Unquenching the three-gluon vertex

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Hadron properties from first principles

- Hadron properties (masses, decay constants ...) depend on strong interaction between their constituents and can be described by DSE + BSE

- Rainbow-Ladder approximation uses effective interaction (only tree-level structure of quark-gluon vertex)

  Beyond Rainbow-Ladder

- For example:

  ![Diagram]

  Taken from C.S. Fischer and R. Williams, Phys. Rev. Lett. 103, 122001

- Three-gluon vertex required. YM sector results in (Blum et al. 2014, Eichmann et al. 2014)

  Unquenching
The Dyson-Schwinger approach to correlation functions within a self-consistent calculation (in Landau gauge)

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Picture of Contents
I. The three-gluon vertex DSE in Yang-Mills Theory
The three-gluon vertex DSE

- thick blobs: dressed vertices
- thin blobs: bare vertices
- all internal lines are dressed!
The three-gluon vertex DSE

truncation
The three-gluon vertex DSE
The three-gluon vertex DSE

- neglect all 2-loop diagrams + vertices without tree-level counterpart
The three-gluon vertex DSE

- neglect all 2-loop diagrams + vertices without tree-level counterpart
- dressed 4-gluon vertex model
neglect all 2-loop diagrams + vertices without tree-level counterpart

dressed 4-gluon vertex model

dressed ghost-gluon vertex input from solved gh-gl vertex DSE

(M.Q. Huber and L. von Smekal JHEP 1304 (2013) 149)
The three-gluon vertex DSE

- neglect all 2-loop diagrams + vertices without tree-level counterpart

- dressed 4-gluon vertex model

- dressed ghost-gluon vertex input from solved gh-gl vertex DSE

(M.Q. Huber and L. von Smekal JHEP 1304 (2013) 149)
Dressed propagators and vertices in Landau gauge

dressed propagators:

- ghost propagator: \( \mathcal{D}^G(p^2) = - \frac{G(p^2)}{p^2} \)

- gluon propagator: \( \mathcal{D}^{\mu\nu}(p^2) = \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2} \)

\( G(p^2), Z(p^2) = \) dressing functions
Dressed propagators and vertices in Landau gauge

**dressed vertices:**

In Landau gauge the full dynamics of the theory are described by the transverse part\(^1\)

**ghost-gluon vertex:**  
\[ \Gamma^{A\bar{c}}_{\mu} (k; p, q) = ig \ A(k; p, q) p_{\mu} + \text{long. terms} \]

**three-gluon vertex:**  
4 transverse + 10 longitudinal tensors  
\[ \Gamma^{A^3}_{\mu\nu\rho} (p, q, k) = \sum_{i=1}^{4} F_i (p, q, k) \tau^{(i)}_{\mu\nu\rho} + \text{long. terms} \]

**four-gluon vertex:**  
we employ a model

---

Dressed propagators and vertices in Landau gauge

**the four-gluon vertex model:**

- cancellations between gluon-triangle and swordfish diagrams
- model must take into account the balance between these diagrams especially strength in *midmomentum* regime important

- we make the following ansatz:

\[
\Gamma_{\mu\nu\rho\sigma}^{A^4,abcd} (p, q, k, r) = (a \tanh(b / \bar{p}_{A^4}^2) + 1)D_{\mu\nu\rho\sigma}^{A^4,UV} (p, q, k, r) \Gamma_{\mu\nu\rho\sigma}^{(0)A^4,abcd} (p, q, k, r)
\]

\[
\bar{p}_{A^4}^2 = (p^2 + q^2 + k^2 + r^2)/2
\]

- parameters a,b can be varied produces a band of solutions
comparison with lattice data\textsuperscript{1}

\[ F_1(p^2, p^2, \frac{2\pi}{3}) \]

upper band: \( a=1.5, b=(1.48-1.97) \text{ GeV}^2 \)

lower band: \( a=1.5-2, b=1.97 \text{ GeV}^2 \)

Results are in very good agreement with lattice data
Hints at dominance of tree-level dressing
\text{sinh}^{-1}(x) \approx x \text{ for } x \leq 1

