Exploring the Physics Frontier with Circular Colliders, Aspen,
January 28, 2015

Seung J. Lee (KAIST)

Have we really discovered a SM-like Higgs boson?

1. Study of SM Higgs boson partial widths and branching fractions

2. Higgs potentials and more: Higgs from a quantum phase transition

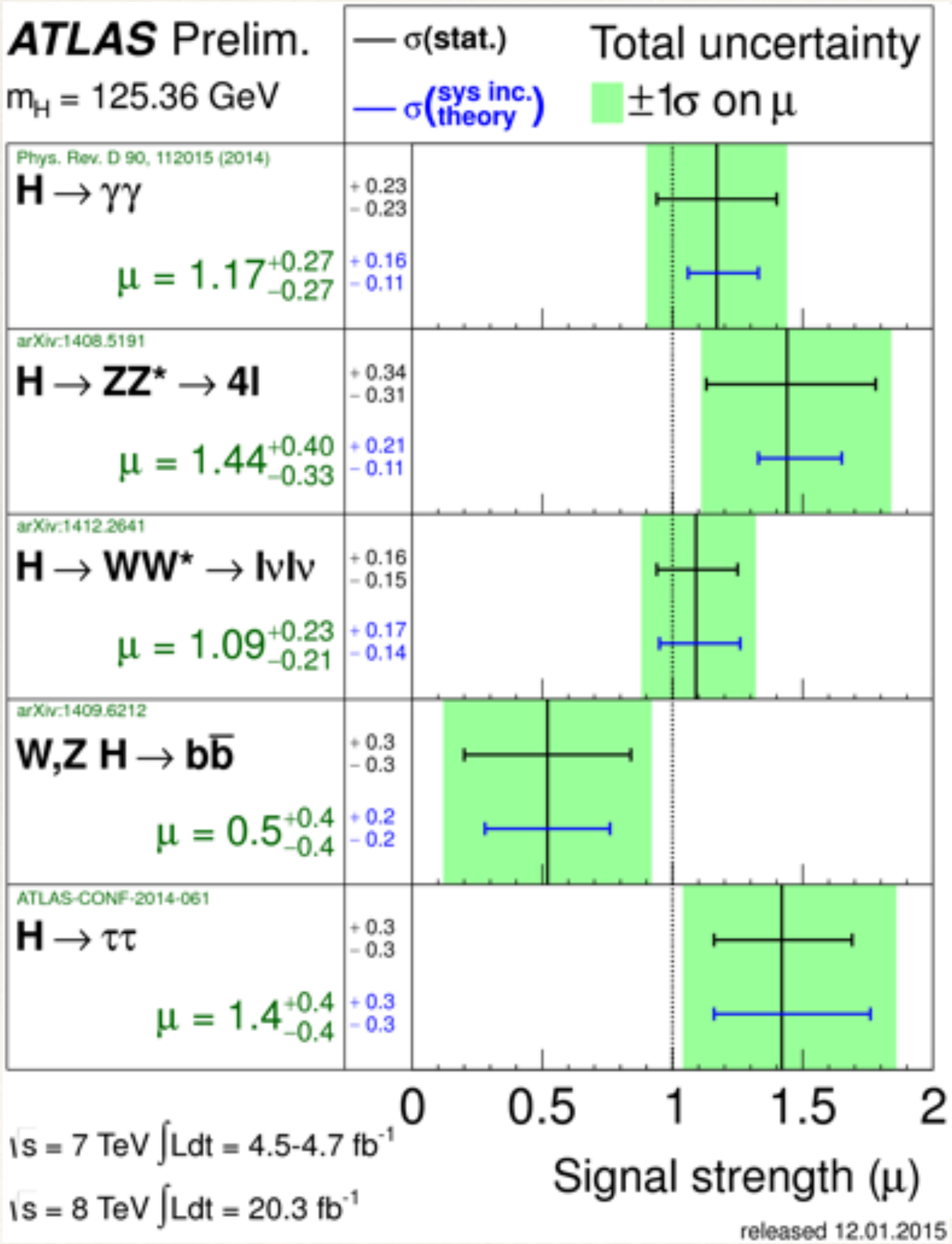
1. With L. Almeida, S.L., S. Porkorski, J. Wells
arXiv:1311.6721v3

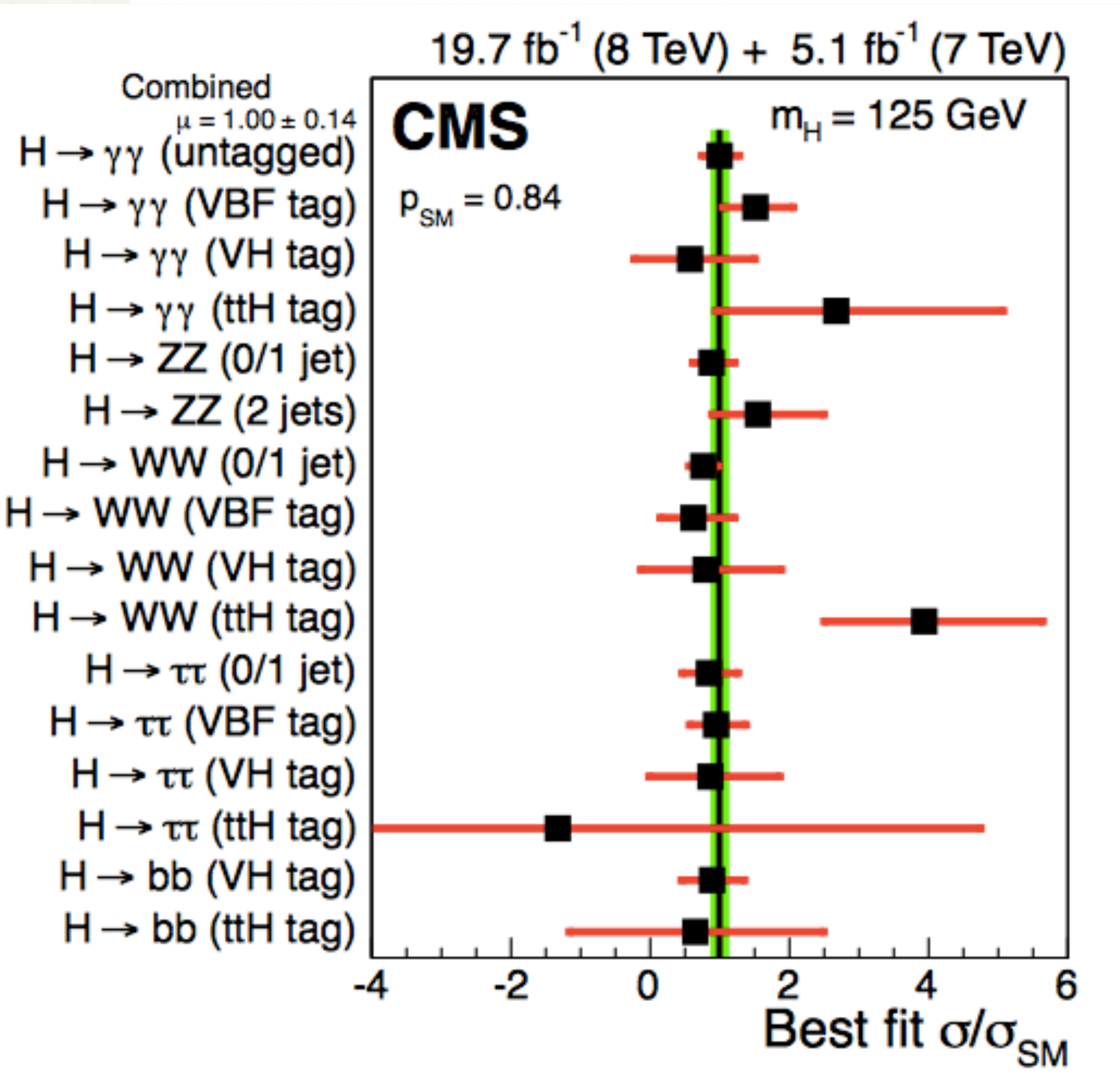
2. With B. Bellazzini, C. Csaki, J. Hubisz, S.L., J. Serra, J. Terning
work in progress (to appear)

Nothing but Higgs, with ~ 10-20% Precision for Higgs couplings so far

$m_H = 125.02\,^{+0.26}_{-0.27}\,(\mathrm{stat})\,^{+0.14}_{-0.15}\,(\mathrm{syst})\ \mathrm{GeV}$

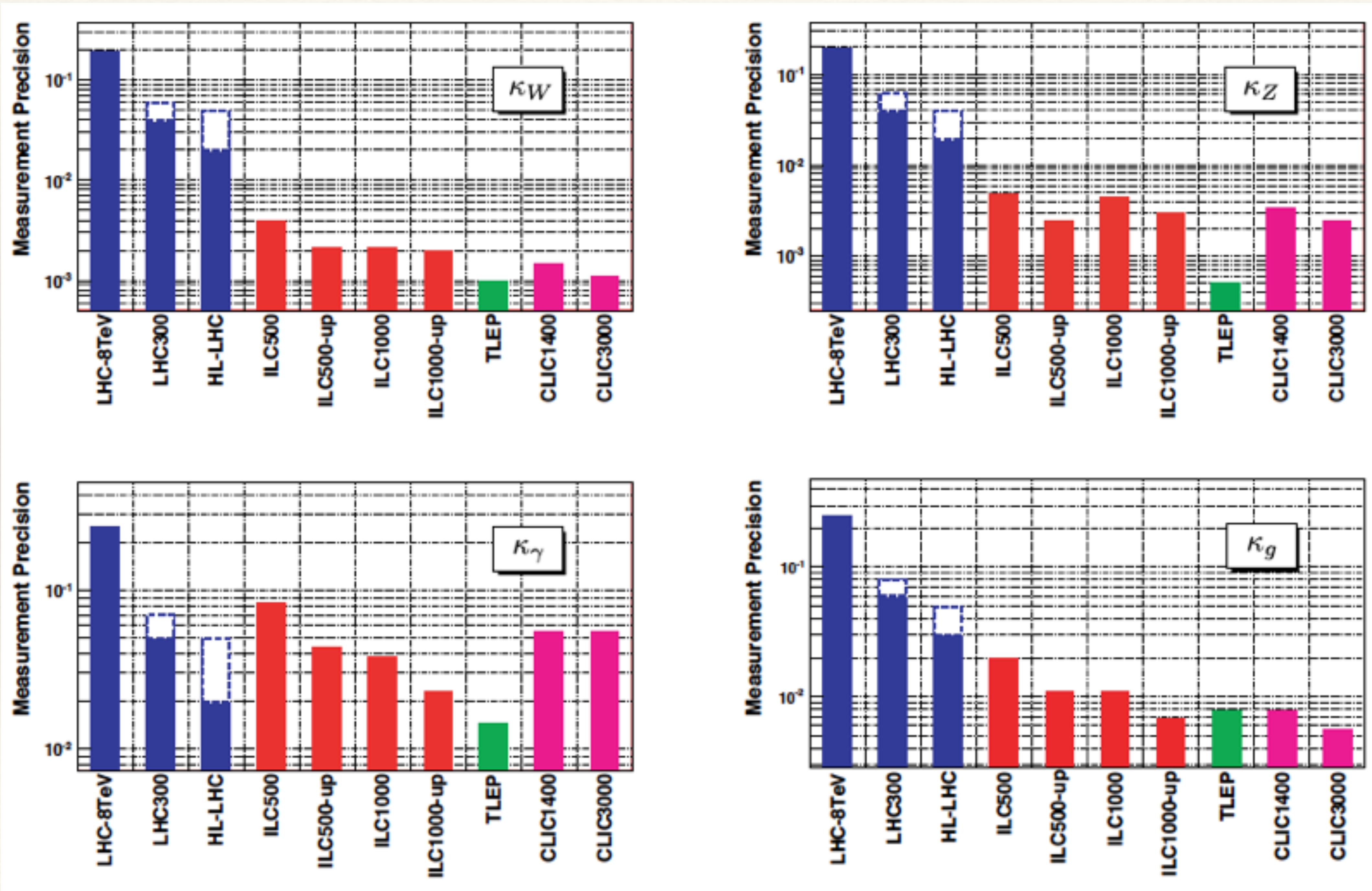
$\mu = 1.00 \pm 0.09\,(\mathrm{stat})\,^{+0.08}_{-0.07}\,(\mathrm{theo}) \pm 0.07\,(\mathrm{syst})$





