The anomalous magnetic moment of the muon from lattice QCD

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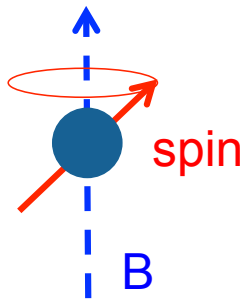
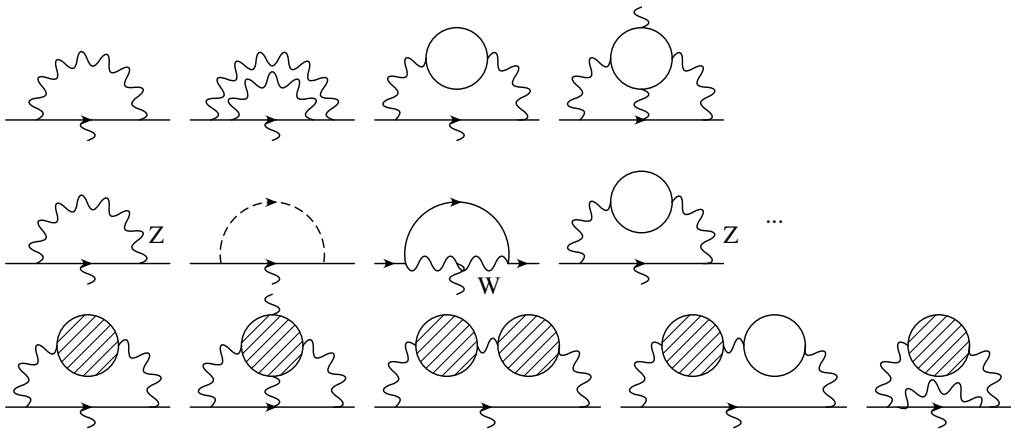
The magnetic moment of the muon

- Magnetic moment of a muon : $\vec{\mu} = g\frac{Qe}{2m}\vec{s}$.
- Prediction from free Dirac theory : $g = 2$.
- In interacting quantum field theory: g gets corrections.





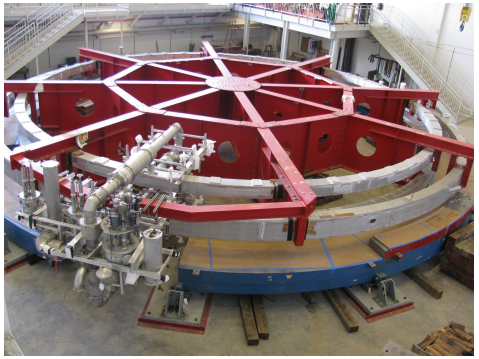
- Anomalous magnetic moment of muon: $a_\mu = \frac{g-2}{2}$.
- $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$; Only hadronic contribution can be calculated on lattice.
- Sub-ppm measurement is sensitive to corrections beyond the Standard Model.

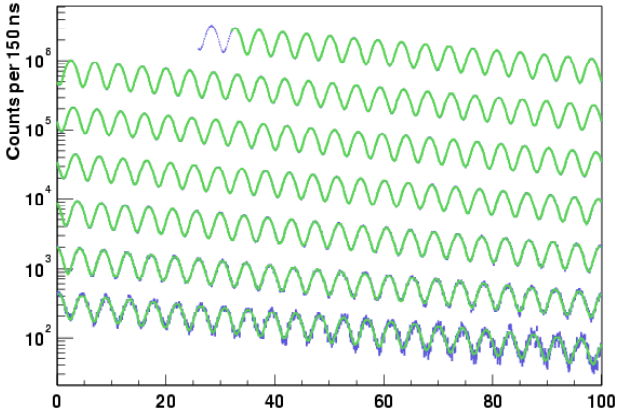
Current status of muon g-2

- BNL E821 (2001) : a_μ^{exp} = 116592089(63) 10^{-11} (0.54 ppm).

