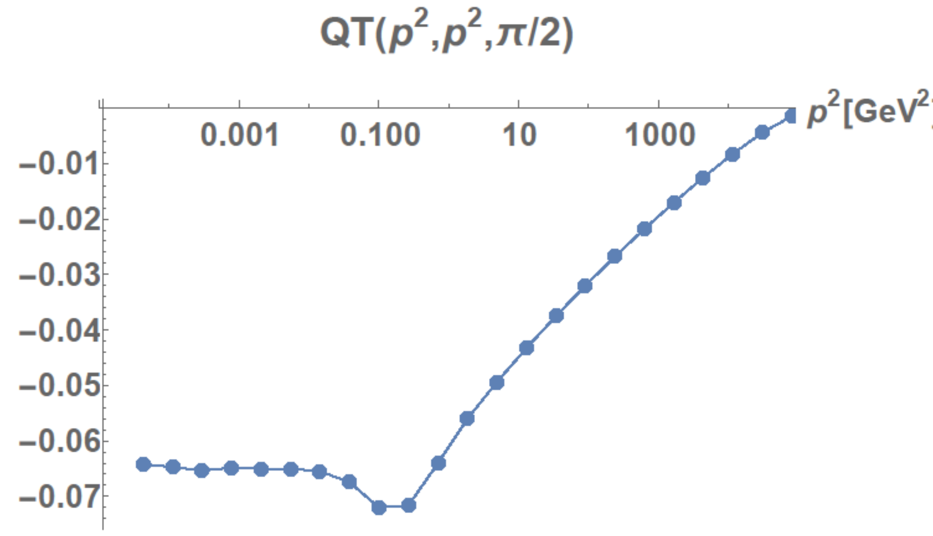
+
+ 1/2
+ 1/2
+ 1/2
-
-

Quark-Triangle: Preliminary Results

- only tree-level dressing function g_0 taken into account

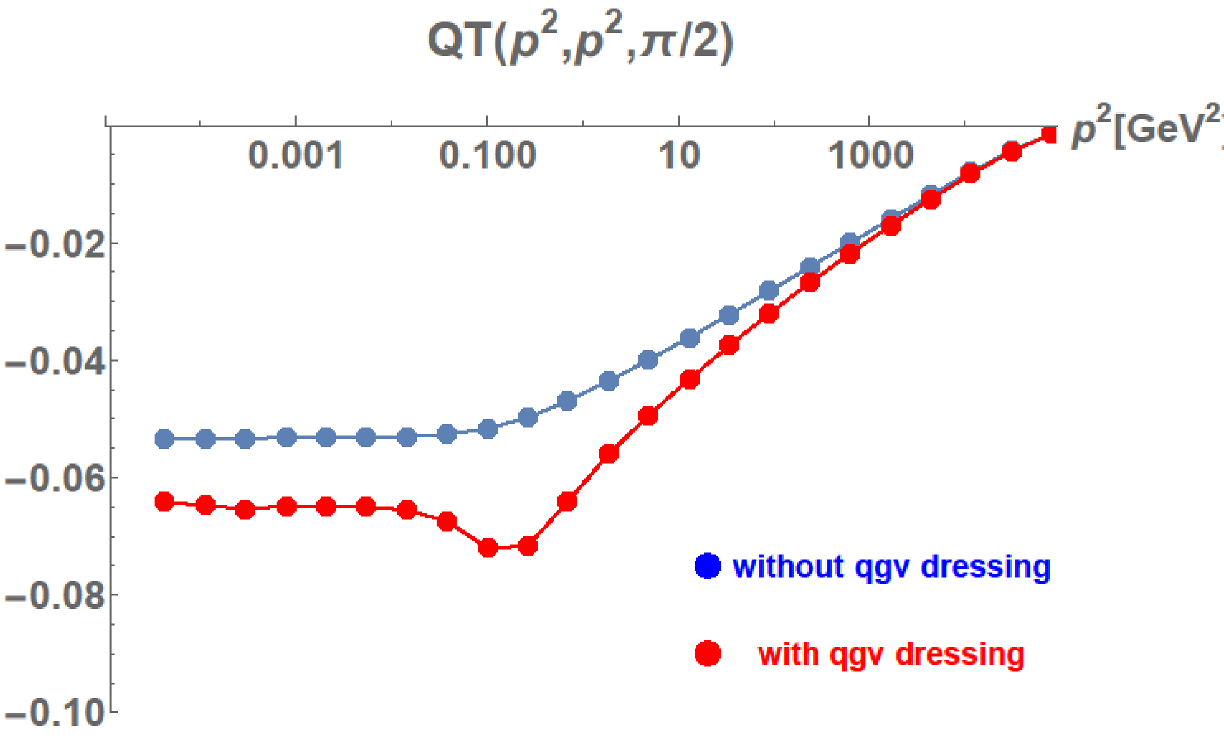




- contribution of quark-triangle to three-gluon vertex very small

Impact of quark-gluon vertex on three-gluon vertex

- set dressing function g_0 equal to 1 in order to study its impact
- only the midmomentum is affected
- no qualitative change in infrared regime



Summary and Outlook

- calculation of three-gluon vertex and quark-gluon vertex is under control

  ➡ allows for unquenching

- impact of three-gluon vertex on quark-gluon vertex has been seen in previous studies (→ zero-crossing), but with a modelled three-gluon vertex

- impact of quark sector on three-gluon vertex has not been studied yet
  seems to be very small (so far)

<u>What remains to be done:</u>

- add all dressing functions

- add quark-swordfish diagram to employed truncation

- couple unquenched three-gluon vertex back to quark-gluon vertex DSE

Back up

RG improvement terms:

- for correct anomalous dimensions: replace Z_1, Z_4 by RG improvement terms

$$Z_1 \longrightarrow D^{A^3,UV}(p,q,k) = G(\overline{p}_{A^3}^{\,2})^{\alpha_{A^3}} \; Z(\overline{p}_{A^3}^{\,2})^{\beta_{A^3}} \qquad \overline{p}_{A^3}^{\,2} = (p^2+q^2+k^2)/2$$

$$Z_4 \longrightarrow D^{A^4,UV}(p,q,k,r) = G(\overline{p}_{A^4}^{\,2})^{\alpha_{A^4}} \; Z(\overline{p}_{A^4}^{\,2})^{\beta_{A^4}} \qquad \overline{p}_{A^4}^{\,2} = (p^2+q^2+k^2+r^2)/2$$

- α, β constructed to give correct anomalous dimension + IR finiteness

decoupling solution:

$\alpha_{A^3} = 3 + 1/\delta,\ \beta_{A^3} = 0$

$\alpha_{A^4} = 4 + 1/\delta,\ \beta_{A^4} = 0$

scaling solution:

$\alpha_{A^3} = -2 - 6\delta,\ \beta_{A^3} = -1 - 3\delta$

$\alpha_{A^4} = -2 - 8\delta,\ \beta_{A^4} = -1 - 4\delta$