New era of precision studies of the Higgs sector

(from Snowmass Higgs working group report): Higgs precision will approach that of EWP



Part I: **Precision Higgs Analysis:** expansion formalism of the Higgs boson partial widths and branching fractions

L. Almeida, S.L., S. Porkorski, J. Wells
arXiv:1311.6721v3

- The sub-percent-level determination of the Higgs boson mass now enables a complete set of input observables whereby any perturbative high-energy observable involving the Higgs boson can be predicted.
  - careful exposition of the decay partial widths and branching fractions of a SM Higgs boson with mass near 125 GeV.
  - state-of-the-art formulas that can be used in any precision electroweak analysis to investigate compatibility of the data with the SM predictions in these most fundamental and sensitive observables

What's new in our expansion formalism?

- Other calculations exist in the literature, mostly notably from the computer program HDECAY; however, we wish to provide an independent calculation that includes the latest advances and allows us to vary the renormalization scale in all parts of the computations. This flexibility will be useful in discussions regarding uncertainties
- We also aim to detail the errors that each input into the computation propagates to the final answer for each observable

Our Expansion Formalism of Partial Widths and Uncertainties

Taylor expand the full expressions for partial width around the input observables. This expansion is made possible by the fact that with the discovery of the Higgs boson, and knowledge of its mass, all input observables are now known to good enough accuracy to render an expansion of this nature useful and accurate.

We represent the partial width expansion by

$$\Gamma_{H\to X} = \Gamma_X^{(\mathrm{ref})}\left(1+\sum_i a_{\tau_i,X}\overline{\delta\tau_i}\right) \tag{7}$$

where

$$\overline{\delta\tau_i} = \frac{\tau_i - \tau_{i,ref}}{\tau_{i,ref}}, \tag{8}$$

and τ_i are the $\left\{m_H, M_Z, \Delta\alpha_{had}^{(5)}, \alpha_S(M_Z), m_f\right\}$ for the calculation. The total width is the sum of all the partial widths and for convenience we present dedicated expansion parameters for that as well:

$$\Gamma_{\mathrm{tot}} = \sum_X \Gamma_{H\to X} = \Gamma_{\mathrm{tot}}^{(\mathrm{ref})}\left(1+\sum_i a_{\tau_i,\mathrm{tot}}\overline{\delta\tau_i}\right). \tag{9}$$

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$$\Gamma_{H\to X} = \Gamma_X^{(\mathrm{ref})}\left(1 + \sum_i a_{\mathcal{T}_i,X}\overline{\delta\mathcal{T}_i}\right) \tag{7}$$

where

$$\overline{\delta\mathcal{T}_i} = \frac{\mathcal{T}_i - \mathcal{T}_{i,ref}}{\mathcal{T}_{i,ref}}, \tag{8}$$

	$\Gamma_X^{(\mathrm{Ref})}$/GeV	$a_{m_t,X}$	$a_{m_H,X}$	$a_{\alpha(M_Z),X}$	$a_{\alpha_S(M_Z),X}$	$a_{m_b,X}$	$a_{M_Z,X}$	$a_{m_c,X}$	$a_{m_\tau,X}$	$a_{G_F,X}$
total	4.17×10^{-3}	-3.3×10^{-2}	4.34	8.35×10^{-1}	-5.05×10^{-1}	1.32	-3.21	7.80×10^{-2}	1.24×10^{-1}	8.49×10^{-1}
gg	3.61×10^{-4}	-1.62×10^{-1}	2.89	0.	2.48	-6.51×10^{-2}	3.76×10^{-1}	0.	0.	1.00
$\gamma\gamma$	1.08×10^{-5}	-2.69×10^{-2}	4.32	2.56	1.80×10^{-2}	8.29×10^{-3}	-1.86	0.	0.	7.24×10^{-1}
$b\bar{b}$	2.35×10^{-3}	8.07×10^{-3}	8.09×10^{-1}	3.76×10^{-2}	-1.12	2.36	-2.72×10^{-1}	0.	0.	9.53×10^{-1}
$c\bar{c}$	1.22×10^{-4}	-4.52×10^{-2}	7.99×10^{-1}	1.02×10^{-2}	-3.10	0.	-4.89×10^{-1}	2.67	0.	9.70×10^{-1}
$\tau^+\tau^-$	2.58×10^{-4}	4.71×10^{-2}	9.95×10^{-1}	-2.09×10^{-2}	-2.14×10^{-3}	0.	-1.61×10^{-2}	0.	2.01	1.02
WW^*	9.43×10^{-4}	-1.13×10^{-1}	1.37×10^{1}	3.66	9.04×10^{-3}	0.	-1.21×10^{1}	0.	0.	2.49×10^{-1}
ZZ^*	1.17×10^{-4}	2.27×10^{-2}	1.53×10^{1}	-7.37×10^{-1}	-1.82×10^{-3}	0.	-1.12×10^{1}	0.	0.	2.53
$Z\gamma$	6.89×10^{-6}	-1.52×10^{-2}	1.11×10^{1}	8.45×10^{-1}	0.	-7.93×10^{-3}	-4.82	0.	0.	2.62
$\mu^+\mu^-$	8.93×10^{-7}	4.82×10^{-2}	9.92×10^{-1}	-4.31×10^{-2}	-2.19×10^{-3}	0.	-1.62×10^{-2}	0.	0.	1.02

Input Parameters for our expansion

- $$\text{Inputs}: \left\{ m_H, M_Z, \Delta\alpha^{(5)}_{had}, \alpha_S(M_Z), m_f \right\}, \qquad (1)$$

- Now that we have established our convention that M_W is an output observable, when the W mass appears in formulas below, we should view it as a short-hand notation for the full computation of the W mass within the theory in terms of our agreed-upon inputs. In the SM this substitution is

$$M_W \xrightarrow{SM} (80.368\,\text{GeV})\,(1 + 1.42\,\delta M_Z + 0.21\,\delta G_F - 0.43\,\delta\alpha + 0.013\,\delta M_t - 0.0011\,\delta\alpha_S - 0.00075\,\delta M_H)\,. \quad (3)$$

$$\delta\tau \equiv (\tau - \tau_{ref})/\tau_{ref}$$

-

m_H	125.7(4)	pole mass m_t	173.07(89)
$\overline{\text{MS}}$ mass m_c	1.275(25)	$\overline{\text{MS}}$ mass m_b	4.18(3)
pole mass m_τ	1.77682(16)	$\alpha_S(M_Z)$	0.1184(7)
$\alpha(M_Z)$	1/128.96(2)	$\Delta\alpha^{(5)}_{had}$	0.0275(1)

Expansion of BR and ratio of BRs

$$\mathrm{B}(H \to \mathrm{X}) = \mathrm{B}(\mathrm{X})^{(\mathrm{ref})} \left(1 + \sum_{\mathrm{i}} \mathrm{b}_{\tau_{\mathrm{i}},\mathrm{X}} \overline{\delta \tau_{\mathrm{i}}}\right), \qquad (12)$$

where τ_i represents the $\left\{m_H, M_Z, \Delta\alpha^{(5)}_{had}, \alpha_S(M_Z), m_f\right\}$. Expansion parameters $b_{\tau_i,X}$ are related to $a_{\tau_i,X}$ by

$$b_{\tau_i,X} = a_{\tau_i,X} - a_{\tau_i,tot}. \qquad (13)$$

$$\frac{\mathrm{B}(H \to \mathrm{X})}{\mathrm{B}(H \to \mathrm{Y})} = \frac{\mathrm{B}(\mathrm{X})^{(\mathrm{ref})}}{\mathrm{B}(\mathrm{Y})^{(\mathrm{ref})}} \left(1 + \sum_{i} r_{\tau_i,X,Y} \overline{\delta \tau_i}\right),$$

where τ_i represent the $\left\{m_H, M_Z, \Delta\alpha^{(5)}_{had}, \alpha_S(M_Z), m_f\right\}$. The expansion parameters $r_{\tau_i,X,Y}$ is related to $a_{\tau_i,X}$ by

$$r_{\tau_i,X,Y} = a_{\tau_i,X} - a_{\tau_i,Y}. \qquad (17)$$

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The table of expansion coefficients enables us to compute the uncertainty in a final state branching ratio due to each input parameter. The percent uncertainty Δ_i^X on branching fraction $B(X)$ due to input parameter τ_i is

$$\Delta_i^X = (100\%) \times |b_{\tau_i,X}| \frac{\Delta\tau_i}{\tau_i^{ref}} \qquad (14)$$

where $\Delta\tau_i$ are the current experimental uncertainties in input parameter τ_i. For example, the percentage uncertainty in the $H \to gg$ branching fraction is

$$\Delta_b^{gg} = (100\%)(1.389)\frac{0.03\,\mathrm{GeV}}{4.18\,\mathrm{GeV}} = 1.00\%. \qquad (15)$$

Expansion of BR and ratio of BRs

$$\mathrm{B}(H \to \mathrm{X}) = \mathrm{B}(\mathrm{X})^{(\mathrm{ref})}\left(1 + \sum_{\mathrm{i}} \mathrm{b}_{\tau_{\mathrm{i}},\mathrm{X}}\overline{\delta\tau_{\mathrm{i}}}\right), \tag{12}$$

The table of e...
the uncertainty ...
input parameter ...
fraction $B(X)$ d...

where $\Delta\tau_i$ are th...
parameter τ_i. Fo...
$H \to gg$ branc...