Overview of measurement technique

- Muons fed into a storage ring having precise $\vec{B}$
- Muons continually decay into positrons and neutrinos



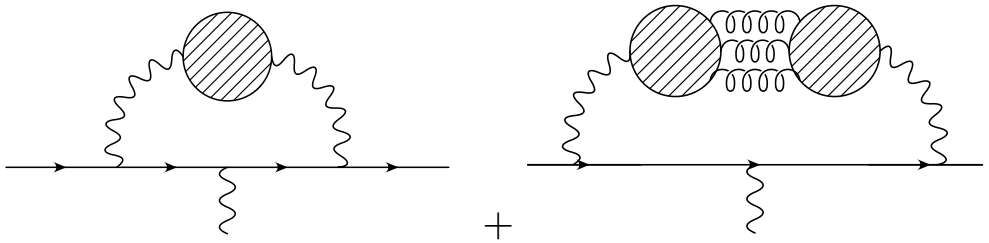


- Weak decay : Positron direction follows muon spin
- Muon's spin precession obtained from the time distribution of decay positrons

- g-2 discrepancy of 3.6σ between the standard model and the experiment: $a_\mu^{exp} - a_\mu^{SM} = 25(9)x10^{-10}$.
- Discovery of new physics (if 5σ): Supersymmetric particles, dark photon, multi-Higgs model, extra dimentions ???

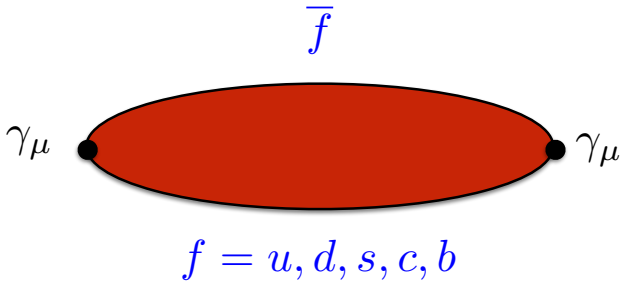
Next goals

- Future experiment: Fermilab E989 aims to reduce experimental error in g-2 to $\pm 1.6x10^{-10}$ (four-fold improvement).
- Theoretical goal to get SM uncertainty of 3.5 x 10^{-10} (.3-.4%) before results from Fermilab E989 (2017/18).
- a_μ^{QED} calculated phenomenologically with same precision as in experiment; a_μ^{EW} very small.
- The current theoretical uncertainty dominated by that from the lowest order hadronic vacuum polarization (HVP) contribution (connected + disconnected).



- HVP contribution from dispersion relation + cross section for e^+e^- (and τ) $\rightarrow$ hadrons around 700 x 10^{-10} with a 1% error.
- Our aim : To achieve uncertainty of less than 1% in $a_\mu^{HVP,lo}$ using first principle lattice QCD calculations.

Our method for calculating HVP from full lattice QCD



$$a^{(\mathrm{f})}_{\mu,\mathrm{HVP}} = \frac{\alpha}{\pi}\int dq^2 f(q^2)(4\pi\alpha Q_{\mathrm{f}}^2)\hat{\Pi}_{\mathrm{f}}(q^2).$$

(Lautrup,'72, T. Blum,'02)

- Integrand strongly peaked at $q^2 \sim m_\mu^2/4 \sim 0.003\mathrm{GeV}^2$.
- Extrapolating from higher values of q^2 leads to model uncertainties.

Our (HPQCD) method (B. Chakraborty et al.,Phys. Rev. D 89, 114501 (2014)):

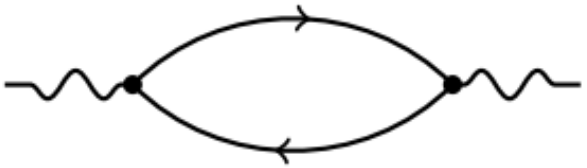
- HVP contribution expressed as a Taylor series of small number of derivatives of the vacuum polarization function $\hat{\Pi}$ evaluated at $q^2 = 0$.
- Defining $\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j}\Pi_j$.
- And then $\Pi_j = (-1)^{j+1}\frac{G_{2j+2}}{(2j+2)!}$ where $G_{2n} = (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2\hat{\Pi}(q^2)\Big|_{q^2=0}$.
- The derivatives at $q^2 = 0$ of $\hat{\Pi}$ readily and accurately given by $t^n\times$ vector meson correlators (time moments) for n = 4,6,8,10:

$$\begin{aligned} G_{2n} &\equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^i(\vec{x},t) j^i(0)\rangle \\ &= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2\hat{\Pi}(q^2)\Big|_{q^2=0}. \end{aligned}$$

- Using Padé approximants instead of Taylor approximation allows us to deal with high momenta.