\text{sinh}^{-1}(x) \approx \log(x) \text{ for large } |x|
\[ \sinh^{-1}(x) \approx x \text{ for } x \leq 1 \]
\[ \sinh^{-1}(x) \approx \log(x) \text{ for large } |x| \]
\[ S_0 = \frac{1}{3}(p^2 + q^2 + \sqrt{p^2 q^2} \cos \alpha) \]
\[
\sinh^{-1}(x) \approx x \text{ for } x \leq 1
\]
\[
\sinh^{-1}(x) \approx \log(x) \text{ for large } |x|
\]

\[
S_0 = \frac{1}{3} (p^2 + q^2 + \sqrt{p^2} \sqrt{q^2} \cos \alpha)
\]


two families of solutions: decoupling and scaling
Features of the three-gluon vertex

• three-gluon vertex features a zero crossing

• dominant contribution stems from dressing function \( F_1 \)
  \( F_1 \) corresponds to tree-level tensor structure

• calculation of \( F_1 \) in good agreement with lattice data

• truncation scheme reliable
II. The quark-gluon vertex
The quark-gluon vertex DSE

based on the work by Andreas Windisch and Markus Hopfer
The quark-gluon vertex DSE

based on the work by Andreas Windisch and Markus Hopfer

\[ \text{DSE} \]

The contribution of two-quark-two-gluon scattering kernel is sizeable
The quark-gluon vertex DSE

- The quark-gluon vertex can be decomposed into 12 basis tensors:

\[
\overline{\Gamma}_\mu^q_{\alpha\nu}(p, q; p - q) = \sum_{i=1}^{12} g_i (p, q; p - q) \lambda^{(i)}_\mu
\]

- Naive Basis:
  - simple
  - all 12 tensors have to be calculated

- Ball-Chiu Basis:
  - free of kinematic singularities
  - too complex
Instead:

- use relative momentum $\Delta = p - q$ and total momentum $\Sigma = \frac{1}{2} (p+q)$
- in Landau gauge only transversely projected vertex relevant

$$\Gamma^{qg\nu}_\mu (q,p;\Delta) = P_{\mu\nu}(\Delta) \, \bar{\Gamma}^{qg\nu}_\nu (q,p;\Delta)$$

Transversely Projected Basis:

$$\mathcal{G} = P_{\mu\nu}(\Delta) \begin{pmatrix} 1 \\ \Sigma \\ \Delta \end{pmatrix} \otimes \begin{pmatrix} \gamma_\nu \\ \Sigma_\nu \\ \Delta_\nu \end{pmatrix}$$
Finding a basis for the quark-gluon vertex

Instead:

- use relative momentum $\Delta = p - q$ and total momentum $\Sigma = \frac{1}{2} (p+q)$

- in Landau gauge only transversely projected vertex relevant
  
  $\Gamma^q g^v_{\mu}(q, p; \Delta) = P_{\mu \nu}(\Delta) \bar{\Gamma}^{q g v}_{\nu}(q, p; \Delta)$

Transversely Projected Basis:

$\mathcal{G} = \{ \gamma^T_\mu, i \hat{\Sigma}_\mu^T, i \hat{\sigma} \gamma^T_\mu, \hat{\sigma} \hat{\Sigma}_\mu^T, i \hat{T}^T \gamma^T_\mu, \hat{\gamma}^T \hat{\sigma} \gamma^T_\mu, \hat{T}^T \hat{\sigma} \hat{\Sigma}_\mu^T \}$
Finding a basis for the quark-gluon vertex

- second projection with $P_{\mu\nu}(\hat{\Sigma}^T)$ onto $\gamma_{\nu}^T$ allows for construction of orthonormal basis $\mathcal{F}$

- Externally: use orthonormal basis $\mathcal{F}$

- Internally: use transversal basis $\mathcal{G}$

- convert from one basis set to the other in each iteration step
The quark propagator DSE

dressed quark propagator: \( S(p) = \frac{1}{-i\not{p} A(p^2) + B(p^2)} = Z_f(p^2) \frac{i\not{p} + M(p^2)}{p^2 + M^2(p^2)} \)

quark wave function renormalization: \( Z_f(p^2) = 1/A(p^2) \)

quark mass function: \( M(p^2) = B(p^2)/A(p^2) \)

Solve coupled system of quark propagator + quark-gluon vertex DSE
all calculations are performed in the chiral limit