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the uncertainty in the b-quark mass input observable constitutes the largest uncertainty in the branching ratio computations.

The large uncertainty of the charm quark mass is the decisive contributor to H ->cc uncertainty as well

How well can we predict SM observables?

Percent relative uncertainty, P_Q: $Q = Q_0\,(1 + 0.01\,P_Q)$.

	$P_\Gamma^\pm$(par.add.)	$P_\Gamma^\pm$(par.quad.)	$(P_\Gamma^+, P_\Gamma^-)(\mu)$
total	2.82 (1.79)	1.71 (1.07)	(0.08,0.10)
gg	2.52 (1.83)	1.74 (1.49)	(0.05,0.03)
$\gamma\gamma$	1.45 (0.42)	1.38 (0.35)	(1.31,0.60)
$b\bar{b}$	2.62 (2.43)	1.84 (1.82)	(0.29,0.01)
$c\bar{c}$	7.34 (7.15)	5.55 (5.54)	(0.45,0.35)
$\tau^+\tau^-$	0.36 (0.12)	0.32 (0.08)	(0.01,0.01)
WW^*	4.41 (1.17)	4.97 (1.25)	(0.25,0.31)
ZZ^*	4.90 (1.25)	4.42 (1.11)	(0.,0.)
$Z\gamma$	3.56 (0.92)	3.52 (0.88)	(0.56,0.23)
$\mu^+\mu^-$	0.34 (0.11)	0.32 (0.08)	(0.03,0.03)

The meaning of "$P_\Gamma^\pm$(par.add.)" is that all input parameters have been allowed to range over their 1σ errors and the maximum percent relative errors are recorded. The meaning of "$P_\Gamma^\pm$(par.quad.)" is that the uncertainties of each parameter are added in Gaussian quadrature. In other words, $P_{\Gamma_i}^\pm(\text{par.quad.}) = 100\,\Delta\Gamma_i/\Gamma_i$, where

$$(\Delta\Gamma_i)^2 = \left(\frac{\partial\Gamma_i}{\partial m_t}\right)^2 (\Delta m_t)^2 + \left(\frac{\partial\Gamma_i}{\partial \alpha_s}\right)^2 (\Delta\alpha_s)^2 + \cdots. \quad (11)$$

The uncertainties in varying the scale parameter μ in the calculation, attempts to capture the uncertainty in not knowing higher order corrections. A full calculation at all orders would give a result that does not depend on μ but a finite-order calculation does, and the uncertainty of dropping the higher order calculations are assumed to be approximated reasonably well by noting how much the result changes by varying μ by a factor of two upward and downward: $m_H/2 < \mu < 2m_H$. The meaning of "$P_\Gamma^\pm(\mu)$" in Table 4 concerns the relative percent uncertainties associated with this scale dependence algorithm.

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

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quark	at $\mu = m_H$ $(m_H/2,\, 2m_H)$	$P_m(\Delta m)$
$m_c(\mu)$	0.638 (0.675, 0.603) GeV	2.62%
$m_b(\mu)$	2.79 (2.96, 2.64) GeV	0.85%
$m_t(\mu)$	166.5 (176.8,157.8) GeV	0.56%

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SM vs. New Physics? Uncertainties in BRs

	Δ_{m_t}	Δ_{m_H}	$\Delta_{\alpha(M_Z)}$	$\Delta_{\alpha_S(M_Z)}$	Δ_{m_b}	Δ_{M_Z}	Δ_{m_c}	Δ_{m_τ}	Δ_{G_F}
gg	0.07	0.46 (0.12)	0.01	1.77	1.00	0.01	0.15	-	-
$\gamma\gamma$	-	0.01 (-)	0.03	0.31	0.94	-	0.15	-	-
$b\bar{b}$	0.02	1.13 (0.28)	0.01	0.36	0.74	0.01	0.15	-	-
$c\bar{c}$	0.01	1.13 (0.28)	0.01	1.53	0.95	0.01	5.08	-	-
$\tau^+\tau^-$	0.04	1.07 (0.27)	0.01	0.30	0.95	0.01	0.15	0.02	-
WW^*	0.04	2.97 (0.74)	0.04	0.30	0.95	0.02	0.15	-	-
ZZ^*	0.03	3.48 (0.87)	0.02	0.30	0.95	0.02	0.15	-	-
$Z\gamma$	0.01	2.14 (0.53)	-	0.30	0.96	-	0.15	-	-
$\mu^+\mu^-$	0.04	1.07 (0.27)	0.01	0.30	0.95	0.01	0.15	-	-

$$\Delta_i^X = (100\%) \times |b_{\tau_i,X}| \frac{\Delta\tau_i}{\tau_i^{ref}}$$

	$P^{\pm}_{\rm BR}$(par.-add.)	$P^{\pm}_{\rm BR}$(par.-quad.)	$(P^{+}_{\rm BR}, P^{-}_{\rm BR})(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
$\gamma\gamma$	1.45 (1.44)	1.01 (1.01)	(1.81,1.83)
$b\bar{b}$	2.43 (1.58)	1.41 (0.89)	(0.21,0.)
$c\bar{c}$	8.72 (7.87)	5.51 (5.40)	(0.54,0.44)
$\tau^+\tau^-$	2.55 (1.75)	1.47 (1.04)	(0.09,0.07)
WW^*	4.48 (2.26)	3.13 (1.25)	(0.10,0.08)
ZZ^*	4.96 (2.34)	3.63 (1.33)	(0.10,0.08)
$Z\gamma$	3.56 (1.96)	2.36 (1.15)	(0.83,0.80)
$\mu^+\mu^-$	2.53 (1.73)	1.47 (1.04)	(0.07,0.06)

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$$\Delta_i^X = (100\%) \times |b_{\tau_i,X}| \frac{\Delta\tau_i}{\tau_i^{ref}}$$

For example, if the data at a later stage of the LHC, or ILC, or CLIC suggests that the branching fraction into b quarks can be determined to better than 1%, this does not mean that we are sensitive to new physics contributions of 1% to $H \to b\bar{b}$. The reason can be seen from Tables 6 and 7 that the SM uncertainty in computing $B(H \to b\bar{b})$ is presently 2.4% (sum of absolute values of all errors) and expected to not get better than 1.6%, with most of that coming from uncertainty of the bottom Yukawa coupling determination stemming from the uncertainty of the measured bottom quark $\overline{\text{MS}}$ mass

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$b\bar{b}$	2.43 (1.58)	1.41 (0.89)	(0.21,0.)
$c\bar{c}$	8.72 (7.87)	5.51 (5.40)	(0.54,0.44)
$\tau^+\tau^-$	2.55 (1.75)	1.47 (1.04)	(0.09,0.07)
WW^*	4.48 (2.26)	3.13 (1.25)	(0.10,0.08)
ZZ^*	4.96 (2.34)	3.63 (1.33)	(0.10,0.08)
$Z\gamma$	3.56 (1.96)	2.36 (1.15)	(0.83,0.80)
$\mu^+\mu^-$	2.53 (1.73)	1.47 (1.04)	(0.07,0.06)

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

SM vs. New Physics: Uncertainties in BRs

	Δ_{m_t}	
gg	0.07	0.4
$\gamma\gamma$	-	0.0
$b\bar{b}$	0.02	1.1
$c\bar{c}$	0.01	1.1
$\tau^+\tau^-$	0.04	1.0
WW^*	0.04	2.9
ZZ^*	0.03	3.4
$Z\gamma$	0.01	2.1
$\mu^+\mu^-$	0.04	1.0

Thus, without reducing this error, any new physics contribution to the $b\bar{b}$ branching fraction that is not at least a factor of three or four larger than 1% cannot be discerned from SM. Thus, a deviation of at least 3% is required of detectable new physics.

However, the lattice QCD calculation could improve it to match the experimental improvement on time. (arXiv:1404.0319v1, Lepege, Mechenzie, Peskin)

For example, if the data at a later stage of the LHC or ILC, or CLIC suggests that the branching fraction into b quarks can be determined to better than 1%, this does not mean that we are sensitive to new physics contributions of 1% to $H \to b\bar{b}$. The reason can be seen from Tables 6 and 7 that the SM uncertainty in computing $B(H \to b\bar{b})$ is presently 2.4% (sum of absolute values of all errors) and expected to not get better than 1.6%, with most of that coming from uncertainty of the bottom Yukawa coupling determination stemming from the uncertainty of the measured bottom quark $\overline{\text{MS}}$ mass

	$P^{\pm}_{\text{BR}}$(par.-add.)	$P^{\pm}_{\text{BR}}$(par.-quad.)	$(P^{+}_{\text{BR}}, P^{-}_{\text{BR}})(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
$\gamma\gamma$	1.45 (1.44)	1.01 (1.01)	(1.81,1.83)
$b\bar{b}$	2.43 (1.58)	1.41 (0.89)	(0.21,0.)
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Summary I

- Higgs Precision can be reaching at the level of EWP
- With improved theoretical tools (e.g. expansion formalism), SM will be tested at per mille level
- SM Higgs vs. BSM Higgs can be tested @ FCC (and ILC) beyond the typical direct search limit
- Recently a further study on low energy observables were done (see arXiv:1501.02803v1 by Petrov, Porkoski, Wells, Zhang)

Part II: Quantum Critical Higgs

B. Bellazzini, C. Csaki, J. Hubisz, S.L., J. Serra, J. Terning
work in progress (to appear)

- With the observation of a scalar with properties close to the SM Higgs, we are now confident that the interactions of the Higgs boson with gauge bosons and fermions are mainly dictated by its kinetic term and Yukawa coupling

$$\mathcal{L}_H = |D_\mu H|^2 - \bar{\psi}^\alpha H \psi^\beta$$

- Higher-dimensional operators predict relations between the mass of a given particle and its coupling to the Higgs that deviate O(1) from the ones derived from the above Lagrangian.

$$|D_\mu H|^2 |H|^2,\ F_{\mu\nu}^2 |H|^2,\ \text{or}\ \bar{\psi} H \psi |H|^2$$

- The observation at the LHC of Higgs couplings consistent with the above Lagrangian implies that such higher-dimensional operators must be treated as small perturbations.

- The situation is however different in what regards the last part of the SM Lagrangian, the Higgs potential,

$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4$$

$\mu \sim 88$ GeV $\qquad \lambda \sim 0.13$

Higgs Potential: a toy example

- Let's consider the following potential

$$\tilde{V}(H) = -\lambda|H|^4 + c_6|H|^6 \qquad c_6 \sim \Lambda^{-2}$$

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- With this potential the VEV and mass of the Higgs are given by,

$$v^2 = \frac{4}{3}\frac{\lambda}{c_6}\,, \qquad \frac{m_h^2}{v^2} = 2\lambda$$

$\xrightarrow{\text{experimentally}}$ $\lambda \approx 0.13$

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- defining $c_6 = 1/f^2$, we find f ~ 600 GeV, so new physics effects from such scale might have escaped detection with $\Lambda \sim 4\pi f$

This is just one simple example that shows that the form of the Higgs potential is completely undetermined

Higgs Potential: a toy example

- However, FCC can potentially distinguish between the two possibilities just presented.