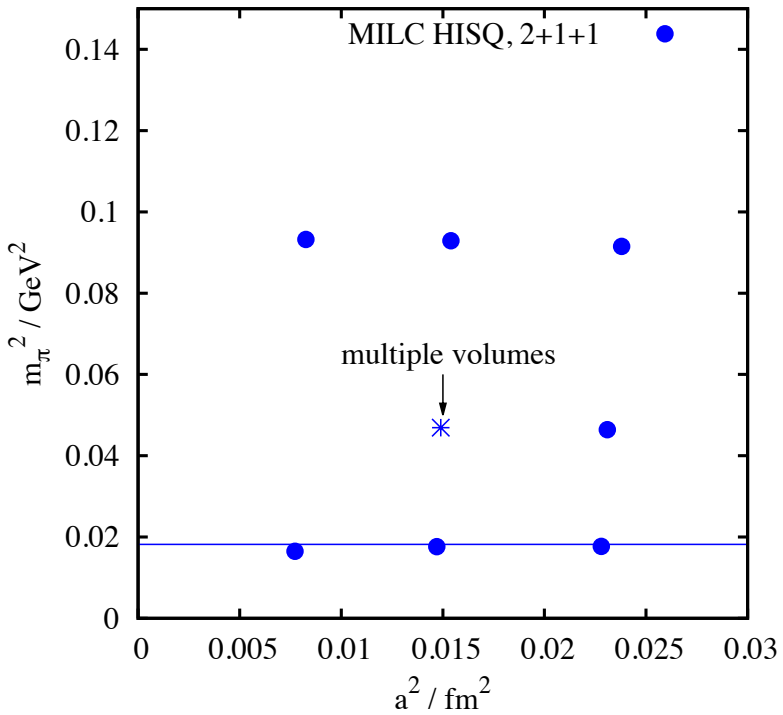
Calculating meson correlator (connected) on lattice

- Continuum QCD action with gluons and quarks; QCD vacuum with gluonic background and sea quarks, but no valence quarks.
- Expectation values of observables as a path integral over fields: Infinite number of integrals.
- 4-dimensional space-time lattice: Finite number of integrals.
- Discretise the QCD lagrangian: Different formalisms.
- Take an ensemble of lattice gauge configurations (few hundreds) as QCD vacuum.
- A meson generated with a Dirac bilinear and allowed to propagate resulting in a 2-point correlator (Quark-line connected and disconnected).
- Quark-line connected meson correlator: $G_{conn}(x_0 - y_0) = \langle(\sum_{\vec{x}} Tr[\gamma_k D^{-1}(x,y)\gamma_k D^{-1}(y,x)])\rangle$.



- $D^{-1}(x,y)$ is the inverse of lattice Dirac matrix or propagator
- Calculate renormalisation factor Z to match lattice and continuum currents.
- Averaging over configurations extract meson properties.
- At the end, finite volume extrapolation, continuum extrapolation, extrapolation to chiral limit.