• important contribution from chirally broken dressing function $g_2$

• in contrast to the three-gluon vertex tensor structures beyond tree-level contribute significantly

Taken from M. Hopfer, PhD thesis, Karl-Franzens-Universität 2014
Quark-gluon vertex: Impact on quark propagator

- mass generation starts at $p \approx 1\ GeV$

- behaviour of vertex dressing functions in IR (below $p \approx 0.1\ GeV$) has almost no effect on $M(0)$ (or the chiral condensate)

Taken from M. Hopfer, PhD thesis, Karl-Franzens-Universität 2014
Quark-gluon vertex: Results

Importance of different contributions to the quark-gluon vertex

- the importance of different dressing functions can be extracted from their impact on $M(0)$ and the chiral condensate

- only 5 out of 8 dressing functions are necessary for good results (3 dressing functions for a minimal setup)

- the Abelian diagram is suppressed by the color factor $N_c^2$ as well as dynamically

- dynamical suppression can be investigated in the adjoint representation
Impact of three-gluon vertex on quark-gluon vertex

- for the dressed three-gluon vertex a model was employed

- the zero-crossing of the three-gluon vertex induces a zero crossing in (most of) the quark-gluon vertex dressing functions

- the IR-behaviour of the three-gluon vertex has only small impact on $M(0)$ and chiral condensate, but behaviour in midmomentum + UV crucial

- to achieve self-consistency:

  include three-gluon vertex dynamically

![Graphs showing the impact of three-gluon vertex on quark-gluon vertex](image)
III. Unquenching the three-gluon vertex
The unquenched three-gluon vertex DSE

- employ the same truncation as for the YM three-gluon vertex

- contribution from quark-swordfish diagram may be included in future investigations by modelling the two-quark-two-gluon scattering kernel
The unquenched three-gluon vertex DSE

- employ the same truncation as for the YM three-gluon vertex

- contribution from quark-swordfish diagram may be included in future investigations by modelling the two-quark-two-gluon scattering kernel
Quark-Triangle: Preliminary Results

- only tree-level dressing function $g_0$ taken into account

- contribution of quark-triangle to three-gluon vertex very small
Impact of quark-gluon vertex on three-gluon vertex

- set dressing function $g_0$ equal to 1 in order to study its impact
- only the midmomentum is affected
- no qualitative change in infrared regime

![Graph showing the impact of quark-gluon vertex on three-gluon vertex](image)
Summary and Outlook

- calculation of three-gluon vertex and quark-gluon vertex is under control
  allows for unquenching

- impact of three-gluon vertex on quark-gluon vertex has been seen in previous studies (→ zero-crossing), but with a modelled three-gluon vertex

- impact of quark sector on three-gluon vertex has not been studied yet seems to be very small (so far)

What remains to be done:

- add all dressing functions

- add quark-swordfish diagram to employed truncation

- couple unquenched three-gluon vertex back to quark-gluon vertex DSE
Back up
RG improvement terms:

• for correct anomalous dimensions: replace $Z_1$, $Z_4$ by RG improvement terms

\[
Z_1 \rightarrow D^{A^3,UV} (p, q, k) = G \left( \bar{p}_{A^3}^2 \right)^{\alpha_{A^3}} \left( \bar{p}_{A^3}^2 \right)^{\beta_{A^3}} \bar{p}_{A^3}^2 = (p^2 + q^2 + k^2)/2
\]

\[
Z_4 \rightarrow D^{A^4,UV} (p, q, k, r) = G \left( \bar{p}_{A^4}^2 \right)^{\alpha_{A^4}} \left( \bar{p}_{A^4}^2 \right)^{\beta_{A^4}} \bar{p}_{A^4}^2 = (p^2 + q^2 + k^2 + r^2)/2
\]

• $\alpha, \beta$ constructed to give correct anomalous dimension + IR finiteness


decoupling solution:

\[
\alpha_{A^3} = 3 + 1/\delta, \ \beta_{A^3} = 0
\]

\[
\alpha_{A^4} = 4 + 1/\delta, \ \beta_{A^4} = 0
\]

scaling solution:

\[
\alpha_{A^3} = -2 - 6\delta, \ \beta_{A^3} = -1 - 3\delta
\]

\[
\alpha_{A^4} = -2 - 8\delta, \ \beta_{A^4} = -1 - 4\delta
\]