- In particular, via double Higgs production, the triple Higgs coupling can be probed: i.e.

$$\lambda_{hhh} = 3\frac{m_h^2}{v} \quad \Longleftrightarrow \quad \tilde{\lambda}_{hhh} = 7\frac{m_h^2}{v}$$

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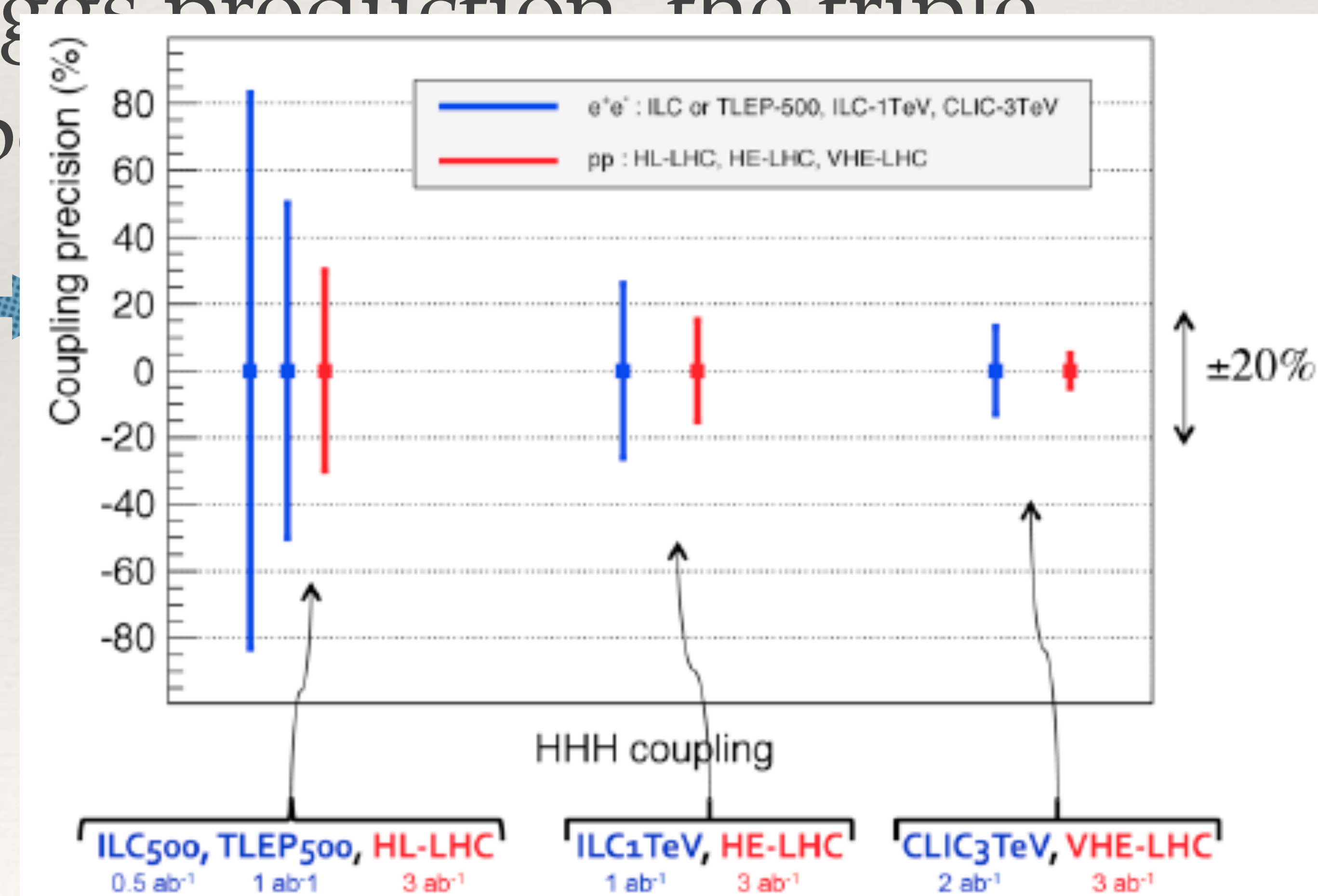
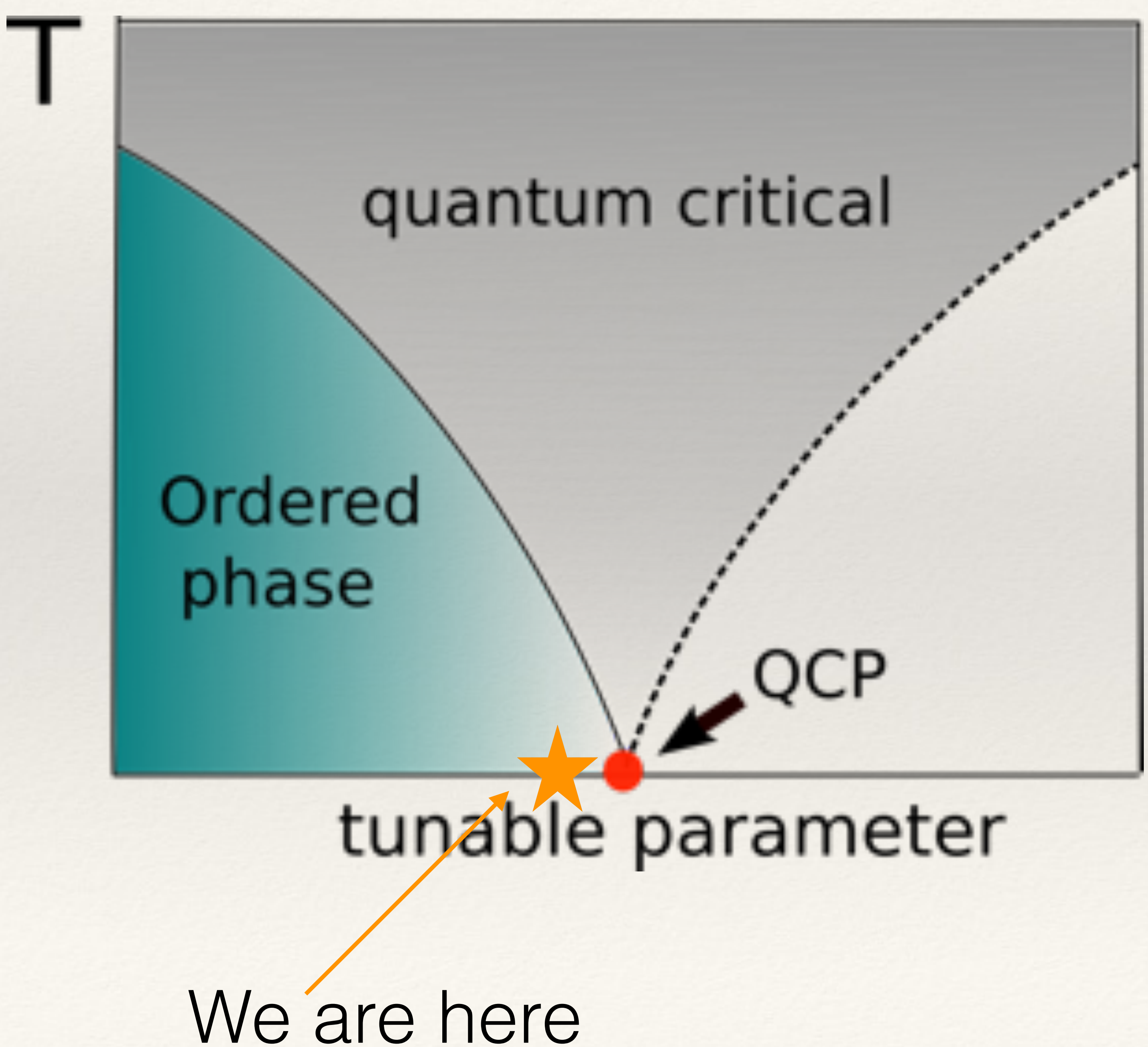


Quantum Phase Transition



Higgs field near a second order Quantum Phase Transition

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- Phase transitions happening at zero temperature are usually referred to as quantum phase transitions (QPT). It is thus natural to use a condensed matter analogy and describe the Higgs in terms of a second order QPT. At the second order QPT all masses vanish and the theory is scale invariant, characterized by the scaling dimensions of the field

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If the SM is correct up to the Planck scale, then a change in 1 part in 10^{30} can push us through the phase transition, so we are very, very close to the critical point.

- Th… Hi… ma…
- Wh… condensed matter systems … (like temperature) close to a critical value where a continuous (second order) phase transition occurs.
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H… …tion

If there is new physics beyond the SM then the relevant questions are: Does the underlying theory also have a QPT?”

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e.g., if the system is in the domain of attraction of a non-trivial fixed point then we find non-trivial critical exponents.

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We assume the field is not free field (assuming that the theory is close to an interacting fixed point), such that scaling dimension do not add up => our theory is not corresponding to generalized free fields theory

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Quantum Critical Higgs

- The assumption that the theory is close to a QPT implies that the Higgs field should be characterized by its scaling dimension Δ, where $1 \leq \Delta < 2$. In general, the two point function of a scalar with caling dimension Δ in a CFT is

$$G_{\rm CFT}(p) = -\frac{i}{(-p^2 + i\epsilon)^{2-\Delta}}$$

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- This however gives a large contribution to the potential in the p ➔ 0 limit, removing all light degrees of freedom. A light pole can be reintroduced (while leaving the cut starting at μ by subtracting the mass term:

$$-\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H}$$

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we are neglecting small SM couplings such as gauge couplings and Yukawas that represent an external deformation of the CFT: treat such deformations as small perturbations that are accounted by higher loops involving insertions of these small couplings.