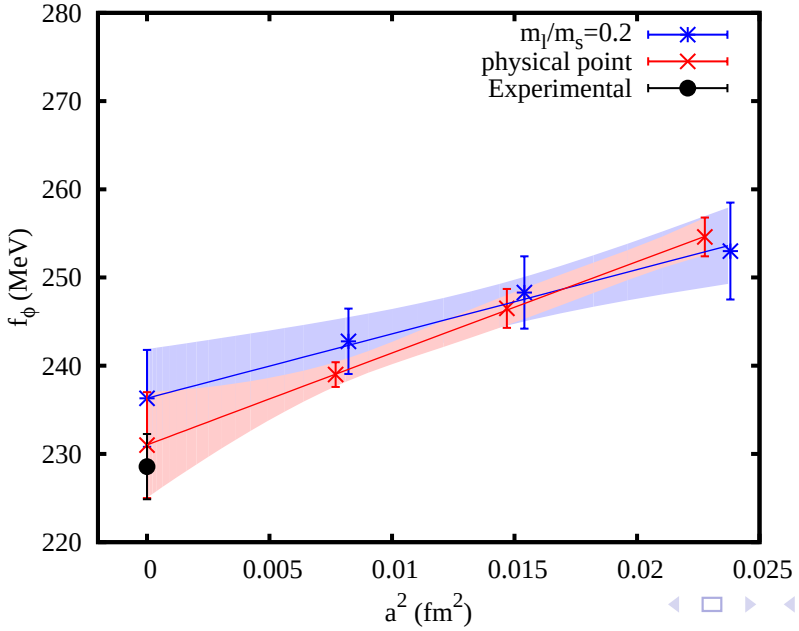
Lattice Configurations and Parameters



- Lattice configurations by MILC using Highly Improved Staggered Quark (HISQ) formalism : Small discretisation error.
- Light (up/down), strange, charm dynamical sea quarks included.
- Valence strange and charm quark masses precisely tuned, but valence light quark mass taken to be same as in the sea.
- Large box size: 5.6 fm on the finest lattices.
- Multiple volumes on a particular light sea quark mass and a particular lattice spacing.
- 1000 gluon configurations: High statistics.

Our correlators for vector meson made of strange valence quarks: ϕ

- Precise determination of mass and decay constant (annihilation amplitude) of $s\bar{s}$ vector meson ϕ from large time behaviour of correlator.
- For a_μ^{HVP} we are interested in small time behaviour.
- Renormalization factor $Z_{V,\bar{s}s}$ calculated precisely with 0.1% uncertainty completely non-perturbatively (B.Chakraborty et al., PoS LATTICE2013, 309 (2013)).
- Finite volume effect is negligible.
- Our result of f_ϕ in the continuum limit agree with the experimental result related to $\Gamma(\phi \to e^+e^-)$.
- Disconnected diagrams are not included; but very small contribution expected.

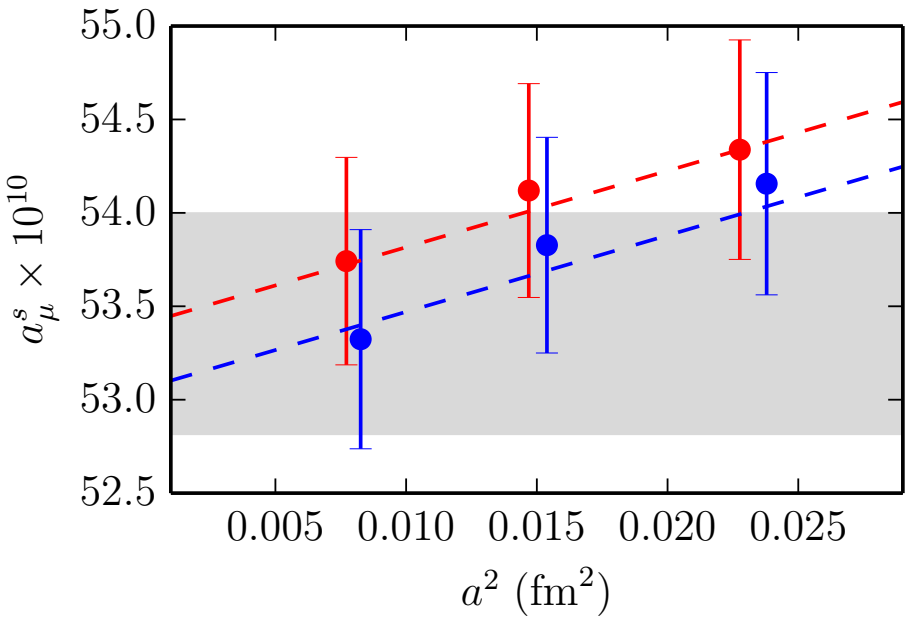


Our results: Connected contributions to a_μ^s

Our final result for the connected contribution for s quarks to $g-2$ is :

$a_\mu^s = 53.41(59) \times 10^{-10}$ (B.Chakraborty et al.,Phys. Rev. D 89, 114501);

with $m_{u/d}^{\rm lat} = m_{u/d}^{\rm phys}$, after extrapolation to $a = 0$.

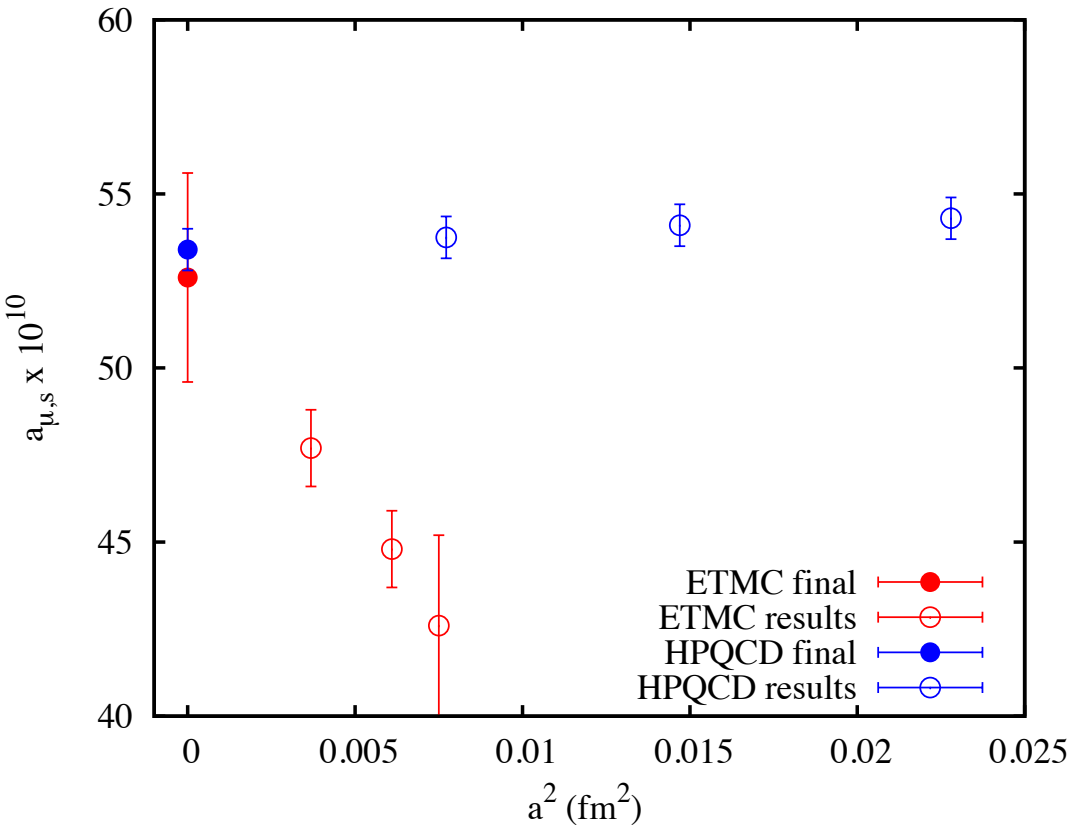


- The lower blue points represent $m_{u/d}^{\rm lat} = m_s/5$ and the upper red points represent $m_{u/d}^{\rm lat} = m_{u/d}^{\rm phys}$.
- Precision obtained at 1.1% level in a_μ^s, the best so far.
- Major part of the uncertainty comes from lattice spacing error (1%), can be improved if better precision required.