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Quantum Critical Higgs

- The upshot is that there is a CFT (QPT), it has non-trivial dynamics that will have a non-trivial 4-point function H^4 which will have momentum dependence
- the effective potential will be the p→0 limit of the momentum space 4-pt function

$$-\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H} - V(|\mathcal{H}|)$$

- We will parameterize the Higgs VEV as $$\langle \mathcal{H} \rangle = \frac{\tilde{v}^\Delta}{\sqrt{2}}$$

completely free parameter

$$V(\tilde{h}) = \tilde{v}^{2\Delta} \tilde{m}^{4-2\Delta} \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{\tilde{h}}{\tilde{v}^\Delta}\right)^n$$

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The momentum space propagator for the physical Higgs scalar

$$G_h(p)=-\frac{i}{(-p^2+\mu^2+i\epsilon)^{2-\Delta}-\mu^{4-2\Delta}+\tilde{m}^{4-2\Delta}}$$

Location of pole (Higgs mass of 125 GeV)

$$m_h^2=\mu^2-\left[\mu^{4-2\Delta}-\tilde{m}^{4-2\Delta}\right]^{\frac{1}{2-\Delta}}\approx\frac{\tilde{m}^2}{2-\Delta}\left(\frac{\tilde{m}}{\mu}\right)^{2-2\Delta}\quad \text{for } \mu\gg\tilde{m}$$

Quantum Critical Higgs: modified propagator

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- Using Cauchy's integral theorem, the above propagator can be rewritten as a single pole plus a contribution of the continuum, corresponding to the usual spectral decomposition

$$-iG_h(p) = \frac{Z_h}{p^2 - m_h^2} + \int_{\mu^2}^{\infty} \frac{\rho_h(M^2)dM^2}{p^2 - M^2}$$

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- Rescaling field to go into a basis corresponding to canonical normalization

$$\mathcal{H} \to \frac{1}{\sqrt{2-\Delta}}\mu^{\Delta-1}H$$

$$Z_h = \left(\frac{\mu^2}{\mu^2 - m_h^2}\right)^{1-\Delta} = 1 - (\Delta - 1)\frac{m_h^2}{\mu^2} + \mathcal{O}\left(\frac{m_h^4}{\mu^4}\right)$$

$$\rho_h(M^2) = \theta(M^2-\mu^2)\frac{(2-\Delta)\mu^{2-2\Delta}}{\pi}\frac{\sin(\pi(2-\Delta))(M^2-\mu^2)^{2-\Delta}}{2\cos(\pi(2-\Delta))(M^2-\mu^2)^{2-\Delta} - (\mu^{4-2\Delta} - \tilde{m}^{4-2\Delta})^2}$$

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For the propagators of the Nambu-Goldstone bosons of EWSB (the longitudinal polarization of the W and Z), also contain a pole, at zero momentum, and a continuum => set m̃=0

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Non-local operator

$$-\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta}\mathcal{H}^\dagger\mathcal{H} - V(|\mathcal{H}|)$$

$$[\partial^2 - \mu^2]^{2-\Delta}\delta(x-y)$$

similar to SCET!

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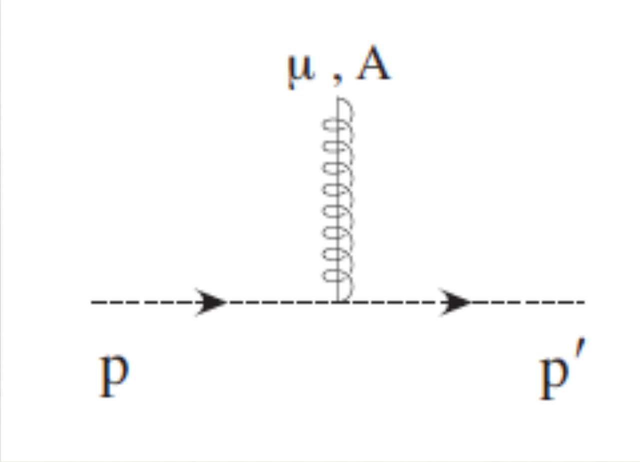

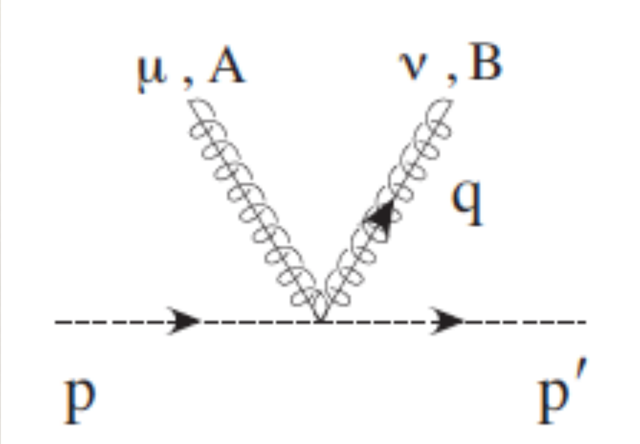

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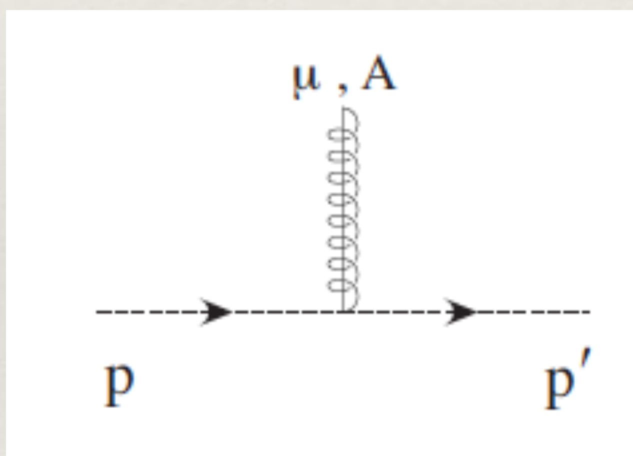
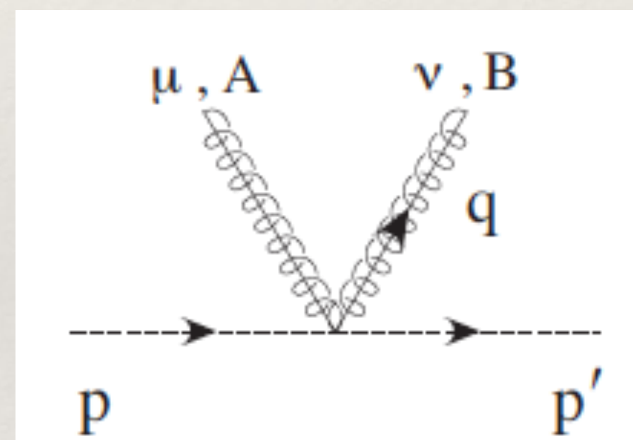




- e.g. for the trilinear interaction in momentum space: $\mathcal{H}^\dagger(p+q)A_\mu^a(q)\mathcal{H}(p)\Gamma^{\mu,a}(p,q)$

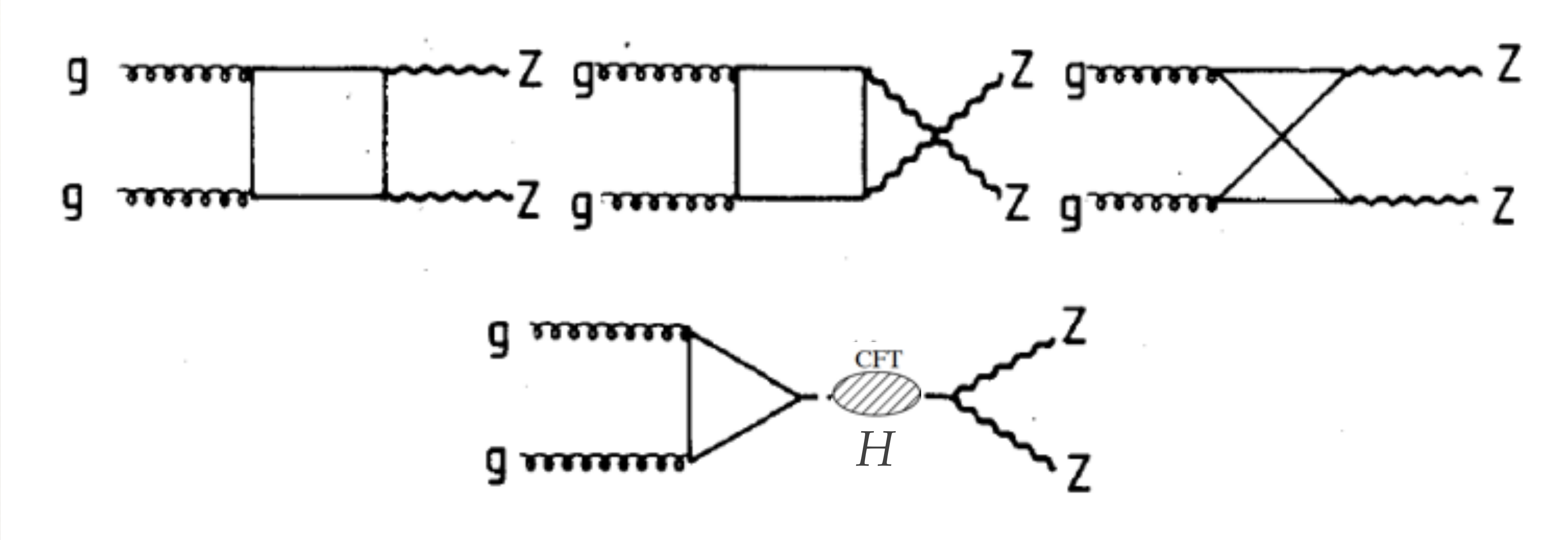
$$\Gamma^{\mu,a}(p,q)=gT^a\left(2p^\mu+q^\mu\right)F(p,q)\ ,$$

$$F(p,q)=-\frac{\left(\mu^2-(p+q)^2\right)^{2-\Delta}-\left(\mu^2-p^2\right)^{2-\Delta}}{2p\cdot q+q^2}$$

similar to SCET!