Comparison of our results for a_μ^s with other results

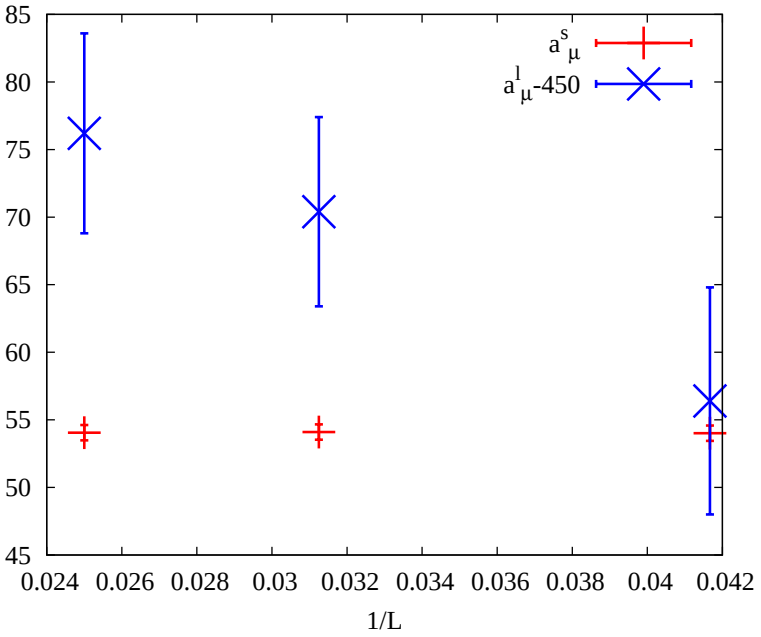
- Comparison of our results for a_μ^s with results from another lattice groups.

HPQCD	ETMC (preliminary)	RBC/UKQCD (preliminary)
53.41(59)x10^{-10}	53(3)x10^{-10}	52.4(21)x10^{-10}



Preliminary results of the connected contribution to $a_\mu^{u/d}$

- Most significant contribution, challenging due to poor Signal-to-noise ratio at larger timeslices.
- Time-moments calculated from correlators reconstructed from the best fit parameters improves precision.
- Other techniques used namely Gaussian smearing etc.
- $a_\mu^{u/d}$ shows significant finite volume effects compared to a_μ^s, needs finite volume extrapolation.



- Indicative (preliminary) result: $a_\mu^{u/d}$ = 631(9) x 10^{-10} (~1.4% uncertainty)
- Need to look more carefully at finite volume dependence.
- ETMC preliminary from full QCD (1311.3885) : $a_\mu^{u/d}$ = 567(11) x e^{-10} (2%), no finite volume effect seen.

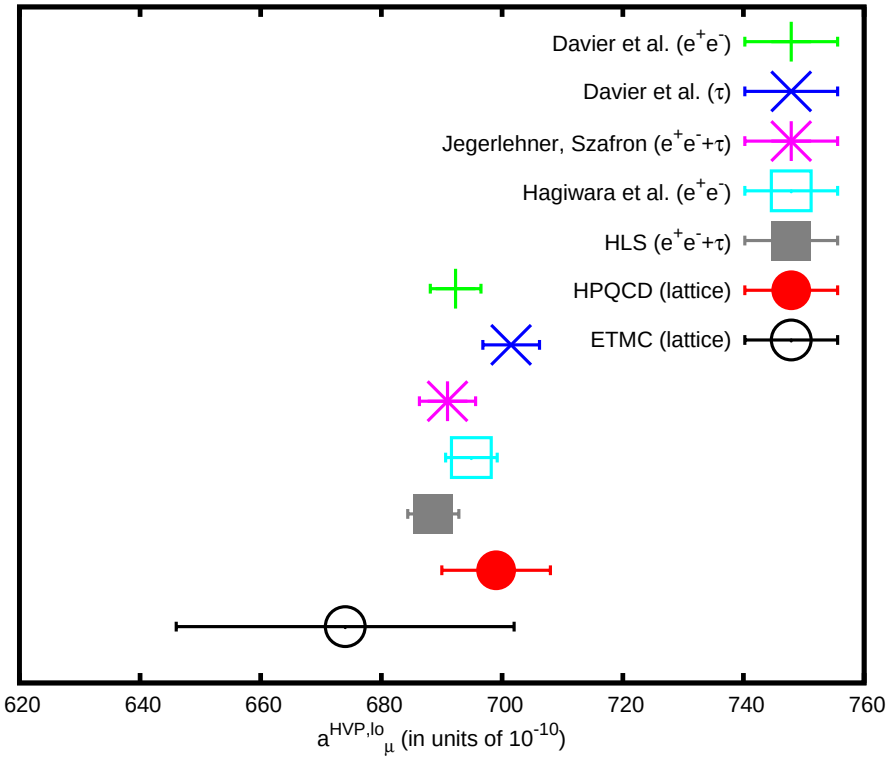
Preliminary lattice results for full four flavour estimation of $a_\mu^{HVP,lo}$

- Lattice can provide full four-flavor estimation.
- Our (HPQCD) preliminary estimation:

$$\begin{aligned} a_\mu^{HVP,lo} &= a_\mu^{light} + a_\mu^{s} + a_\mu^{c} + a_\mu^{b} \\ &= (596(7) + 53.41(59) + 14.42(39) + 0.27(4)) \times 10^{-10} \\ &\sim 699(9) \times 10^{-10} (\sim 1.3\%) \end{aligned}$$

- ETMC preliminary number (1311.3885): $a_\mu^{HVP,lo}$ = 674(28) $\times 10^{-10} (\sim 4\%)$

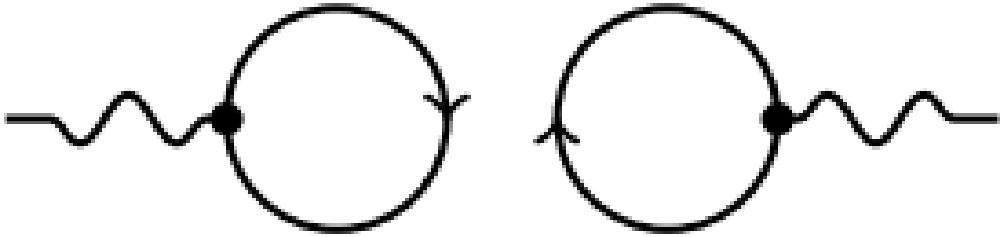
Comparison between lattice and phenomenology



- Accuracy still doesn't match with phenomenological result.
- Still result from QCD only without experimental input or model assumption.
- Our (HPQCD) result hints at no significant disconnected piece (still needs consistency check!)

Leading order disconnected contribution to a_μ^{HVP}

- Expected to be very small, may be a significant source of systematic error, in partially quenched ChPT as large as -10% of the connected one.



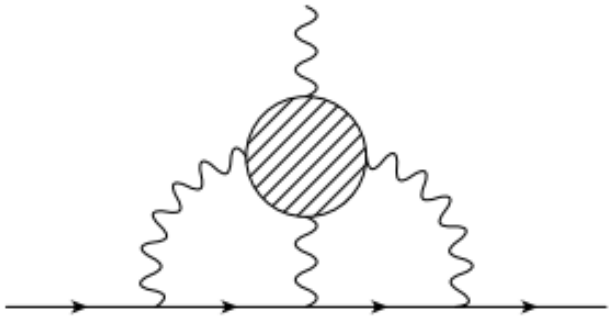
- On lattice we calculate:

$$G_{disc}(x_0 - y_0) = -Z_V\langle(\sum_{\vec{x}} Tr[\gamma_k D^{-1}(x,x)])(\sum_{\vec{y}} Tr[\gamma_k D^{-1}(y,y)])\rangle$$

- All-to-all propagator.
- Poor signal-to noise ratio : challenging.
- We aim to calculate a conservative upper bound for a systematic error from neglecting the disconnected contribution.
- Currently existing upper bound : 4-5% (Mainz, 1411.7592).

The Hadronic light-by-light contribution

- Small compared to HVP, but important for getting less than 1% theoretical errors.



- Blob: All possible hadronic states.
- Model estimates: about (10-12)x10^{-10} with a 25-40% uncertainty (0901.0306).
- Lattice calculation involves four-point functions: direct calculations not possible.
- First attempt on lattice includes electromagnetism in simulations (Tom Blum and collaborators, 1407.2923).
- Problem reduces to difference of 3-point functions.
- Calculated for unphysical quark and muon masses: Still looks encouraging.

Conclusion

To summarize:

- $a_\mu^{HVP,lo,conn}$ = 699(9) $\times 10^{-10}$($\sim$ 1.1%) (HPQCD preliminary using HISQ), work in progress on u/d quark piece, finite volume effects; First lattice result at physical quark and pion masses and to take care of finite volume effects.
- A conservative upperbound for not including $a_\mu^{HVP,lo,disc}$ 4-5% (Mainz).
- $a_\mu^{HLbL,conn}$ calculated with unphysical quark and muon mass which looks promising (Tom Blum and collaborators).