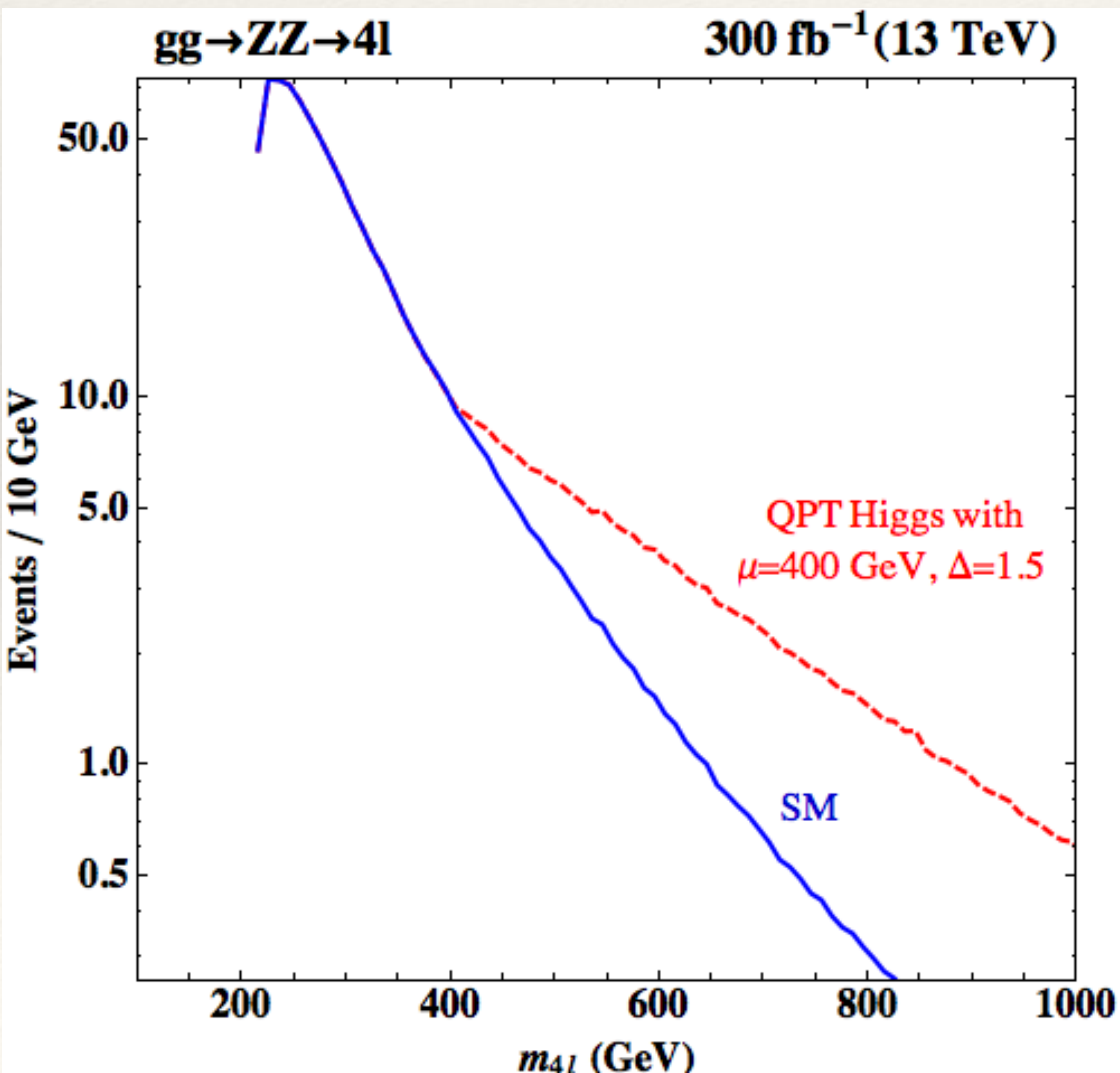
Direct Signals

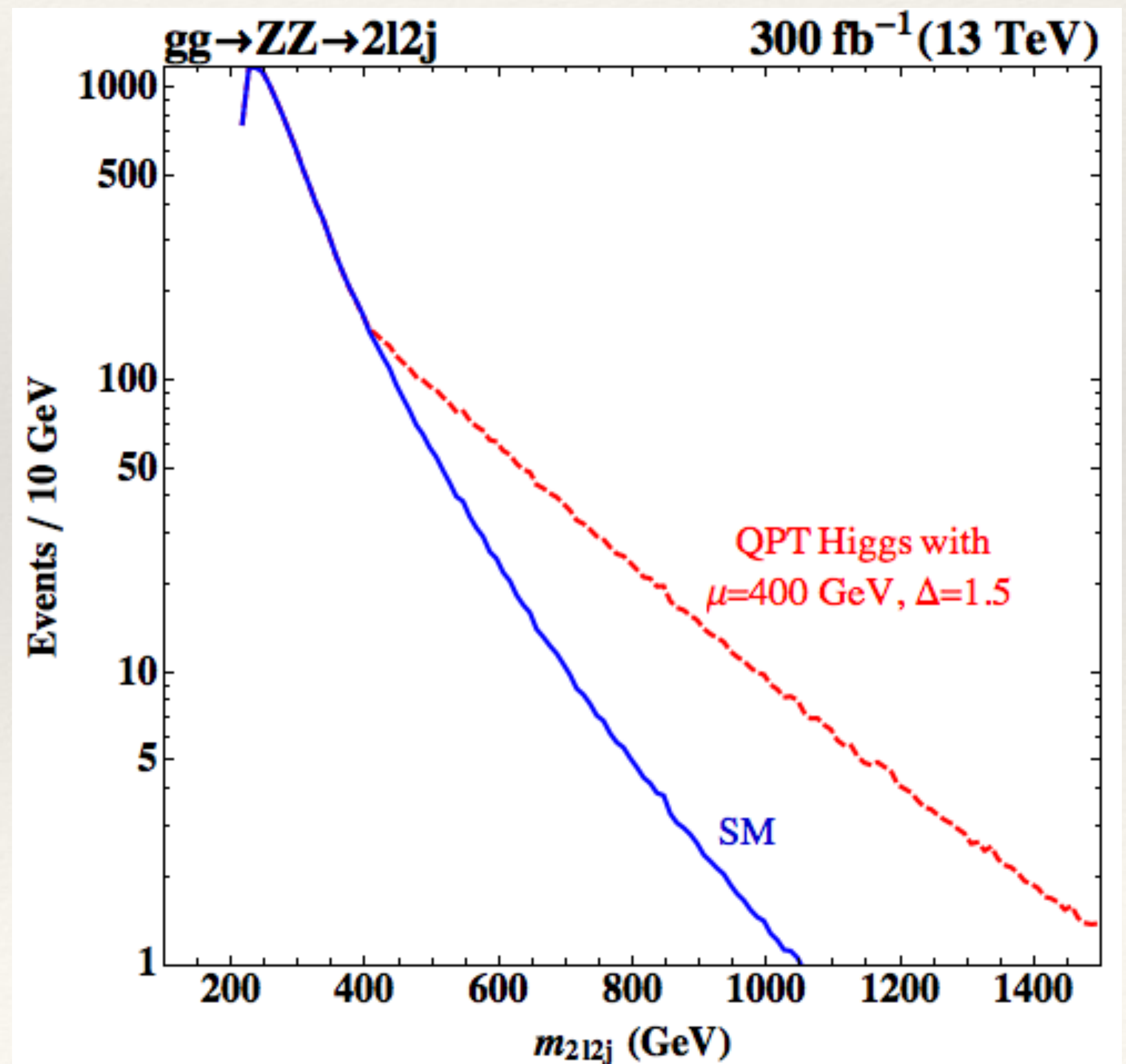
- Off-shell Higgs can be tested via interference.



Direct Signals

- Single Higgs production: Production of the cut modifies Higgs cross sections for energies above μ => modifies any cross sections that involve the (tree-level) exchange of the components of Higgs





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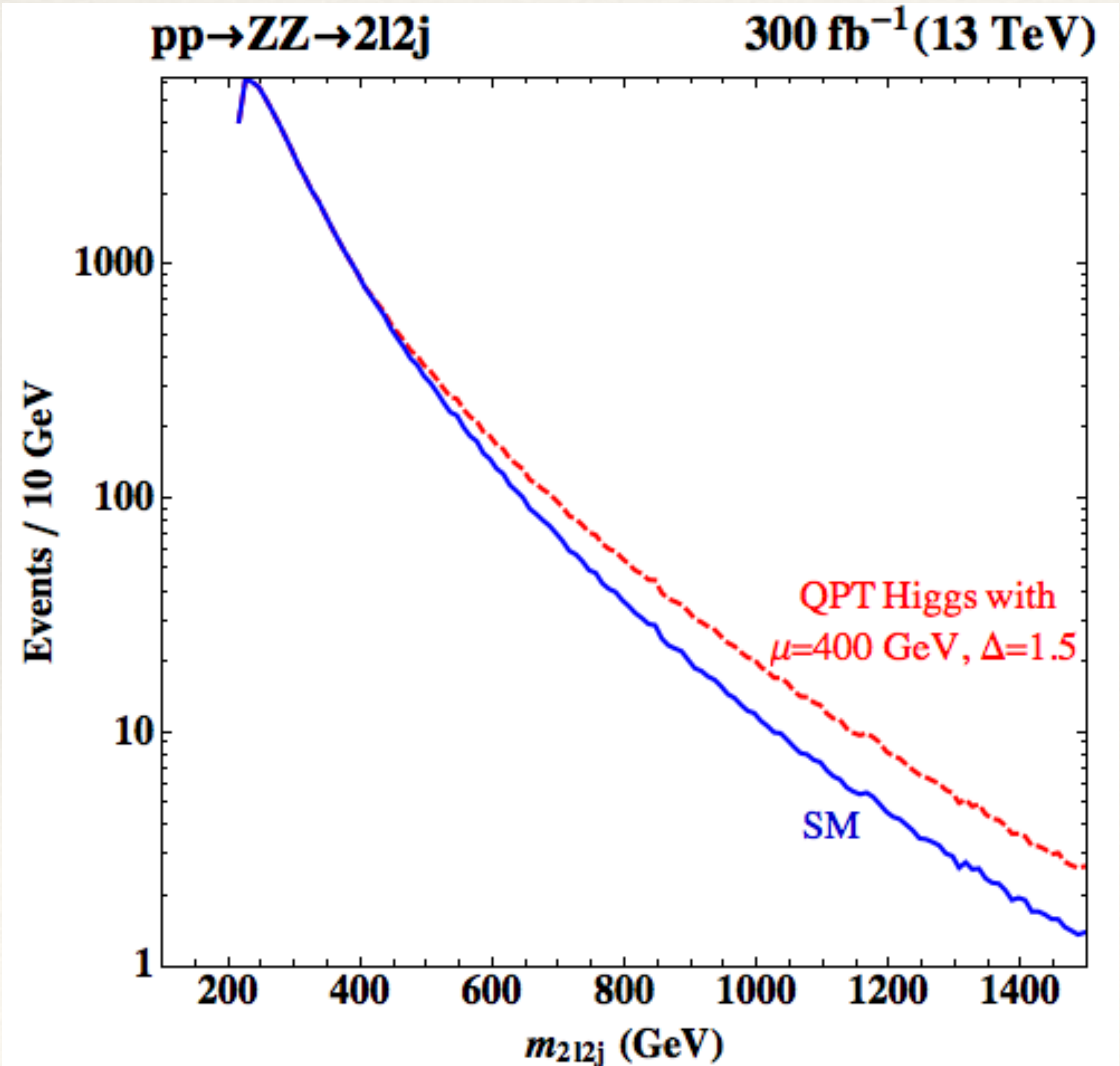