Future:

- Ensemble size of 10,000 (on a$\sim$ 0.15fm, 0.12fm lattices) being made by MILC to be used by MILC and HPQCD for a_μ^{light} and $a_\mu^{HVP,lo,disc}$ calculations, 3-fold improvement in the uncertainty expected.
- Our (HPQCD) work in progress; highly optimistic about obtaining uncertainty in $a_\mu^{HVP,lo}$ at less than 1% level by 2017.
- HLbL (connected) on physical quark and muon masses: Finite volume correction and other systematic improvements needed. HLbL disconnected piece tiny, but to be calculated.
- Situation seems to be under control!

Back up : Our method for calculating HVP from full lattice QCD (B. Chakraborty et al.,Phys. Rev. D 89, 114501 (2014))

- For spatial currents at zero spatial momentum: $\Pi^{ii}(q^2) = q^2\Pi(q^2) = a^4 \sum_t e^{iqt} \sum_{\vec{x}} \langle j^i(\vec{x},t) j^i(0)\rangle$ with q the Euclidean energy.
- We need the renormalized vacuum polarization function: $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$.
- Time moments of the correlator give the derivatives at $q^2 = 0$ of $\hat{\Pi}$ readily and accurately.

$$\begin{aligned} G_{2n} &\equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^i(\vec{x},t) j^i(0)\rangle \\ &= (-1)^n \left.\frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2)\right|_{q^2=0} . \end{aligned}$$

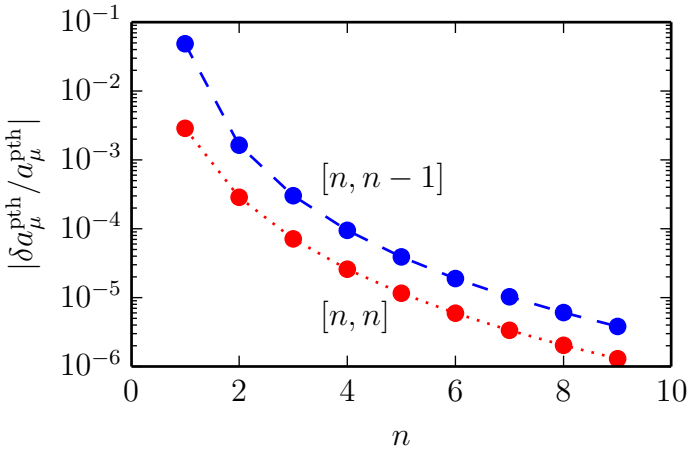
- Defining $\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j$.
- Then $\Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$.

Back up: Our method for calculating HVP from full lattice QCD (B. Chakraborty et al.,Phys. Rev. D 89, 114501 (2014))

- We replace $\hat{\Pi}(q^2)$ with its $[n, n]$ and $[n, n-1]$ Padé approximants derived from the Π_j.
- The q^2 integral $\int dq^2 f(q^2)(4\pi\alpha Q_{\rm f}^2)\hat{\Pi}_{\rm f}(q^2)$ is done numerically.
- Using Padé approximants instead of Taylor approximation allows us to deal with high momenta, say $q \geq 1$ GeV which is important when better than 1% precision is desired.
- The precision of different Padé approximants tested by comparing with the exact results for a_μ from the perturbation theory.
- The Padés converge exponentially quickly to the correct result, achieving better than 1% precision after only two terms are included.

Back up: Discussions about our method of approximation

- In calculating G_{2n} the error due to the finite T is exponentially suppressed and is negligible near the edge of the lattice at $T/2$ contributing to higher order moments only.
- Thus, the finite T effect becomes important for higher order moments, which we do not need in our present calculation of a_μ^s.
- The finite T approximation gives an error of 0.002% in our analysis for configuration set 10.
- To test the power of Padé approximants, we compare the precision of diffrent [n,n] and [n,n-1] approximants with exact result for the one-loop quark vacuum polarization function from perturbation theory.



- The quark mass is set equal to the kaon mass so that the Taylor expansion has the same radius of convergence as the physical s-quark vacuum polarization.
- The Padés converge exponentially quickly to the correct result, achieving better than 1% precision after only two terms are included.
- The finite precision of exact moments from a simulation limits the precision of the final results for