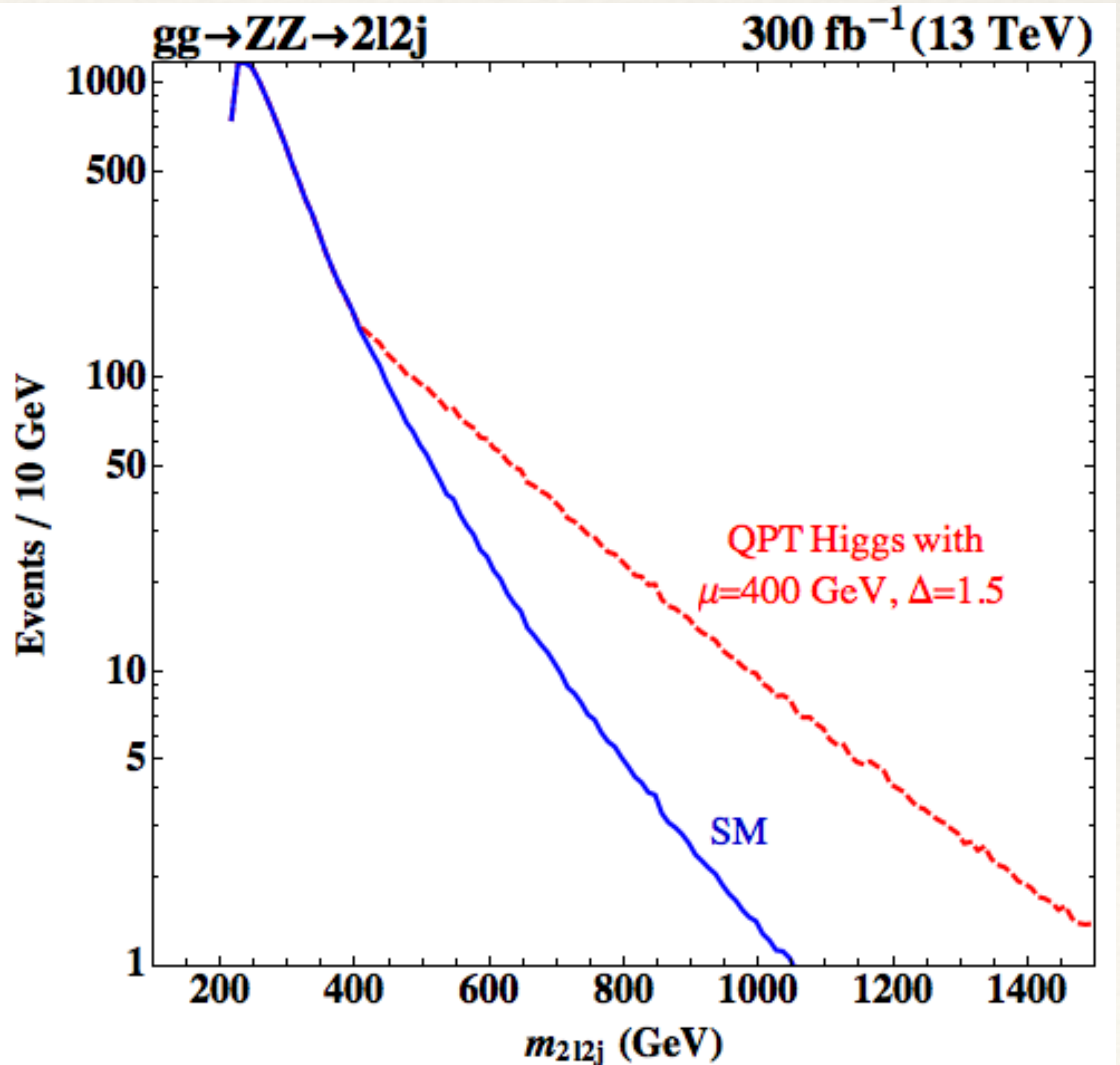

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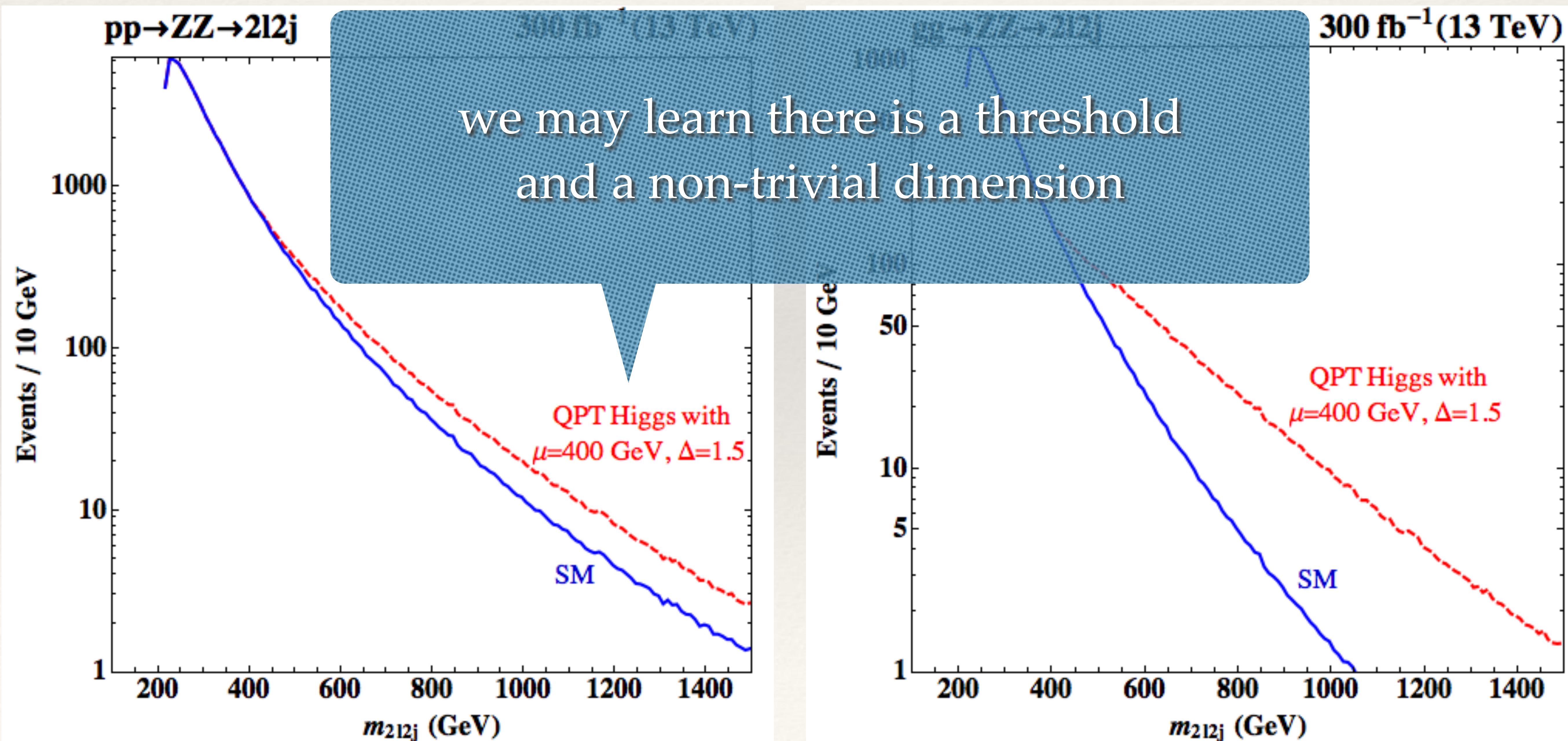
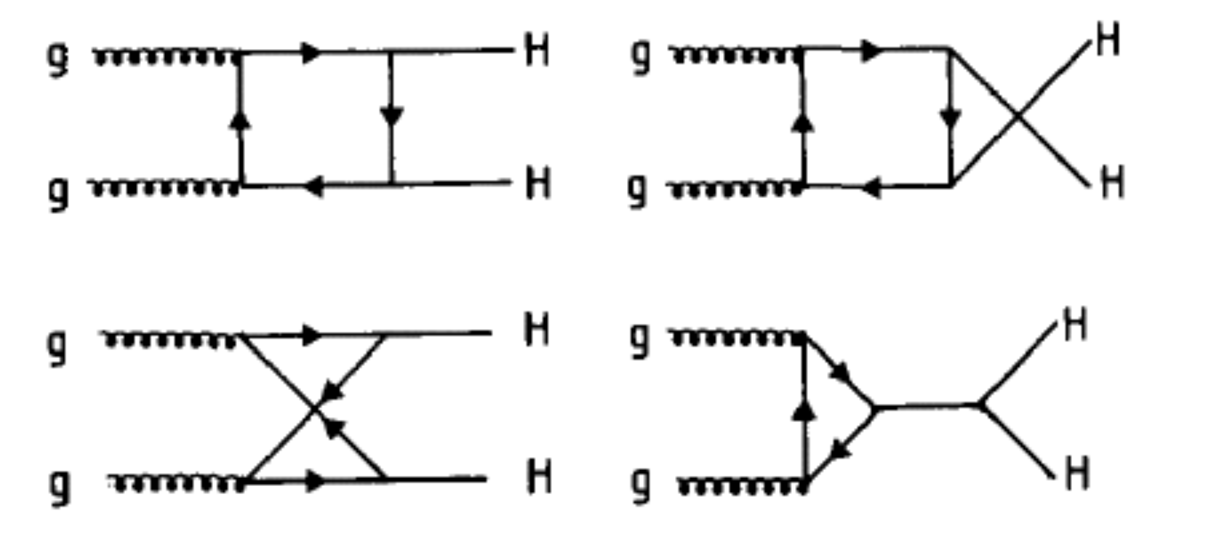


Direct Signals

- Double Higgs production



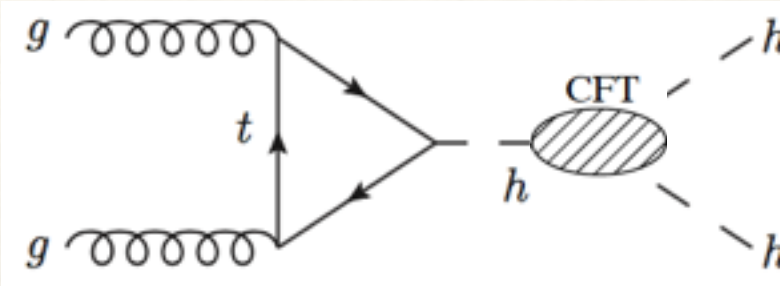
$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\alpha_W^2 \alpha_s^2}{2^{15}\pi M_W^4 \hat{s}^2}\left(|\mathrm{gauge1}|^2 + |\mathrm{gauge2}|^2\right)$$

gauge1 = box + triangle (negative interference)

gauge2 = box (largest contribution)

Direct Signals

- Double Higgs production: correction on triangle diagram



$$\mathcal{L}_{\tilde{h}\tilde{h}\tilde{h}} = -C_3 \frac{\sqrt{2-\Delta}}{2v} \mu^{1-\Delta} \left(\mu^{4-2\Delta} - \left(\mu^2 - m_{\tilde{h}}^2\right)^{2-\Delta} \right) \tilde{h}^3$$

C3=1 if the first two terms dominate we get SM-like potential (if there is small parameter expansion: unlikely that the dimensions differ from 1 very much)

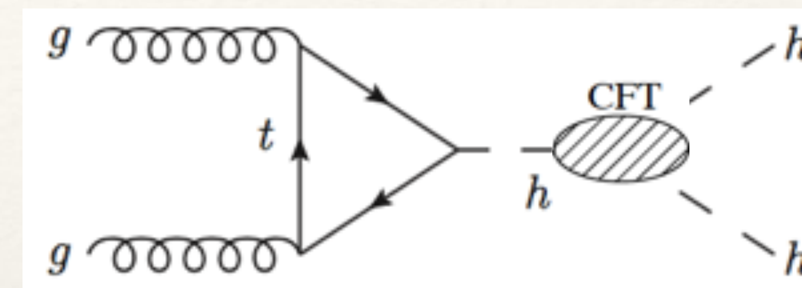
$$V(|\mathcal{H}|) = -m_{\mathcal{H}}^{4-2\Delta} |\mathcal{H}|^2 + \frac{\lambda_{\mathcal{H}}}{\Lambda_V^{4(\Delta-1)}} |\mathcal{H}|^4$$

C3≠1 we can have an arbitrary potential ((or an arbitrary function of |H|^2)) if the coupling that controls the various terms is not small (presumably some CFT coupling)

$$V(\tilde{h}) = \tilde{v}^{2\Delta} \tilde{m}^{4-2\Delta} \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{\tilde{h}}{\tilde{v}^{\Delta}} \right)^n$$

Direct Signals

- Double Higgs production: correction on triangle diagram



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$C_3=1$ if the fi... SM-like potential (if there is small parameter... expansion, unlikely that the dimensions differ from 1 very much)

$$V(\mathcal{H}) = \ldots |\mathcal{H}|^2 + \frac{\lambda_{\mathcal{H}}}{\Lambda_V^{4(\Delta-1)}} |\mathcal{H}|^4$$

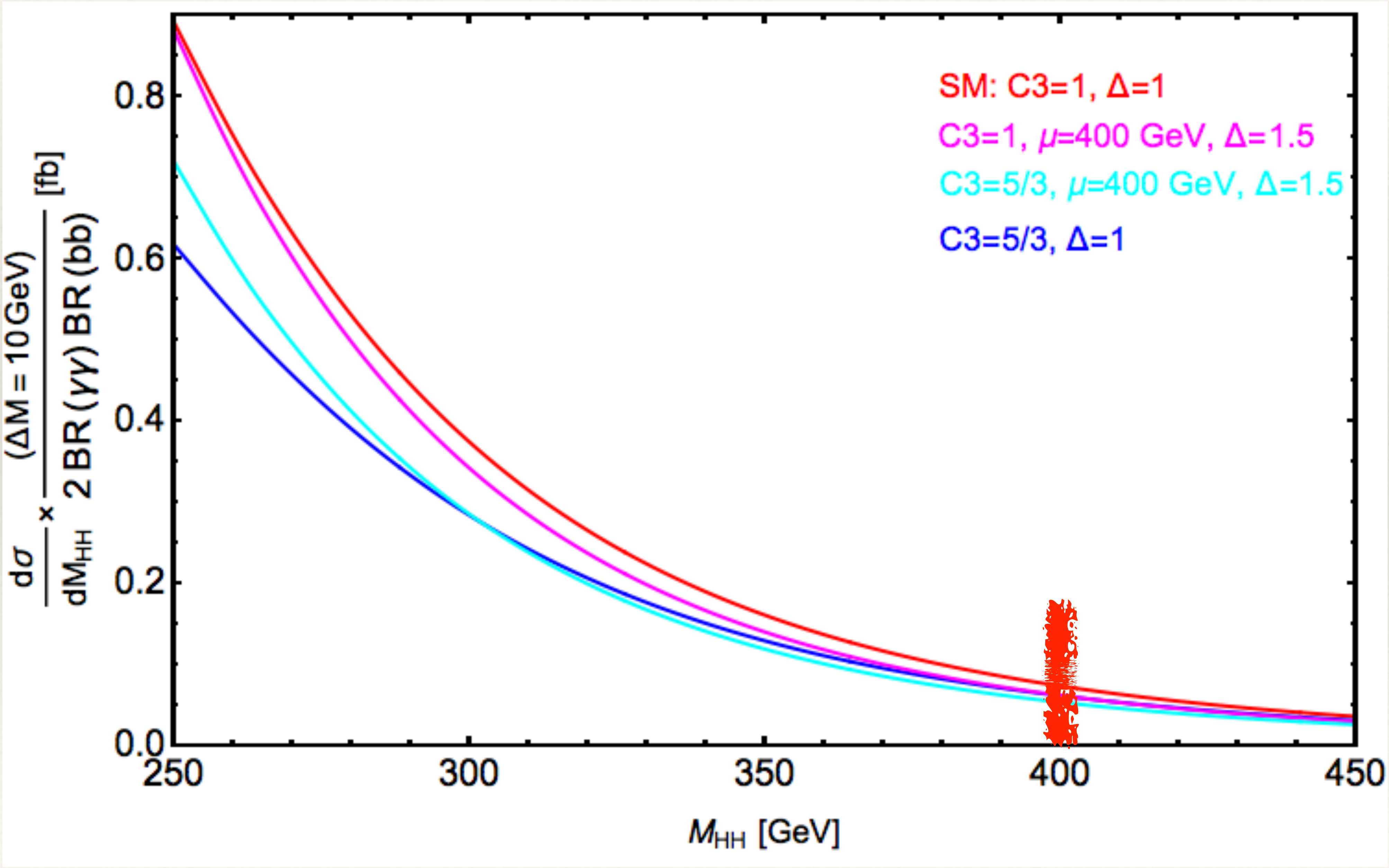
These expansion can be trusted only for $p < \mu$: above $\sim\mu$, momentum dependent terms from CFT will appear

$C_3 \neq 1$ we can have an arbitrary potential ((or an arbitrary function of |H|^2)) if the coupling that controls the various terms is not small (presumably some CFT coupling)

$$V(\tilde{h}) = \tilde{v}^{2\Delta} \tilde{m}^{4-2\Delta} \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{\tilde{h}}{\tilde{v}^{\Delta}} \right)^n$$

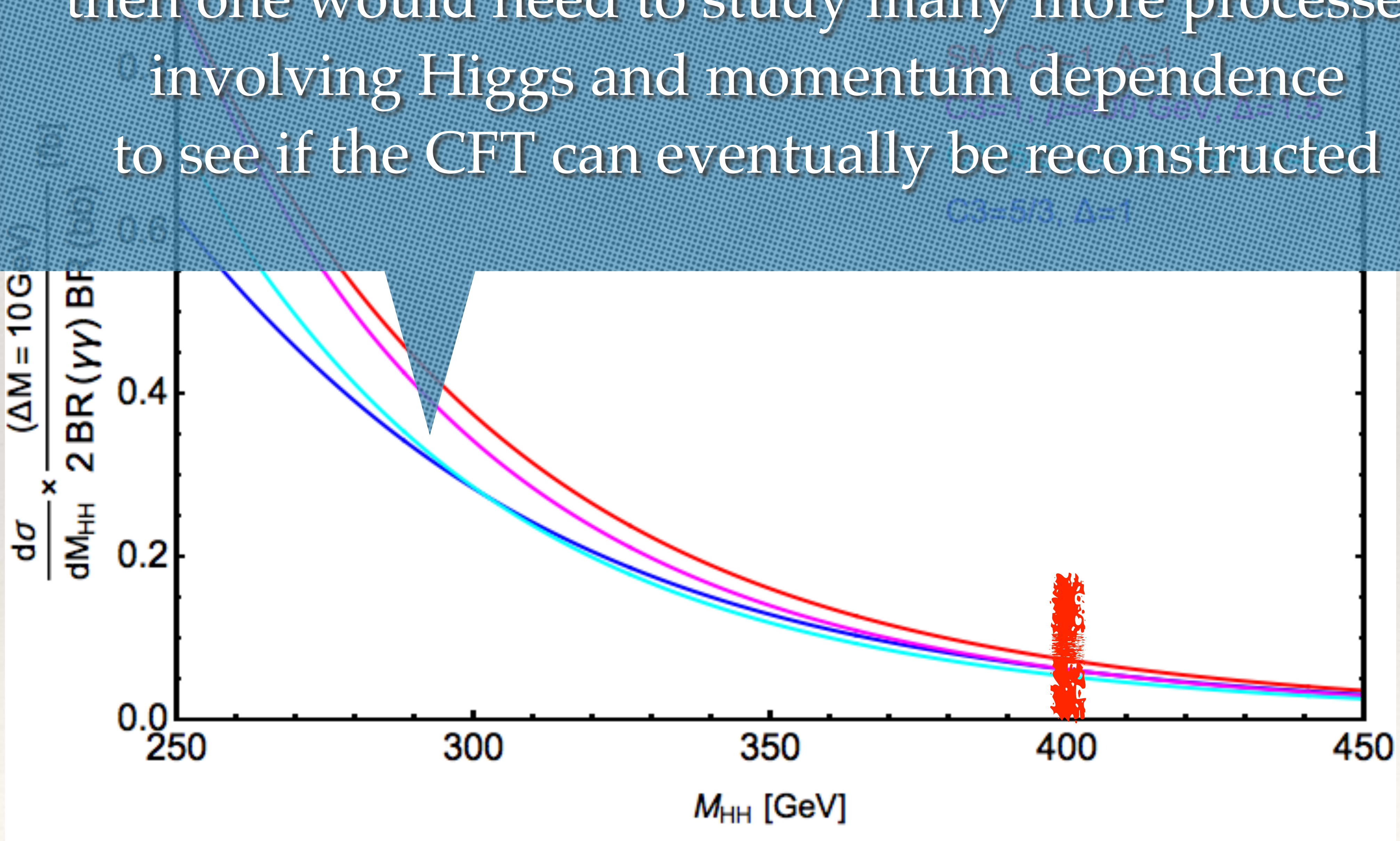
Direct Signals

- Double Higgs production



We don't know what the CFT is:
We want to learn as much as possible about it
The quatities {μ , Δ, C_3} would be a good start

but if it turns out this is indeed the right direction
then one would need to study many more processes
involving Higgs and momentum dependence
to see if the CFT can eventually be reconstructed



Summary II: Have we really discovered a SM-like Higgs boson?

- With improved theoretical tools, SM will be tested at per mille level.
- Meanwhile we are quite ignorant about the form of Higgs potential, or even kinetic term.
- It's interesting whether the Higgs sector is close to a quantum critical point with non-mean-field behavior, that is with non-trivial critical exponents and scaling dimensions
- LHC and FCC will explore these questions!